A Stochastic Model of an Industrial Control System Kill Chain

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ABSTRACT
In this paper, we present a Semi-Markov Process (SMP) model of an Industrial Control System (ICS) Kill Chain. We develop the steady state probability equations by first examining the embedded Discrete Time Markov Chain (DTMC) sojourn times. Based on published reports of ICS vulnerabilities and an actual case study of a cyber-attack on a number of power stations on the Ukraine power grid, we derive the parameter values for our SMP model. Using these values, we calculate the steady state probabilities of the model and provide insights on the results particularly on the top two ICS security attributes: availability and integrity.

Categories and Subject Descriptors
• Security and privacy–Formal security models

General Terms
Measurement, Performance, Security, Theory

Keywords
Industrial Control Systems, SCADA, ICS Kill Chain, Stochastic Modelling, Semi-Markov Chain, Security Model

1. INTRODUCTION
Industrial Control Systems (ICS) have been widely employed to supervise and control critical infrastructures in various sectors such as supplying and/or controlling essential energy, water treatment, transport, chemicals, and disparate manufacturing processes. ICS typically consist of a combination of software, hardware and operators. The most common components found in ICS are Systems Control and Data Acquisition (SCADA), Distributed Control Systems (DCS) and Programmable Logic Controllers (PLC).

ICS are originally designed as air-gapped networks, but have now become increasingly interconnected with other IT systems and external networks. Whilst offering efficient communication and high throughput, such evolution has also exposed ICS to a growing number of malicious cyber-attacks. Cyber-attacks on ICS could result in unexpected disruption to controlling critical infrastructures and bring harmful physical damage to all living creatures and the environment. There were 245 cyber-attacks on ICS that were reported to ICS-CERT in 2014 [16]; 295 incidents were reported in 2015 [17].

Stuxnet was disclosed in 2010 as the first cyber weapon causing havoc to a large nuclear plant in Iran. Until September 2010, Stuxnet infected approximately 100,000 hosts across over 155 countries. Two more recent examples are the security breach to a German steel mill causing massive damage to the whole plant in 2014 [18] and the cyber-attack against Ukrainian power companies in 2015 leading to power outage and affecting approximately 225,000 customers[6]. Currently the cyber security of ICS has become an increasing concern for government, industry and academia all over the world.

In this paper, we propose a stochastic model to formally express the complete life-cycle of an ICS-targeted attack. We follow the key phases of the ICS Kill Chain [2], which are then formally represented as a Semi-Markov Chain process. The key contribution of this paper is a novel formal model of an ICS Kill Chain which can be used to derive key security and performance indicators for system integrity, resiliency, survivability, availability, and failure.

The rest of the paper is organized as follows: the ICS Kill Chain and a brief introduction to stochastic modeling are described in section 1; section 2 introduces the transition model of the ICS Kill Chain; section 3 shows the formal semi-Markov model and the development of its steady state equations; section 4 presents the derivation of the parameters values; and finally, in section 5, we provide concluding remarks and some future research directions.

1.1 Industrial Control Systems Kill Chain
In [2], an Industrial Control System (ICS) Kill Chain is introduced. This model is adopted from the seminal work by Hutchins, Cloppert and Amin [4] on Cyber Kill Chain™. The Cyber Kill Chain™ is patterned after the military concept of kill chains to gain a better understanding of the adversary’s campaign. To better appreciate the relationship between the original Cyber Kill Chain™ and the ICS Kill Chain, we superimposed them in Figures 1, 2, and 3. A description of each phase is attributed to Assante and Lee [2].

1.1.1 Stage One
Planning Phase. In this phase reconnaissance is performed to gauge the strength of the system defenses as well as gather information that may be used to create attack vectors. In ICS, it may also include researching specific control system vulnerabilities that are endemic to the type of infrastructure that is of interest.

Preparation Phase. This phase will include weaponization or targeting. While weaponization involves the crafting of seemingly innocuous files with exploit code to facilitate the advancement of
the malicious objective, targeting is the process of prioritizing targets and matching harmful actions that are appropriate to those targets.

**Cyber Intrusion Phase.** This phase includes the Delivery step, which is used to induce interaction with the system or the user; the Exploit step, which is the process of gaining unauthorized access to the system; the Install step, wherein the adversary installs applications such as backdoors to be able to gain unimpeded access to the system; and the Modify step, wherein the adversary changes the system environment using existing tools such as PowerShell to be able to escalate system privileges.

**Management and Enablement Phase.** In this phase, the adversary establishes command and control (C2) using the tools and applications that were successfully installed or built in the previous phase. More often than not, the established C2 will be configured with stealth communication features to prevent detection.

**Sustainment, Entrenchment, Development, and Execution.** This phase is when malicious actions start to take place. Covert capture of user credentials, lateral movement, data collection and exfiltration, installation of advanced tools, and anti-forensic activities are initiated.

### 1.1.2 Stage Two

**Attack Development and Tuning.** This phase may take longer than the other phases. This is the time in which the adversary would take time in developing the suitable attack method and tools based on the exfiltrated data gathered during the previous phases.

**Validation Phase.** This is when the attacker would test the developed capabilities on a similar system. Activities in this phase would reveal the extent of how much the attack could inflict on the target system.

**ICS Attack.** In this step, the capability is delivered, installed, and executed.

### 1.2 Stochastic Modelling

A stochastic model expresses the uncertainty of security posture due to incomplete knowledge by actors on both sides: the adversary and the system designer or operator. In developing the model, probabilities and cumulative distribution functions are used to describe the events that trigger transitions to different states.

#### 1.2.1 Markov Chain

The application of Markov chain model requires the stochastic process to be discretized into a number of states and determining the transition probabilities between two states. In [19], a formal definition of a Markov chain is provided as follows:

Let $X(t)$ be a discrete-state stochastic process and let $P_r(X(t_0) = j)$ be the probability that the process is in state $j$ at the time $t_0$. $X(t)$ is a Markov chain if, for any ordered times $t_1 < t_2 < \ldots < t_n$, the conditional probability of being in any state $j$ such as that $P_r(X(t_n) = j | \{X(t_{n-1}) = i_{n-1}, X(t_{n-2}) = i_{n-2}, \ldots, X(t_0) = i_0\})$. Essentially, it asserts that any state depends on the state immediately prior to it and cannot depend on any state before that prior state.

In real cyber-attack events, the sojourn time in a particular state may not at all be exponentially distributed but may be described by any arbitrary distribution function. Thus, we use a semi-Markov process in which the rate of transition from one state to another may depend on the sojourn time in the source state but not on anything that happened prior to reaching that source state.

Because of the immense variety of attacks, an assortment of distribution functions may need to be considered. In [8], Madan et al. suggest the appropriate distribution function that is suited to a specific threat situation as follows: a hypo-exponential distribution to model threats involving multi-stage activities and threat situations that may cause monotonically increasing failure rate; a hyper-exponential distribution function to model threats that exhibit a monotonically decreasing failure rate; a Weibull distribution function to model constant failure rate, a monotonically increasing failure rate, or a monotonically decreasing failure rate; and a log-logistic type of distribution function for a combination of decreasing rate of success initially followed by an increasing rate (or vice-versa)[7].

### 2 The State Transition Model for the ICS Kill Chain

Using the ICS Kill Chain as a basis, we derive a state transition model as shown in Figure 4. Each node represents a state which may comprise of one or more steps on the kill chain. The labelled arcs represent a transition between states. This model will later be transformed into a formal Semi-Markov process chain with steady-state and transition probabilities.

The Normal state, $G$, represents a condition in which the ICS is in normal operating condition. The transition to the Detect state, $D$, denotes the detection/notification of abnormal system behavior; a system patch, or firmware update which may or may not cause any disruption. At some point in time, the ICS transitions to the Recon state, $R$, this is the first step on the ICS Kill Chain.
Thus, the prospect of bringing back the system to the Attack state is enabled.

3 SEMI-MARKOV PROCESS MODELLING

Formally, we describe the Semi-Markov Process (SMP) as follows: Let \((X(t) : t \geq 0)\) be the base stochastic process with a set of discrete state space \(X = \{A, C, D, F, G, R\}\) with sojourn time distributions, \(H_i(t)\) and parameters: \(h_i\), as the sojourn time in state \(i \in X\), and the transition probabilities \(p_{ij}\) between states \(i\) and \(j\), where \(i, j \in X\). At the instants of state transitions, a semi-Markov chain behaves like a Markov chain. Hence, at those instants we have an embedded discrete-parameter Markov chain [10].

| Table 1 The ICS Kill Chain Semi-Markov Model Parameters |
|---------------------------------|---------------------------------|
| \(h_A\) | Mean time for the ICS to stay in the state of being attacked |
| \(h_C\) | Mean time for the ICS to remain in a compromised state |
| \(h_D\) | Mean time for the ICS to remain in detection, mitigation, and repair state |
| \(h_F\) | Mean time for the ICS to remain in the failed state |
| \(h_G\) | Mean time for the ICS to stay in a normal state |
| \(h_R\) | Mean time for the ICS to stay in the reconnaissance, planning, and preparation state |
| \(P_A\) | Probability of being in the attack state |
| \(P_C\) | Probability of being compromised |
| \(P_F\) | Probability of system failure |
| \(P_R\) | Probability of moving to the reconnaissance/planning state |
| \(P_U\) | Probability of an unmitigated/un-repaired vulnerability |
| \(I - P_A\) | Probability of detecting a cyber intrusion |
| \(I - P_C\) | Probability of detecting a reconnaissance or system mapping |
| \(I - P_F\) | Probability of detecting or mitigating a vulnerability/impending attack |
| \(I - P_U\) | Probability of successful mitigation/patch/repair |

Using a similar analysis on the semi-Markov process found in [10], we compute the steady state probability vector \([\pi_A, \pi_C, \pi_D, \pi_F, \pi_G, \pi_R]\) by first computing, for each state, the mean sojourn time

\[
h_i = \int_0^\infty (1 - H_i(t)) \, dt
\]

Next we find the steady state \(\Gamma = [\gamma_A, \gamma_C, \gamma_D, \gamma_F, \gamma_G, \gamma_R]\) for the embedded discrete-parameter Markov chain by solving the linear system of Equations:

\[
\Gamma = \Gamma \times P
\]

\(\Gamma \times e = 1\) or simply \(\sum_i \gamma_i = 1, \forall i \in \{A, C, D, F, G, R\}\) (1)

Where \(P\) is the discrete Markov-chain probability matrix and \(e = (1, 1, 1, 1, 1, 1)^T\) . Finally, we can compute the steady-state probabilities for the SMP using...
\[
\pi_i = \frac{\gamma_i \times h_i}{\sum_j \gamma_j \times h_j} \quad \forall \, i, j \in \{A, C, D, F, G, R\}
\]

where the semi-Markov transition is from state \( j \) to state \( i \). For a detailed derivation of the preceding equations, the interested reader is referred to the works of Sahner [10] and Trivedi [19].

The parameters for the SMP are enumerated in Table 1. The transition probability matrix \( P \) describes the state transition probabilities between the embedded DTMC states shown in Figure 4. The steady-state probabilities of the DTMC, \( \Gamma = [\gamma_0, \gamma_c, \gamma_d, \gamma_f, \gamma_r, \gamma_u] \), are calculated using the following system of linear equations from (1):

\[
\begin{align*}
\gamma_A &= \gamma_c \gamma_A + \gamma_D \gamma_U \\
\gamma_C &= \gamma_c \gamma_C \\
\gamma_D &= \gamma_k (1 - \gamma_c) + \gamma_c (1 - \gamma_A) + \gamma_A (1 - \gamma_F) + F \\
\gamma_F &= \gamma_c \gamma_F \\
\gamma_G &= \gamma_D (1 - \gamma_F) \\
\gamma_R &= \gamma_G \gamma_B \\
\gamma_A + \gamma_C + \gamma_D + \gamma_F + \gamma_R + \gamma_B &= 1 \quad (\text{Equations 3-9})
\end{align*}
\]

Solving for the steady-state probabilities of the DTMC in terms of the transition probabilities using Matlab’s Symbolic Math Toolbox [9] yields equations (10)-(15) as shown in the Appendix. Using equations (10)-(15), we can now calculate the SMP steady-state probability for each state. Henceforth, equations (16)-(21), as shown in the Appendix, are derived.

3.2 Passage Time Analysis

Another measure of interest is that on passage-time distributions. A passage time is a random variable describing the amount of time it takes to reach a state \( j \), given that the process starts in states \( i \). Hence, for the analysis we must assume that target state \( j \) is an absorbing state, i.e. states with no outgoing transitions, so that we actually stop measuring the time of a trajectory once we reached the target state. From passage-time distributions we can derive metrics such as mean time to detection (MTTD) which is the average passage time from the Normal state, \( G \), to the Detect state, \( D \). Mean time to compromise (MTTC) is the average passage time from Normal state, \( G \), to Compromise state, \( C \). Mean time to failure (MTTF) is the average passage time from Normal state, \( G \), to Fail state, \( F \). Mean time to recover (MTTR) is the average passage time from Compromise state, \( C \), to Normal state, \( G \). In section 4.4, we present the results of computing the passage-time distributions by gathering samples from multiple simulation runs.

4 DERIVING PARAMETER VALUES

In order to create a model that will represent the system as realistically as possible, we derive values for the sojourn time value for each state \( i \) and provide justification based on an actual event and other published reports.

4.1 The Cyber Attack on the Ukrainian Power Grid: A Case Study

In December, 2015 a regional electricity distribution company in Ukraine was subjected to a cyber-attack resulting in several power outages that lasted for three hours [6]. The Industrial Control Systems Cyber Emergency Response Team (ICS-CERT) cited public reports that the BlackEnergy (BE) malware was found on the companies’ network but would not confirm its role in the attack [5].

The Ukrainian Power Grid (UPG) cyber-attack demonstrated an adversary that is extremely capable and highly resourced. Several evidence materials point to the fact that the adversary has been conducting long-term planning and reconnaissance, lasting at least six months [13], to be able to execute a highly coordinated and effective attack. Among the technical components used by the adversary are spear phishing, theft of credentials, use of Virtual Private Networks (VPN), use of remote access tools to manipulate Human Machine Interfaces (HMI), firmware level disruption of devices, utilization of KillDisk, and telephone denial-of-service [6]. The ICS Kill Chain mapping to the UPG cyber-attack is well articulated in [6] and is recapitulated in the following:

The **Reconnaissance** in stage 1 took place at least six months before the actual attack. Evidence reveals that this is a directed attack and that the levels of automation on the distribution systems made them attractive targets.

In the **Weaponization** and/or **Targeting** step at stage 1, the adversary crafted Microsoft Office documents with embedded BlackEnergy malware as attack vectors.

During the **Cyber Intrusion** step at stage 1, the weaponized documents were delivered to individuals working in the administrative and IT sectors of the companies. When the documents were opened, the embedded macros were unleashed which enabled the installation of the BlackEnergy malware. The malware facilitated the command and control communication between the adversary at a remote site and the infected systems within the companies’ premises. It is also at this step when credentials were harvested and lateral movement within the IT infrastructure was carried-out.

The **Develop** step in stage 2 occurred mostly within the adversary environment to minimize detection. This step included the design and implementation of malicious tools, both software and firmware, to gain control of the Distribution Management Systems (DMS) and the serial-to-ethernet devices. The **Validation** step in stage 2 was conducted on the adversary site to evaluate and test the malicious tools before the actual attack. These fine-tuned malicious tools were then delivered to the compromised systems before the execution of the actual attack.

During the last step of the ICS Kill Chain, the **ICS Attack**, the adversary utilized the HMIIs to manipulate the breakers in the SCADA environment. This opening of breakers enabled 27 substations to be taken offline causing power interruption to more than 225,000 customers. At the same time, the malicious firmware was uploaded to the serial-to-ethernet devices and thereby disabling them for remote control. To exacerbate the situation, a telephonic denial of service on the companies’ call centers was initiated to prevent customers from contacting customer support. The entire Kill Chain operation transpired commencing in March 2015 until December 2015.

The purpose of describing the above case study is to provide the reader a grasp of the timeline in which each of the steps in the Kill Chain has occurred. The discussion also facilitates a segue to the derivation of the values for the DTMC sojourn times.

4.2 Industrial Control Systems Vulnerability Statistics

In a 2016 report, Kaspersky lab published a report [1] indicating that there are a total of 189 vulnerabilities in ICS components in 2015. Out of this total number of vulnerabilities, 26 have associated
exploits. The report also includes an alarming statistic that patches and/or new firmware are available to only 85% of the published vulnerabilities. Of the remaining 15%, five percent were partially fixed, two percent were unpatched and removed, three percent were unpatched and declared obsolete and five percent remained unpatched. Using this data and using Polityuk’s model [11], we calculate a mean sojourn time in the Compromised (C) state of our SMP model to be 24 hours.

4.3 The Parameter Values
Tables 2 and 3 depict a summary of our findings.

### Table 2. The Mean Sojourn Time

<table>
<thead>
<tr>
<th>Sojourn Time</th>
<th>Description</th>
<th>Value (days)</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_A$</td>
<td>Mean time for the ICS to stay in the state of being attacked</td>
<td>180</td>
<td>Actual duration of attack (6 months) as reported in the “Analysis of the Cyber Attack on the Ukrainian Power Grid” [6].</td>
</tr>
<tr>
<td>$h_C$</td>
<td>Mean time for the ICS to remain in a compromised position</td>
<td>1</td>
<td>McQueen’s model [11] and data from Kaspersky lab report [1]</td>
</tr>
<tr>
<td>$h_D$</td>
<td>Mean time for the ICS to remain in detection/mitigation/patching state</td>
<td>54</td>
<td>2016 Cost of Cyber Crime Study &amp; the Risk of Business Innovation [14]</td>
</tr>
<tr>
<td>$h_F$</td>
<td>Mean time for the ICS to remain in the failed state</td>
<td>0.125</td>
<td>Actual duration of failure reported on the “Analysis of the Cyber Attack on the Ukrainian Power Grid” [6].</td>
</tr>
<tr>
<td>$h_G$</td>
<td>Mean time for the ICS to stay uncompromised in the presence of vulnerability</td>
<td>365</td>
<td>Estimated as 365 days.</td>
</tr>
<tr>
<td>$h_R$</td>
<td>Mean time for the ICS to stay in the reconnaissance, planning, and preparation state</td>
<td>90</td>
<td>Estimated based on the report “Analysis of the Cyber Attack on the Ukrainian Power Grid” [6].</td>
</tr>
</tbody>
</table>

### Table 3. Transition Probabilities of the SMP

<table>
<thead>
<tr>
<th>Transition Probability</th>
<th>Description</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_A$</td>
<td>Probability of being attacked</td>
<td>0.70</td>
<td>Miscellaneous published reports</td>
</tr>
<tr>
<td>$P_C$</td>
<td>Probability of being compromised</td>
<td>0.10</td>
<td>Estimated using the Analysis of the “Cyber Attack on the Ukrainian Power Grid” [6].</td>
</tr>
<tr>
<td>$P_F$</td>
<td>Probability of failure</td>
<td>0.15</td>
<td>Estimated using the “Analysis of the Cyber Attack on the Ukrainian Power Grid” [6] and the 2016 Kaspersky lab report [1].</td>
</tr>
<tr>
<td>$P_R$</td>
<td>Probability of being targeted</td>
<td>0.99</td>
<td>Estimated using the “Analysis of the Cyber Attack on the Ukrainian Power Grid” [6].</td>
</tr>
<tr>
<td>$P_U$</td>
<td>Probability of an unmitigated/un-repaired vulnerability</td>
<td>0.05</td>
<td>Estimated using the “Analysis of the Cyber Attack on the Ukrainian Power Grid” [6].</td>
</tr>
</tbody>
</table>

4.4 Interpretation of Results
Using the derived steady-state equations (16)-(21) for the SMP and the data gathered in Tables 2 and 3, we calculated the following steady state probabilities:

\[
\pi_A = 0.03912 \\
\pi_C = 0.000176 \\
\pi_D = 0.10132 \\
\pi_F = 0.0000176 \\
\pi_G = 0.68481 \\
\pi_R = 0.1588113
\]

Furthermore, we also obtained from the simulations of passage time distributions the following results: $MTTD = 459\pm7$, $MTTC=5117\pm99$, $MTTF=29009\pm555$, $MTTR=194\pm3$ with 95% confidence interval using 10,000 simulation runs.

Based on the above results, we provide the following observations:

a. The steady state probability values suggest the dominant stay of the ICS in the Normal (G), Recon (R) and Detect (D) states. This result clearly validates the Ukraine Power Grid report [6].

b. The almost negligible probability and yet, highly consequential effect, of being in the failed state indicates a highly sophisticated adversary that can produce a significant loss within a short duration.

c. The System Availability, calculated as $1 - \pi_F = 0.9999996$, remains at a high level despite the fact that the ICS is, at times, in compromised and attack states. The metric
indicates the level with which the system is delivering its mission free from degradation or impairment.

d. The System Integrity, calculated as \( \pi_G + \pi_R = 0.8436 \), indicates a good amount of time that the system is performing its intended functions without being compromised or manipulated.

5 CONCLUSIONS and FUTURE WORKS

A stochastic model of an industrial control system Kill Chain is designed and applied using parameter values derived from data gathered from an actual case study and currently available published reports. Using the results that were gleaned from calculations using equations derived from the model, we were able to produce the following important ICS security and performance metrics: system availability, system integrity, MTTF, MTTD, MTTC, and MTTR.

A meta-model of cyber physical system attacks referred to as a cyber-physical kill-chain is introduced in [3]. It would be an interesting extension to this study the application of the SMP model to that cyber-physical chain. Further, we recognize that the stochastic model described above requires additional validation using empirical data and robust simulation. Thus, we offer the following future research directions:

- Perform a sensitivity analysis on the model using various sets of parameter values;
- Continue to gather data from actual field reports and use those to validate the model; and
- Design and implement computer simulations to study the effect of various distribution functions on the model.

6 ACKNOWLEDGMENTS

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7 REFERENCES


8 APPENDIX

Equations (10-15):

\[
\begin{align*}
\gamma_G &= \frac{1 - P_U}{2 + P_R + P_U[P_F - P_R] + P_C P_R[1 - P_U] + P_R P_C P_R[1 + P_F - P_U - P_R P_U]}
\gamma_R &= \frac{P_R(1 - P_U)}{2 + P_R + P_U[P_F - P_R] + P_C P_R[1 - P_U] + P_R P_C P_R[1 + P_F - P_U - P_R P_U]}
\gamma_C &= \frac{P_C P_R (1 - P_U)}{2 + P_R + P_U[P_F - P_R] + P_C P_R[1 - P_U] + P_R P_C P_R[1 + P_F - P_U - P_R P_U]}
\gamma_A &= \frac{P_U + P_A P_C P_R - P_R P_C P_R P_U}{2 + P_R + P_U[P_F - P_R] + P_C P_R[1 - P_U] + P_R P_C P_R[1 + P_F - P_U - P_R P_U]}
\gamma_F &= \frac{P_F [P_U + P_A P_C P_R - P_R P_C P_R P_U]}{2 + P_R + P_U[P_F - P_R] + P_C P_R[1 - P_U] + P_R P_C P_R[1 + P_F - P_U - P_R P_U]}
\gamma_D &= \frac{1}{2 + P_R + P_U[P_F - P_R] + P_C P_R[1 - P_U] + P_R P_C P_R[1 + P_F - P_U - P_R P_U]}
\end{align*}
\]

Equations (16-21):

\[
\begin{align*}
\pi_G &= \frac{h_c[1 - P_U]}{h_D + h_G[1 - P_U] + h_R P_R (1 - P_U) + h_c P_C P_R (1 - P_U) + h_A[P_U + P_A P_C P_R - P_R P_C P_R P_U] + h_F P_F [P_U + P_A P_C P_R - P_R P_C P_R P_U]}
\pi_c &= \frac{h_c [P_C P_R (1 - P_U)] \pi_G}{h_G}
\pi_A &= \frac{h_A[P_U + P_A P_C P_R - P_R P_C P_R P_U] \pi_G}{h_G}
\pi_D &= \frac{h_D \pi_G}{h_G}
\pi_F &= \frac{h_F P_F [P_U + P_A P_C P_R - P_R P_C P_R P_U] \pi_G}{h_G}
\pi_R &= \frac{h_R P_R[1 - P_U] \pi_G}{h_G}
\end{align*}
\]