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MAE 499: UAH Moonbuggy: Analysis and Recommendations

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Abstract

A study has been conducted to determine the cause of failure of the previous UAH moonbuggy design and to evaluate and provide recommendations for new designs. Several points of failure on the old design were explored. The proposed new design was analyzed, and recommendations made as to its construction. An additional new design was also proposed.
Introduction / Background

The UAH moonbuggy is a human-powered vehicle designed and built by UAH undergraduates for an annual competition at the U.S. Space and Rocket Center in Huntsville. The relevant requirements are that the buggy must be storable in a 4’ cube, have 2 occupants, be carried for a certain distance by the same 2 people, and complete a course filled with various obstacles. The time required for conversion (or assembly) from the storage configuration to the racing configuration and the time required to complete the course are the main part of the competition. For the foregoing reasons, it is desirable to keep the vehicle as light as possible and keep assembly to a minimum. To allow quick navigation of the course by keeping bumps and jolts to a minimum, a suspension system is installed on the buggy. This was a point of failure on the previous design.

The old moonbuggy (from Spring 2008) is shown in Figures 1 and 2. The suspension is essentially a 4-bar linkage. The link to which the wheel mounts is the upright, and the upper and lower links are A-arms. Another member (with an almost question-mark shape) pulls on the lower A-arm to hold the vehicle up. This piece is called a pushrod, though there is probably a better term for it, especially since it is in tension. It is connected to the shock absorber through a crank.
In the competition, there were several points of failure. One was the insufficient ground clearance of the pushrods, which scraped against an obstacle and broke. Another was the lower A-arms, which bent on impacts until they became useless (Figure 3). Another was the rod ends that were installed between the upright and the A-arms; these also broke (Figure 4). The last was the driveshaft, which at the front wheels (Figure 2) had too much freedom of motion and tended to bind up. The first objective in the analysis is to estimate the forces involved so as to
understand why failure occurred in the A-arms and rod ends. A degree-of-freedom analysis is conducted to confirm the cause of trouble in the driveshaft and to show how to fix it.

**Figure 3. Bending of Lower A-arm.**

**Figure 4. Broken Rod End.**
The first proposed new design for spring 2009 (Figures 5 and 6) involves a flexible member that acts as a spring. It corrects many of the issues of the old design. There are no rod ends that can suffer bending. The lower A-arm flexes like a spring, eliminating the need for shock absorbers and greatly simplifying the system. The upright still swivels, but on an axis defined by revolute joints on the top and bottom. These joints are connected to “upright tips” that attach via more revolute joints to the (upper) A-arm and flexible member. A hollow axle is mounted to the upright, and the wheel rests on this axle, held in place by a pin. Bearings in the wheel allow it to rotate freely on the axle. The drive shaft passes through the hollow axle and mounts to the wheel on the outside. Thus, the drive shaft theoretically does not have to support the vehicle’s weight. All this had been designed before the study began. The drive train will include 2 universal joints or 2 constant velocity (CV) joints with a telescoping section in between. This is not studied in detail because it is assumed that universal joints (the fallback plan at this point) will work reasonably well, and because time is limited. Only the rigid components of the suspension system are analyzed; the flexible member is not a focus.
Figure 5. New Flexible Member Design.

Figure 6. Cross-section of New Flexible Member Design.
It remains an open question whether the flexible member will actually work. Calculations done by the moonbuggy team in Patran/Nastran are encouraging, but it is wise to have a backup plan. Therefore, a traditional shock absorber suspension (called the “mechanical design”) is also under development (Figure 7). This is to interface with the same moonbuggy chassis, uprights, and drive train as the flexible member design. As with the other new design, lessons learned from the previous failure have been incorporated. A link connects the top upright tip to an assembly of 3 links. These 3 links form a triangular lever, which allows the wheels to have a greater range of movement than the shock absorber.

Figure 7. New Mechanical Design.
Methods / calculations

The Old Design

Lower A-arm stress:

The first part of the analysis on the old buggy is to estimate the forces on the lower A-arm. This is to show why it failed and possibly to show what kind of forces the next design will have to withstand. It is estimated that the buggy weighed 100 lb and each occupant weighed 150 lb, so the total weight was 400 lb. This is divided between the 4 wheels to get 100 lb per wheel. A free-body diagram is shown for the lower A-arm (Figure 8). The two rods are called A and D, and the points at the ends of the rods are also called A and D.

Figure 8. Lower A-arm Free-body Diagram.

For the analysis in Mathcad, further assumptions are made. Since the joints at A, B, and D are all spherical bearings (i.e., ball and socket joints), there are no applied moments there. The only applied moment can be at C, and only in the x and y directions since it is a revolute joint.
Due to the symmetry of the A-arm, it is assumed that these moments are zero. The applied force at the wheel is vertical and the pushrod force $F$ has no $z$-component. Therefore, all forces in the $z$-direction are zero. The angle $\theta$ at which the force $F$ is applied could be measured from the actual moonbuggy (it was around -15 degrees, meaning that $F$ pointed down). The equations, from statics, are as follows:

Forces, $x$-direction:
\[ \sum F_x = 0 = D_x + A_x + F_x \]  \hspace{1cm} (1)

Forces, $y$-direction:
\[ \sum F_y = 0 = D_y + A_y + F_y + B_y \]  \hspace{1cm} (2)

Moments about point $C$:
\[ \sum M_C = 0 = \vec{r}_{BC} \times \vec{B} + \vec{r}_{AC} \times \vec{A} + \vec{r}_{DC} \times \vec{D} \]  \hspace{1cm} (3)

The known information about the direction of force $F$, as well as the assumptions noted above, results in the following equations:

\[ \frac{F_y}{F_x} = \tan \theta \]  \hspace{1cm} (4)

\[ D_z = A_z = F_z = 0 \]  \hspace{1cm} (5)

Since the $z$-components of force are thus known, there remain 6 equations with 6 unknowns (equation 3 is a vector equation). These are placed in a Mathcad solve block. Once the force at $A$ is found, it is transformed into Rod A coordinates to find the internal reactions for rod A (due to symmetry, rod D was identical):

\[ \vec{A}_{2x} = \vec{A} \begin{bmatrix} \cos(\phi / 2) \\ 0 \\ \sin(\phi / 2) \end{bmatrix} \]  \hspace{1cm} (6)

\[ \vec{A}_{2y} = \vec{A} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \]  \hspace{1cm} (7)
\[
\bar{A}_{2z} = \bar{A} \begin{pmatrix} -\sin(\varphi/2) \\ 0 \\ \cos(\varphi/2) \end{pmatrix}
\]  

where \(\varphi\) is the angle between rods A and D. Basically, this transformation rotates the coordinates about the y-axis so that they are aligned with the rod: \(x\) is now the axial direction, \(z\) is now the horizontal transverse direction, and \(y\) remains vertical. The resultant of \(A_{2y}\) and \(A_{2z}\) is the force that causes bending in the rod, and the stress caused by this force is calculated using a standard formula for round cross-sections:

\[
A_{2\text{trans}} = \sqrt{A_{2y}^2 + A_{2z}^2}
\]

\[
I = \frac{\pi}{4} \left( \frac{d}{2} \right)^4
\]

\[
\sigma_{\text{bend}} = \frac{M \cdot c}{I} = \frac{(A_{2\text{trans}} \cdot L)(d/2)}{I}
\]

where \(d\) is the diameter of the rod (0.5") and \(L\) is the length of the rod (~13"). The axial component is added to the bending component to get maximum normal stress (transverse shear is neglected):

\[
\sigma = \sigma_{\text{bend}} + \frac{\text{axial force}}{\text{area}} = \sigma_{\text{bend}} + \frac{A_{2x}}{\pi(d/2)^2}
\]

It was desired to conduct a parametric analysis, but Mathcad was not very easy to use in that respect. Therefore, the statics equations were arranged by hand into matrix form and coded into Matlab. A function was written that calculated key values (stress, force, etc.) based on certain inputs (dimensions, loads). Driver programs were used to run this function with various inputs. Some of the assumptions changed: there is now an applied force \(B_z\), resulting in reactions in the \(z\)-direction at A and D. The summation of forces in the \(z\)-direction is now
\[ \sum F_z = 0 = D_z + A_z + B_z \quad (13) \]

These 2 reactions are assumed equal. The resulting equations, solved using matrix inversion, are as follows (the lowercase letters are components of the position vectors for points \( A, B, \) and \( D, \) respectively):

\[
\begin{bmatrix}
0 & a_z & 0 & a_z - d_z \\
a_z & 0 & a_z - d_z & 0 \\
a_y & -a_x & a_y - d_y & d_y - a_x \\
\tan \theta & -1 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
F_x \\
F_y \\
D_x \\
D_y
\end{bmatrix}
= \begin{bmatrix}
-B_x a_z + B_z (a_y / 2 + d_y / 2 - b_y) \\
-B_x a_z + B_z (a_y / 2 + d_y / 2 - b_y) \\
B_z (b_y - a_y) + B_y (a_x - b_x) \\
0
\end{bmatrix} \quad (14)
\]

Once this is solved, the rest of the forces can be calculated:

\[ A_x = -F_x - B_x - D_x \quad (15) \]
\[ A_y = -F_y - B_y - D_y \quad (16) \]
\[ A_z = -B_z / 2 \quad (17) \]
\[ D_z = A_z \quad (18) \]

The hand solution is included in Appendix B. The stresses are calculated the same as in Mathcad. The results are given in the discussion section below.

Rod ends:

Another point of failure was the rod ends, which broke during the competition. These were each a \( \frac{1}{4}'' \) diameter steel threaded rod with a spherical bearing on the end. A typical rod end is shown in Figure 9 with a force \( F \) applied as on the lower A-arm.
Since the rod ends broke during the competition, the ultimate strength of the steel must have been exceeded. The critical point is where the bending moment is highest, which is where the threaded rod attaches to the A-arm. The “stress area” (area of the cross-section in the threaded rod) is a function of diameter, and is provided by Juvinall and Marshek (387). From this stress area, an equivalent radius is calculated; this is used to find moment of inertia:

\[ r = \sqrt{\frac{\text{stress area}}{\pi}} \]  \hspace{1cm} (19)

\[ I = \frac{\pi}{4} r^4 \]  \hspace{1cm} (20)

The bending stress equation is rearranged to calculate force required to break the rod end:

\[ S_u = \frac{Mc}{I} = \frac{F L r}{I} \rightarrow F = \frac{S_u I}{rL} \]  \hspace{1cm} (21)

Degree-of-freedom analysis:

Another problem for the old moonbuggy was the drive train. As shown in Figure 10, the drive train had 3 universal joints and a telescoping section. The number of degrees of freedom for a system is equal to the total initial number of degrees of freedom of all the bodies minus the degrees of freedom removed by the joints. Since this is a 3-dimensional situation, each body
starts with 6 degrees of freedom (3 rotational, 3 translational). A grounded joint (fixed joint) removes all 6. A revolute (pin) joint allows only rotation, and only about a single axis, so it removes 5. The telescoping joint is a prismatic joint that removes all but one translation axis, so it also removes 5. The number of degrees of freedom of the system is therefore

$$DOF = 6N_{bodies} - 6N_{ground} - 5N_{prismatic} - 5N_{revolute}$$  \hspace{1cm} (22)

where $N_{bodies}$ is the number of bodies in the system (9 in this case), $N_{ground}$ is the number of grounded bodies (1), $N_{prismatic}$ is the number of prismatic joints (1), and $N_{revolute}$ is the number of revolute joints (7). Figure 10 labels all the bodies and joints. Note that bodies 2 and 9 are connected to ground (body 1) via revolute joints. The number of degrees of freedom for this system is 3.

A similar calculation is performed for the more ideal system shown in Figure 11. It is the same but with 1 less universal joint. It has 1 degree of freedom.

**Figure 10.** Degree-of-freedom Analysis.
The Flexible Member Design

For the new flexible member design, all calculations are done in Mathcad. The first step is to solve for the linkage geometry. To simplify the math, the flexible member is modeled as a straight link that pivots roughly about the moonbuggy frame’s bottom tube (shown in Figure 6). On the left side of Figure 12 is shown the vector polygon used. Vectors D and E are constant, and the lengths of A (the A-arm), B (the upright and upright tips), and C (the flexible member) are known. The angle \( \theta \) is given and the angles \( \alpha \) and \( \varphi \) are unknown. The imaginary number method is used; the following equation is placed in a Mathcad solve block:

\[
C_{\text{len}}e^{i\alpha} + D_{\text{len}}i + E_{\text{len}} = B_{\text{len}}e^{i\varphi} + A_{\text{len}}e^{i\theta}
\]

where the “len” subscript indicates that the quantities are lengths, not forces.

Alternatively, this could easily have been done with sines and cosines, but the imaginary number notation is simpler. Angles \( \alpha \) and \( \varphi \) are found for multiple values of \( \theta \). As a sanity check, the solutions are plotted and animated, with \( \theta \) as the time-varying quantity. One frame of the animation is shown in Figure 13. This proves that the results actually make sense and are trustworthy.
Statics in 2 dimensions is used to calculate the forces on the upright. The free body diagram is shown in Figure 12, on the right. The length $d$ is the distance from the upright to the center of the wheel. Note also that

$$\rho = 90^\circ - \varphi$$

(24)

Figure 12. 2D Analysis of Flexible Member Design.

Figure 13. Sanity Check for Vector Polygon Solution.

In order to calculate the forces, certain assumptions have to be made:

- The wheels are not steered to any angle (i.e., the vehicle is not turning)
- The vehicle and occupants weigh 400 lb total.
- 1/4 of the vehicle's weight is on each wheel.
The statics equations are as follows:

Forces in x-direction

\[ A \cos \theta + F_x + B_x = 0 \]  
\[
(25)
\]

Forces in y-direction

\[ A \sin \theta + F_y + B_y = 0 \]
\[
(26)
\]

Moments about A

\[
B_x B_{\text{len}} \cos \rho - B_y B_{\text{len}} \sin \rho + F_x \left( \frac{1}{2} B_{\text{len}} \cos \rho + d \sin \rho \right) \\
+ F_y \left( - \frac{1}{2} B_{\text{len}} \sin \rho + d \cos \rho \right) + M_F = 0
\]
\[
(27)
\]

Where \( M_F \) is an extra moment applied where the force \( F \) is applied (when the buggy is stationary, \( F_y \) and \( M_F \) are zero; other situations are discussed later). The next step is a force balance on the A-arm. The free-body diagram is shown in Figure 14 and the dimensions used in Equations 28-30 below are defined in Figure 15.

![Free Body Diagram of A-arm](image)

**Figure 14. Free Body Diagram of A-arm.**
The statics equations used are as follows:

Forces in y-direction

\[ P_y + Q_y - A = 0 \]  \hspace{1cm} (28)

Moments about P

\[ a_4 Q_y - a_3 A = 0 \]  \hspace{1cm} (29)

Moments about A

\[ -a_1(2P_x) - a_3 P_y + a_2 Q_y = 0 \]  \hspace{1cm} (30)

It is assumed in Equation 30 that \( P_x = P_y \) because it is impossible to solve for each independently. The next step is to find the forces on the upright tips. The loads at A and B are converted to upright tip coordinates as follows:

\[
B_{side} = \begin{bmatrix} B_x \\ B_y \end{bmatrix} \cdot \begin{bmatrix} \cos \rho \\ -\sin \rho \end{bmatrix}
\]  \hspace{1cm} (31)

\[
B_{up} = \begin{bmatrix} B_x \\ B_y \end{bmatrix} \cdot \begin{bmatrix} \sin \rho \\ \cos \rho \end{bmatrix}
\]  \hspace{1cm} (32)
If the upright is vertical, then $B_{\text{side}}$ and $B_{\text{up}}$ are horizontal and vertical, respectively. The upright is usually not quite vertical, however, which is why the transformation is used.

The preceding analysis is for straight-ahead driving. However, since the buggy is a 4-wheeled vehicle, going around curves at significant speeds will produce an additional force at each wheel, pushing the vehicle toward the center of curvature (Figure 16). This force has to be estimated, so the location of the center of mass was assumed and the wheelbase was measured in the CAD model, then a simple dynamics analysis resulted in the maximum cornering force applied to the outside wheels without tipping the vehicle:

![Diagram showing forces and variables involved in the maximum turning force calculation.]

**Figure 16. Maximum Turning Force.**

\[
f = \frac{mg l_{\text{wheel}}}{h}
\]  

where $m$ is the mass of the moonbuggy, $g$ is acceleration due to gravity, $l_{\text{wheel}}$ is half the wheel track, and $h$ is the height of the center of mass. Each outside wheel takes half this force, as well as half the weight of the buggy (since the vehicle is on the verge of tipping, the inside wheels have no weight on them). Of course, at the axle there is not only this force but also a
bending moment equal to the cornering force times the wheel radius, and this is also included in
the force calculations on the upright:

$$M_p = \frac{f}{2} \frac{d_{\text{wheel}}}{2}$$  \hspace{1cm} (34)

An impact factor is required to get a good estimate of the real abuse that the suspension
components face. The impact factor is given by

$$C = 1 + \sqrt{1 + \frac{2h}{\delta}}$$  \hspace{1cm} (35)

where $C$ is the impact factor, $h$ is a drop height, and $\delta$ is the static deflection of the
suspension. This gives a multiplier for the equivalent peak force if the buggy were to be dropped
from a height $h$. At Dr. Wallace’s suggestion, 2 feet was used for the height. Eric Becnel
provided the static deflection estimate of 4 inches. The impact factor is therefore about 4.5.
Using this impact factor by multiplying it by the calculated forces, the yield stress for the axle
tube required to get a factor of safety greater than 1 is estimated.

It should be noted that the extra force required to compress the suspension beyond the
normal static compression is not directly calculated. Loading of 100 lbf is assumed, which is
realistic when the suspension is deflected for normal ride height. However, when the buggy hits
a bump and the suspension deflects further, larger forces exist because the flexible member acts
like a spring, so the further it deflects, the more force is involved. This would require a dynamic
analysis and cannot be calculated with statics. Therefore, all results are multiplied by the impact
factor to get a more realistic estimate of the forces involved. This is done also for the
mechanical suspension, which is discussed later.

To get an estimate for lateral forces on the upper A-arm and upright tips, steering effects
are taken into account. When the wheels are steered, the 100 lbf load is still acting vertically but
it now causes a bending moment in a new direction, which is opposed by forces from the upper A-arm acting in the longitudinal direction of the vehicle. Figure 17 below shows the free-body diagram of the rotated upright. Points A and C are where the upright tips connect to the A-arms. The angle $\theta$ is the steering angle. There are only 2 moments at A and C because there are revolute joints between the upright and the upright tips, so it is free to swivel about the y-axis (see Figure 6 for a better picture).

![Figure 17. Steering Effects.](image)

Forces and moments are summed about the upright. The knowns are steering angle $\theta$, which is varied from zero to 45 degrees, and the forces and moment applied at the wheel ($F$ and $M_{Fz}$). That leaves 10 unknowns; more than the 6 that can be solved using statics. Additional constraints are applied: since the joints between the upright tips and A-arms are also revolute,
there cannot be a moment in the direction of those joints. That leaves 8 equations and 10 unknowns, so $A_y$ is assumed zero (most vertical force is applied by the flexible member at the bottom). In addition, since there is no applied force $F_z$ (it is neglected in this analysis), then $C_z$ and $A_z$ have to be zero. The remaining 7 equations for the remaining 7 unknowns (with $A_y = 0$) are as follows:

Forces in x-direction

\[ A_x + C_x + F_x = 0 \]  \hspace{1cm} (36)

Forces in y-direction

\[ A_y + C_y + F_y = 0 \]  \hspace{1cm} (37)

Moments about C in yz-plane

\[ M_{C_x} + M_{A_x} = 0 \]  \hspace{1cm} (38)

Moments about C in xy-plane

\[ M_{A_x} + M_{C_x} + M_{F_z} - A_x B_{ten} - F_z \frac{B_{ten}}{2} + F_y d = 0 \]  \hspace{1cm} (39)

Moments about F (the place where force $F$ is applied) in xy-plane

\[ M_{A_z} + M_{C_z} + M_{F_z} + d \left( -A_y - C_y \right) + \frac{B_{ten}}{2} \left( C_x - A_x \right) = 0 \]  \hspace{1cm} (40)

A-arm revolute joint constraints

\[ M_{A_x} \sin \theta + M_{A_x} \cos \theta = 0 \]  \hspace{1cm} (41)

\[ M_{C_x} \sin \theta + M_{C_x} \cos \theta = 0 \]  \hspace{1cm} (42)

It was found that with 2 unknown moments in the same direction ($M_{A_x}$ and $M_{C_x}$), there were an infinite number of solutions. Therefore, $M_{C_x}$ was assumed zero such that all the moment would be applied at $A$, and this caused all the unknown moments to become zero. Only the forces have to be considered subsequently. These forces are transformed into coordinates that can be applied to the upright tips and upper A-arm. This is necessary because all the statics is
done in upright coordinates, and the upright is rotated with respect to the rest of the vehicle. The maximum forces are noted.

All maximum forces for the upright tips and upper A-arm were sent to a moonbuggy team member (Patrick Giddens) who ran stress analysis with finite-element analysis (FEA) software.

The stress on the axle tube is calculated in Mathcad. There is a vertical force and an applied moment at the wheel, and the axle is cantilevered from the upright, so maximum stress occurs where the axle meets the upright. This is calculated according to:

$$\sigma_{\text{max}} = \frac{M(D_o/2)}{I}$$ (43)

$$M = M_F + Fd \quad I = \frac{\pi}{64}(D_o^4 - D_i^4)$$ (44, 45)

where $D_o$ is the outer diameter, $D_i$ is the inner diameter, and $d$ is the distance from the upright to the point of application of the force, which is at the center of the wheel. Stress-concentration is ignored because the part should be made of a ductile material. Fatigue is not included in any of the stress analysis.

The New Mechanical Design

The calculations for the mechanical design are similar to those for the flexible design; the same methods are used. The vector polygon for the system as a whole and the free-body diagram for the upright are shown below. Notice that vector $C$ is in the opposite direction.
As can be seen, the vector polygon comprised of A through E is nearly unchanged from the flexible model. What is added is link N, a 2-force member (and therefore not subjected to bending); G and H, a triangular link; and I, the shock absorber. The force balance on the upright is also similar to the flexible design, except now C is a 2-force member and a new force N is added. The vector polygon equations are as follows:

\[ D_{len}i + E_{len} = C_{len}e^{i\phi} + B_{len}e^{i\theta} + A_{len}e^{i\theta} \]  \( (46) \)

\[ A_{len}e^{i\theta} + K_{len}i + J_{len} + G_{len}e^{i\alpha} = N_{len}e^{i\beta} \]  \( (47) \)

\[ K_{len}i + J_{len} + H_{len}e^{i(\alpha+\rho)} = M_{len}i + L_{len} + I_{len}e^{i\gamma} \]  \( (48) \)

Once the geometry is solved, a sanity check graph is created as before:
Statics is done on the upright as before. The following are the results of summing moments about the bottom of the upright and summing forces, respectively.

\[
- B_{\text{len}} \sin \varphi (A \cos \theta + N \cos \beta) + B_{\text{len}} \cos \varphi (A \sin \theta + N \sin \beta) + \left(\frac{1}{2} B_{\text{len}} \sin \varphi - d \sin \delta\right)(-F_x) - \left(\frac{1}{2} B_{\text{len}} \cos \varphi - d \cos \delta\right)(F_y) + M_F = 0
\]  

(49)

\[
-C e^{i\omega} + A e^{i\theta} + N e^{i\psi} + F_x + F_y i = 0
\]  

(50)

Statics is also done on the triangle link; the free body diagram is shown below:

![Free Body Diagram of Triangle Link](image-url)
For the triangle, the static equations are:

\[ H_{ten}/(\sin(\alpha + \rho) \cos \gamma - \cos(\alpha + \rho) \sin \gamma) + G_{ten}N(\sin \alpha \cos \beta - \cos \alpha \sin \beta) = 0 \quad (51) \]

\[ AA_x + AA_yi - Ne^{ip} - Ie^{ir} = 0 \quad (52) \]

As usual, these are put into Mathcad solve blocks. As before, the forces on the upright tips are converted to upward and sideways forces.

Once all the geometry and forces are calculated, it is possible to tweak dimensions here and there to improve the system. The primary concern was suspension travel/ride height. For static loading, assuming the shock absorber behaves as a spring, the force balance can be idealized according to Figure 21.

By summing moments about the hinge on the left, an appropriate ratio \( b/a \) can be calculated (Equation 53). It is assumed that \( F = 200 \text{lbf} \), \( k = 372 \text{lbf/in} \), and \( x = 2.25 \text{in} \). The spring constant \( k \) and maximum deflection \( x \) are based on the shock absorbers used on the old moonbuggy. It is found that the wheel travel (Equation 54) is maximized if the force \( F \) is twice the static loading (hence its value of 200 lbf). For the simple system shown in Figure 21, Equation 54 is only accurate for small deflections; it was used on the more sophisticated suspension linkage to get ballpark numbers.

\[ \frac{b}{a} = \frac{F}{kx} \quad \text{wheel travel} = x \frac{b}{a} \quad (53, 54) \]
All the analysis in the Mathcad model is done as a function of $\theta$, the angle of the upper A-arm. Maximum and minimum values of $\theta$ are specified and a vector is created that contains all the angles in between. Thus, the results from kinematics and statics are also vectors. The applied vertical load at the wheel is 100lbf for every case, the theory being (as explained before) that the impact factor will suffice to model the larger forces present in dynamic loading.

To adjust the Mathcad model until it meets the physical constraints of the shock absorber and the criteria that the static deflection is half the total deflection, the following procedure is used (Appendix I contains the Mathcad sheet):

- The maximum and minimum lengths of the shock absorber (vector $I$ in the polygon) are calculated.

- The maximum and minimum values of angle $\theta$ (angle of the upper A-arm) are adjusted until the limits of $I$ are within the physical limits of the real shock absorber.

- The magnitude of the force $I$ for 100lbf loading, and the magnitude of $kx$, are plotted against $\theta$ to see where they are equal. This point corresponds to the static equilibrium position. The vector index of the element of $\theta$ for which equilibrium occurred is noted.

- The wheel travel is calculated by taking the difference in the vertical coordinate of the vector $C$ between zero deflection and static deflection, where the static deflection is the element in $C$ with the same vector index as noted in the previous step.
- Wheel travel beyond static deflection is calculated by subtracting the maximum value of the vertical coordinate of \( C \) from the same static deflection value used above.

- Desired changes in geometry are made, and the above procedure is repeated as needed.

This is nearly the end of analysis for the study. To aid in future development of the design, forces on the main bracket are calculated. The applied forces are at the shock absorber's and the triangle's mounting locations. These are known already. The reaction forces are at the points where the bracket mounts to the tubes in the frame. These reaction forces are shown in Figure 22. It is assumed that the force \( BA_{xR} \) is the only horizontal reaction force, because it is impossible to calculate an individual horizontal reaction force for each side.

Subscripts "R" and "L" are used to differentiate between different sides of the bracket, in case future analysis requires the loads to be unbalanced. Points \( AA_R \) and \( AA_L \) refer to the triangle pivot points and points \( BA_R \) and \( BA_L \) refer to the frame tube mounting locations. There are no applied or reaction moments because all joints are modeled as pin joints. The equations for static equilibrium are given below.
\[ BA_{\text{st}} + AA_{\text{sl}} - AA_{\text{sr}} + I_R \cos \gamma_R - I_L \cos \gamma_L = 0 \]  
(55)

\[ BA_{\text{yr}} + BA_{\text{yl}} - AA_{\text{yr}} - AA_{\text{yl}} + I_R \sin \gamma_R + I_L \sin \gamma_L = 0 \]  
(56)

\[ BA_{\text{yr}} b_1 - AA_{\text{yr}} (b_1 - b_2 + J_{\text{len}} - L_{\text{len}}) + AA_{\text{sr}} (b_3 + K_{\text{len}} - M_{\text{len}}) - I_R \cos \gamma_R b_3 + I_R \sin \gamma_R (b_1 - b_2) + I_L \cos \gamma_L b_3 + I_L \sin \gamma_L b_2 + AA_{\text{yl}} (-b_2 + J_{\text{len}} - L_{\text{len}}) + AA_{\text{sl}} (b_3 + K_{\text{len}} - M_{\text{len}}) = 0 \]  
(57)

Some of these dimensions were defined earlier as part of the vector polygon (Figure 18). The others are defined below in Figure 23. Note that these calculations are for the brackets as a pair; each bracket sees only half these forces.

Figure 23. New Dimensions.
Results / Interpretation

The Old Design

One might wonder why the pushrod on the old suspension was applied at such a small angle. The answer: there was no other simple way to do it. The force from the shock absorber had to be applied somewhere, and it could not be applied at the upright because the upright was free to swivel for steering (applying a force here probably would have made it hard to steer—a very bad thing). It was decided to apply the force to the lower A-arm from the bottom, pulling down, since the drive axle’s location prevented the force from being applied from the top of the A-arm, pushing down. To keep ground clearance as large as possible, this necessitated an almost horizontal force application, causing very high forces. Due to geometry, this caused a bending moment in the lower A-arm, resulting in failure of the aluminum rods.

For the old moonbuggy’s lower A-arms, it turned out that the axial stress in the rods was negligible compared to the bending stress. The maximum bending moment occurred at the end of the rod farthest from A. The maximum normal stress (including axial compression) was \(-105\) ksi. The aluminum used was 6061 alloy with probably around 40 ksi yield strength (Juvinall, Marshek 796), so the factor of safety was 0.4.

The results of the parametric analysis in Matlab are presented in the following figures. The idea was to see if the failure could have been avoided by varying the geometry and rod diameters. The Mathcad static loading analysis had revealed that the force in the pushrod had been about 580 lbf of tension, which caused the reactions at A and D to be of a high magnitude. If the pushrod had acted in a more vertical direction, then it would have had a much smaller magnitude. The following figure shows this trend; the force is a magnitude so it is always positive. For \(\theta = 0\), the pushrod is horizontal and force is infinity. For the pushrod angled down
at -15 degrees, the force is 580 lbf. If angled up at 35 degrees or down at -35 degrees, the pushrod tension is a much more reasonable value. Negative 35 degrees would not be practical, however. Note that this figure represents static loading with no horizontal force applied at the wheel.

![Force in Pushrod as a Function of Pushrod Angle.](image)

**Figure 24. Force in Pushrod as a Function of Pushrod Angle.**

The next figure shows the corresponding stress in rod A, which follows the same trends. Thirty-five (or -35) degrees is about the smallest angle for which the rods do not yield.
Figure 25. Maximum Normal Stress in Aluminum Rod as a Function of Pushrod Angle.

Another subject of interest was the stress in the rods when the buggy struck an obstacle and a force was applied in the z-direction (see Figure 8). This required changing the assumptions: now $B_z$ was not zero and $A_z$ and $D_z$ were equal. The resulting stress for rod A is shown in the figure below. The graph for rod C would be the mirror image of this graph. Although this horizontal force makes a difference, it is small compared to the stress already present in the rod.
Figure 26. Maximum Normal Stress in Aluminum Rod as a Function of Horizontal (Impact) Force, Holding Pushrod Angle at \(-15\) degrees.

The next graph shows the same thing, but with the pushrod angle at 35 degrees instead of \(-15\) degrees. The graph has the opposite slope probably because the pushrod is applying force in a different direction. Note that the rods remain below yield stress.
Another topic of interest was what would happen if the A-arm were reconfigured to change the location the pushrod force was applied. The position of C was varied relative to the rest of the A-arm, and the resulting rod A stress is shown in the figure below. On the existing A-arm, the distance is 1 inch. As can be seen, applying the force lower, such that the line of action passes closer to point B, causes a significantly lower stress in the rods. If one were to imagine summing moments about point B (see Figure 8), then if $F$ was pointed at B, the forces at A and D would be significantly lower. This explains the trend in the graph.
Figure 28. Maximum Normal Stress in Aluminum Rod as a Function of Vertical Position of Pushrod Applied Force, with Pushrod Angle at -15 degrees.

The next figure shows how the maximum stress would vary if the diameter of the aluminum rods was varied. As can be seen, increasing the diameter to 1" would have brought the stress down below the yield stress.
Figure 29. Maximum Normal Stress in Aluminum Rod as a Function of Rod Diameter, with Pushrod Angle at $-15$ degrees.

The next figure shows the same thing, but with the pushrod at $35$ degrees. A diameter of $\frac{3}{4}''$ would have resulted in about $\frac{1}{4}$ the yield stress.
Figure 30. Maximum Normal Stress in Aluminum Rod as a Function of Rod Diameter, with Pushrod Angle at 35 degrees.

It should be noted that the pushrod was situated as it was because of geometric constraints (the drive shaft was in the way), so changing it to 35 degrees would not be easy. It would have required redesigning a large number of parts; probably the only way to accomplish it would be to have 2 pushrods connected to the lower A-arm, one on each side of the drive shaft.

It should also be noted that an impact loading was not considered except for horizontal forces, so all the vertical-load-induced stresses correspond to static loading.

The rod ends were also considered. Assuming a tensile strength of 80 ksi (the exact type of steel was unknown), and using appropriate stress areas, a diameter of at least 5/16" would be required (for the threaded rod) to avoid failure under static loading. For impact loading, a much larger diameter would be required. The fact that the rod ends did not fail until impact indicates that they had a larger tensile strength. This may have resulted from the cold-working the steel experienced in the thread rolling process.
The reason for the driveshaft’s design is as follows. The upper and lower A-arms were different lengths, so the axle had to telescope. Two universal joints were also required: one at the differential and one at the upright. Unfortunately, the differential was not aligned with the upright because the shock absorber was in the way, resulting in an excessively large angle when the wheels were steered (see Figure 2). This required a third universal joint. Because of all this, the drive shaft had 3 degrees of freedom. A 3-DOF drive axle is not a good design, and it tended to bind up and break periodically. A much better design would be to move the shock absorber somewhere else and just use 2 universal joints and a telescoping section (as in Figure 11). This would have had 1 degree of freedom, which is as it should be.

The Flexible Member Design

It was found that for the axle, a yield strength of 170 ksi is required for a factor of safety greater than 1. This means that it must be made of heat-treated steel. One option is oil-quenched 4140 steel, which can achieve a yield strength of up to about 250 ksi while still remaining relatively ductile (~15% elongation at break) (Juvinall, Marshek 791). The problem is that once heat-treated, the part will need to be ground to size, since heat-treating tends to deform the part. The UAH shop does not have the capability to do the grinding, so it would have to be sent to some other shop. An alternative is to make the inner diameter smaller, but this would reduce the size of the drive shaft, which was determined by the moonbuggy team after calculating maximum torque output. If the drive shaft diameter was reduced, then it would need to be made of heat-treated steel as well, thus defeating the purpose. Another option is to make the axle’s outer diameter larger, but this would eliminate the use of a commercially-made bicycle hub. It would require making a custom wheel hub, but the outer rim and tire could of course be
commercially made. Either heat treating or custom making the hub are reasonable options; each has its pros and cons.

The forces calculated for the upright tips were handed to Patrick Giddens on the moonbuggy team. His FEA results, which include the impact factor on all forces, are shown in Figure 31. There are one or maybe two small spots where the stress reaches about 30 ksi, but since these are small points it is not a big deal. Since the yield stress of 6061-T6 aluminum is around 40 ksi, the factor of safety is greater than 1 anyway. When the various loads are applied individually, the maximum stress is lower than shown here.

Figure 31. Upright Tip Stress Analysis by Patrick Giddens.

Below is shown the FEA results from the upper A-arm. The deflection shown here and in the upright tip image is greatly exaggerated. The A-arm is loaded in compression with a
sideways force applied as calculated from steering effects. The maximum stress is under 10 ksi, so the part is plenty strong; it is actually a little heavier than it needs to be.

Figure 32. Upper A-arm Stress Analysis by Patrick Giddens.

The New Mechanical Design

The mechanical design offers a back-up plan in case the flexible design does not work. The primary lesson from the old design is to apply the force as close to the wheel as possible, and as vertically as possible. The only known feasible way to do this is to apply the force from above the upright, so that is what is done. The use of rod ends is also avoided. Another lesson learned is to keep the A-arms as 2-force members. The connecting link and the 3 links of the triangle are also 2-force members, greatly simplifying the analysis and avoiding the devastating bending stresses.

The design uses the same shock absorbers as the old buggy: 7.875" uncompressed length, maximum compression 2.25", and spring constant 370 lbf/in. As noted previously, an iterative
procedure was used to get the desired wheel travel. Figure 33 shows the current range of motion. Due to geometry, the wheel travel was not linear with respect to applied load (or weight), so the system was configured for about 3.9 in of travel to the static loading position and an additional 4.1 in of travel beyond that. This was for a spring constant of about 370 lbf/in and a triangle ratio (long lever to short lever) of 3.46. This allows the use of shock absorbers from the old moonbuggy.

![Figure 33. Wheel Travel (front bracket not shown).](image)

Due to time constraints, no stress analysis was done on the new mechanical system as a part of this study. However, a Mathcad file has been developed that calculates forces (as noted earlier), and this can be used to do stress analysis. This is highly recommended for the moonbuggy team to pursue in the future if the mechanical design is used. The design uses the same upper A-arm, uprights, upright tips, axle, and drive system as the flexible member design, but some of these components are under significantly different stresses for this system, so stress analysis should precede their use. Forces applied to the upper A-arm are about 3 times as high,
and those applied to the upright tips are also larger. It might be necessary to make some parts thicker so they can be used on either model. But that all depends on future analysis.
Summary

Typically when rods are used in a suspension, they are designed to be 2-force members; either in tension or compression. The old design did not achieve this, which is why the rods failed in bending. Based on the parametric analysis, if the force had been applied at a steeper angle, then the failure might have been avoided. Increasing the diameter of the aluminum rods would have greatly helped as well. For the rod ends, it was found that if the diameter was increased, they might not have failed. A rather large diameter would be required to sustain impact loading. It would probably have been better, however, to use some other kind of connector, because rod ends are really designed for tension/compression forces, not bending. Revolute joints are used on the new designs, and they do not have these problems. To solve the degree-of-freedom problem, all that is needed is to eliminate one of the universal joints. The flexible member design, on which the new mechanical design is based, was developed by the team before the study began and features a differential aligned with the wheel, eliminating the “need” for a third U-joint.

Based on the analysis, the flexible member design will work. It will be necessary to make the axle tube out of heat-treated steel or custom-fabricate larger wheel hubs to withstand estimated stresses. The FEA results show that the upright tips and upper A-arm are satisfactorily strong.

The new mechanical design shows promise of being an acceptable system. Forces on all components have been calculated but stress analysis is yet to be done. The range of motion seems satisfactory, and if it needs to be adjusted, the length of the triangle links can be changed.

A collection of Mathcad files used in the analysis will be provided to the team for continued use on the current project and for future reference.
References

Aurora Bearing Company. “Aurora Bearing Commercial Catalog.”

Appendix A

3D static analysis of the moonbuggy lower A-arm for buggy at rest with passengers seated.

\[ x := 0 \quad y := 1 \quad z := 2 \quad \text{(for array indexes)} \]

Given information (all dimensions approximate)

- Angle between rods: \( \varphi := 40\deg \)
- Diameter of rod: \( d := 0.5\text{in} \)
- Angle of force \( F \): \( \theta := -15\deg \)
- Length of rod: \( L := (12 + 0.9)\text{in} \)

Point vectors

\[
CC := \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad \text{in} \\
AA := \begin{pmatrix} 14.75 \text{in} \\ -1 \text{in} \\ \frac{11 + \frac{5}{16}}{2} \text{in} \end{pmatrix} \\
\text{AA} = \begin{pmatrix} 14.75 \\ -1 \\ 5.656 \end{pmatrix} \text{in}
\]

\[
BB := \begin{pmatrix} -1.75 \\ -1 \\ 0 \end{pmatrix} \quad \text{in} \\
DD := \begin{pmatrix} AA_x \\ AA_y \\ -AA_z \end{pmatrix} \\
DD = \begin{pmatrix} 14.75 \\ -1 \\ -5.656 \end{pmatrix} \text{in}
\]

Forces

\[
B := \begin{pmatrix} 0 \\ 100 \text{lbf} \\ 0 \end{pmatrix} \quad B = \begin{pmatrix} 0 \\ 100 \text{lbf} \\ 0 \end{pmatrix}
\]

Guesses

\[
F := \begin{pmatrix} 50 \text{lbf} \\ -50 \text{lbf} \\ 0 \end{pmatrix} \quad A := \begin{pmatrix} 10 \text{lbf} \\ 10 \text{lbf} \\ 0 \end{pmatrix} \quad D := \begin{pmatrix} 10 \text{lbf} \\ 10 \text{lbf} \\ 0 \end{pmatrix}
\]

Solve block

Given

\[
D_x + A_x + F_x = 0 \quad \text{sum of forces, x-direction}
\]

\[
B_y + F_y + A_y + D_y = 0 \quad \text{sum of forces in y-direction}
\]

\[
(BB - CC) \times B + (AA - CC) \times A + (DD - CC) \times D = 0 \quad \text{sum of moments about C}
\]

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Appendix A

\[ \begin{align*}
D_z &= 0 & A_z &= 0 & F_z &= 0 \\
\tan(\theta) &= \frac{F_y}{F_x} \\
\begin{pmatrix}
A \\
D \\
F
\end{pmatrix} &= \text{Find}(A, D, F)
\end{align*} \]

\[
A = \begin{pmatrix}
-279.448 \\
24.878 \\
0
\end{pmatrix} \text{ lbf} \\
D = \begin{pmatrix}
279.448 \\
24.878 \\
0
\end{pmatrix} \text{ lbf} \\
F = \begin{pmatrix}
558.896 \\
-149.756 \\
0
\end{pmatrix} \text{ lbf}
\]

\[ |F| = 578.611 \text{ lbf} \]

Verify answers

\[
A + B + D + F = \begin{pmatrix}
0 \\
-2.556 \times 10^{-14} \\
0
\end{pmatrix} \text{ lbf} \]

sum of forces

\[
(DD - AA) \times D + (CC - AA) \times F + (BB - AA) \times B = \begin{pmatrix}
0 \\
0 \\
2.516 \times 10^{-13}
\end{pmatrix} \text{ lbf-in}
\]

Now, need to convert to rod coordinates (x along rod axis and z normal to rod axis; y is unchanged). The origin of this reference frame is at the end of the rod.

\[
x_B := \begin{pmatrix}
\cos\left(\frac{\varphi}{2}\right) \\
0 \\
\sin\left(\frac{\varphi}{2}\right)
\end{pmatrix} \\
y_B := \begin{pmatrix}
0 \\
1 \\
0
\end{pmatrix} \\
z_B := \begin{pmatrix}
-\sin\left(\frac{\varphi}{2}\right) \\
0 \\
\cos\left(\frac{\varphi}{2}\right)
\end{pmatrix}
\]

\[
A2 := \begin{pmatrix}
A \cdot x_B \\
A \cdot y_B \\
A \cdot z_B
\end{pmatrix}
\]

\[ A2 = \begin{pmatrix}
-262.595 \\
24.878 \\
95.577
\end{pmatrix} \text{ lbf} \]

\[ \sqrt{(A2_x)^2 + (A2_z)^2} = 279.448 \text{ lbf} \]
Appendix A

Now I can break the load down into an axial and transverse force

\[ A_{2\text{axial}} := A_{2x} \quad A_{2\text{trans}} := \sqrt{(A_{2y})^2 + (A_{2z})^2} \]

\[ A_{2\text{axial}} = -262.595 \cdot \text{lb} \] (compression)

\[ A_{2\text{trans}} = 98.761 \cdot \text{lb} \]

Calculate stress

\[ l := \frac{\pi}{4} \left( \frac{d}{2} \right)^4 \quad \text{area} := \pi \left( \frac{d}{2} \right)^2 \quad l = 3.068 \times 10^{-3} \cdot \text{in}^4 \quad \text{area} = 0.196 \cdot \text{in}^2 \]

\[ \sigma_{\text{axial}} := \frac{A_{2\text{axial}}}{\text{area}} \quad \sigma_{\text{axial}} = -1.337 \times 10^3 \, \text{psi} \]

\[ A_y = 24.878 \cdot \text{lb} \]

\[ \sigma_{\text{bend}}(x) := \frac{A_{2\text{trans}}(-x)}{l} \left( \frac{d}{2} \right) \left( \sigma = \frac{M \cdot y}{l} \right) \]

\[ \sigma_{\text{tot}}(x) := -\sigma_{\text{bend}}(x) + \sigma_{\text{axial}} \quad \text{Bending + compressive stress (the minus is to get the maximum compressive force)} \]

\[ x_1 := 0 \text{in}, -0.1 \text{in} \ldots -L \]

Maximum compression is:

\[ -\sigma_{\text{bend}}(-L) + \sigma_{\text{axial}} = -105.154 \cdot \text{ksi} \]

Maximum tension is:
Appendix A

\[ \sigma_{\text{bend}}(-L) + \sigma_{\text{axial}} = 102.479 \text{ ksi} \]

Maximum shear stress along cross-sectional plane is:

\[ \frac{4}{3} \cdot \frac{A_{\text{trans}}}{\text{area}} = 0.671 \text{ ksi} \]

Tensile yield strength of 6061 aluminum is 40 ksi
elastic modulus is 10,000 ksi,
shear modulus is 3770 ksi

Appendix B: Static Equilibrium Equations for Matlab

\[ \begin{align*}
\text{Given:} \quad & F_y = \tan \theta, \quad A_x = D_x \sqrt{b_x}, \quad b_x = 0 \\
\sum F_z &= A_x + D_z + B_z = 0 \quad \Rightarrow 2A_z = -B_z \quad \Rightarrow A_z = -\frac{B_z}{2} \\
\sum F_y &= 0 = B_y + F_y + D_y + A_y \quad \Rightarrow A_y = -B_y - F_y - D_y \\
\sum F_x &= 0 = F_x + B_x + D_x + A_x \quad \Rightarrow A_x = -F_x - B_x - D_x \\
\sum M_c &= 0 = \tau_{alc} \times \vec{A} + \tau_{bxc} \times \vec{B} + \tau_{alc} \times \vec{B} \\
\end{align*} \]

\[ \begin{vmatrix}
\vec{\tau} \\
\vec{a}_z \\
\vec{A}_x \\
\vec{A}_y \\
\vec{A}_z \\
\vec{D}_x \\
\vec{D}_y \\
\vec{D}_z \\
\vec{B}_x \\
\vec{B}_y \\
\vec{B}_z \\
\end{vmatrix} + \begin{vmatrix}
\vec{\tau} \\
\vec{a}_x \\
\vec{a}_y \\
\vec{a}_z \\
\vec{D}_x \\
\vec{D}_y \\
\vec{D}_z \\
\vec{B}_x \\
\vec{B}_y \\
\vec{B}_z \\
\end{vmatrix} + \begin{vmatrix}
\vec{\tau} \\
\vec{a}_x \\
\vec{a}_y \\
\vec{a}_z \\
\vec{D}_x \\
\vec{D}_y \\
\vec{D}_z \\
\vec{B}_x \\
\vec{B}_y \\
\vec{B}_z \\
\end{vmatrix} = 0 \]

\[ \begin{align*}
\vec{F}_x &= F_x \tan \theta \\
\vec{F}_y &= \frac{B_y}{2} \\
\vec{F}_z &= \frac{B_z}{2} \\
\vec{F}_r &= F_r \sec \theta \\
\end{align*} \]

\[ \begin{align*}
\dot{x} &= \dot{a}_x (-\frac{B_z}{2}) - a_z (-B_y - F_y - D_y) + d_y (-\frac{B_z}{2}) - d_z D_z + b_x B_z - b_z B_y \\
\dot{y} &= \dot{b}_y B_x \\
\dot{z} &= \dot{b}_z B_x \\
\dot{r} &= \dot{x} \frac{B_x}{2} - \dot{y} \frac{B_x}{2} - \dot{z} \frac{B_x}{2} \\
\end{align*} \]

\[ \begin{align*}
\vec{F}_x(a_x) + D_x(a_x - d_x) + B_x(-\frac{a_x}{2} + \frac{d_x}{2}) + B_y(a_x) &= 0 \\
\vec{F}_y(a_y) + D_y(a_y - d_y) + B_y(-\frac{a_y}{2} + \frac{d_y}{2}) + B_x(a_y) &= 0 \\
\vec{F}_z(a_z) + D_z(a_z - d_z) + B_z(-\frac{a_z}{2} + \frac{d_z}{2}) + B_y(a_z) &= 0 \\
\end{align*} \]
Appendix C: Matlab Code

Part 1: Force and Stress Calculation Program

function [sigma_max, max_shear, A2_axial, A2_trans, F] = A_arm_iter_03( aa, bb, dd, theta, d, B )

% David Agnew, MAE 499: Matlab program for iteratively calculating stresses
% in A-arm of suspension
% Version 3 is the same as version 2 except output is suppressed.
% -- ALL LENGTHS IN INCHES, ALL FORCES IN POUNDS, ALL ANGLES IN DEGREES, ALL
% STRESSES IN KSI --
% sigma_max = maximum tensile stress (2-element array)
% max_shear = maximum shear stress
% A2_axial = axial force in rod A
% A2_trans = transverse force (causing bending) in rod A
% F = force in suspension 2-force member
% aa = position of point A (where force A is applied)
% bb = position of point B (where force B is applied)
% dd = position of point D (where force D is applied)
% The above positions are 3-element vectors, either row or column.
% It is assumed that force F is applied at C, which is at the origin.
% theta = angle at which F is applied
% d = diameter of rods
% B = force B (3-element vector, row or column)
% NOTE: It is assumed that bz = 0 and that Fx = 0.

x = 1; y = 2; z = 3; % array indexes

% GIVENS
ax = aa(x); ay = aa(y); az = aa(z);

bx = bb(x); by = bb(y); bz = bb(z);
if( bz ~= 0 )
  disp('Warning: the program assumes that bz = 0, so the results may not be accurate.');
end

dx = dd(x); dy = dd(y); dz = dd(z);

phi = 40; % angle between rods, degrees
L = 12.09; % approximate length of each rod, inches

Bx = B(x);
By = B(y);
Appendix C: Matlab Code

Bz = B(z); 

theta = theta*pi/180.0; % convert to radians  
phi = phi*pi/180.0;  

% Solve system of equations  

matrix = [ 0, az, 0, az-dz;  
az, 0, az-dz, 0;  
ay, -ax, ay-dy, dx-ax;  
tan(theta), -1, 0, 0 ];  

vector = [ -By*az + Bz*(ay/2 + dy/2 - by);  
-az*Bx + Bz*(ax/2 + dx/2 - bx);  
Bx*(by-ay) + By*(ax-bx);  
0 ];  

answers = (matrix^-1) * vector; % solve system of linear equations  

% Process results  
Fx = answers(1);  
Fy = answers(2);  
Dx = answers(3);  
Dy = answers(4);  
Ax = -Fx - Bx - Dx;  
Ay = -Fy - By - Dy;  
Az = -Bz/2;  
Dz = Az;  
A = [ Ax; Ay; Az ];  
D = [ Dx; Dy; Dz ];  
F = [ Fx; Fy; 0 ];  

% Find stresses  

xB = [ cos(phi/2); 0; sin(phi/2) ];  
yB = [ 0; 1; 0 ];  
zB = [-sin(phi/2); 0; cos(phi/2) ];  

A2 = [ A'*xB; A'*yB; A'*zB ];  

A2_axial = A2(x);  
A2_trans = sqrt( A2(y)^2 + A2(z)^2 );  

I = (pi/4)*(d/2)^4; % area moment of inertia, inches^4  
area = pi*(d/2)^2; % area, in^2  

% disp('Maximum stresses (ksi)');  
sigma_max = [ A2_axial/area + A2_trans*(-L)*(d/2)/I;  
A2_axial/area - A2_trans*(-L)*(d/2)/I; ]/1000;  

max_shear = (4/3)*abs(A2_trans/area)/1000;  

end
Appendix C: Matlab Code

Part 2: Driver Program for Analysis by Varying Force Angle (θ)

% File for calling A-arm calculation file, with results as a function of theta.

aa = [14.75, -1, 5.656]; % point A
bb = [-1.75, -1, 0]; % point B
dd = [14.75, -1, -5.656]; % point D
dia = 0.5; % diameter of rods
B = [0, 100, 0]; % force B

sigma = []; % maximum normal stress in rod A
tau = []; % maximum shear stress in rod A
A2_ax = []; % axial force in rod A
A2_tr = []; % transverse force (causing bending moment) in rod A
F = []; % force F (applied at C, which is at the origin)

theta = []; % the independent variable at this time (angle of force F)
numPts = 50; % number of points
min_ = -40;
max_ = 90;

for k = 1:numPts
    theta = [theta, min_ + (k-1)*(max_-min_)/(numPts-1)]; % get next value for independent variable
    [a, b, c, d, e] = A_arm_iteration03(aa, bb, dd, theta(k), dia, B); % get forces/stresses from program
    sigma = [sigma, max(abs(a(1)), abs(a(2)))]; % add new values to vectors
    tau = [tau, b];
    A2_ax = [A2_ax, c];
    A2_tr = [A2_tr, d];
    F = [F, norm(e)];
end

disp(' ');
disp(' ');
disp('theta, sigma, tau, A2_ax, A2_tr, F');

figure1 = figure('name','Maximum Stress as a Function of Force Angle');
axes1 = axes('parent',figure1,'ylim',[0,100]);
box('on');
hold('all');
title('with B = 100lbf vertical', 'parent',axes1);
xlabel('Angle theta (degrees)', 'parent',axes1);
ylabel('Normal Stress (ksi)', 'parent',axes1);
plot1 = plot(theta, sigma, 'parent', axes1);

figure2 = figure('name','Force F as a Function of Force Angle');
axes2 = axes('parent',figure2,'ylim',[0,10000]);
box('on');
Appendix C: Matlab Code

hold('all');
title('with B = 100lbf vertical', 'parent', axes2);
xlabel('Angle theta (degrees)', 'fontweight', 'bold', 'parent', axes2);
ylabel('Force (lbf)', 'fontweight', 'bold', 'parent', axes2);
plot2 = plot( theta, F, 'parent', axes2);

Part 3: Driver Program for Analysis by Varying Horizontally Applied Force (Bz)

% File for calling A-arm calculation file, with results as a function of Bz.

aa = [15.2, -1, 5.6]; % point A
bb = [-1.75, -1, 0]; % point B
dd = [15.2, -1, -5.6]; % point D
dia = 0.5; % diameter of rods
theta = -15; % angle of F

sigma = []; % normal stress
tau = []; % shear stress
A2_ax = []; % axial force in rod A
A2_tr = []; % transverse force in rod A
F = []; % force F
Bz = []; % the independent variable at this time (force at B in z-direction)
umPts = 50; % number of points
min_ = -200; % minimum value
max_ = 200; % maximum value

for k = 1:numPts
    Bz = [Bz, min_ + (k-1)*(max_-min_)/(numPts-1)]; % get next value for independent variable
    [a, b, c, d, e] = A_arm_iteration03(aa, bb, dd, theta, dia, [0,100,Bz(k)]); % get forces/stresses from program
    sigma = [sigma, max(abs(a(1)), abs(a(2)))]; % add new values to vectors
    tau = [tau, b];
    A2_ax = [A2_ax, c];
    A2_tr = [A2_tr, d];
    F = [F, norm(e)];
end

disp(' ');
disp(' ');
disp('Bz, sigma, tau, A2_ax, A2_tr, F');
[Bz',sigma',tau',A2_ax',A2_tr',F']

figure1 = figure('name', 'Maximum Stress as a Function of Bz');
axes1 = axes('parent', figure1, 'ylim', [0,120]);
box('on');
hold('all');
title(['(with theta = ',num2str(theta),')', 'degrees']),'parent', axes1);
xlabel('Bz (lbf)', 'fontweight', 'bold', 'parent', axes1);
ylabel('Normal Stress (ksi)', 'fontweight', 'bold', 'parent', axes1);
Appendix C: Matlab Code

```matlab
plot1 = plot( Bz, sigma, 'parent', axes1 );

figure2 = figure('name','Axial Force in rod A as a Function of Bz');
axes2 = axes('parent',figure2);
box('on');
hold('all');
title(['with theta = ',num2str(theta), ' degrees']);
xlabel('Bz (lbf)', 'fontweight', 'bold', 'parent', axes2);
ylabel('Force in rod A (lbf)', 'fontweight', 'bold', 'parent', axes2);
plot2 = plot( Bz, A2_ax, 'parent', axes2 );

% REDO with different angle theta
theta = 35;  % angle of F

sigma = []; % normal stress
tau = []; % shear stress
A2_ax = []; % axial force in rod A
A2_tr = []; % transverse force in rod A
F = []; % force F
Bz = [];  % the independent variable at this time
for k = 1:numPts
    Bz = [Bz, min_ + (k-1)*(max_-min_)/(numPts-1)];
    [a, b, c, d, e] = A_arm_iteration03( aa, bb, dd, theta, dia, [0,100,Bz(k)] );
    sigma = [sigma, max( abs( a(1) ), abs( a(2) ) )];
    tau = [tau, b];
    A2_ax = [A2_ax, c];
    A2_tr = [A2_tr, d];
    F = [F, norm(e)];
end

disp(' ');
disp(' ');
disp(['Bz, sigma, tau, A2_ax, A2_tr, F']);
[Bz',sigma',tau',A2_ax',A2_tr',F']

figure3 = figure('name','Maximum Stress as a Function of Bz');
axes3 = axes('parent',figure3,'ylim',[0,120]);
box('on');
hold('all');
title(['with theta = ',num2str(theta), ' degrees']);
xlabel('Bz (lbf)', 'fontweight', 'bold', 'parent', axes3);
ylabel('Normal Stress (ksi)', 'fontweight', 'bold', 'parent', axes3);
plot3 = plot( Bz, sigma, 'parent', axes3);

Part 4: Driver Program for Analysis by Varying Rod Diameter

% File for calling A-arm calculation file, with results as a function of rod diameter.
```

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Appendix C: Matlab Code

aa = [ 15.2, -1, 5.6 ]; % point A
bb = [-1.75, -1, 0 ]; % point B
dd = [ 15.2, -1, -5.6 ]; % point C
B = [ 0, 100, 0 ]; % Force B
theta = -15; % angle of F

sigma = []; % normal stress
tau = []; % shear stress
A2_ax = []; % axial force on rod A
A2_tr = []; % transverse force on rod A
F = []; % force F

dia = []; % the independent variable at this time (rod diameter)
numPts = 50; % number of points
min_ = 0.4; % minimum value
max_ = 1.5; % maximum value

for k = 1:numPts
    dia = (dia, min_ + (k-1)*(max_-min_)/(numPts-1)); % get next value for independent variable
    [a, b, c, d, e] = A_armIteration03(aa, bb, dd, theta, dia(k), B); % get forces/stresses from program
    sigma = [sigma, max(abs(a(1)), abs(a(2)))]; % add new values to vectors
    tau = [tau, b];
    A2_ax = [A2_ax, c];
    A2_tr = [A2_tr, d];
    F = [F, norm(e)];
end

disp(' ');
disp(' ');
disp('dia, sigma, tau, A2_ax, A2_tr, F');
[dia',sigma',tau',A2_ax',A2_tr',F']

figure1 = figure('name','Maximum Stress as a Function of Rod Diameter');
axes1 = axes('parent',figure1,'ylim',[0,120]);
box('on');
hold('all');
title(['(with theta = ',num2str(theta),') degrees']);
xlabel('Diameter (in)','fontweight','bold','parent',axes1);
ylabel('Normal Stress (ksi)','fontweight','bold','parent',axes1);
plot1 = plot(dia, sigma, 'parent', axes1);

% REDO with different angle of F

theta = 35; % angle of F

sigma = []; % normal stress
tau = []; % shear stress
A2_ax = []; % axial force on rod A
A2_tr = []; % transverse force in rod A
Appendix C: Matlab Code

\[ F = [ ]; \text{ \% force } F \]

dia = [ ]; \text{ \% the independent variable at this time (rod diameter) } 

numPts = 50; \text{ \% number of points}

min_ = 0.4; \text{ \% minimum value}

max_ = 1.5; \text{ \% maximum value}

for k = 1:numPts
    dia = (dia, min_ + (k-1)* (max_-min_)/(numPts-1)); \text{ \% get next value for independent variable}
    [a, b, c, d, e] = A_arm_iteration03( aa, bb, dd, theta, dia(k), B ); \text{ \% get forces/stresses from program}
    sigma = [sigma, max( abs( a(1) ), abs( a(2) ) )]; \text{ \% add new values to vectors}
    tau = [tau, b];
    A2_ax = [A2_ax, c];
    A2_tr = [A2_tr, d];
    F = [F, norm(e)];
end

disp( ' ');
disp( ' ');

disp('dia, [sigma], tau, A2_ax, A2_tr, F');
[dia',sigma',tau',A2_ax',A2_tr',F']

figure2 = figure('name','Maximum Stress as a Function of Rod Diameter');
axes2 = axes('parent',figure2,'ylim',[0,120]);
box('on');
hold('all');
title(['

Part 5: Driver Program for Analysis by Varying Geometry

\% File for calling A-arm calculation file, with results as a function of force F location (y-direction).
\% Based on the resulting plot, it probably would have been better had the \% force been applied to the upper A-arm rather than the lower one.

aa = [15.2, -1, 5.6 ]; \text{ \% point A}
bb = [ -1.75, -1, 0 ]; \text{ \% point B}
dd = [15.2, -1, -5.6]; \text{ \% point D}
B = [0, 100, 0 ]; \text{ \% Force B}
theta = -15; \% angle of F

dia = 0.5; \% diameter of rods

sigma = [ ]; \% normal stress

tau = [ ]; \% shear stress
A2_ax = [ ]; \% axial force in rod A

A2_tr = [ ]; \% transverse force in rod A

F = [ ]; \% force F
Appendix C: Matlab Code

h = [];  % the independent variable at this time (height of A, B, and D BELOW C)
numPts = 50;  % number of points
min_ = -2.0;  % minimum value
max_ = 2.0;  % maximum value

for k = 1:numPts
    h = [h, min_ + (k-1)*(max_-min_)/(numPts-1)];  % get next value for independent variable
    aa(2) = h(k);  % increment heights
    bb(2) = h(k);
    dd(2) = h(k);
    [a, b, c, d, e] = A_arm_iteration03( aa, bb, dd, theta, dia, B );  % get forces/stresses from program
    sigma = [sigma, max( abs( a(1) ), abs( a(2) ) )];  % add new values to vectors
    tau = [tau, b];
    A2_ax = [A2_ax, c];
    A2_tr = [A2_tr, d];
    F = [F, norm(e)];
end

disp(' ');
disp(' ');
disp('[h',sigma', tau', A2_ax', A2_tr', F']);
[h',sigma',tau',A2_ax',A2_tr',F']

figure1 = figure('name','Maximum Stress as a Function of Height of point C');
axes1 = axes('parent',figure1,'ylim',[0,160]);
box('on');
hold('all');
title([['with theta = ',num2str(theta),' degrees']], 'parent',axes1);
xlabel('Height Difference of C above A, B, and D (in)', 'fontweight','bold','parent',axes1);
ylabel('Normal Stress (ksi)', 'fontweight','bold','parent',axes1);
plot1 = plot( -h, sigma, 'parent', axes1 );  % h is negative here in order to % get the height of C ABOVE A, B, and D
Appendix D
Stress on Rod Ends

From "Fundamentals of Machine Component Design," page 387:

\[
\text{Diameter} := \left(\frac{1}{4}, \frac{5}{16}, \frac{3}{8}, \frac{7}{16}, \frac{1}{2}\right) \text{-in} \\
\text{StressArea} := \left(0.0318, 0.0524, 0.0775, 0.1063, 0.1419\right) \text{-in}^2
\]

Don't know exactly what the ultimate stress is because steel type is unknown. Assume for now that \( S_u := 80\text{ksi} \)

\[
P = \frac{F}{A} \quad F = P \cdot A
\]

For pure tensile stress, the maximum is:

\[
F_{\text{axial}} := (S_u \cdot \text{StressArea}) \\
F_{\text{axial}} = \left(2.544 \times 10^3, 4.192 \times 10^3, 6.2 \times 10^3, 8.504 \times 10^3, 1.135 \times 10^4\right) \text{-lbf}
\]

The length from the end of the rod to the eye of the rod end, minus the thickness of the nut, was around \( L := (0.9 - 0.2)\text{-in} \)

\[
A = \pi \cdot r^2 \\
r = \frac{A}{\pi} \\
r := \sqrt{\frac{\text{StressArea}}{\pi}} \\
I := \left(\frac{\pi}{4} \cdot r^2\right)
\]

So, based on bending moment equation:

\[
\sigma = \frac{M \cdot y}{I} = \frac{L \cdot F \cdot y}{I} \\
F_{\text{bend}} := \frac{S_u \cdot l}{L \cdot r} \\
F_{\text{bend}} = \left(91.411, 193.354, 347.784, 558.672, 861.649\right) \text{-lbf}
\]

\[
\tau_u := S_u \cdot 0.577 \\
\tau_u = 46.16 \text{-ksi}
\]

\[
\tau_{\text{max}} = \frac{4 \cdot P}{3 \cdot A} \\
P = \frac{3}{4} \cdot \tau_{\text{max}} \cdot A
\]

\[
F_{\text{shear}} := \frac{3}{4} \cdot \tau_u \cdot \text{StressArea} \\
F_{\text{shear}} = \left(1.101 \times 10^3, 1.814 \times 10^3, 2.683 \times 10^3, 3.68 \times 10^3, 4.913 \times 10^3\right) \text{-lbf}
\]

Based on these numbers, the most likely cause of failure was bending. The images from the wreckage show that the rod end broke near where it connected to the rod, which is where the bending moment would be highest.

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Appendix E

Degree of Freedom Analysis

B = body
R = revolute joint
P = prismatic joint

\[ \text{DOF} = 6N_{\text{bodies}} - \text{DOF}_{\text{removed}} \]

Degrees of freedom removed

revolute := 5 only 1 rotation allowed
Appendix E

\text{prismatic} := 5 \quad \text{only 1 translation allowed (rotation not allowed)}

\text{fixed} := 6 \quad \text{no movement allowed (a grounded link)}

\textbf{First system:}

\begin{align*}
N_{\text{bodies}} & := 7 \\
N_{\text{revolute}} & := 6 \\
N_{\text{prismatic}} & := 1 \\
N_{\text{fixed}} & := 1 \\
\text{DOF} & := 6 \cdot N_{\text{bodies}} - \text{revolute} \cdot N_{\text{revolute}} - \text{prismatic} \cdot N_{\text{prismatic}} - \text{fixed} \cdot N_{\text{fixed}}
\end{align*}

\text{DOF} = 1

\textbf{Second system:}

\begin{align*}
N_{\text{bodies}} & := 9 \\
N_{\text{revolute}} & := 8 \\
N_{\text{prismatic}} & := 1 \\
N_{\text{fixed}} & := 1 \\
\text{DOF} & := 6 \cdot N_{\text{bodies}} - \text{revolute} \cdot N_{\text{revolute}} - \text{prismatic} \cdot N_{\text{prismatic}} - \text{fixed} \cdot N_{\text{fixed}}
\end{align*}

\text{DOF} = 3
Appendix F

Kinematic model of new suspension design (2D) with forces. I have simplified it by considering the flexible member as a rigid link (see diagram below).

Dimensions highlighted in green, other parameters in blue; answers in yellow.

When referring to lengths, the letters have the subscript "len"; otherwise they refer to forces.

Most of the graphs are just sanity checks; the really important ones are highlighted.

Range variable definition: \( \text{numPts} := 60 \quad k := 0 \ldots \text{numPts} - 1 \)

Dimensions

- Length of upper A-arm is \( A_{len} := 9.75\text{in} \)
- Height of upright is \( B_{len} := 6\text{in} \)
- Length of flexible member is \( C_{len} := 16\text{in} \)
- Height of upper A-arm above flexible member at frame mounts is \( D_{len} := 5\text{in} \)

The flexible member is approximated as a simple link that pivots about the bottom tube of the frame.
Appendix F

Horizontal difference between the mounts of the above 2 members is \( E_{\text{len}} = 6.75 \text{in} \)

Driving variables

\[
\theta_{\text{min}} := 130^\circ \quad \theta_{\text{max}} := 200^\circ
\]

Angles of upper A-arm

\[
\theta_k := \theta_{\text{min}} + \left( \theta_{\text{max}} - \theta_{\text{min}} \right) \frac{k}{\text{numPts} - 1}
\]

Guesses

\( \varphi_k := 90^\circ \quad \alpha_k := 140^\circ \)

Solve block

Given

\[
\left( C_{\text{len}} e^{i\alpha} + D_{\text{len}} e^{i\varphi} + E_{\text{len}} \right) = \left( B_{\text{len}} e^{i\varphi} + A_{\text{len}} e^{i\theta} \right)
\]

\[\begin{align*}
\varphi &:= \text{Find}(\varphi, \alpha) \\
\alpha &:= \text{Find}(\varphi, \alpha)
\end{align*}\]
Graph of assembly (movie)

\[ j := \text{FRAME} \]

\[ \alpha := 10 \]

\[
\begin{align*}
P_{0x} &:= 0 & P_{1x} &= P_{0x} + \text{Re}(C_{\text{len}} e^{i\phi_j}) & P_{2x} &= P_{1x} & P_{3x} &= P_{2x} + E_{\text{len}} & P_{4x} &= P_{3x} - \text{Re}(A_{\text{len}} e^{i\theta_j}) & P_{5x} &= P_{4x} \\
P_{0y} &:= 0 & P_{1y} &= P_{0y} + \text{Im}(C_{\text{len}} e^{i\phi_j}) & P_{2y} &= P_{1y} + D_{\text{len}} & P_{3y} &= P_{2y} & P_{4y} &= P_{3y} - \text{Im}(A_{\text{len}} e^{i\theta_j}) & P_{5y} &= P_{4y} \\
X &:= (P_{0x} P_{1x} P_{2x} P_{3x} P_{4x} P_{5x}) & Y &:= (P_{0y} P_{1y} P_{2y} P_{3y} P_{4y} P_{5y})
\end{align*}
\]
Appendix F

Force calculations for upright based on kinematic model:

**Mass of the buggy with riders is** \( \text{m}_b = 400 \text{lbm} \)

If center of mass is located a distance \( \text{com}_y = 2.5 \text{ft} \) off the ground, and the distance between the vehicle center plane and the outside wheels is \( l_{\text{wheel}} = 17 \text{in} \), then max force at turn without tipping is given by

\[
F_{\text{turn}} = \frac{1}{2} \frac{\text{m}_b \cdot g \cdot l_{\text{wheel}}}{\text{com}_y}
\]

\[
F_{\text{turn}} = 113.333 \text{lb} \quad \text{divided by 2 because each wheel sees half}
\]

**Diameter of the wheel with tire is** \( d_{\text{wheel}} = 28 \text{in} \)

\[
M_{F_k} := -F_{\text{turn}} \frac{d_{\text{wheel}}}{2}
\]

\[
M_{F_k} = -1.587 \times 10^3 \text{lb} \cdot \text{in}
\]

**Moment applied by wheel to upright**
forces applied by wheel

\[ x_k := -F_{\text{turn}} \]
\[ y_k := 200\text{lb} \]

\[ d := 4.8\text{in} \]

distance from upright to center of wheel where load is applied
(it's a large number because of the spacers at the front wheels)

\[ \rho := 90\text{deg} - \varphi \]

angle of upright from the vertical

**Guesses**

\[ A_k := 0\text{lb} \]
\[ B_x := 0\text{lb} \]
\[ B_y := 0\text{lb} \]

**Given**

\[ (A - \cos(\theta) + F_x + B_x) = 0 \]

sum of forces, \( y \)

\[ (A - \sin(\theta) + F_y + B_y) = 0 \]

sum of forces, \( x \)

\[
\begin{align*}
B_x - B_{\text{len}} \cos(\rho) - B_y - B_{\text{len}} \sin(\rho) &+ F_x \left( \frac{B_{\text{len}} \cos(\rho)}{2} + d \cdot \sin(\rho) \right) \\
&+ F_y \left( \frac{B_{\text{len}} \sin(\rho)}{2} + d \cdot \cos(\rho) \right) + M_F = 0
\end{align*}
\]

sum of moments about \( A \) (the top)

\[
\begin{pmatrix} A \\ B_x \\ B_y \end{pmatrix} := \text{Find}(A, B_x, B_y)
\]

Find the external forces applied to the upright

\[
\text{max}(A) = 87.687\text{-lb} \]
\[
\text{min}(A) = 47.867\text{-lb} \]
Now, do statics to find the forces at the other joints of the A-arm
Appendix F

\[ a_1 = 9.75 \text{ in} \quad a_3 = 2.688 \text{ in} \quad a_4 = 11.375 \text{ in} \quad a_2 = a_4 - a_3 \]

**Guesses:**
- \( P_{y_k} = 0 \text{ lbf} \)
- \( P_{x_k} = 0 \text{ lbf} \)
- \( Q_{y_k} = 0 \text{ lbf} \)

**Given**

\[
\begin{align*}
(P_y + Q_y - A) &= 0 \\
(a_4 Q_y - a_3 A) &= 0 \\
[-a_1 (2P_x) - a_3 P_y + a_2 Q_y] &= 0
\end{align*}
\]

Sum of forces

Sum of moments about \( P \)

Sum of moments about \( A \)

\[
\begin{align*}
P_x \\
P_y \\
Q_y
\end{align*}
:= \text{Find}(P_x, P_y, Q_y)
\]

\[
\begin{align*}
\text{min}(P_y) &= 36.555 \text{ lbf} \\
\text{max}(P_y) &= 66.966 \text{ lbf} \\
\text{min}(Q_y) &= 11.311 \text{ lbf} \\
\text{max}(Q_y) &= 20.721 \text{ lbf}
\end{align*}
\]

**Qx is assumed equal to Px**

---

**Force at P is greater, so use P and ignore Q**

**Force components perpendicular (side) and parallel (up) to the upright, at base of**
Appendix F

upright

\[ E_{\text{side}, k} := \begin{pmatrix} Bx_k \\ By_k \end{pmatrix} \begin{pmatrix} \cos(\rho_k) \\ -\sin(\rho_k) \end{pmatrix} \]

\[ E_{\text{up}, k} := \begin{pmatrix} Bx_k \\ By_k \end{pmatrix} \begin{pmatrix} \sin(\rho_k) \\ \cos(\rho_k) \end{pmatrix} \]

**Force components perpendicular (side) and parallel (up) to the upright, at top of upright**

\[ A_{\text{side}, k} := \begin{pmatrix} A_k \cdot \cos(\theta_k) \\ A_k \cdot \sin(\theta_k) \end{pmatrix} \begin{pmatrix} \cos(\rho_k) \\ -\sin(\rho_k) \end{pmatrix} \]

\[ A_{\text{up}, k} := \begin{pmatrix} A_k \cdot \cos(\theta_k) \\ A_k \cdot \sin(\theta_k) \end{pmatrix} \begin{pmatrix} \sin(\rho_k) \\ \cos(\rho_k) \end{pmatrix} \]
Appendix F

Maximums:

**horizontal forces**

\[
\begin{pmatrix}
\max(B_{side}) \\
\min(B_{side}) \\
\max(A_{side}) \\
\min(A_{side})
\end{pmatrix} = \begin{pmatrix}
179.891 \\
143.318 \\
-47.577 \\
-49.669
\end{pmatrix} \text{ lbf}
\]

**vertical forces**

\[
\begin{pmatrix}
\max(B_{up}) \\
\min(B_{up}) \\
\max(A_{up}) \\
\min(A_{up})
\end{pmatrix} = \begin{pmatrix}
-180.292 \\
-282.202 \\
72.263 \\
-18.104
\end{pmatrix} \text{ lbf}
\]

\[
\frac{179.891}{2} = 89.945
\]

\[
\frac{282.202}{2} = 141.101
\]
Appendix G

Steering Effects

Assumptions / dimensions

Assume upright is vertical.

Height of upright is \( B_{cen} = 6\text{in} \)

\( d = 4.8\text{in} \)

distance from upright to center of wheel where load is applied
(it's a large number because of the spacers at the front wheels)

Diameter of the wheel with tire is \( d_{wheel} = 28\text{in} \)

Mass of the buggy with riders is \( m_b = 400\text{lbm} \)

If center of mass is located a distance \( y_{com} = 2.5\text{ft} \) off the ground, and the distance between the vehicle center plane and the outside wheels is \( l_{wheel} = 17\text{in} \), then max force at turn without tipping is given by

\[
F_{turn} := \frac{1}{2} \frac{m_b g l_{wheel}}{y_{com}}
\]

\[
F_{turn} = 113.333\text{lb} \]

I have decided to ignore forces in the z-direction because they would merely be superimposed on the results. It would not cause a moment reaction at the pivot points because they are revolute joints (the steering system would counter the moment), and extra shear force is most likely not a really big deal in this case.

\[
M_{Fz} := 0.5 \left( -\frac{F_{turn} d_{wheel}}{2} \right)
\]

\[
M_{Fz} = -793.333\text{lb-ft} \]

\( F_x := -F_{turn} \)

\( F_y := 200\text{lb} \)

Steering angle is \( \theta_k \) and \( k := \frac{45\text{deg}}{30} \)

This actually has no impact on the force solve block

Guesses

\[
A_x := 10\text{lb}
\]

\[
A_y := 10\text{lb}
\]

\[
C_x := 10\text{lb}
\]

\[
C_y := 10\text{lb}
\]

\[
M_{A_x} := 10\text{lb-ft}
\]

\[
M_{A_y} := 10\text{lb-ft}
\]

\[
M_{C_x} := 10\text{lb-ft}
\]

\[
M_{C_y} := 10\text{lb-ft}
\]

Given
Appendix G

**Sum of forces**

\[ 0 = (Ax + Cx + Fx) \]
\[ 0 = (Ay + Cy + Fy) \]

**Sum of moments**

\[ 0 = (M_{Cx} + M_{Ax}) \]
\[ 0 = \left( M_{Az} - M_{Cz} + M_{Fz} - Ax \cdot Blen - Fx \cdot \frac{B_{len}}{2} + Fy \cdot d \right) \]
\[ 0 = \left[ M_{Az} + M_{Cz} + M_{Fz} + d \cdot (-Ay - Cy) + \frac{B_{len}}{2} \cdot (Cx - Ax) \right] \]

**Assumptions**

\[ Ay = 0 \quad M_{Cx} = 0 \]

**Additional constraints due to joints above and below**

\[ (M_{Ax} \cdot \sin(\theta) + M_{Az} \cdot \cos(\theta)) = 0 \]
\[ (M_{Cx} \cdot \sin(\theta) + M_{Cz} \cdot \cos(\theta)) = 0 \]
Appendix G

\[
\begin{pmatrix}
Ax \\
Ay \\
Cx \\
Cy \\
M_{Ax} \\
M_{Az} \\
M_{Cx} \\
M_{Cz}
\end{pmatrix}
:= \text{Find}(Ax, Ay, Cx, Cy, M_{Ax}, M_{Az}, M_{Cx}, M_{Cz})
\]

\[
\theta \text{ \ deg}
\]

\[
\frac{M_{Ax}}{\text{lb}} \quad \frac{M_{Az}}{\text{lb}} \quad \frac{M_{Cx}}{\text{lb}} \quad \frac{M_{Cz}}{\text{lb}}
\]

\[
R_{Ax} := (Ax \cdot \cos(\theta) - Ax \cdot \sin(\theta))
\]

\[
R_{Cx} := (Cx \cdot \cos(\theta) - Cz \cdot \sin(\theta))
\]

\[
A_{z} := 0 \text{ lb}
\]

\[
C_{z} := 0 \text{ lb}
\]

transform into moonbuggy coordinates
Appendix G

\[
\begin{align*}
R_{Ay} &= Ay \\
R_{Az} &= (Ax \cdot \sin(\theta) + Az \cdot \cos(\theta)) \\
R_{Cy} &= Cy \\
R_{Cz} &= (Cx \cdot \sin(\theta) + Cz \cdot \cos(\theta))
\end{align*}
\]

![Graph with axes and labeled lines showing force variation with respect to angle.]

\[
\begin{align*}
\max(R_{Az}) &= 59.711 \text{ lbf} \\
\min(R_{Az}) &= 0 \text{ lbf} \\
\max(R_{Cz}) &= 20.428 \text{ lbf} \\
\min(R_{Cz}) &= 0 \text{ lbf}
\end{align*}
\]

This is for a max turning angle of 45 deg

For 1/2 max. turning force, the force is

\[R_{Az} = 39.677 \text{ lbf}\]

For just sitting on ground with wheels turned:

\[R_{Az} = 56.569 \text{ lbf}\]
Axle Tube (supports wheel; drive shaft is inside)

**Outer diameter is** \( D_o := 20\text{mm} \quad D_o = 0.787\text{-in} \)

**Inner diameter is** \( D_i := 0.625\text{in} \)

**Length of tube (from bottom of fillet to end of tube) is** \( L := 4.637\text{in} \)

**Distance from bottom of fillet to wheel side of upright is** \( a := 0.625\text{in} \)

**Applied load from wheel (either distributed or applied at midpoint between edge of upright and end of tube) is** \( F := 200\text{lbf} \)

**Moment applied by wheel due to forces from rounding a turn is** \( M_F := -1.587 \times 10^3 \text{lbf-in} \)

**Moment of inertia is** \( I := \frac{\pi}{64} \left( D_o^4 - D_i^4 \right) = 0.011\text{-in}^4 \)

\[
M := \left( \frac{L-a}{2} + a \right) F + M_F = \frac{D_o}{2} \quad \frac{L-a}{2} + a = 2.631\text{-in} \\
\]

\[
\sigma_{\text{max}} := \frac{M}{I} = \frac{D_o}{2} \quad \sigma_{\text{max}} = -36.702\text{-ksi} \\
\]

**Impact factor:**

\[
\text{height} := 2\text{ft} \quad \text{static_deflection} := 4\text{in} \\
\text{IF} = 1 + \sqrt{1 + \frac{2 \cdot \text{height}}{\text{static_deflection}}} = 4.606 \\
\]

**Assume that the yield stress is** \( \sigma_y := 170\text{ksi} \)

**Factor of safety:**

\[
\frac{\sigma_y}{\text{IF} \cdot \sigma_{\text{max}}} = 1.006 \\
\]

With the original dimensions, using a steel with \( Sy = 240 \text{ ksi} \) (oil quenched 4140 steel) for both the axle and drive shaft, a factor of safety of 2.0 for both axle and drive shaft can be achieved.

Or, we could make our own wheels with larger diameters, thus allowing regular steel to be used and still achieve a factor of safety of 3.5. The bearings could be press-fitted onto the axle, thus eliminating having a pin on the end, and screwed into the wheel. Or they could be press-fit into both.
Appendix H
Appendix I

Kinematic model of new suspension design (2D) with forces

Dimensions highlighted in green, other parameters in blue; answers in yellow.

Vector diagram of model:

When referring to lengths, the letters have the subscript "len"; otherwise they refer to forces.

Most of the graphs are just sanity checks; the really important ones are highlighted.

Range variable definition:  \( \text{numPts} := 50 \quad k := 0..\text{numPts} - 1 \)

Dimensions

The flexible member is approximated as a simple link that
Appendix I

Length of upper A-arm is \( A_{len} := 9.75 \text{in} \)
Height of upright is \( B_{len} := 6 \text{in} \)
Length of lower A-arm is \( C_{len} := 15 \text{in} \)
Height of upper A-arm's mount above lower A-arm's mount is \( D_{len} := 5.5 \text{in} \)
Horizontal difference between the mounts of the above 2 members is \( E_{len} := 5.25 \text{in} \)
Length of diagonal suspension 2-force member is \( N_{len} := 5 \text{in} \)
Shock absorber side of triangle is \( H_{len} := 2.125 \text{in} \)
Other side of triangle is \( G_{len} := 7.35 \text{in} \)
Horizontal distance between A-arm mount and triangle pivot is \( J_{len} := 2.5 \text{in} \)
Horizontal distance between A-arm mount and shock absorber mount is \( L_{len} := -4 \text{in} \)
Vertical distance between A-arm mount and triangle pivot is \( K_{len} := 3 \text{in} \)
Vertical distance between A-arm mount and shock absorber mount is \( M_{len} := 2.5 \text{in} \)
Angle of triangle piece is \( \rho := 79 \text{deg} \)
Axle tube length (to center of wheel) is \( \text{axle} := 2.6 \text{in} \)

Driving variables

\( \theta_{\text{min}} := 155 \text{deg} \quad \theta_{\text{max}} := 203 \text{deg} \)

Angles of upper A-arm

\[
\theta_k := \theta_{\text{min}} + \left( \theta_{\text{max}} - \theta_{\text{min}} \right) \frac{k}{\text{numPts} - 1}
\]

Guesses

\( \varphi_k := 90 \text{deg} \quad \psi_k := -10 \text{deg} \quad \alpha_k := 45 \text{deg} \quad \beta_k := 135 \text{deg} \quad \gamma_k := 30 \text{deg} \)

\( I_{len_k} := 10 \text{in} \) (length of shock absorber)

Solve block 1

Given

\[
\left( D_{len} e^{i \psi} + E_{len} \right) = (C_{len} e^{i \varphi} + B_{len} e^{i \psi} + A_{len} e^{i \theta})
\]

\( \text{Im}(\varphi) = 0 \quad \text{Im}(\psi) = 0 \)

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Appendix I

\[
\begin{bmatrix} \varphi \\ \psi \end{bmatrix} := \text{Find}(\varphi, \psi)
\]

Solve block 2

Given

\[
\begin{bmatrix} A_{\text{len}} e^{i \theta} + K_{\text{len}} e^{i \phi} + J_{\text{len}} + G_{\text{len}} e^{i \alpha} \\ N_{\text{len}} e^{i \beta} \end{bmatrix} = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}
\]

\[
\text{Im}(\alpha) = 0 \quad \text{Im}(\beta) = 0
\]

\[
\begin{bmatrix} \alpha \\ \beta \end{bmatrix} := \text{Find}(\alpha, \beta)
\]

Solve block 3

Given

\[
\begin{bmatrix} J_{\text{len}} + K_{\text{len}} e^{i \phi} + H_{\text{len}} e^{i (\theta + \phi)} \\ I_{\text{len}} + M_{\text{len}} e^{i \gamma} \end{bmatrix} = \begin{bmatrix} \gamma \\ I_{\text{len}} \end{bmatrix}
\]

\[
\text{Im}(\gamma) = 0 \quad \text{Im}(I_{\text{len}}) = 0
\]

\[
\begin{bmatrix} \gamma \\ I_{\text{len}} \end{bmatrix} := \text{Find}(\gamma, I_{\text{len}})
\]

Make animation

\[
j := \text{FRAME} \quad j := 59
\]

First loop

\[
P0x := 0 \quad P1x := P0x + \Re\left(C_{\text{len}} e^{i \psi} \right) \quad P2x := P1x + \Re\left(B_{\text{len}} e^{i \psi} \right) \quad P3x := P2x + \Re\left(A_{\text{len}} e^{i \theta} \right) \quad P4x := P3x - E_{\text{len}} \quad P5x := P4x
\]

\[
P0y := 0 \quad P1y := P0y + \Im\left(C_{\text{len}} e^{i \psi} \right) \quad P2y := P1y + \Im\left(B_{\text{len}} e^{i \psi} \right) \quad P3y := P2y + \Im\left(A_{\text{len}} e^{i \theta} \right) \quad P4y := P3y \quad P5y := P4y - D_{\text{len}}
\]

\[
X := (P0x \quad P1x \quad P2x \quad P3x \quad P4x \quad P5x) \quad Y := (P0y \quad P1y \quad P2y \quad P3y \quad P4y \quad P5y)
\]

Second loop

\[
Q0x := P2x \quad Q1x := Q0x + \Re\left(N_{\text{len}} e^{i \beta} \right) \quad Q2x := Q1x - \Re\left(G_{\text{len}} e^{i \alpha} \right) \quad Q3x := Q2x + \Re\left(H_{\text{len}} e^{i (\alpha + \phi)} \right) \quad Q4x := Q3x - \Re\left(I_{\text{len}} e^{i \gamma} \right)
\]
Appendix I

\[ Q_0y := P_2y \quad Q_{1y} := Q_{0y} + \text{Im} \left( N_{\text{len}} e^{i\cdot3j} \right) \quad Q_{2y} := Q_{1y} - \text{Im} \left( G_{\text{len}} e^{i\cdot0j} \right) \quad Q_{3y} := Q_{2y} + \text{Im} \left[ H_{\text{len}} e^{i\cdot(\alpha_j+p)} \right] \quad Q_{4y} := Q_{3y} - \text{Im} \left( I_{\text{len}} e^{i\cdot\gamma_j} \right) \]

\[ X_2 := (Q_{0x} \quad Q_{1x} \quad Q_{2x} \quad Q_{3x} \quad Q_{4x}) \quad Y_2 := (Q_{0y} \quad Q_{1y} \quad Q_{2y} \quad Q_{3y} \quad Q_{4y}) \]

**Axle piece**

\[ R_{0x} := \text{mean}(P_{1x}, P_{2x}) \quad R_{1x} := R_{0x} + \text{axle}\cdot\cos(\phi_j - 90\text{deg}) \]

\[ R_{0y} := \text{mean}(P_{1y}, P_{2y}) \quad R_{1y} := R_{0y} + \text{axle}\cdot\sin(\phi_j - 90\text{deg}) \]

\[ X_3 := (R_{0x} \quad R_{1x}) \quad Y_3 := (R_{0y} \quad R_{1y}) \]

**Shock absorber travel**

\[ \text{min}(I_{\text{len}}) = 5.634\text{ in} \]

\[ \text{max}(I_{\text{len}}) = 7.869\text{ in} \]

\[ \text{max}(I_{\text{len}}) - \text{min}(I_{\text{len}}) = 2.235\text{ in} \]

**Wheel travel:**

\[ \text{max}(C_{\text{len}}\cdot\sin(\psi)) - \text{min}(C_{\text{len}}\cdot\sin(\psi)) = 7.941\text{ in} \]

\[ \text{shock\_travel} := \text{max}(I_{\text{len}}) - \text{min}(I_{\text{len}}) \]

\[ \text{shock\_travel} = 2.235\text{ in} \]

\[ \text{wheel\_travel} := \text{max}(C_{\text{len}}\cdot\sin(\psi)) - \text{min}(C_{\text{len}}\cdot\sin(\psi)) \]

\[ \text{wheel\_travel} = 7.941\text{ in} \]

**Forces applied by wheel to upright**

\[ F_{x_k} = 113.333\text{ lbf} \]

\[ F_{y_k} = 200\text{ lbf} \]

\[ M_{F_k} = -1.587 \times 10^3\text{ lbf}\cdot\text{in} \]

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Appendix I

Solve for forces at upright

\[ \delta := 90\degree - \varphi \]

Guesses

\[ A_k = -50\text{lb} \quad N_k = -50\text{lb} \quad C_k := 20\text{lb} \]

Given

sum of moments about bottom of upright

\[
0 = \left[ -B_{\text{len}} \cdot \sin(\varphi) \cdot (A \cdot \cos(\theta) + N \cdot \cos(\beta)) + B_{\text{len}} \cdot \cos(\varphi) \cdot (A \cdot \sin(\theta) + N \cdot \sin(\beta)) + \left( \frac{B_{\text{len}}}{2} \cdot \sin(\varphi) - \text{axle} \cdot \sin(\delta) \right) \cdot (-F_x) - \left( \frac{B_{\text{len}}}{2} \cdot \cos(\varphi) + \text{axle} \cdot \cos(\delta) \right) \cdot F_y + M_F \right]
\]

sum of forces

\[
0 = \left( -C \cdot e^{i \psi} + A \cdot e^{i \theta} + N \cdot e^{i \beta} + F_x + F_y \cdot i \right)
\]

\[
\begin{pmatrix}
A \\
C \\
N
\end{pmatrix} := \text{Find}(A, C, N)
\]

\[
\text{Im}(A) = 0 \\
\text{Im}(C) = 0 \\
\text{Im}(N) = 0
\]
Appendix I

max(A) = 345.694·lbf
min(A) = 286.625·lbf
max(C) = -388.84·lbf
min(C) = -421.67·lbf
max(N) = -159.89·lbf
min(N) = -193.847·lbf

Forces on the top upright tip

\[
\begin{align*}
\min [(A \cdot \cos(\theta) + N \cdot \cos(\beta))] &= -295.735\cdot\text{lbf} \\
\max [(A \cdot \cos(\theta) + N \cdot \cos(\beta))] &= -265.844\cdot\text{lbf} \\
\min [(A \cdot \sin(\theta) + N \cdot \sin(\beta))] &= -286.144\cdot\text{lbf} \\
\max [(A \cdot \sin(\theta) + N \cdot \sin(\beta))] &= -70.191\cdot\text{lbf}
\end{align*}
\]

vertical

Forces on lower upright tip

\[
\begin{align*}
\min [(C \cdot \cos(\psi))] &= -409.068\cdot\text{lbf} \\
\max [(C \cdot \cos(\psi))] &= -379.177\cdot\text{lbf} \\
\min [(C \cdot \sin(\psi))] &= -86.144\cdot\text{lbf} \\
\max [(C \cdot \sin(\psi))] &= 129.809\cdot\text{lbf}
\end{align*}
\]

horizontal

Solve for forces on triangle

Guesses
\[
\begin{align*}
A_{A_{x}} &= 0\text{lbf} \hspace{1cm} A_{A_{y}} &= 0\text{lbf} \hspace{1cm} I_{k} &= 100\text{lbf}
\end{align*}
\]

Given

sum of moments about pivot of triangle
Appendix I

\[ 0 = \left[ H_{\text{len}} \cdot \mathrm{i} \cdot (\sin(\alpha + \rho) \cdot \cos(\gamma) - \cos(\alpha + \rho) \cdot \sin(\gamma)) + G_{\text{len}} \cdot N \cdot (\sin(\alpha) \cdot \cos(\beta) - \cos(\alpha) \cdot \sin(\beta)) \right] \]

sum of forces

\[ 0 = \left( A_{A_x} + A_{A_y} \cdot \mathrm{i} - N \cdot e^{i\beta} - I \cdot e^{i\beta} \right) \]

\[ \text{Im}(A_{A_x}) = 0 \quad \text{Im}(A_{A_y}) = 0 \quad \text{Im}(I) = 0 \]

\[
\begin{pmatrix}
A_{A_x} \\
A_{A_y} \\
i
\end{pmatrix} = \text{Find}(A_{A_x}, A_{A_y}, I)
\]

These values do not reflect the actual compressive force in the shock for the angle theta specified.

\[
\min(I) = -947.655 \cdot \text{lbfc}
\]

\[
\max(I) = -497.446 \cdot \text{lbfc}
\]

Determine travel based on shock absorber constraints

Static deflection is where the force in I is equal to -kx

\[
k_s := 373 \frac{\text{lbfc}}{\text{in}} \quad x_k = 7.875 \text{in} - l_{\text{len}}
\]
Appendix I

Travel for static loading:

\[ \delta_{\text{static}} := \left( C_{\text{len}} \sin(\psi) \right)_{\text{element}} - \min\left( C_{\text{len}} \sin(\psi) \right) = 3.878\text{-in} \]

Additional travel past static loading:

\[ \delta_{\text{extra}} := \max\left( C_{\text{len}} \sin(\psi) \right) - \left( C_{\text{len}} \sin(\psi) \right)_{\text{element}} = 4.062\text{-in} \]

\[ \delta_{\text{static}} + \delta_{\text{extra}} = 7.941\text{-in} \]

Force applied at static loading:

\[ k_s \left( 7.875\text{in} - I_{\text{len}_{\text{element}}} \right) \frac{H_{\text{len}}}{G_{\text{len}}} = 103.937\text{-lbf} \]

Vertical force applied at max deflection:

\[ k_s \left( 7.875\text{in} - 2.25\text{in} \right) \frac{H_{\text{len}}}{G_{\text{len}}} \cos(\alpha_{\text{numPts}-1}) = 399.224\text{-lb} \]

Approximate equivalent drop height:

\[ F \cdot d = \frac{1}{2} k \cdot x^2 \]
\[ d = \frac{k \cdot x^2}{2F} \]
\[ \text{drop height} := \frac{k_s \cdot (2.25\text{in})^2}{2 \cdot 100\text{lbf}} = 9.442\text{-in} \]

That's total fall distance, which includes the suspension compressing.
Appendix I

The distance the buggy would fall before wheels touch ground would be:

\[ \text{drop\_height} = \delta_{\text{extra}} - \delta_{\text{static}} = 1.501 \text{-in} \]

**Force calculations for mounting bracket**

The following formulas take into account the forces on both sides of the buggy (from left and right wheels). Therefore, for symmetric loading, just set both sides equal. If loading is not symmetric, the forces at the frame tubes will adjust to maintain equilibrium. Only 1 horizontal force is calculated at the tubes; for stress analysis, apply this to either tube. The equations are set up so that the forces on one side can be from a past set of calculations while on the other side can be for the current set of calculations.

- **Distance between top two frame tubes**: \( b_1 = 10.5 \text{in} \)
- **Horizontal distance between shock absorber mount and frame tube**: \( b_2 = 2.5 \text{in} \)
- **Vertical distance between shock absorber mount and frame tube**: \( b_3 = 1.5 \text{in} \)
- **Shock absorber force, left side**: \( l_k \) or \( l_k \) 
- **Shock absorber angle, left side**: \( \gamma_k \)
- **Vertical triangle pivot force, left side**: \( AA_{y_k} \) or \( AA_{y_k} \)
- **Horizontal triangle pivot force, left side**: \( AA_{x_k} \) or \( AA_{x_k} \)
- **Shock absorber force, right side**: \( r_k \) or \( r_k \)
- **Shock absorber angle, right side**: \( \gamma_k \)
- **Vertical triangle pivot force, right side**: \( AA_{y_k} \) or \( AA_{y_k} \)
- **Horizontal triangle pivot force, right side**: \( AA_{x_k} \) or \( AA_{x_k} \)

The triangle pivot forces were obtained above by doing statics on the triangle. They are applied here to the bracket.
Appendix I

Solve for forces on the bracket piece

**Guesses**

\[ \text{BA}_{yL} = 0 \text{lb} \quad \text{BA}_{xR} = 0 \text{lb} \quad \text{BA}_{yR} = 0 \text{lb} \quad \text{A}_{1k} = 1 \text{in-lbf} \quad \text{A}_{2k} = 1 \text{in-lbf} \]

"BA\_yL" and "BA\_yR" are the names of the locations of the left and right frame tubes, respectively.

Given
Appendix I

\[ (B_{xR} + A_{xL} - A_{xR} + I_{R} \cos(\gamma_{R}) - I_{L} \cos(\gamma_{L})) = 0 \]

\[ (B_{yR} + B_{yL} - A_{yR} - A_{yL} + I_{R} \sin(\gamma_{R}) + I_{L} \sin(\gamma_{L})) = 0 \]

Split up sum of moments because it's long

\[ \left[ B_{yR} b_{1} - A_{yR} (b_{1} - b_{2} + J_{len} - L_{len}) + A_{xR} (b_{3} + K_{len} - M_{len}) - I_{R} \cos(\gamma_{R}) b_{3} + I_{R} \sin(\gamma_{R}) (b_{1} - b_{2}) \right] = A_{1} \]

\[ \left[ I_{L} \cos(\gamma_{L}) b_{3} + I_{L} \sin(\gamma_{L}) b_{2} + A_{yL} (J_{len} - L_{len} - b_{2}) - A_{xL} (b_{3} + K_{len} - M_{len}) \right] = A_{2} \]

\[
\begin{pmatrix}
B_{yL} \\
B_{xR} \\
B_{yR} \\
A_{1} \\
A_{2}
\end{pmatrix} = \text{Find}(B_{yL}, B_{xR}, B_{yR}, A_{1}, A_{2})
\]
Appendix I

\[
\begin{align*}
\left( BA_{XR} + A A_{xL} - A A_{xR} + I_R \cdot \cos(\gamma_R) - I_L \cdot \cos(\gamma_L) \right) &= 0 \\
\left( BA_{yR} + BA_{yL} - A A_{yR} - A A_{yL} + I_R \cdot \sin(\gamma_R) + I_L \cdot \sin(\gamma_L) \right) &= 0
\end{align*}
\]

Split up sum of moments because it's long

\[
\begin{align*}
\left[ BA_{yR} \cdot b_1 - A A_{yR} \left( b_1 - b_2 + J_{len} - I_{len} \right) + A A_{xR} \left( b_3 + K_{len} - M_{len} \right) - I_R \cdot \cos(\gamma_R) \cdot b_3 + I_L \cdot \sin(\gamma_R) \cdot \left( b_1 - b_2 \right) \right] &= A_1 \\
\left[ I_L \cdot \cos(\gamma_L) \cdot b_3 + I_L \cdot \sin(\gamma_L) \cdot b_2 + A A_{yL} \left( J_{len} - I_{len} - b_2 \right) - A A_{xL} \left( b_3 + K_{len} - M_{len} \right) \right] &= A_2
\end{align*}
\]

\[
\begin{bmatrix}
BA_{yL} \\
BA_{xR} \\
BA_{yR}
\end{bmatrix}
:= \text{Find}(BA_{yL}, BA_{xR}, BA_{yR}, A_1, A_2)
\]

\[
\begin{bmatrix}
A_1 \\
A_2
\end{bmatrix}
\]
Appendix I

\[
\begin{align*}
\min(B_{yL}) &= -192.011 \text{ lbf} & \max(B_{yL}) &= -151.071 \text{ lbf} \\
\min(B_{xR}) &= 0 \text{ lbf} & \max(B_{xR}) &= 0 \text{ lbf} \\
\min(B_{yR}) &= -192.011 \text{ lbf} & \max(B_{yR}) &= -151.071 \text{ lbf}
\end{align*}
\]

**IMPORTANT!**
These forces are shared by the 2 brackets, so each bracket sees half of these forces.
Appendix J
New Mechanical Design Drawings

TOP ASSEMBLY

SHOCK ABSORBER
TRIANGLE LINK 2
BRACKET 2x
TRIANGLE LINK 1
CONNECTING LINK
LOWER A-ARM
TRIANGLE LINK 3
UAH
2009
Appendix J

LOWER A-ARM
Appendix J

CONNECTING LINK

[Diagram of a connecting link with dimensions and annotations]
Appendix J

TRIANGLE LINK I
Appendix J

TRIANGLE LINK 2

TRIANGLE ASSEMBLY
Appendix J

TRIANGLE LINK 3