Satellite Periods and the Gravitational Constant

Edgar W. Woolard

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By Edgar W. Woolard
assistant director,
nautical almanac office
u. s. naval observatory

EDITOR'S INTRODUCTION: Earth satellite problems, such as those encountered with Project Vanguard, require an exact knowledge of the relationship between the satellite periods of revolution around the earth, distance from the earth, size and shape of the earth, and distribution of the earth's mass. Of fundamental importance is the value of the Gaussian Constant of Gravitation. Will it have the same value as that used in solving planetary problems, or will it be different, and why? There is no disagreement among astronomers that one and only one constant, k^2, exists. There is, however, a very real question concerning the auxiliary constant and units that should be employed with k^2. One school of thought*, that of Herrick, Samuel, Baker and Hilton, favors different values of k^2 depending upon the case in question: whether planetary or earth-satellite. Another school, represented by Dr. Woolard and C. M. Clemence, Directors of the Nautical Almanac Office, favors use of just one constant in all calculations. For those readers who are curious as to how k^2 enters Kepler's Third, or Harmonic Law, a numerical example is appended at the end of this article. Dr. Woolard's views now follow.**

IN MY JUDGMENT, there is no method that could be relied upon to give better value of the periods of close earth satellites, or that would be more advantageous in any respect, than the conversion of the Gaussian constant to the most convenient units by means of conversion factors obtained from the standard system of astronomical constants used in the national ephemerides.

In undisturbed elliptic motion, the mean motion $\underline{n}$ of a mass $\underline{m}$ (from which the period 2pi/n is immediately obtained), and the mean distance $\underline{a}$ from the primary mass $\underline{M}$, are rigorously connected by the relation

$$n^2a^3 = (\text{const. of grav.}) \times (M + m.).$$

In terms of the astronomical system of units of length, mass and time, the numerical value of the constant of gravitation is $\underline{k^2}$, where $\underline{k}$ is the Gaussian constant which, as explained in the AMERICAN EPHEM-

*Herrick, Samuel, Baker and Hilton are co-authors of "Units and Constants for Geocentric Orbits" and members of the Department of Astronomy, University of California, and Systems Laboratories Corp., Los Angeles. They also serve as consultants for the International Geophysical Year Satellite Program, through the Smithsonian Astrophysical Observatory.

**From a letter dated 18 May 1956.

ERIS (page 611 of the 1956 volume) is fixed by definition at exactly 0.017 202 098 95.

To obtain the value of the constant of gravitation in any other system of units, the factors required are in some cases only definitions, such as the ratio of the day to the second, but in general they depend upon actual measurements; in particular, the value in cgs units can be obtained only by direct measurement. As illustrated by various examples, many different methods may be used, depending upon different quantities and different relations among them; and in the case of the quantities that are determined by measurement, it is necessary to decide which of the determinations on record in the literature to adopt. However, either in different methods or in any particular method, the values of the different quantities which may have to be used cannot always be assigned independently. Numerical values must be used which are consistent with all the theoretical relations that may exist among the quantities, and which are in strict accordance with the exact definitions of these quantities. If this is done, and if the number of significant figures required for computational consistency is retained in each arithmetical operation, irrespective of whether all of them are physically significant, the same results will be obtained by any method. In selecting numerical values to adopt for any particular set of quantities, there is no basis at present for not using the internationally accepted system of astronomical constants.

For example, in units of the kilometer, the hour, and the mass of the earth, the constant of gravitation may be obtained by converting either $\underline{k^2}$, or, less conveniently, the measured laboratory value $\underline{G}$, to these units. The conversion of $\underline{G}$ requires the mass of the earth in grams, which may be derived from $\underline{G}$ by two or three different methods; but in any method, some of the same constants that must be used are likewise either required in converting $\underline{k^2}$ or are related to constants involved in converting $\underline{k^2}$. With Heyl's value $G = 6.673 \times 10^{-8}$ cgs, and solar parallax 8″.80, equatorial radius of the earth 6378388 meters, mass of sun/mass of earth 333 432, both methods when correctly applied give,

$$5.148\ 649\ 3 \times 10^{12}.$$
$$[\mathrm{Km}^3 / \mathrm{Hr}^2 \text{ earth masses}]$$

Nothing would be gained by basing the calculation on the motion of the moon. In the disturbed motion of any body, the value of $\underline{a}$ calculated from the observed mean motion is the semi-major axis of an elliptic reference orbit which necessarily is conventional and is in a mathematically arbitrary relation to the actual motion. The distance of the moon, given on page xvi of the AMERICAN EPHEMERIS, at which the parallax is 57′02″.70, is not this elliptic mean distance, but is derived from the $\underline{a}$ calculated from the mean motion with the Gaussian constant; its relation to $\underline{a}$, by means of which it is determined, depends upon the lunar perturbations.

EDITOR'S EXAMPLE: If the mass of the earth plus the mass of the satellite, $(M+m)$, is unity, then Kepler's Third Law becomes:

$$k^2p^2 = (2\text{pi})^2a^3$$

when, for the earth:

$k^2 = 5.148\ 649\ 3 \times 10^{12}$

p = sidereal period of revolution around earth (hrs)

a = distance from earth's center (km) (equatorial diameter plus altitude)

If $a = 6778.386$ km (6378.388 km + 400 km altitude)

Then: $p = 1.545\ 336$ hrs.

Therefore, the satellite revolves around the earth in 1.545 hours with respect to the stars.