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## Microwave Propagation through Plasma Media UARI Research Report No. 34

W. R. Garrett

F. H. Mitchelll Jr.

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UARI Research Report No. 34

MICROWAVE PROPAGATION THROUGH PLASMA MEDIA

by

W. R. Garrett

F. H. Mitchell, Jr.

Final Technical Report

This research work was supported by the Army Missile Command  
Contract DA-AMC-01-021-64-G1, Task B

University of Alabama Research Institute

Huntsville, Alabama

January 1966

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## PART I

### MICROWAVE PROPAGATION IN INHOMOGENEOUS MEDIA

#### A. Introduction

In two previous reports<sup>1,2</sup> on wave propagation in plasma type media, the problem of microwave propagation in plasma adjoining a conducting surface has been described in detail. In this study a new perturbation technique for handling inhomogeneous permittivity gradients was derived as described fully in Reference 1. The details of this work will not be given here in this final report since the earlier reports<sup>1,2</sup> are self-contained. A brief summary of results obtained in this phase of the study of wave propagation in plasma will be given as it pertains to further results which are to be described in Part II.

#### B. Summary of Earlier Results

A new method was developed for treating wave propagation in inhomogeneous media next to a conducting surface containing a radiating element. The method of solution is analogous to the partial wave technique which is often used in the quantum mechanical treatment of particle scattering. The analysis leads to an integral recursion relation where no restrictions are necessary with regard to the thickness of the medium layer. More general application of the same techniques may be made to the study of transmission and scattering properties of similar configurations with a propagating wave incident from infinity.

In the work reported<sup>1,3</sup> three symmetries have been considered using analytical methods. The first was an infinite plasma-clad conducting plane with periodic electric or magnetic sources on the surface. The second, an infinite plasma-clad right circular cylinder with a source parallel to the symmetry axis or with periodic circumferential sources. The third case was a plasma-clad conducting sphere with a circumferential antenna. In each case the permittivity may be allowed to vary arbitrarily about a



constant value in a direction perpendicular to the conducting surface. Details for the planar case with a strip antenna were used to illustrate the method.

Further extension of this work was reported<sup>2,4</sup> where an array of line sources was treated by the same technique. Furthermore, a computer program has been described in Reference 2 which allows the computation of the complex phase shifts to be made for an arbitrary perpendicular variation of the permittivity. Calculations were made of the attenuation and the radiation pattern of an array radiating into a lossy plasma medium.

In summary a method of partial waves for analyzing wave propagation in inhomogeneous media has been developed in this study and has been applied to problems of plasma-clad conductors of different symmetries. Detailed calculations for specific problems have been made whereby the modified radiation patterns and the attenuation due to a plasma type medium were calculated. The utility of the technique for other similar type problems has been indicated.

In the following section a further extension of the methods developed heretofore will be treated, the extension being made to include both inhomogeneous and nonlinear properties of the plasma medium.

The actual and approximate fields  $F_n^{II}$  and  $\tilde{F}_n^{II}$  outside the medium in free space ( $z \geq z_0$ ) must have the same functional forms and can only be different by a constant factor,

$$F_n^{II} = \tilde{F}_n^{II} \cdot \Delta_n^0 \quad (2)$$

The fields must be continuous at the boundary  $z_0$  so that  $\Delta_n^0(z_0) = \Delta_n^0$ . A knowledge of  $\Delta_n^0$  then enables one to predict the complete properties of the radiation field and it is this quantity that the perturbation method provides. The discussion will be held to the radiation problem and the restrictions given by (6) through (9), Reference 1, are no longer necessary.



PART II

PROPAGATION IN NONLINEAR INHOMOGENEOUS MEDIA

A. Perturbation Treatment of Microwaves in Inhomogeneous and Nonlinear Media

1. Introduction

A perturbation technique for treating microwave propagation in inhomogeneous media adjoining a conducting surface has been developed in earlier work.<sup>1-3</sup> In the following discussion this technique will be presented in considerable detail without some of the restrictions previously noted and it will be extended to include field-dependent (nonlinear) perturbations.

The electric or magnetic field  $F_n^I$  inside the inhomogeneous or nonlinear medium ( $\xi_i \leq \xi \leq \xi_e$ ) is found in terms of an approximate solution  $\bar{F}_n^I$  and a perturbation term  $\Delta_n(\xi)$ , i.e.,

$$F_n^I(\xi) = \bar{F}_n^I(\xi) e^{\Delta_n(\xi)}. \quad (1)$$

The actual and approximate fields  $F_n^{II}$  and  $\bar{F}_n^{II}$  outside the medium in free space ( $\xi \geq \xi_e$ ) must have the same functional forms and can only be different by a constant factor,

$$F_n^{II} = \bar{F}_n^{II} e^{\Delta_n^0}. \quad (2)$$

The fields must be continuous at the boundary  $\xi_e$  so that  $\Delta_n(\xi_e) = \Delta_n^0$ . A knowledge of  $\Delta(\xi_e)$  then enables one to predict the complete properties of the radiation field and it is this quantity that the perturbation method provides. The discussion will be held to the radiation problem and the restrictions given by (6) through (9), Reference 1, are no longer necessary.

## 2. Integration of the Differential Equations

From (12) of Reference 1 the approximate and exact fields must obey the relation

$$\left[ \bar{y}_n \frac{dy_n}{d\xi} - y_n \frac{d\bar{y}_n}{d\xi} \right]_{\xi_1}^{\xi_2} = \int_{\xi_1}^{\xi_2} (\bar{\phi}_0 - \phi) y_n \bar{y}_n d\xi \quad (3)$$

where

$$y_n = F_n^I \exp \left[ \frac{1}{2} \int_{\xi}^{\xi} x_1 [n, t, \epsilon_r(t)] dt \right] \quad (4)$$

$$\bar{y}_n = \bar{F}_n^I \exp \left[ \frac{1}{2} \int_{\xi}^{\xi} x_1 [n, t, \epsilon_a] dt \right] \quad (5)$$

The permittivity  $\epsilon_r(\xi)$  of the medium can be a function of position and of the absolute value of the field intensity.

Substituting from (4) and (5) into (3),

$$\begin{aligned} G(\xi) \left[ \bar{F}_n^I \frac{dF_n^I}{d\xi} - F_n^I \frac{d\bar{F}_n^I}{d\xi} - \frac{1}{2} F_n^I \bar{F}_n^I (x_1 [n, \xi, \epsilon_a] - x_1 [n, \xi, \epsilon_r(\xi)]) \right]_{\xi_1}^{\xi_2} \\ = \int_{\xi_1}^{\xi_2} (\bar{\phi}_0 - \phi) F_n^I \bar{F}_n^I G(\xi) d\xi \end{aligned} \quad (6)$$

where

$$G(\xi) = \exp \left[ \frac{1}{2} \int_{\xi}^{\xi} (x_1 [n, t, \epsilon_a] + x_1 [n, t, \epsilon_r(t)]) dt \right] \quad (7)$$

Now choose  $\xi_2 = \xi_e$  and  $\xi_1 = \xi$  (arbitrary) and note that since  $\Delta_n(\xi) = \Delta_n^0$ ,

$$\left[ \bar{F}_n^I \frac{dF_n^I}{d\xi} - F_n^I \frac{d\bar{F}_n^I}{d\xi} \right]_{\xi_e} = 0, \quad (8)$$



(6) becomes

$$\left[ \bar{F}_n^I \frac{dF_n^I}{d\xi} - F_n^I \frac{d\bar{F}_n^I}{d\xi} - \frac{1}{2} F_n^I \bar{F}_n^I (x_1[n, \xi, \epsilon_a] - x_1[n, \xi, \epsilon_r(\xi)]) \right]_{\xi_e} \quad (9)$$

$$= -\frac{1}{G(\xi)} \left[ \int_{\xi}^{\xi_e} e^{(\Phi_0 - \Phi)} F_n^I \bar{F}_n^I G(\xi) d\xi + \frac{1}{2} \left\{ F_n^I \bar{F}_n^I G(\xi) (x_1[n, \xi, \epsilon_a] - x_1[n, \xi, \epsilon_r(\xi)]) \right\}_{\xi_e} \right]$$

which can be written in the form

$$\frac{d^2 F_n^I}{d\xi^2} + F_n^I \alpha(\xi) + \beta(\xi) = 0 \quad (10)$$

and the general solution to this first-order differential equation is

$$F_n^I = -e^{-\int \alpha d\xi} \left[ \int \beta e^{\int \alpha d\xi} - C \right]. \quad (11)$$

Substituting for  $\alpha$  and  $\beta$ ,

$$\frac{F_n^I}{\bar{F}_n^I} = -P(\xi) \left\{ \int \frac{d\xi'}{\Gamma(\xi')} \left[ \int_{\xi'}^{\xi_e} e^{(\Phi_0 - \Phi)} F_n^I \bar{F}_n^I G(\xi) d\xi + \left( \frac{A(\xi) F_n^I}{P(\xi) \bar{F}_n^I} \right)_{\xi_e} \right] - C \right\} \quad (12)$$

$$P(\xi) = \exp \left[ \frac{1}{2} \int_{\xi}^{\xi_e} (x_1[n, t, \epsilon_a] - x_1[n, t, \epsilon_r(t)]) dt \right] \quad (13)$$

$$A(\xi) = \frac{1}{2} \Gamma(\xi) (x_1[n, \xi, \epsilon_a] - x_1[n, \xi, \epsilon_r(\xi)]) \quad (14)$$

$$\Gamma(\xi) = G(\xi) P(\xi) \bar{F}_n^{I2}(\xi). \quad (15)$$

The constant of integration C is evaluated from one of the following boundary conditions on the conductor surface:

Case I:  $F_n$  represents an E field

$$F_n^I(\xi_i) = \bar{F}_n^I(\xi_i) \quad (16)$$

Case II:  $F_n$  represents an H field

$$\frac{1}{\epsilon_r(\xi_i)} \left( \frac{dF_n^I}{d\xi} \right)_{\xi_i} = \frac{1}{\epsilon_a} \left( \frac{d\bar{F}_n^I}{d\xi} \right)_{\xi_i} \quad (17)$$

### 3. $F_n$ as an Electric Field

For Case I,

$$C_E = \frac{1}{P(\xi_i)} + \int_{\xi_i}^{\xi_e} \frac{d\xi'}{\Gamma(\xi')} \left[ \int_{\xi'}^{\xi_e} (\Phi_0 - \Phi) E_n^I \bar{E}_n^I G(\xi) d\xi + \left( \frac{A(\xi) E_n^I}{P(\xi) \bar{E}_n^I} \right)_{\xi_e} \right] \quad (18)$$

and

$$\frac{E_n^I}{\bar{E}_n^I} = P(\xi) \left\{ \frac{1}{P(\xi_i)} - \int_{\xi_i}^{\xi} \frac{d\xi'}{\Gamma(\xi')} \left[ \int_{\xi'}^{\xi_e} (\Phi_0 - \Phi) E_n^I \bar{E}_n^I G(\xi) d\xi + \left( \frac{A(\xi) E_n^I}{P(\xi) \bar{E}_n^I} \right)_{\xi_e} \right] \right\} \quad (19)$$

The left-hand side of (19) is evaluated at  $\xi_e$  and terms rearranged so that

$$\left( \frac{E_n^I}{\bar{E}_n^I} \right)_{\xi_e} \left[ 1 + A(\xi_e) \int_{\xi_i}^{\xi_e} \frac{d\xi'}{\Gamma(\xi')} \right] = P(\xi_e) \left[ \frac{1}{P(\xi_i)} - \int_{\xi_i}^{\xi_e} \frac{d\xi'}{\Gamma(\xi')} \int_{\xi_i}^{\xi_e} (\Phi_0 - \Phi) E_n^I \bar{E}_n^I G(\xi) d\xi \right] \quad (20)$$



Now define  $\theta(\xi)$

$$\frac{E_n^I(\xi)}{\bar{E}_n^I(\xi)} = e^{\Delta(\xi)} = \frac{P(\xi)}{P(\xi_i)} [1 + \theta(\xi)] \quad (21)$$

and substitute into (20):

$$\theta(\xi_e) = \frac{-\int_{\xi_i}^{\xi_e} \frac{d\xi'}{\Gamma(\xi')} \int_{\xi_i}^{\xi_e} (\bar{\Phi}_0 - \bar{\Phi}) \Gamma(\xi) [1 + \theta(\xi)] d\xi}{1 + A(\xi_e) \int_{\xi_i}^{\xi_e} \frac{d\xi'}{\Gamma(\xi')}} \quad (22)$$

This is an exact relationship between the actual and approximate fields, as expressed through the variable  $\theta(\xi)$ .

At this point  $\theta(\xi)$  and  $(\bar{\Phi}_0 - \bar{\Phi})/\bar{\Phi}_0$  are restricted to values much smaller than one so that to first order the integral over the product of these functions may be neglected.  $\theta(\xi)$  now represents a perturbation on the original system and

$$\theta(\xi_e) \approx \frac{-\int_{\xi_i}^{\xi_e} \frac{d\xi'}{\Gamma(\xi')} \int_{\xi_i}^{\xi_e} (\bar{\Phi}_0 - \bar{\Phi}) \Gamma(\xi) d\xi}{1 + A(\xi_e) \int_{\xi_i}^{\xi_e} \frac{d\xi'}{\Gamma(\xi')}} \quad (23)$$

$$\Delta_n^0 = \Delta_n(\xi_e) = \frac{P(\xi_e)}{P(\xi_i)} [1 + \theta(\xi_e)] - 1 \quad (24)$$

If  $X_1[n, \xi_e, \epsilon_a] = X_1[n, \xi_e, \epsilon_r(\xi_e)]$ , then  $A(\xi_e) = 0$  and (23) assumes a simplified form.



4.  $F_n$  as a Magnetic Field

For Case II,

$$C_H = \int_{\xi_i}^{\xi_e} \frac{d\xi'}{\Gamma(\xi')} \left[ \int_{\xi_i}^{\xi_e} (\Phi_0 - \Phi) H_n^I \bar{H}_n^I G(\xi) d\xi + \left( \frac{A(\xi) H_n^I}{P(\xi) \bar{H}_n^I} \right)_{\xi_e} \right]$$

$$+ \frac{1}{B(\xi_i)} \left\{ \frac{\epsilon_r(\xi_i)}{\epsilon_a} \Gamma(\xi_i) \left( \frac{1}{\bar{H}_n} \frac{d\bar{H}_n}{d\xi} \right)_{\xi_i} + P(\xi_i) \int_{\xi_i}^{\xi_e} (\Phi_0 - \Phi) H_n^I \bar{H}_n^I G(\xi) d\xi \right.$$

$$\left. + \frac{P(\xi_i)}{P(\xi_e)} A(\xi_e) \left( \frac{H_n^I}{\bar{H}_n^I} \right)_{\xi_e} \right\} \quad (25)$$

$$B(\xi_i) + P(\xi_i) \left[ \frac{\Gamma(\xi)}{\bar{H}_n} \frac{1 d\bar{H}_n^I}{d\xi} + A(\xi) \right]_{\xi_i} \quad (26)$$

and

$$\frac{H_n^I}{\bar{H}_n^I} = P(\xi) \left[ \frac{1}{B(\xi_i)} \left\{ \frac{\epsilon_r(\xi_i)}{\epsilon_a} \Gamma(\xi_i) \left( \frac{1}{\bar{H}_n} \frac{d\bar{H}_n^I}{d\xi} \right)_{\xi_i} + P(\xi_i) \int_{\xi_i}^{\xi_e} (\Phi_0 - \Phi) H_n^I \bar{H}_n^I G(\xi) d\xi \right. \right.$$

$$\left. \left. + \frac{P(\xi_i)}{P(\xi_e)} A(\xi_e) \left( \frac{H_n^I}{\bar{H}_n^I} \right)_{\xi_e} \right\} - \int_{\xi_i}^{\xi} \frac{d\xi'}{\Gamma(\xi')} \left\{ \int_{\xi_i}^{\xi_e} (\Phi_0 - \Phi) H_n^I \bar{H}_n^I G(\xi) d\xi + \left( \frac{A(\xi)}{P(\xi)} \right) \frac{H_n^I}{\bar{H}_n^I} \right\} \right] \quad (27)$$

The left-hand side of (27) is evaluated at  $\xi_e$  and the resultant terms can be rearranged to yield

$$\left(\frac{H_n^I}{\bar{H}_n^I}\right)_{\xi_e} \left[ 1 - \frac{P(\xi_i)}{B(\xi_i)} A(\xi_e) + A(\xi_e) \int_{\xi_i}^{\xi_e} \frac{d\xi'}{\Gamma(\xi')} \right]$$

$$= P(\xi_e) \left[ \frac{1}{B(\xi_i)} \left\{ \frac{\epsilon_r(\epsilon_i)}{\epsilon_a} \Gamma(\xi_i) \left( \frac{1}{\bar{H}_n^I} \frac{d\bar{H}_n^I}{d\xi} \right)_{\xi_i} P(\xi_i) \int_{\xi_i}^{\xi_e} (\Phi_0 - \Phi) H_n^I \bar{H}_n^I G(\xi) d\xi \right\} \right.$$

$$\left. - \int_{\xi_i}^{\xi_e} \frac{d\xi'}{\Gamma(\xi')} \int_{\xi'}^{\xi_e} (\Phi_0 - \Phi) H_n^I \bar{H}_n^I G(\xi) d\xi \right]. \quad (28)$$

Defining

$$\frac{H_n^I}{\bar{H}_n^I} = e^{\Delta_n(\xi)} = \frac{P(\xi_e)}{B(\xi_i)} \frac{\epsilon_r(\epsilon_i)}{\epsilon_a} \Gamma(\xi_i) \left( \frac{1}{\bar{H}_n^I} \frac{d\bar{H}_n^I}{d\xi} \right)_{\xi_i} [1 + \theta(\xi)] \quad (29)$$

(28) becomes

$$\theta(\xi_e) = \frac{- \int_{\xi_i}^{\xi_e} \frac{d\xi'}{\Gamma(\xi')} \int_{\xi'}^{\xi_e} (\Phi_0 - \Phi) \Gamma(\xi) [1 + \theta(\xi)] d\xi + P(\xi_i) \int_{\xi_i}^{\xi_e} (\Phi_0 - \Phi) \Gamma(\xi) d\xi}{1 - \frac{P(\xi_i)}{B(\xi_i)} A(\xi_e) \int_{\xi_i}^{\xi_e} \frac{d\xi'}{\Gamma(\xi')}} \quad (30)$$

This is again an exact relationship.  $\theta(\xi)$  and  $(\Phi_0 - \Phi)/\Phi_0$  are again assumed much smaller than one so that the integral over their product can be neglected:

$$\theta(\xi_e) \approx \frac{\int_{\xi_i}^{\xi_e} \frac{d\xi'}{\Gamma(\xi')} \int_{\xi'}^{\xi_e} (\Phi_0 - \Phi) \Gamma(\xi) d\xi + P(\xi_i) \int_{\xi_i}^{\xi_e} (\Phi_0 - \Phi) \Gamma(\xi) d\xi}{1 - \frac{P(\xi_i)}{B(\xi_i)} A(\xi_e) + A(\xi_e) \int_{\xi_i}^{\xi_e} \frac{d\xi'}{\Gamma(\xi')}} \quad (31)$$



$$\Delta_n^0 = \Delta_n(\xi_e) = \frac{P(\xi_e)}{B(\xi_i)} \frac{\epsilon_r(\xi_i)}{\epsilon_a} \Gamma(\xi_i) \left( \frac{1}{\bar{H}_n} \frac{d\bar{H}_n}{d\xi} \right) [1 + \theta(\xi_e)] - 1 \quad (32)$$

If  $X_1[n, \xi, \epsilon_a] = X_1[n, \xi_e, \epsilon_r(\xi_e)]$ , then  $A(\xi_e) = 0$  and (32) takes on the simplified form used in References 1 and 2.

### 5. Inhomogeneous Plasma Sheaths

The functions  $X_1$  and  $\Phi$  are known functions of  $\epsilon_r(\xi)$  and  $\epsilon_a$ , which for a plasma are in turn functions of the ionization density  $n$  and collision frequency  $\nu$  of the medium:

$$\begin{aligned} \epsilon_r(\xi) &= \epsilon_0 - \frac{n(\xi) e^2}{m\omega[\omega - i\nu(\xi)]} \\ \epsilon_a &= \epsilon_0 - \frac{n_0 e^2}{m\omega[\omega - i\nu_0]} \end{aligned} \quad (33)$$

$n_0$  and  $\nu_0$  are the averaged values that lead to the approximate solution  $\bar{F}_n^I$ . The field perturbation  $\Delta_n^0$  is a function of the difference between  $n(\xi)$  and  $n_0$ ,  $\nu(\xi)$  and  $\nu_0$ , integrated over the sheath. The only restrictions on  $\epsilon_r(\xi)$  are that  $(\Phi_0 - \Phi)/\Phi_0 \ll 1$ ,  $\Delta_n^0 \ll 1$ .

### 6. Nonlinear Perturbations in a Plasma

The same perturbation technique can also be applied to study nonlinear interactions in a plasma sheath. This nonlinear "self-modulation" occurs because the effective collision frequency is a function of the field intensity for strong fields. Ginzburg<sup>5</sup> demonstrates that

$$\nu'(\xi) = \nu(\xi) \sqrt{\frac{T_e}{T}} \quad (34)$$

where  $\nu'$  is the modified collision frequency,  $\nu$  the small-field value and  $T_e$  and



$T_e$  and  $T$  are the electron and ion temperatures in the plasma. The equipartition of energy requires that  $T_e = T$  when  $\bar{E}$  is negligible.

For  $\omega \gg \nu$  (the high frequency limit),

$$T_e - T = \frac{e^2 |\bar{E}|^2}{3mk\delta\omega^2} \quad (35)$$

$k$  being the Boltzman constant and  $\delta$  the average relative fraction of energy imparted by an electron to a heavy particle in one collision. Combining (34) and (35), one obtain

$$\nu' = \nu \sqrt{1 + \frac{e^2 |\bar{E}|^2}{3mkT\delta\omega^2}} \quad (36)$$

The approximate (linear) permittivity  $\epsilon_a$  is then perturbed according to (33), where (36) is the modified collision frequency.

## B. Effect of Field Dependent Conductivity on Radiation Through a Plasma

### 1. Introduction

In the problems considered below, the microwave source is pictured as an antenna on the surface of a missile, and it is desired to find how the radiation fields on this antenna are affected by the plasma sheath that coats it.

In principle one could combine appropriate boundary conditions with an adequate description of the plasma and obtain a general solution to any problem from Maxwell's equations. However, this approach turns out to be mathematically intractable and various approximations must be introduced in any analysis. These approximations lead to a system model that includes the reentry vehicle itself, the surrounding plasma sheath and various microwave sources on the vehicle (Figure 1).



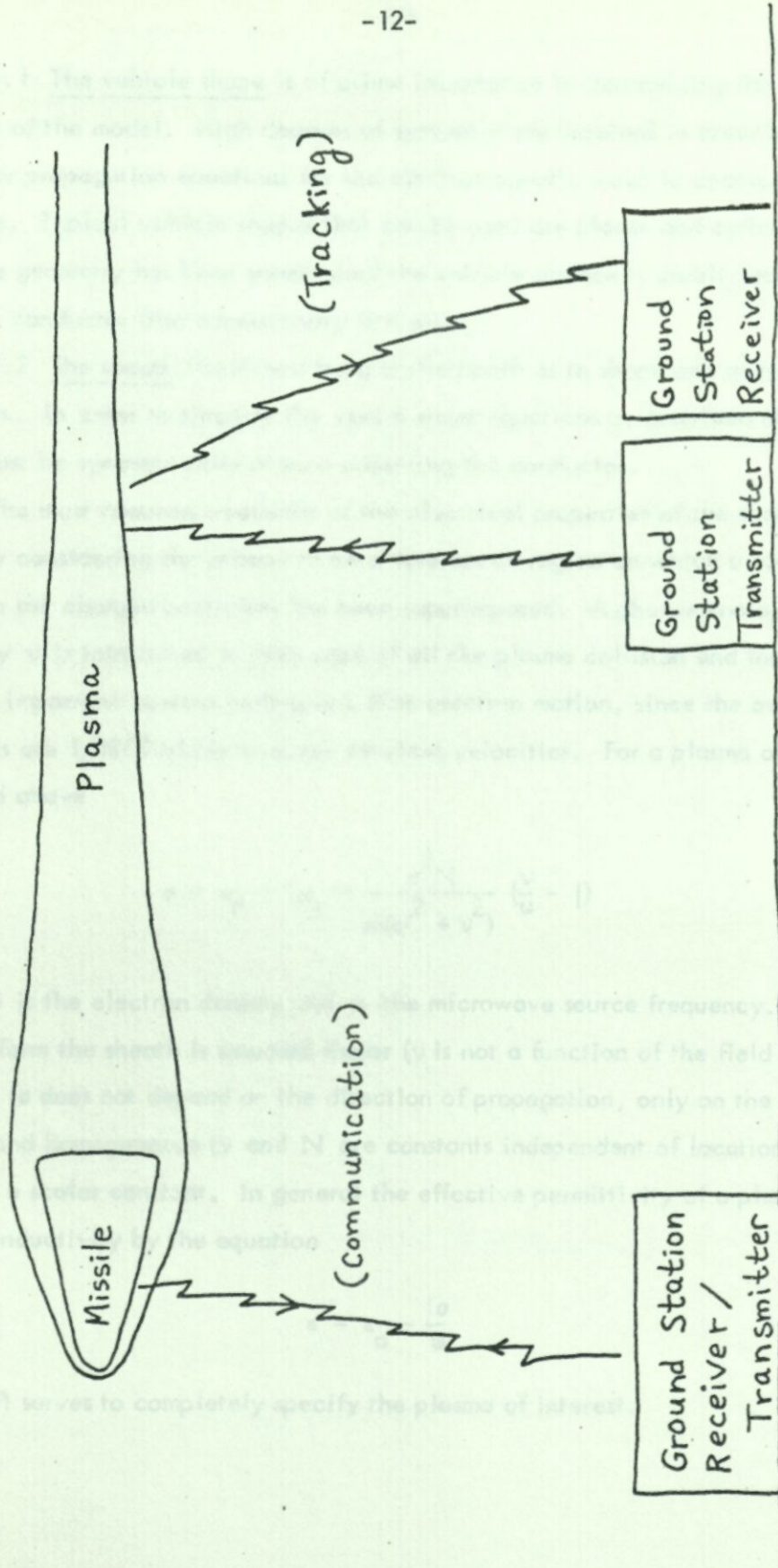


Fig. 1 The basic communications and/or tracking problem

1.1 The vehicle shape is of prime importance in determining the geometrical symmetry of the model. High degrees of symmetry are required in order to reduce the vector propagation equations for the electromagnetic wave to uncoupled scalar equations. Typical vehicle shapes that can be used are planes and cylinders (Figure 2). Once the geometry has been established the vehicle surface is usually assumed to be a perfect conductor (the conductivity  $\sigma = \infty$ ).

1.2 The sheath itself must be specified both as to shape and as to electrical properties. In order to simplify the vector wave equations as described above, the sheath must be symmetrically placed adjoining the conductor.

The most common treatment of the electrical properties of the sheath is obtained by considering the plasma to be a free-space region on which a current density  $\vec{J}$  (due to the charged particles) has been superimposed. A phenomenological collision frequency  $\nu$  is introduced to take care of all the plasma collision and loss processes. The most important current term arises from electron motion, since the average ion velocities are 1/1860 of the average electron velocities. For a plasma of the type described above

$$\sigma = \sigma_R + i\sigma_I = \frac{e^2 N}{m(\omega^2 + \nu^2)} \left( \frac{\nu}{\omega} - i \right) \quad (37)$$

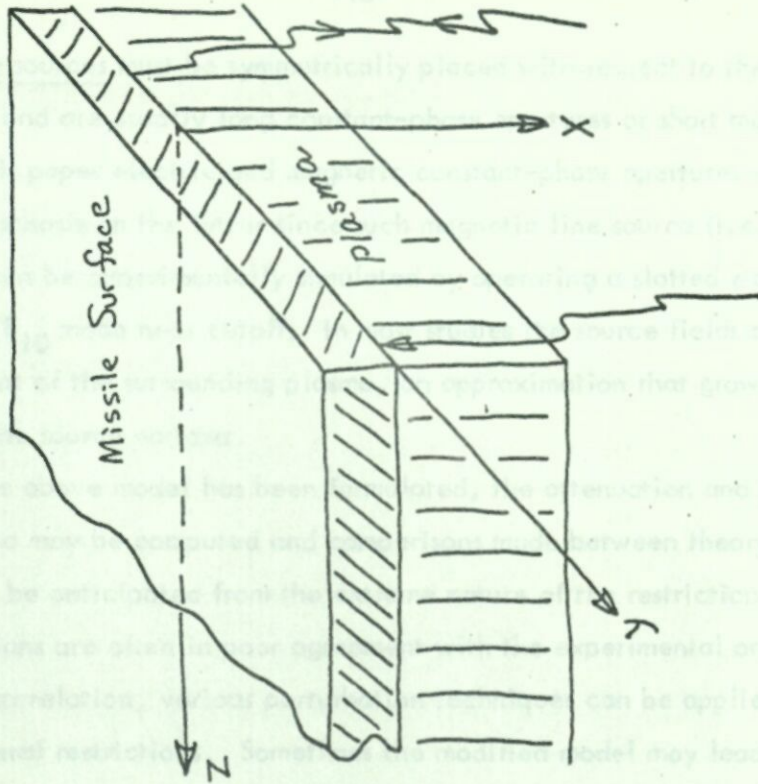
where  $N$  is the electron density and  $\omega$  the microwave source frequency. In its simplest form the sheath is assumed linear ( $\nu$  is not a function of the field intensities), isotropic ( $\sigma$  does not depend on the direction of propagation, only on the location in space), and homogeneous ( $\nu$  and  $N$  are constants independent of location in space);  $\sigma$  is then a scalar constant. In general the effective permittivity of a plasma is related to the conductivity by the equation

$$\epsilon = \epsilon_0 - \frac{i\sigma}{\omega} \quad (38)$$

so that (37) serves to completely specify the plasma of interest.



Plane Model



Cylindrical Model

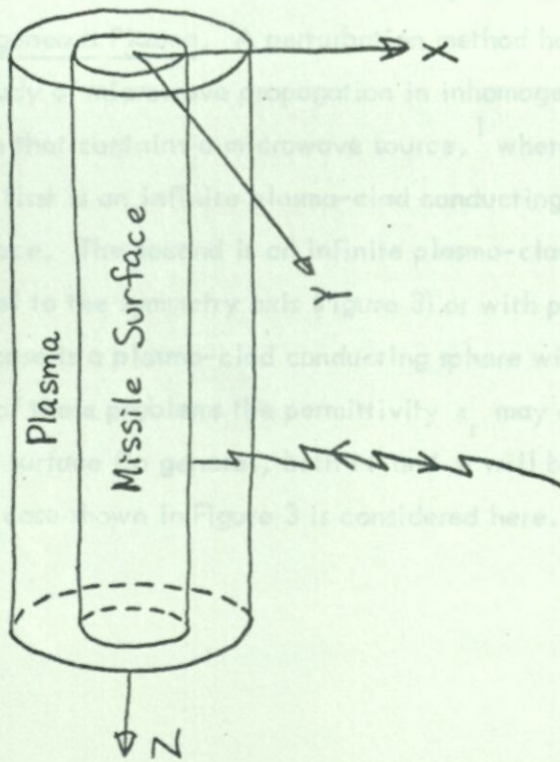


Fig. 2 System models

1.3 The sources must be symmetrically placed with respect to the vehicle and the sheath, and are usually long constant-phase apertures or short multipole antennas. In this paper electric and magnetic constant-phase apertures will be considered, with emphasis on the latter since such magnetic line source (i.e., a very narrow source) can be experimentally simulated by operating a slotted waveguide antenna in the  $TE_{10}$  mode near cutoff. In most studies the source fields are assumed to be independent of the surrounding plasma, an approximation that grows successively better as the source narrows.

Once the above model has been formulated, the attenuation and phase shift due to the plasma may be computed and comparisons made between theory and experiment. As might be anticipated from the extreme nature of the restrictions, the theoretical calculations are often in poor agreement with the experimental ones. In order to improve the correlation, various perturbation techniques can be applied to modify some of the general restrictions. Sometimes the modified model may lead to more physically-observed results.

## 2. Perturbation of the Sheath Properties

2.1 Inhomogeneous Plasma. A perturbation method has been described in prior work for the study of microwave propagation in inhomogeneous plasma adjoining a conducting surface that contains a microwave source,<sup>1</sup> where three symmetries can be considered. The first is an infinite plasma-clad conducting plane with periodic antennas on the surface. The second is an infinite plasma-clad right circular cylinder with a source parallel to the symmetry axis (Figure 3) or with periodic circumferential sources. The third case is a plasma-clad conducting sphere with a circumferential antenna. For each of these problems the permittivity  $\epsilon_r$  may only vary in a direction perpendicular to the surface (in general, both  $N$  and  $\nu$  will be functions of position in the sheath). The case shown in Figure 3 is considered here.



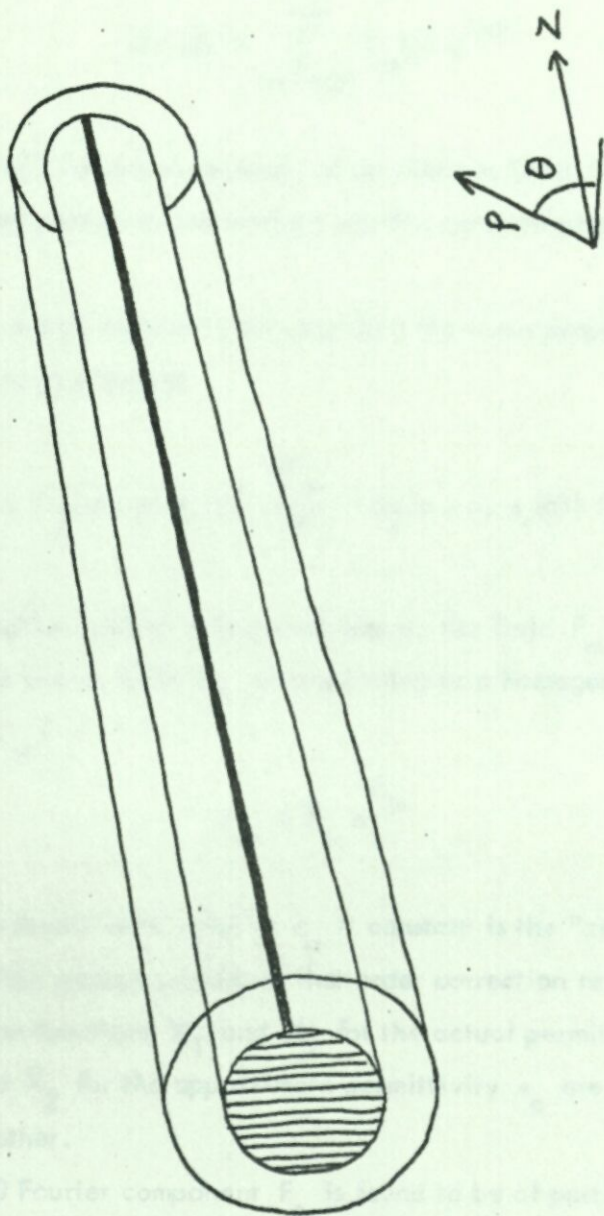


Fig. 3 Conducting cylinder with axial antenna

The  $n = 0$  Fourier component of  $E_z$  is of particular interest, being independent of  $\theta$ . Also, the modification in the  $n = 0$  term due to the presence of the antenna is the same for the planar case and for the case of a large cylinder. The complete field is given by  $E_z$  for an electric dipole at height  $z_0$  above the plane [Eq. (6)] is integrated around the cylinder only the  $n = 0$  term will contribute.

The total field is represented by its Fourier series

$$F(\rho, \theta) = \sum_{m=-\infty}^{+\infty} F_m(\rho) e^{im\theta} \quad (39)$$

where  $F_m$  is the  $m^{\text{th}}$  Fourier component of an electric field  $E$  or magnetic field  $H$ ,  $\rho$  is the coordinate normal to the surface and the constant phase source is in the  $z$  direction.

The differential equation that describes the wave propagation in the inhomogeneous medium may be written

$$\frac{d^2 F_m}{d\rho^2} + X_1[m, \rho, \epsilon_r(\rho)] \frac{dF_m}{d\rho} + X_2[m, \rho, \epsilon_r(\rho)] F_m = 0 \quad (40)$$

Using the perturbation method referenced above, the field  $F_m$  can be found in terms of an approximate known field  $\bar{F}_m$  corresponding to a homogeneous sheath and a correction term  $\Delta_m$ :

$$F_m = \bar{F}_m e^{\Delta_m} \quad (41)$$

The homogeneous sheath with  $\epsilon_r(\rho) = \epsilon_a = \text{constant}$  is the "zero-order" solution, and the perturbation method provide a first order correction term to  $\Delta_m$ . The results are valid when the functions  $X_1$  and  $X_2$  for the actual permittivity  $\epsilon_r(\rho)$  and the functions  $\bar{X}_1$  and  $\bar{X}_2$  for the approximate permittivity  $\epsilon_a$  are first-order approximations to each other.

The  $m = 0$  Fourier component  $F_0$  is found to be of particular interest, having no angular dependence. Also, the modification in the  $m = 0$  term due to the plasma sheath is the same for the planar case and for the case of a large cylinder. The complete field is given by  $F_0$  for an electric or magnetic current cylinder, and if  $|F(\rho, \theta)|^2$  is integrated around the cylinder only the  $m = 0$  term will be nonzero.



\* RELATIVE CONTRIBUTIONS ARE FOR  $\nu = \omega$

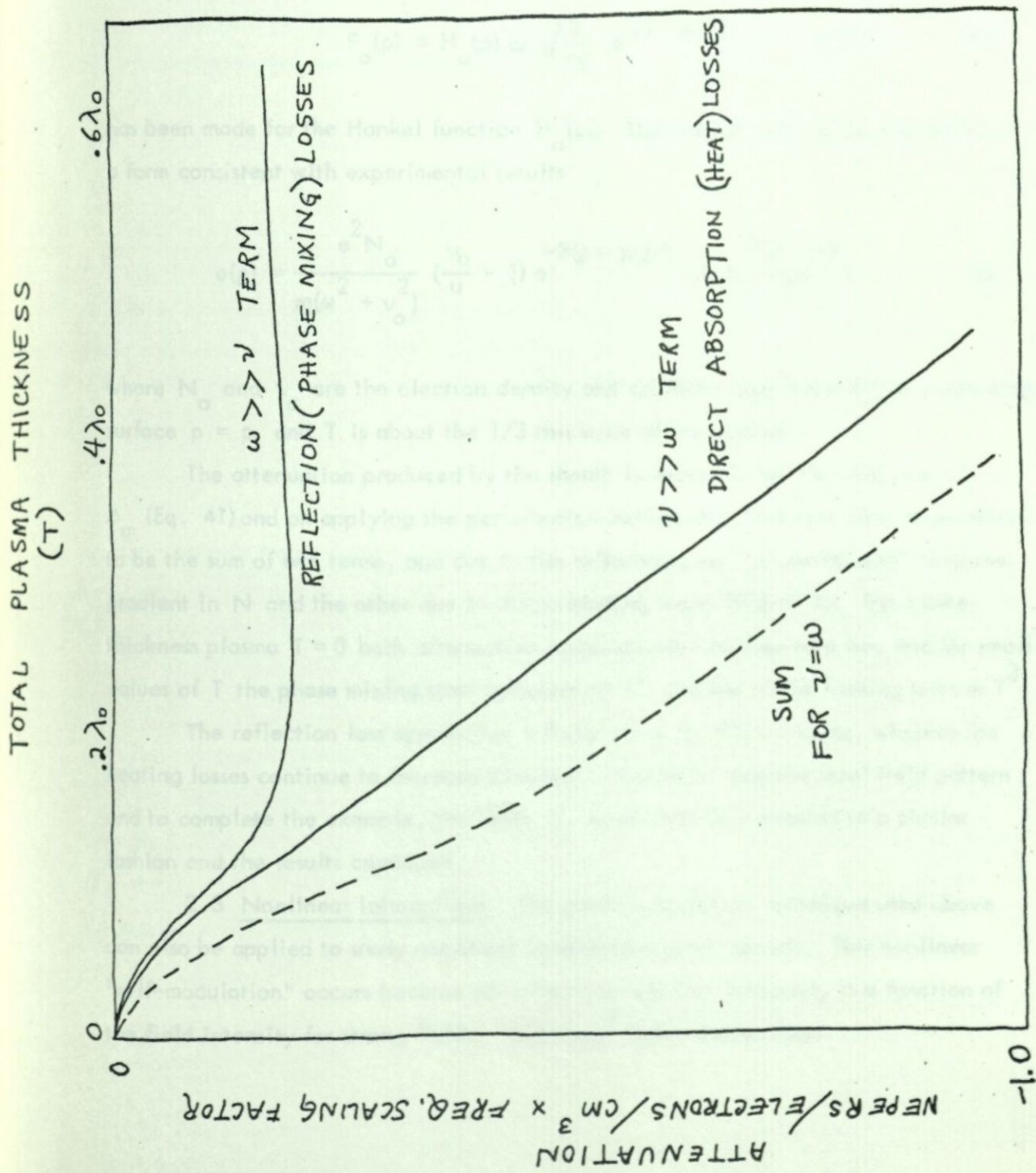


Fig. 4 The  $m=0$  attenuation terms for an exponential sheath on a large cylinder (weak fields)

2.2 Example 1. Now consider a specific example of a large conducting cylinder with a constant phase magnetic source parallel to the cylinder axis and on its surface (Figure 3). The requirement of a "large" cylinder is necessary because for simplicity the expansion

$$F_o(\rho) = H_o(\rho) \approx \sqrt{\frac{2}{\pi\rho}} e^{i(\rho - \pi/4)} \quad \rho > 1 \quad (42)$$

has been made for the Hankel function  $H_o(\rho)$ . The conductivity is assumed to be of a form consistent with experimental results

$$\sigma(\rho) = \frac{e^2 N_o}{m(\omega^2 + \nu_o^2)} \left( \frac{\nu_o}{\omega} - j \right) e^{-2(\rho - \rho_i)/T} \left( 1 + \frac{2(\rho - \rho_i)}{T} \right) \quad (43)$$

where  $N_o$  and  $\nu_o$  are the electron density and collision frequency at the conducting surface  $\rho = \rho_i$  and  $T$  is about the  $1/3$  thickness of the plasma.

The attenuation produced by the sheath in nepers is just the real part of  $\Delta_m$  (Eq. 41) and on applying the perturbation method one finds this total attenuation to be the sum of two terms, one due to the reflections and "phase-mixing" from the gradient in  $N$  and the other due to direct heating losses (Figure 4). For a zero thickness plasma  $T = 0$  both attenuation terms are zero as they must be, and for small values of  $T$  the phase mixing term increases as  $T^2$  and the direct heating term as  $T^3$ .

The reflection loss approaches a fixed value for thick sheaths, whereas the heating losses continue to increase linearly. In order to find the total field pattern and to complete the example, the other  $F_n$  terms must be computed in a similar fashion and the results combined.

2.3 Nonlinear Interactions. The same perturbation technique used above can also be applied to study nonlinear interactions in the sheath. This nonlinear "self-modulation" occurs because the effective collision frequency is a function of the field intensity for strong fields. Ginzburg<sup>5</sup> demonstrates that:



$$\nu' = \nu_0 \sqrt{\frac{T_e}{T}} \quad (44)$$

where  $\nu'$  is the modified collision frequency,  $\nu_0$  the small-field value and  $T_e$  and  $T$  are the electron and ion temperatures in the plasma. The equipartition of energy requires that  $T_e = T$  when  $\bar{E}$  is negligible.

For  $\omega \gg \nu$  (the high frequency limit),

$$T_e - T \approx \frac{e^2 |\bar{E}_0|^2}{3mk\delta\omega^2} \quad (45)$$

$k$  being the Boltzmann constant and  $\delta$  the average relative fraction of energy imparted by an electron to a heavy particle in one collision. Combining (44) and (45),

$$\nu' = \nu_0 \sqrt{1 + \frac{e^2 |\bar{E}_0|^2}{3mkT\delta\omega^2}} \quad (46)$$

and for "intermediate" field intensities

$$\nu' = \nu_0 \left( 1 + \frac{e^2 |\bar{E}_0|^2}{6mkT\delta\omega^2} \right). \quad (47)$$

This last relation is the one that is used in subsequent analysis.

2.4 Example 2. Now consider the same  $m = 0$  term as in the problem of Section 2.2 but allow for intermediate field intensities so that the collision frequency is given by (47). For this model  $|\bar{E}_0|^2 \propto \rho^{-1}$  and substituting  $\nu'$  for  $\nu_0$  in (43) the field attenuation now produced by the sheath in nepers is



atten (nonlinear) = atten (linear)

$$+ \frac{e^4 N_o v_o |\bar{E}_o(\rho = \rho_i)|^2}{6m^2 \epsilon_o kT \omega^2} \operatorname{Im} \left[ \int_0^\infty \frac{d\xi'}{F_o^2(\xi')} \int_{\xi'}^\infty F_o^2(\xi'') e^{-2\xi''/T} \frac{(1 + \frac{2\xi''}{T}) d\xi''}{1 + \gamma \xi''} \right] \quad (48)$$

where  $\xi' = \rho' - \rho_i$ ,  $\xi'' = \rho'' - \rho_i$ , and  $\gamma$  is the reciprocal of  $\rho_i$  times the free space wave number  $k_o$ . The results from (48) are shown in Figure 5 for  $\gamma < 1$ , where it may be noted the  $\gamma = 0$  term (for which the nonlinearity vanishes) is identical in shape to the direct absorption curve in Figure 4. The reflection term is unchanged by the nonlinear interaction. It may be noted that the nonlinear attenuation components have a larger percentage increase in value for thinner sheaths and tend to flatten out sooner than do the linear components.

### 3. General Purpose of the Analytic Studies

The results above provide in themselves interesting clues into the loss mechanism in a plasma sheath. However, the application of these results should also be considered in light of the over-all effort being made in this field of study. Due to the complexities involved any results that closely parallel an actual physical problem of interest will probably be derived from a long, difficult computer program. But establishing the validity of such numerical techniques depends to a large extent on comparison of results with simpler analytic solutions of the kind described in this paper. Any analytic solution that can be obtained then serves double duty, both as a solution in itself and as a check on a more general computer program.



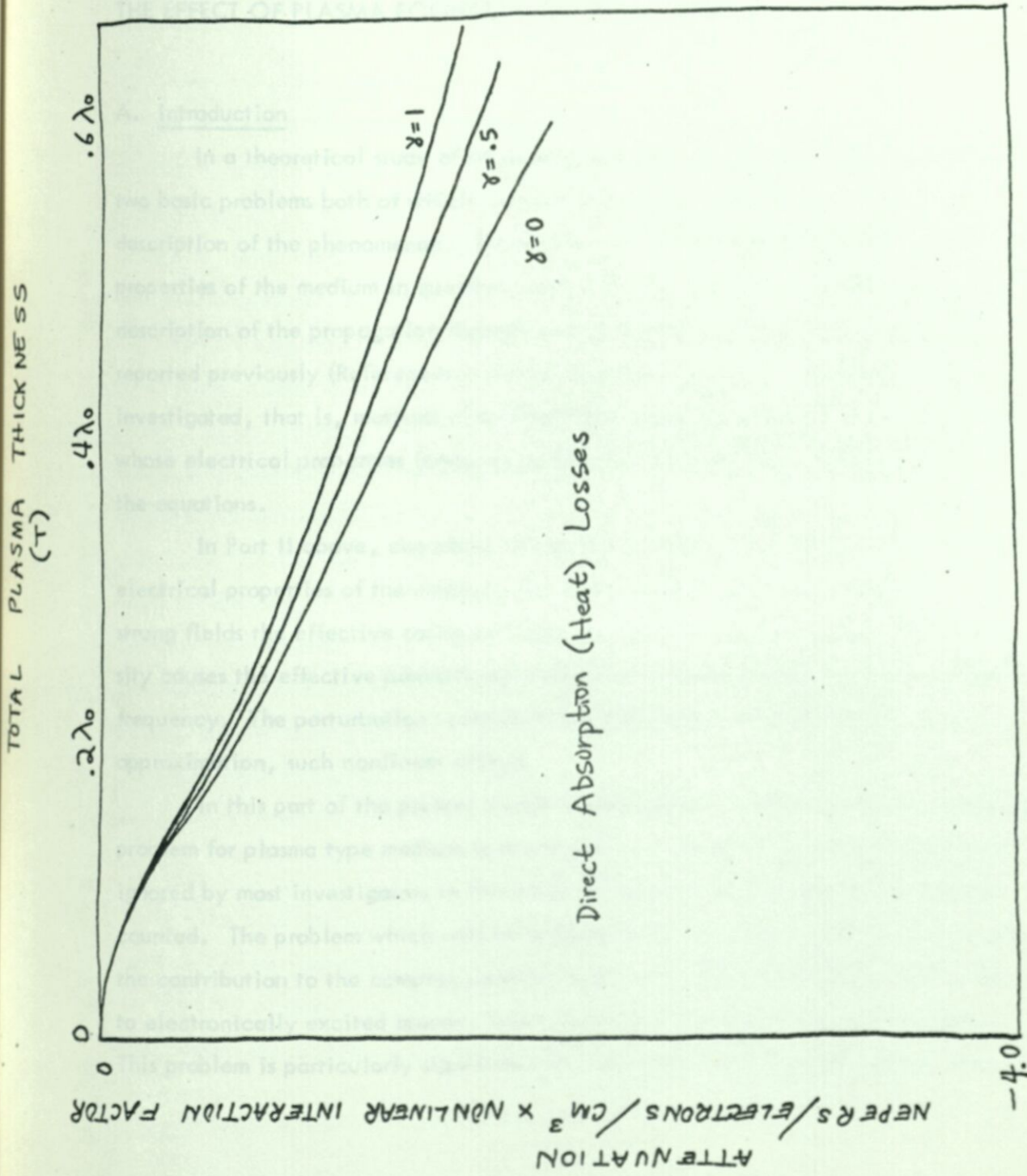


Fig. 5 The  $m = 0$  direct heating term for an exponential sheath on a large cylinder (intermediate fields)

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PART III

THE EFFECT OF PLASMA POLARIZATION ON MICROWAVE PROPAGATION

A. Introduction

In a theoretical study of microwave propagation through plasma there are two basic problems both of which must be adequately solved in any acceptable description of the phenomenon. These are: 1) the description of the electrical properties of the medium in question, and 2) the solution of the wave equation and description of the propagation through a medium of given properties. In the work reported previously (References 1 and 2) only the second of these problems was investigated, that is, methods of solving the propagation problem for a medium whose electrical properties (complex permittivity) are specified for solution of the equations.

In Part II above, one phase of the first problem, that of describing the electrical properties of the medium, has been treated. That is, in the case of strong fields the effective collision frequency being a function of the field intensity causes the effective permittivity to become modified by the change in collision frequency. The perturbation technique was extended to include, in a certain approximation, such nonlinear effects.

In this part of the present report another aspect of the microwave propagation problem for plasma type medium is investigated, this particular aspect having been ignored by most investigators in the subject though its importance has not been discounted. The problem which will be studied in the following is that of determining the contribution to the complex permittivity of an induced polarization vector due to electronically excited atomic, ionic and molecular components of the plasma. This problem is particularly significant for very high frequency propagation where



standard approximations for plasma permittivities become at best, somewhat dubious, and contributions from species other than the free electron distribution may become important.

### B. Statement of Problem

In light of the current interest in very high frequency propagation through plasma, it is necessary to investigate more closely the electrical properties of the plasma medium in frequency ranges where often used approximations begin to break down. In calculating the complex permittivity of a plasma for microwave studies one should consider, in addition to the conductivity due to the presence of free electrons, the possibility of an appreciable plasma polarization term due to the presence of a very large number of atoms, molecules, and ions in highly excited states. Since electrons in atomic or molecular excited states are very weakly bound, the resultant excited atomic or molecular system is very easily polarized by the electric field of the incident electromagnetic wave and thus exhibits a very large induced dipole moment. Since the frequency dependence of the resultant polarization term in the complex permittivity may be quite different from the free electron contribution, the dielectric properties of the medium may be changed appreciably from those predicted by neglecting the polarization terms. Thus the question of the magnitude of a plasma polarization term in high frequency microwave propagation through plasma seems to warrant some attention in order that its possible importance in such problems be ascertained.

#### 1. Plasma Components

The problem as referred to above is very complex for a realistic plasma composed of typical gases, say air for example, at typical plasma temperatures. For the sake of discussion, let us consider air at a density  $\rho$  of  $.1 \rho_0$  ( $\rho_0$  = standard atmospheric density) and in a temperature interval of 5000 to 15,000 °K. The resultant plasma is very complicated in its make-up, consisting of a mixture of



$O_2$ ,  $N_2$ ,  $O$ ,  $N$ ,  $NO$ ,  $N^+$ ,  $O^+$ ,  $NO^+$ , and  $e^-$  in various molar concentrations,<sup>6</sup>  $M_i$  (see Fig. 6). From the curves, the number of particles of a given species per cubic centimeter,  $N_i$ , is given by

$$N_i = 2.55 \times 10^{19} \rho/\rho_0 M_i \quad (3.1)$$

Any or all of the atomic, molecular or ionic species may exist in one of an infinite number of excited states, where the distribution of excited states at equilibrium would be given by the appropriate Boltzmann factor for each state. To calculate the contribution to the frequency dependent permittivity of each of these components in various excited states would involve a great deal of effort, particularly for molecular species. The calculations are meaningful, however, since highly excited electrons become more and more analogous to the free electron component in the plasma as the frequency of a propagating signal is increased, and thus more important in their contribution to the total complex permittivity.

The program which is undertaken in the following sections is not so ambitious as to determine the effect on  $\epsilon$  of all such constituents in a model plasma, rather we will take a much more limited approach and try to answer quantitatively the question as to the importance of the simplest systems (the atomic species) and to infer from the order of magnitude of the results for the simple case the importance of the total contribution from all species and indicate the validity of the problem in microwave studies.

## 2. The Polarization Vector

The problem of describing the dielectric properties of a plasma medium (other than the contribution from the free electron component) can be attacked on a microscopic scale from which a macroscopic description may be derived. Looking at the individual elements of the plasma, each atomic, molecular or ionic component is affected by the electric field of a propagating wave, the charge distribution of each being distorted or "polarized" by the effective field to which it is subjected. The result is that a dipole moment  $\vec{p}$  is induced in each particle, the moment being



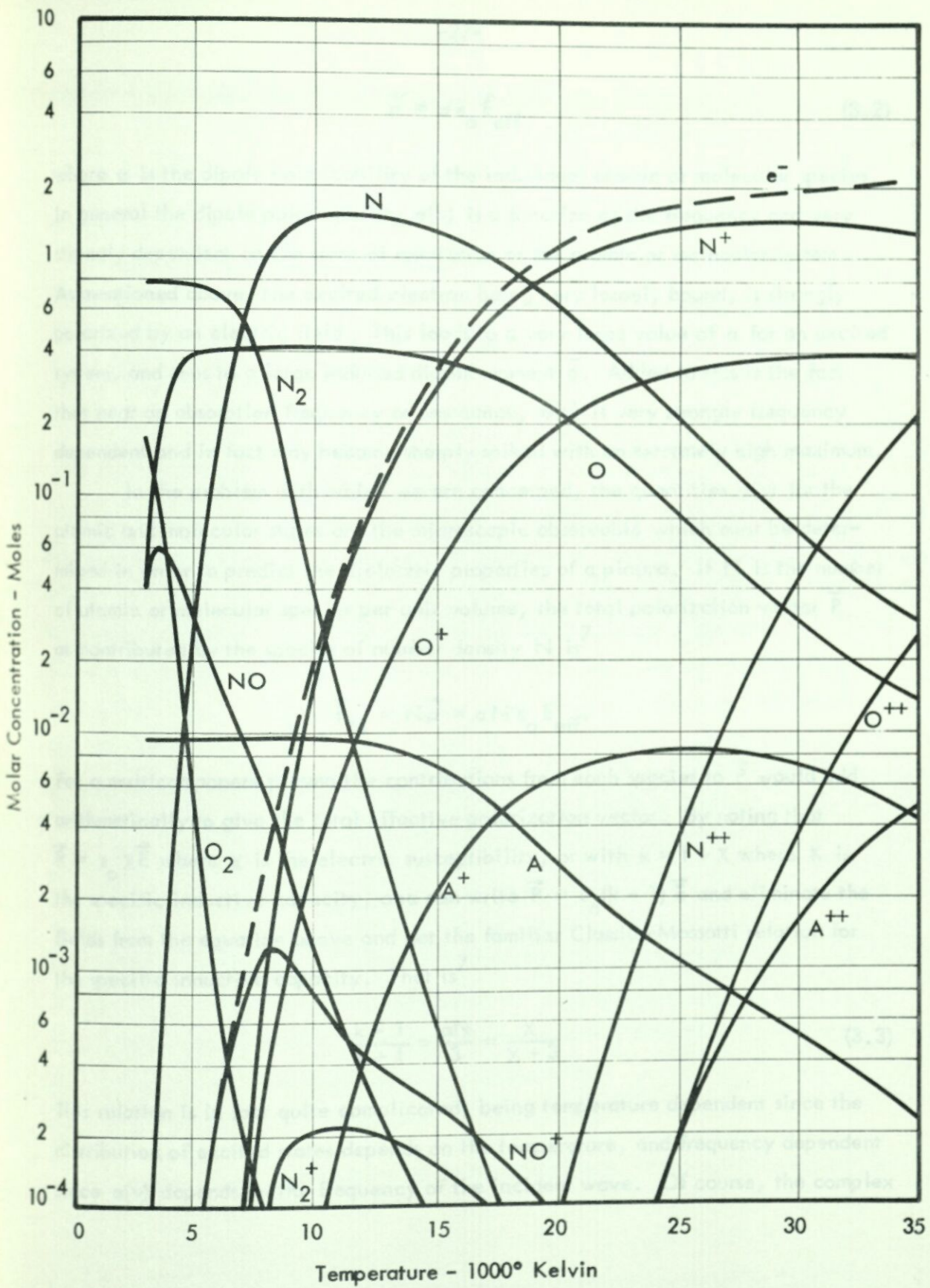


Fig. 6 Equilibrium molar concentration of air at  $\rho/\rho_0 = 10^{-1}$



$$\vec{p} = \alpha \epsilon_0 \vec{E}_{\text{eff}} \quad (3.2)$$

where  $\alpha$  is the dipole polarizability of the individual atomic or molecular species. In general the dipole polarizability  $\alpha(\nu)$  is a function of the frequency and very strongly dependent on the state of excitation of the atomic or molecular system. As mentioned above, the excited electron being very loosely bound, is strongly polarized by an electric field. This leads to a very large value of  $\alpha$  for an excited system, and thus to a large induced dipole moment  $\vec{p}$ . Added to this is the fact that near an absorption frequency or resonance,  $\alpha(\nu)$  is very strongly frequency dependent and in fact may become sharply spiked with an extremely high maximum.

In the problem with which we are concerned, the quantities  $\alpha(\nu)$  for the atomic and molecular states are the microscopic observable which must be determined in order to predict the dielectric properties of a plasma. If  $N$  is the number of atomic or molecular species per unit volume, the total polarization vector  $\vec{P}$  as contributed by the species of number density  $N$  is<sup>7</sup>

$$\vec{P}_N = N\vec{p} = \alpha N \epsilon_0 \vec{E}_{\text{eff}}$$

For a multicomponent plasma the contributions from each species to  $\vec{P}$  would add arithmetically to give the total effective polarization vector. By noting that  $\vec{P} = \epsilon_0 \chi \vec{E}$  where  $\chi$  is the electric susceptibility, or with  $k = 1 + \chi$  where  $k$  is the specific inductive capacity, one can write  $\vec{P} = \epsilon_0 (k - 1) \vec{E}$  and eliminate the fields from the equation above and get the familiar Clausius-Mossotti relation for the specific inductive capacity. That is<sup>7</sup>

$$\frac{k - 1}{k + 1} = \frac{\alpha N}{3} = \frac{\chi}{\chi + 3} \quad (3.3)$$

This relation is in fact quite complicated, being temperature dependent since the distribution of excited states depends on the temperature, and frequency dependent since  $\alpha(\nu)$  depends on the frequency of the incident wave. Of course, the complex



permittivity  $\epsilon$  is given by the relation

$$\epsilon(\omega) = \epsilon_0 (1 + \chi(\omega)) + \frac{i\sigma}{\omega} \quad (3.4)$$

where  $\sigma$  is the plasma conductivity and  $\omega$  is the source frequency.

Summarizing, we note that aside from knowing the number densities  $N_i$  for the plasma components in their various states, knowledge of the real part of the macroscopic parameter  $\epsilon$  depends on knowing the microscopic quantities  $\alpha_i$  for the various plasma components.

### 3. The Dipole Polarizability $\alpha$

On a microscopic scale one should know the distribution of excited states of all plasma species; that is, atomic, molecular, and ionic components, and further should know the polarizability of each in these states. With this knowledge the macroscopic polarization vector  $\vec{P}$  could be determined to a very high degree of accuracy. The problem of calculating dipole polarizabilities  $\alpha$  has been well formulated for systems in the ground state and many specific examples have been worked out which are in good agreement with experiment.<sup>8</sup> However, this statement does not apply to such particles in excited states, nor to more complicated molecules where calculations are very difficult.

In principle, the frequency dependent polarizability tensor is given by<sup>8</sup>

$$\alpha(\nu) = \frac{1}{3} [\alpha_{xx}(\nu) + \alpha_{yy}(\nu) + \alpha_{zz}(\nu)] \quad (3.5)$$

In the limit as  $\nu$  becomes zero,  $\alpha(\nu)$  becomes the usual average polarizability  $\alpha$ , or  $\alpha(0)$ . For any atomic or molecular system in its ground state the quantities  $\alpha_{ii}(\nu)$  may be determined from the oscillator strength sum rules. From time dependent perturbation theory the expression is easily shown to be\*

$$\alpha_{xx}(\nu) = a_0^3 \sum_i \frac{f(x)_{qi} (e^2/a_0)^2}{(\epsilon_q - \epsilon_i)^2 - h^2 \nu^2} \quad (3.6)$$

---

\* Atomic units are utilized throughout this section. Unit of length is the Bohr radius  $a_0$  ( $.53 \times 10^{-8}$  cm).



Here  $f(x)_{qi}$  is the oscillator strength corresponding to a transition from the state  $q$  to the state  $i$  under the influence of x-polarized electric dipole radiation.<sup>8</sup>

$$f(x)_{qi} = 2 \frac{(\epsilon_i - \epsilon_q) \langle \psi_q, \mu_x \psi_i \rangle \langle \psi_i, \mu_x \psi_q \rangle}{(e^2/a_0)(ea_0)^2} \quad (3.7)$$

In the expression for the oscillator strength  $f(x)_{qi}$ , the quantities  $\psi_q$  and  $\psi_i$  are the wave functions for the system in states  $q$  and  $i$  respectively. The term  $\mu_x$  is the x component of the dipole moment,  $\mu_x = \sum_i e_i x_i$ , where  $e_i$  is the charge and  $x_i$  the coordinate of the  $i^{\text{th}}$  particle, and the summation is extended over all of the charged particles in the system including nuclei. Similar expressions obtain for  $\alpha_{yy}$  and  $\alpha_{zz}$ .

The expression (3.1) for the polarizability  $\alpha(\nu)$  of an atomic or molecular system has very limited applicability for the problem of interest to us here. First, as it is written it applies only to systems in their ground state; secondly, the summation is over all the infinite number of excited states and such summations can only be approximated in practice; finally, the oscillator strengths  $f(x)_{qi}$  are difficult to obtain with reliable accuracy, thus placing a severe limitation on the accuracy of calculations made from Eq. (3.7). Thus, we need a more tractable expression for the polarizability if a large number of excited state calculations are to be made. Before leaving this equation, let us note that Eq. (3.6) has a minimum for zero frequency and gradually increases with  $\nu$  to the point where  $h\nu$  is equal to the energy difference between the ground and the first excited state. Thus, for microwave frequencies,  $\alpha(0)$  represents a minimum for the frequency dependent polarizability.

With the observation that the zero frequency value of  $\alpha(\nu)$  represents a minimum in this quantity for microwave frequencies we will make the following simplifications for the purposes of calculation: 1) Calculate the static polarizability  $\alpha(0)$  for certain specific species in several excited states. This problem will be



formulated in the following section in a tractable form. 2) Utilize the results at zero frequency to infer the importance of the polarization vector in plasma for microwave propagation (for simple atomic systems). 3) Estimate the contribution from more complicated systems where calculations become more difficult.

### C. Method of Calculation

The electronic polarizability of an atomic system, whether it be neutral or ionized, may be determined by a perturbation calculation on the Hartree-Fock wave functions for the system in question. This problem has been formulated by Sternheimer<sup>9</sup> and specific examples have been worked out and published in the literature for neutral and ionic species in their ground states.<sup>8</sup> In this section we will consider this method as applied to excited state systems and specific examples will be given in the following section.

Assume that the wave functions for the unperturbed system are known, whether in the ground state or some excited state, and that the Schroedinger equation for each electron of the system is

$$H_0 \psi_0^i = \epsilon_0^i \psi_0^i. \quad (3.8)$$

Here  $H_0$  is the unperturbed Hamiltonian and  $\epsilon_0^i$  the energy of the  $i^{\text{th}}$  electron in the Hartree-Fock equations. Now assume that an electric field is produced by a unit charge  $-e$  at a large distance  $R$  from the nucleus along the positive  $z$ -axis. If  $R$  is the distance in units of the Bohr radius  $a_0$ , the dipole part of the potential energy in Rydberg units is given by the expression

$$H_1 = (2/R^2) r \cos \theta. \quad (3.9)$$

Here  $\theta$  is the angle between the position vector  $r$  of the  $i^{\text{th}}$  electron and the  $z$ -axis. We now consider the potential  $H_1$  as a small perturbation on the Hartree-Fock solutions for the isolated atom. Denote the perturbation of one of the H.F. functions  $\psi_0^i$  by

the symbol  $\chi_i$  and the first order change in energy by  $\epsilon_1$ . The Schroedinger equation to the first order in the perturbation  $H_1$  is given by

$$(H_0 + H_1) (\psi_0^i + \chi_i) = (\epsilon_0^i + \epsilon_1) (\psi_0^i + \chi_i). \quad (3.10)$$

The term  $E_1$ , from standard first order perturbation theory, is given by

$$\epsilon_1 = \int_0^\infty \int_0^\pi \int_0^{2\pi} H_1 \psi_0^i{}^2 r^2 dr \sin \theta d\phi \quad (3.11)$$

Since  $H_1$  is linear in  $\cos \theta$  while  $\psi_0^i{}^2$  is an even function of  $\cos \theta$ , the integral over  $\theta$  vanishes in the above expression, thus yielding  $\epsilon_1 = 0$ . Utilizing equation (3.8), equation (3.2) is reduced to

$$(H_0 - \epsilon_0^i) \chi_i = -H_1 \psi_0^i \quad (3.12)$$

The solution to this equation determines the perturbation  $\chi_i$  of the wave function  $\psi_0^i$ . The unperturbed functions are defined such that

$$\psi_0^i = R(r) Y^m(\theta, \phi)$$

$$\int_0^\infty \int_0^\pi \int_0^{2\pi} |\psi_0^i|^2 r^2 \sin \theta d\theta d\phi = \int_0^\infty u_0^2(r) dr = 1 \quad (3.13)$$

The first order perturbation of the density  $[\psi_0^i + \chi_i]^2$  is given by  $2\psi_0^i \chi_i$  for the  $i^{\text{th}}$  electron. From this, the induced dipole moment in the perturbed atomic system, indicated as  $p_{\text{ind-z}}$ , is given by

$$p_{\text{ind-z}} = 2 e \sum_{nlm} \int_0^\infty \int_0^\pi \int_0^{2\pi} (\psi_0^i \chi_i)_{nlm} r \cos \theta r^2 dr \sin \theta d\theta d\phi \quad (3.14)$$

where we use the subscripts  $nlm$  to denote an orbital of principle quantum number  $n$  having angular momentum quantum numbers  $l$  and  $m$ . The density  $\psi_0^i \chi_i$  depends, of course, on  $nlm$ . The electric field  $E_z$  is given by  $-e^2/R$ , so that the polarizability,



defined as the induced dipole moment per unit field is given by

$$\alpha = p_{\text{ind-z}}/E_z = 2R^2 \sum_{nlm} \int_0^\infty \int_0^\pi \int_0^{2\pi} (\psi_0^i X_i)_{nlm} \cdot r \cos \theta r^2 dr \sin \theta d\theta d\phi \quad (3.15)$$

The integrals in the expression for  $\alpha$  can be reduced to radial integrals, and the inhomogeneous partial differential equation (3.12) may be reduced to a radial equation by expressing the unperturbed functions as in Eq. (3.13) and the perturbation  $X_i$  in a similar fashion, that is, as products of radial functions multiplied by a spherical harmonic. To this end we write Eq. (3.12) in detail.

$$[-\nabla_i^2 + V_{\text{HF}}(r_i) - \epsilon_0^i] X_i = -\frac{2r_i}{R^2} \cos \theta \psi_0^i. \quad (3.16)$$

In this equation  $V_{\text{HF}}$  is the Hartree-Fock potential for the  $i^{\text{th}}$  orbital whose unperturbed eigenvalue is  $\epsilon_0^i$ . In order to separate this equation we write  $\psi_0^i$  and  $X_i$  as

$$\psi_0^i = R_{nl}(r) Y_l^m(\theta, \phi) = \frac{P_{nl}(r)}{r} Y_l^m(\theta, \phi) \quad (3.17)$$

and

$$X_i = \sum_{l', m'} \frac{U_{nl \rightarrow l'}(r)}{r} C_{l \rightarrow l'}^{m \rightarrow m'} Y_{l'}^{m'}(\theta, \phi) \quad (3.18)$$

The above equation then separates into the following radial equation

$$\left[ \frac{d^2}{dr^2} - \frac{l'(l'+1)}{r^2} + V_{\text{HF}}(r) - \epsilon_0^i \right] U_{nl \rightarrow l'}(r) = \frac{2r}{R^2} P_{nl}(r). \quad (3.19)$$

We note that the constants  $C_{l \rightarrow l'}^{m \rightarrow m'}$ , which appear in Eq. (3.18) are determined by the Clebsch Gordon coefficients which occur in the angular integrations. These coefficients vanish unless  $l' = l \pm 1$  for  $l \neq 0$  and  $l' = l + 1$  for  $l = 0$ . Thus, the solutions to two radial equations (3.19) are required in order to determine the radial functions



$U_{n\ell \rightarrow \ell'}$  of the perturbation  $\chi_i$  of a given orbital (for s-states only one). The notation  $U_{n\ell \rightarrow \ell'}$  for the radial functions of the perturbation of an  $n\ell$  orbital is that due to Sternheimer. It indicates an orbital of  $\ell$  character being perturbed to a state of angular momentum  $\ell'$ . The coefficients  $C_{\ell \rightarrow \ell'}^{m \rightarrow m'}$  have been tabulated by Sternheimer.<sup>9</sup> The dipole polarizability due to the contribution from an  $n\ell$  state may now be written

$$\alpha_{n\ell \rightarrow \ell'}^{(m')} = 2R^2 \int_0^\infty \int_0^\pi \int_0^{2\pi} U_{n\ell \rightarrow \ell'}(r) P_{n\ell}(r) Y_{\ell'}^{m'}(\theta, \phi) Y_{\ell}^m(\theta, \phi) r \cos \theta \cdot dr \sin \theta d\theta d\phi = 2R^2 K_{\ell \rightarrow \ell'}^{m'} \int_0^\infty U_{n\ell \rightarrow \ell'}(r) P_{n\ell}(r) r dr \quad (3.20)$$

where  $K_{\ell \rightarrow \ell'}^m$  is a constant which is the product of  $C_{\ell \rightarrow \ell'}^{m \rightarrow m'}$  times the constant which occurs from the angular integral in Eq. (3.20). These are also tabulated by Sternheimer.

If we indicate by the notation  $\alpha(n\ell \rightarrow \ell')$  the contribution to the dipole polarizability of an  $n\ell$  orbital undergoing excitation to state  $\ell'$ , then the total dipole polarizabilities for the atomic system can be written

$$\alpha_{TOT} = \sum_{n\ell\ell'} N^{n\ell} \alpha(n\ell \rightarrow \ell') \quad (3.21)$$

where  $N^{n\ell}$  is the number of electrons in the  $n\ell$  subshell. We note that the above equations give  $\alpha$  in units of  $a_0^3$ . To obtain  $\alpha$  in units of  $\text{\AA}^3$  the results must be multiplied by  $0.529^3 = 0.148$ .

We further note that in considering a system in an excited state one can simplify Eq. (3.21) by noting that for the excited orbital the contribution to the sum is very large as compared to the contribution from other electrons. Consequently, only one term in the total contribution to  $\alpha$  need be considered, namely that of the excited electron, all others being negligible. Thus, unlike the problem of determining the ground state polarizabilities where the contribution to  $\alpha$  from several orbitals is often important, the determination of  $\alpha$  for excited states depends only on determining the unperturbed eigenfunctions for the excited systems and determining the first order perturbation of the excited orbital (from Eq. (3.19)) and performing the integral in Eq. (3.20).



#### D. Results for Oxygen and Nitrogen

Detailed calculations for atomic oxygen and atomic nitrogen have been made for several low lying excited states. We note that the determination of the dipole polarizability of an atomic system in any state  $\Psi$  requires a knowledge of the unperturbed Hartree-Fock wave functions  $\psi_0$  for the system in the given state. With these functions, Eq. (3.19) may be solved and the perturbations  $\chi_i$  of the Hartree-Fock functions thereby determined. With these solutions, both  $\psi_0$  and  $\chi_i$  may be utilized in the expression (3.20) for determining the polarizability  $\alpha$  in the particular state  $n\ell m$ .

##### 1. The Unperturbed Solutions

In Rydberg units the Hartree-Fock equation for the  $i^{\text{th}}$  orbital of the H.F. determinant can be written

$$[-\nabla_i^2 + V(r_i) - A(r_i)] \psi_i(\vec{r}_i) = E_i \psi_i(\vec{r}_i) \quad (3.22)$$

where  $V(r_i)$  is the sum of the nuclear and electronic coulomb potentials

$$V(r_i) = \frac{-2Z}{r_i} + \sum_k \int \psi_k^*(\vec{r}_k) \frac{2}{r_{ik}} \psi_k(r_k) d\tau_k \quad (3.23)$$

and the exchange potential is given by

$$A(r_i) \psi_i(\vec{r}_i) = \sum_k \left[ \int \psi_i^*(\vec{r}_i) \psi_k(\vec{r}_k) \frac{2}{r_{ik}} \psi_i(\vec{r}_i) \psi_k(r_i) d\tau_k \right] \psi_i(\vec{r}_i) \quad (3.24)$$

In the present calculations the exchange integrals  $A(r_i)$  are replaced by the convenient approximation due to Slater which is an average exchange potential for all electrons. One further modification was made on the exchange term, this being to write the Slater exchange term with a variable coefficient; that is,

$$A_s(r_i) \psi_i(r_i) = \lambda_s \cdot 6 \left[ \frac{3}{8\pi} \sum_k \psi_k^*(\vec{r}_i) \psi_k(\vec{r}_i) \right]^{1/3} \psi_i(\vec{r}_i) \quad (3.25)$$

where  $\lambda_s$  is a variable parameter.

The equation can be reduced to radial equation by writing the function

$$\psi_i(\vec{r}_i) = \frac{P_{n\ell}(r_i)}{r_i} Y_{\ell}^m(\theta, \phi) \quad (3.26)$$

Equations (3.22) then separate leading to the radial equation

$$\left[ \frac{-d^2}{dr_i^2} + V(r_i) - \frac{\ell(\ell+1)}{2r_i^2} - A_s(r_i) \right] P_{n\ell}(r_i) = \epsilon_i^0 P_{n\ell}(r_i) \quad (3.27)$$

In solving these equations the coefficient  $\lambda_s$  was varied until the eigenvalues were in agreement with experimental values to within one or two percent. For excited states the experimental values were taken to be the weighted average of the configuration.

## 2. The Perturbation Calculations

In the present calculations the unperturbed wave functions for the atomic system were obtained from a program originally written by Herman and Skillmann,<sup>10</sup> which was modified to yield the variable term in the Hartree-Fock-Slater (H.F.S.) equations as discussed above. The output of the program furnished the functions  $V(r)$ ,  $A_s(r)$ ,  $\epsilon_1^0$  and  $P_{n\ell}(r)$  in equation (3.19), which could then be solved for the perturbations  $U_{n\ell \rightarrow \ell'}$  of a given orbital whose radial function is  $P_{n\ell}$ . For a given value of  $R$ , equation (3.19) was integrated by the Numerov process for inhomogeneous equations as described by Hartree,<sup>11</sup> over the same  $r$  mesh as that of the H.F.S. program which furnished the unperturbed functions. The integration in the inner region was started by noting, as did Sternheimer,<sup>6</sup> that for  $r \sim 0$  the inhomogeneous term on the right side is negligible as compared to the potential terms on the left. The solution may thus be started by a series expansion<sup>11</sup> near the origin and continued by numerical integration. With this procedure there is an arbitrary constant in the starting values, this being the value of  $(U_{n\ell \rightarrow \ell'} / r^{\ell'+1})_{r=0}$ . For the inhomogeneous equation (3.19), this parameter in the starting conditions must be determined in order to satisfy the boundary conditions; that the solution to (3.19) be exponentially



decreasing at infinity. The value of  $(U_{n\ell \rightarrow \ell'} / r^{\ell'+1})_0$  was varied automatically in the coded program until two values were found which enclosed the correct one. The choice was then narrowed by successive solutions until an accuracy of five to six significant figures in the starting value was achieved. The calculations were performed on the Univac 1107 computer at the University of Alabama Research Institute.

The total dipole polarizability for both oxygen and nitrogen in their ground states was calculated from the equations for the two outermost shells of the H.F.S. system. However, as mentioned above, in the case of an excited system only the perturbation of the excited orbital need be considered in the calculation of the polarizability, all other contributions being completely negligible by comparison. With the perturbed functions  $U_{n\ell \rightarrow \ell'}$  determined from Eq. (3.19) the dipole polarizabilities were determined by carrying out the numerical integration of Eq. (3.20).

### 3. Numerical Results

In this section numerical results of dipole polarizability calculations for excited states of atomic oxygen and nitrogen are presented. Calculations have been made for the ground state and for several excited states of both atoms. The unperturbed Hartree-Fock-Slater functions for the various states were obtained as described above by varying the Slater exchange term until the calculated eigenvalues of the excited electron agreed with the experimental value to within one percent. The experimental value in all cases was taken to be the weighted average for the excited configuration. Thus the eigenvalues and the polarizabilities correspond to those for the average of a given excited configuration of the atomic system.

As an illustration of the effect of the perturbing field on an excited state wave function, Figure 7 shows the unperturbed 4p radial function of oxygen along with the radial perturbation  $U_{4p \rightarrow d}$  and  $U_{4p \rightarrow s}$ . The solutions  $U_{n\ell \rightarrow \ell'}$  for the radial equations exhibit a behavior similar to that reported by Sternheimer. Thus the nodes of the radial function  $U_{n\ell \rightarrow \ell'}$  correspond in number to the orbital next higher in energy than  $n\ell$  having  $\ell'$  angular momentum. However, unlike the case

Fig. 7. The radial wave functions for an excited 4p electron of oxygen, along with the 4p-d and 4p-s perturbations,  $U_{4p \rightarrow d}$  and  $U_{4p \rightarrow s}$ .



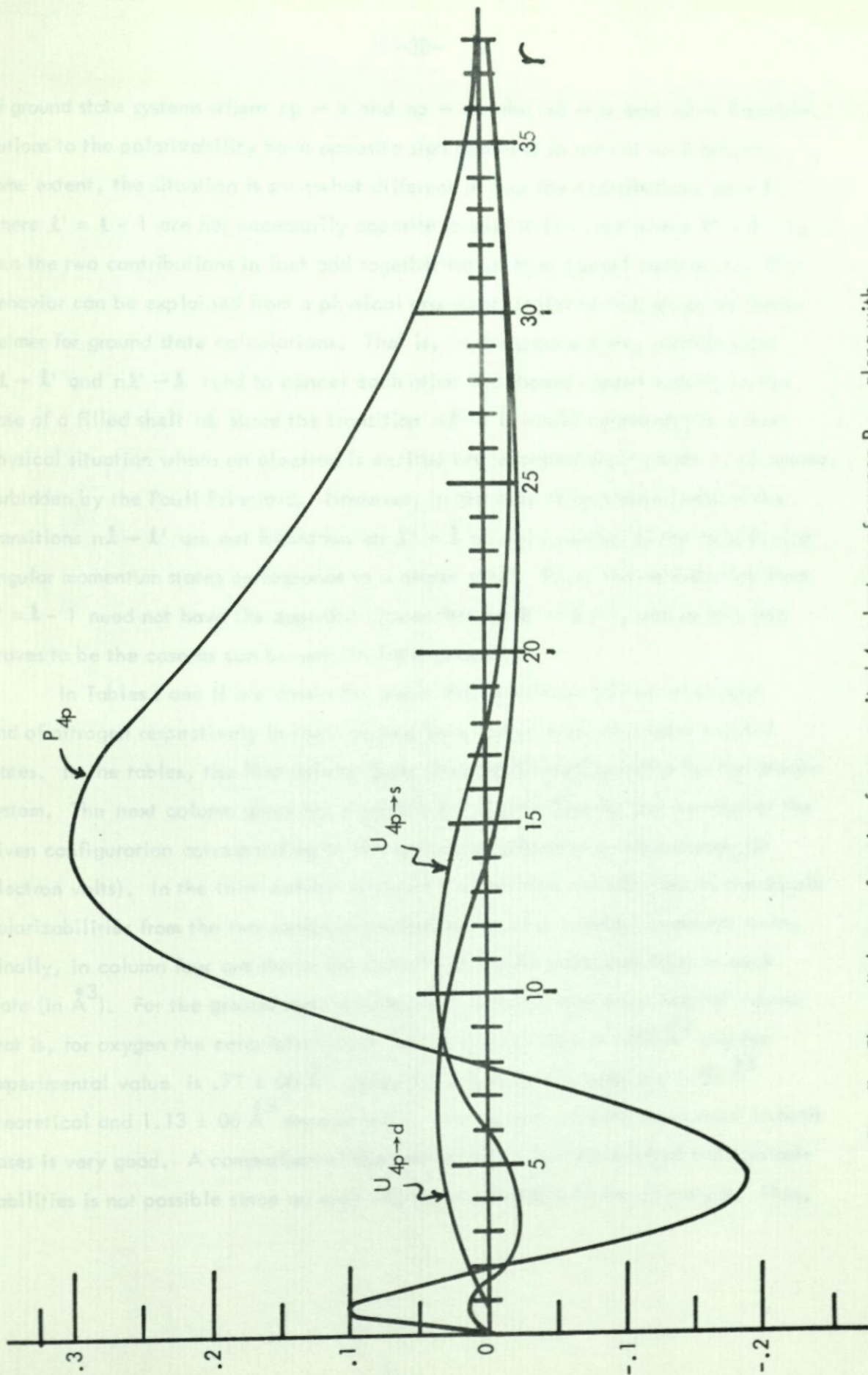


Fig. 7 The radial wave function for an excited 4p electron of oxygen  $P_{4p}$ , along with the  $4p \rightarrow d$  and  $4p \rightarrow s$  perturbations,  $U_{4p \rightarrow d}$  and  $U_{4p \rightarrow s}$



of ground state systems where  $np \rightarrow s$  and  $np \rightarrow d$ , also  $nd \rightarrow p$  and  $nd \rightarrow f$  contributions to the polarizability have opposite sign and tend to cancel each other to some extent, the situation is somewhat different in that the contributions  $n\ell \rightarrow \ell'$  where  $\ell' = \ell - 1$  are not necessarily opposite in sign to the case where  $\ell' = \ell + 1$ , thus the two contributions in fact add together rather than cancel each other. This behavior can be explained from a physical argument similar to that given by Sternheimer for ground state calculations. That is, in the ground state, contributions  $n\ell \rightarrow \ell'$  and  $n\ell' \rightarrow \ell$  tend to cancel each other and should cancel exactly in the case of a filled shell  $n\ell$  since the transition  $n\ell' \rightarrow \ell$  would correspond to a non-physical situation where an electron is excited into a closed shell which is, of course, forbidden by the Pauli Principle. However, in the case of an excited orbital the transitions  $n\ell \rightarrow \ell'$  are not forbidden for  $\ell' = \ell \pm 1$  since neither of the neighboring angular momentum states corresponds to a closed shell. Thus, the contribution from  $\ell' = \ell - 1$  need not have the opposite sign as that for  $\ell' = \ell + 1$ , and in fact this proves to be the case as can be seen in Tables I and II.

In Tables I and II are shown the static dipole polarizabilities of oxygen and of nitrogen respectively in their ground state and in each of several excited states. In the tables, the first column gives the excited configuration for the atomic system. The next column gives the experimental eigenvalues for the average of the given configuration corresponding to the ionization potential as zero energy (in electron volts). In the third column is shown the separate contributions to the dipole polarizabilities from the two modes of excitation (one for  $s$ -states) in atomic units. Finally, in column four are shown the calculated dipole polarizabilities in each state (in  $\text{\AA}^3$ ). For the ground state a comparison is made with experimental values; that is, for oxygen the calculated value for the ground state is  $.762 \text{\AA}^3$  and the experimental value is  $.77 \pm 06 \text{\AA}^3$  while the values for nitrogen are  $1.08 \text{\AA}^3$  theoretical and  $1.13 \pm 06 \text{\AA}^3$  experimental. The agreement with experiment in both cases is very good. A comparison of theoretical values for the excited state polarizabilities is not possible since no such values are available in the literature. Thus,

TABLE I

## OXYGEN POLARIZABILITIES

Config.	$\epsilon_i$ (ev)	Contributions to Polarizability	$\alpha \text{ \AA}^3$
$2P^4$	Ground State		.762 (.77)
$2P^3 3S$	4.3298	472.12	46.58
$2P^3 3P$	2.7833	p-d 1109.7	77.1
$2P^3 4S$	1.7449	7316.8	721.9
$2P^3 3D$	1.5249	d-f 4374.2 d-p 3837.2	410.3
$2P^3 4P$	1.3041	p-d 4969.7 p-s 196.8	333.3
$2P^3 5S$	.9430	53834.5	5311.6
$2P^3 4D$	.8619	d-f 509811. d-p 39605.	31743.



TABLE II

## NITROGEN POLARIZABILITIES

Config.	$\epsilon_i$ (ev)	Contributions to Polarizability	$\alpha \text{ \AA}^3$
$2P^3$	Ground State		1.08 (1.13)
$2P^2 3S$	4.0973	595.65	58.77
$2P^2 3P$	2.6728	p-d 1359.40 p-s 9.95	89.09
$2P^2 4S$	1.6712	9937.38	980.4
$2P^2 3D$	1.5461	d-f 4175.9 2220.2	334.8
$2P^2 4P$	1.2906	p-d 9032.9 2362.2	521.9
$2P^2 5S$	.9165	66715.2	6627.0
$2P^2 4D$	.8713	d-f 408452. d-p 20763.	25005.

an estimate of the accuracy of the calculated values must be obtained by a critical examination of the theoretical method. It is well known that dipole polarizabilities obtained by the Sternheimer method for Hartree-Fock type systems are extremely sensitive to small inaccuracies in the unperturbed H.F. wave functions. The same criticism, of course, applies here. In order to obtain an estimate of the effect of inaccuracies in the H.F. functions on the calculated polarizabilities, the calculations were repeated for a given state with a set of functions obtained by varying the exchange term slightly in order to produce an eigenvalue for the excited orbital which differed from the experimental value by about 3%. It was found that this change in the unperturbed function produced about a 15% change in the calculated value of  $\alpha$ . Thus the method is indeed very sensitive to small changes in  $\psi_0$ . The only recourse under the circumstances is to utilize the most accurate unperturbed system that one can obtain. In the present calculations the criterion which was used to define a good H.F. wave function was that which yielded an eigenvalue which differed by no more than 1% from the weighted average experimental value for a given configuration. This method is, of course, subject to error. The values listed in Tables I and II are estimated to be accurate to about 15%. Though this could possibly lead to an appreciable error in detailed calculations, the accuracy is more than adequate for the purpose of determining the importance of wave propagation, which is our intended purpose here.

Having obtained the microscopic variables  $\alpha_i$  for several states of the two species, neutral atomic oxygen and nitrogen, our objective is to determine the macroscopic variables,  $K$ ,  $\chi$  or  $\epsilon$  which appear in Maxwell's equations and thus to estimate effects in microwave propagation problems. As mentioned earlier, the Clausius-Mossotti relation yields the electric susceptibility  $\chi$  for a simple medium through the relation

$$\frac{\chi}{\chi + 3} = \frac{\alpha N}{3}$$

or the specific induction capacity through  $K = 1 + \chi$ . In this equation  $N$  is the number density of molecules of polarizability  $\alpha$ . For a plasma medium of the type



of interest to us here the problem is complicated by the fact that there are many different species in the medium each of which have a different number density and a different polarizability, these being the various atomic, molecular, and ionic species in their multitude of excited states. Thus, on a macroscopic scale, one must take the various contributions to the specific inductive capacity from the various species present, taking into account their number densities and dipole polarizabilities. For a given plasma component, say atomic oxygen for example, one can calculate the average polarizability  $\bar{\alpha}$  as a function of temperature  $T$ , very simply if thermodynamic equilibrium is assumed. That is, the number density of atoms in an excited state represented by energy  $E_i$  and by ground state energy  $E_0$  at a temperature  $T$  is related to the number  $N_0$  at zero temperature by the appropriate Boltzmann factor

$$N_{E_i} = N_0 e^{-(E_i - E_0)/kT} \quad (3.28)$$

where  $N_{E_i}$  is the number at temperature  $T$ , and  $k$  is Boltzmann's constant. If one knows the polarizability of the state  $E_i$ , one can determine the average polarizability of atomic oxygen at temperature  $T$  by the expression

$$\bar{\alpha}_T = \frac{\sum_i N_{E_i} \alpha_{E_i}}{\sum_i N_{E_i}} \quad (3.29)$$

which is a temperature dependent polarizability for the species. The Clausius-Mossotti relation then yields  $\chi$  or  $K$ , i.e.

$$K = \frac{1 + \frac{2}{3} \bar{\alpha}_T N}{1 - \frac{\bar{\alpha}_T N}{3}} \quad (3.30)$$

For a multi-component mixture of gases this process could be repeated for each



contributing species. In the present work this is done for oxygen and nitrogen. The results can be used to estimate contributions from all species.

In Figure 8 the specific inductive capacity for zero frequency of atomic oxygen and nitrogen is shown as a function of temperature over a range from 6000 to 15,000 degrees Kelvin. Each is assumed to be at standard atmospheric pressure for purposes of comparison. From the figure it can be seen that the electric susceptibility  $\chi$  changes by more than two orders of magnitude over the temperature range shown.

Two facts are very significant about the results shown in Figure 8. These are:

1) The static susceptibilities shown are a lower limit to the frequency dependent susceptibilities which are significant to microwave problems. From Eq. (3.6) for  $\alpha(\nu)$  it is obvious that for certain frequencies resonances will appear which increase the value of  $\chi$  by a very large amount.

2) The second point which should be made is that the static polarizabilities, and thus the resulting susceptibility  $\chi$ , are much smaller, in general, for atomic species such as oxygen and nitrogen than it would be for molecular species which commonly appear in atmospheric plasma. That is, the curves for  $O_2$ ,  $N_2$ ,  $N_2O$ , etc., will be correspondingly larger due to weaker binding in molecular systems.

The frequency dependent polarizability  $\alpha(\nu)$  for oxygen and nitrogen may now be obtained in an approximate way from Eq. (3.6) by utilizing the static value obtained by the method of Section C to determine the sum of the oscillator strengths of Eq. (3.6). That is, one can treat the energy denominators in Eq. (3.6) as being important only in the region near a resonance. Thus, in a region where  $h\nu$  is of the order of the difference between the energy state  $\epsilon_q$  and any one of the higher states  $\epsilon_i$ . As the frequency  $\nu$  increases from zero, the first resonance will exist when  $h\nu$  approaches the difference between  $\epsilon_q$  and the next higher excited state. Taking into account only the first term in Eq. (3.6) for this case we can thus approximate  $\alpha(\nu)$  by

$$\alpha(\nu) = \frac{\alpha(0) (\epsilon_q - \epsilon_i)^2}{(\epsilon_q - \epsilon_i)^2 - h^2 \nu^2} \quad (3.31)$$



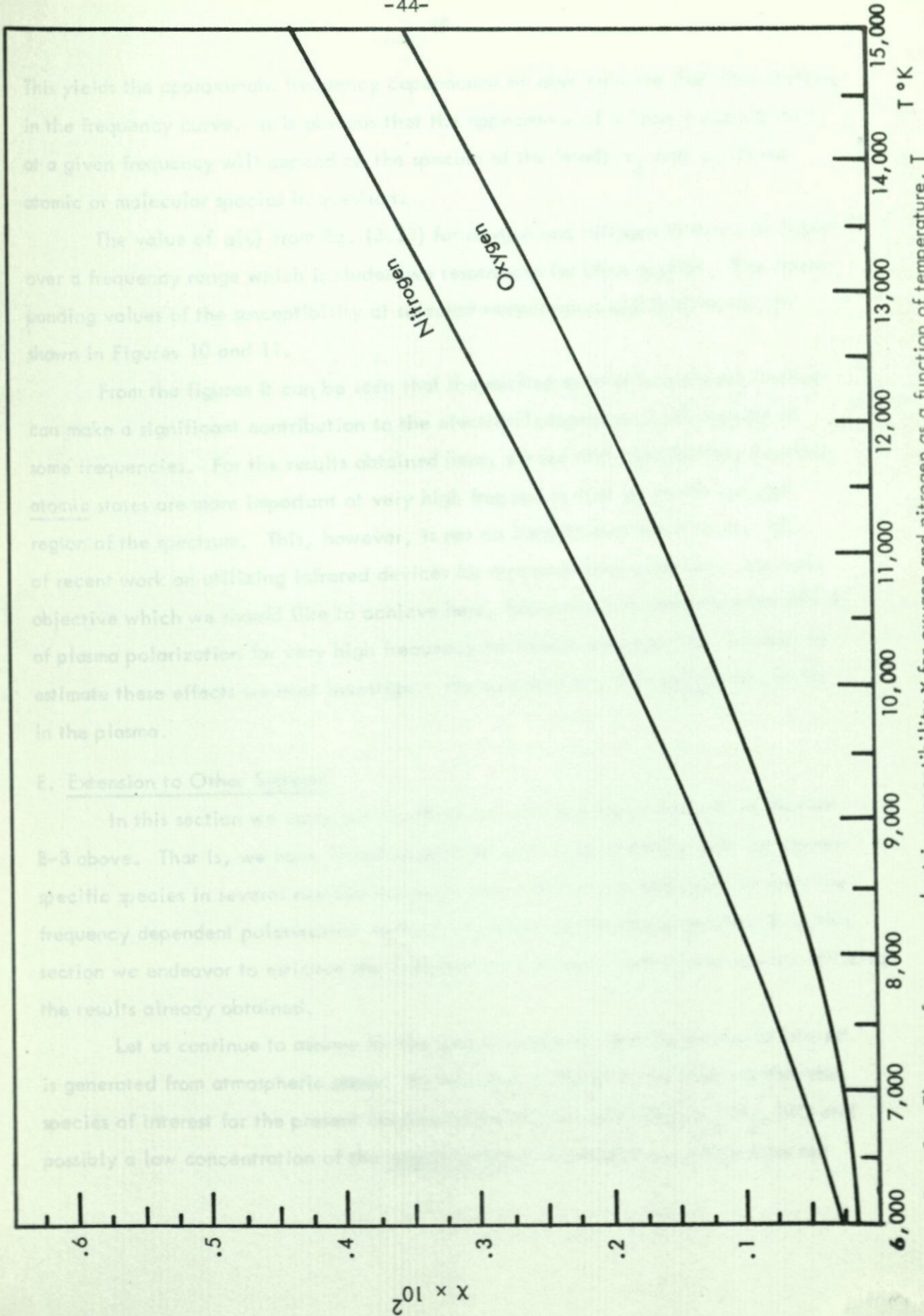


Fig. 8 The zero frequency electric susceptibility X for oxygen and nitrogen as a function of temperature, T

This yields the approximate frequency dependence of the zero frequency susceptibility in the frequency curve, it is obvious that the susceptibility of a given frequency will depend on the species of the body, atomic or molecular species in question.

The value of  $\alpha(\nu)$  from Eq. (12) for a given frequency over a frequency range which includes the resonance frequency corresponding values of the susceptibility of the species is shown in Figures 10 and 11.

From the figures it can be seen that the contribution of the atomic states to the susceptibility is significant at some frequencies. For the results obtained from the atomic states are more important at very high frequency region of the spectrum. This, however, is not the objective of recent work on utilizing infrared devices for the purpose of plasma polarization for very high frequency. To estimate these effects we must consider the contribution of the plasma.

### E. Extension to Other Species

In this section we carry out the same procedure as in B-3 above. That is, we now consider the contribution of specific species in several cases. In the case of the frequency dependent polarization, in this section we endeavor to reproduce the results already obtained.

Let us continue to assume that the plasma is generated from atmospheric air. The species of interest for the present is possibly a low concentration of the species



This yields the approximate frequency dependence of  $\alpha(\nu)$  near the first discontinuity in the frequency curve. It is obvious that the appearance of a large polarizability at a given frequency will depend on the spacing of the levels  $\epsilon_q$  and  $\epsilon_i$  for the atomic or molecular species in question.

The value of  $\alpha(\nu)$  from Eq. (3.31) for oxygen and nitrogen is shown in Figure 9 over a frequency range which includes two resonances for these species. The corresponding values of the susceptibility at selected temperatures and frequencies are shown in Figures 10 and 11.

From the figures it can be seen that the excited species in a plasma medium can make a significant contribution to the electrical properties of the medium at some frequencies. For the results obtained here, we see that contributions from the atomic states are more important at very high frequency; that is, in the infrared region of the spectrum. This, however, is not an insignificant result in the light of recent work on utilizing infrared devices for communication purposes. The main objective which we should like to achieve here, however, is to determine the effect of plasma polarization for very high frequency microwave propagation. In order to estimate these effects we must investigate the contributions from molecular species in the plasma.

#### E. Extension to Other Systems

In this section we carry out the third step in the program outlined in Section B-3 above. That is, we have 1) calculated the static polarizability  $\alpha(0)$  for certain specific species in several excited states; 2) these results have been used to infer the frequency dependent polarization vector in a plasma due to atomic species; 3) in this section we endeavor to estimate the contribution from more complicated systems utilizing the results already obtained.

Let us continue to assume for the sake of argument that the plasma of interest is generated from atmospheric gases. By referring to Figure 6, we then see that the species of interest for the present considerations are the molecules  $O_2$ ,  $N_2$ ,  $NO$  and possibly a low concentration of the negative ions of these species, which were not



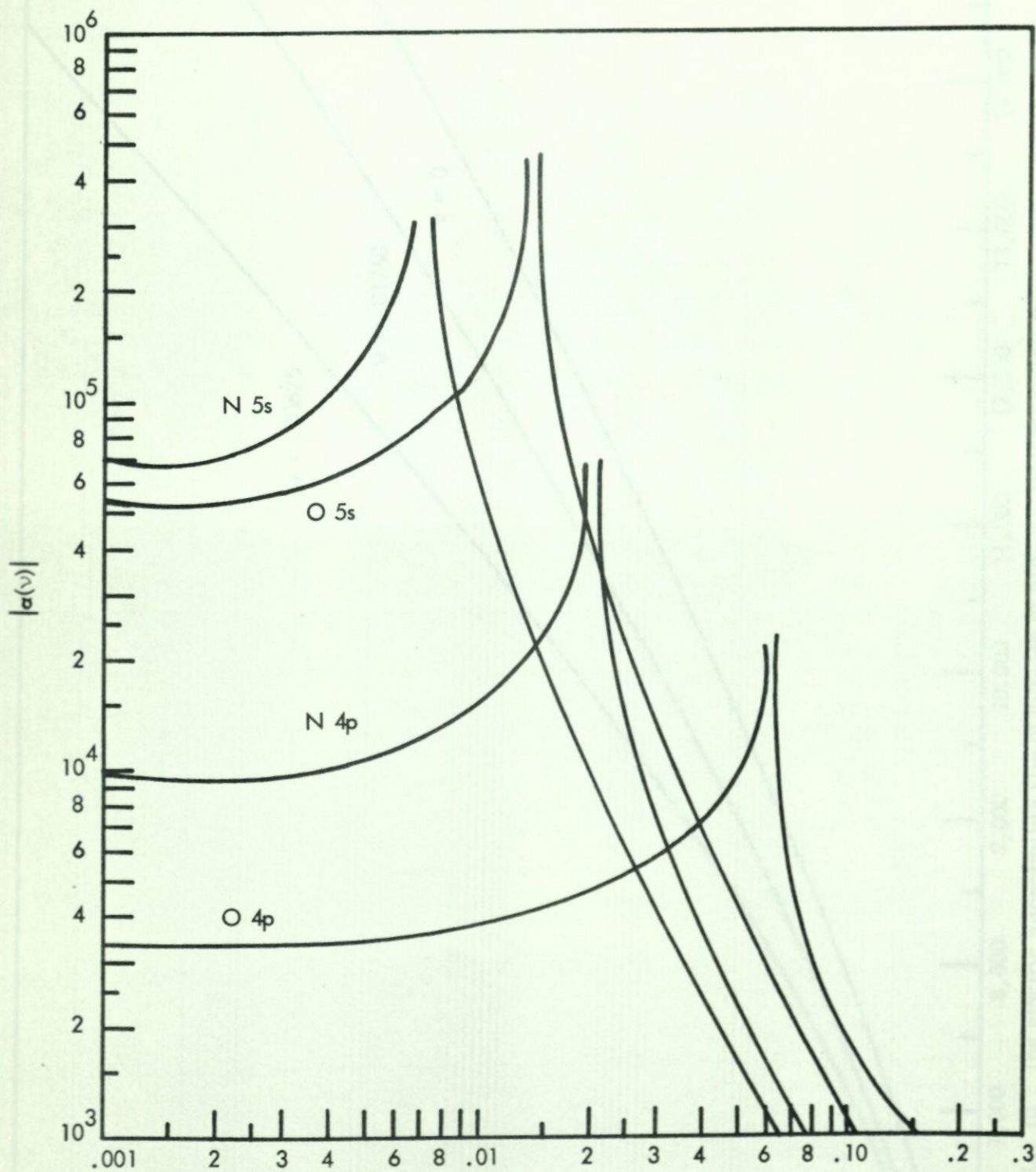


Fig. 9 Frequency dependent polarizability  $\alpha(\nu)$  as a function of frequency  $\nu$  for oxygen and nitrogen in two of their excited states

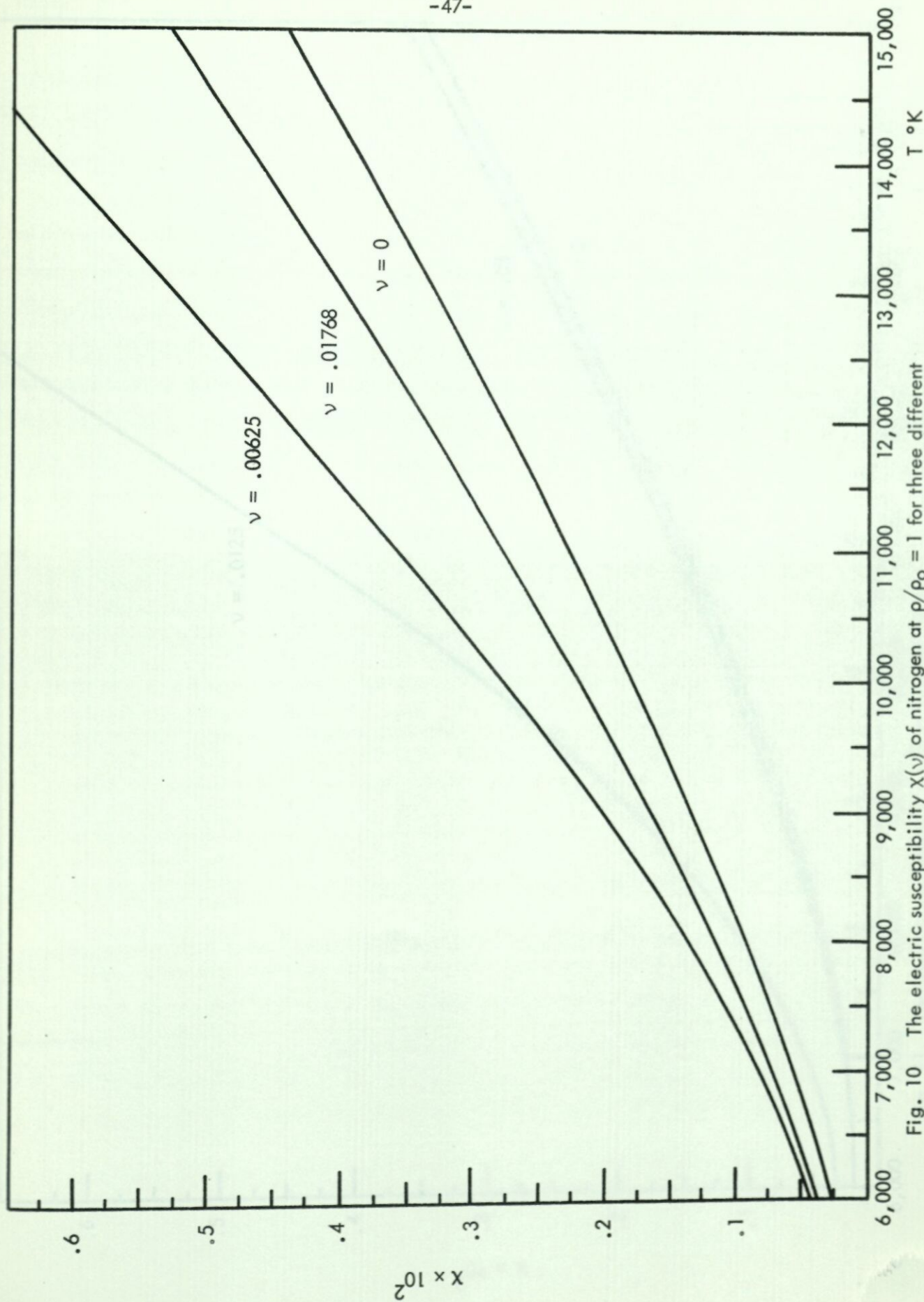


Fig. 10 The electric susceptibility  $X(\nu)$  of nitrogen at  $p/p_0 = 1$  for three different frequencies (atomic units of frequency  $\nu$ ; units of  $\nu = 2.067 \times 10^{16} \text{ sec}^{-1}$ )



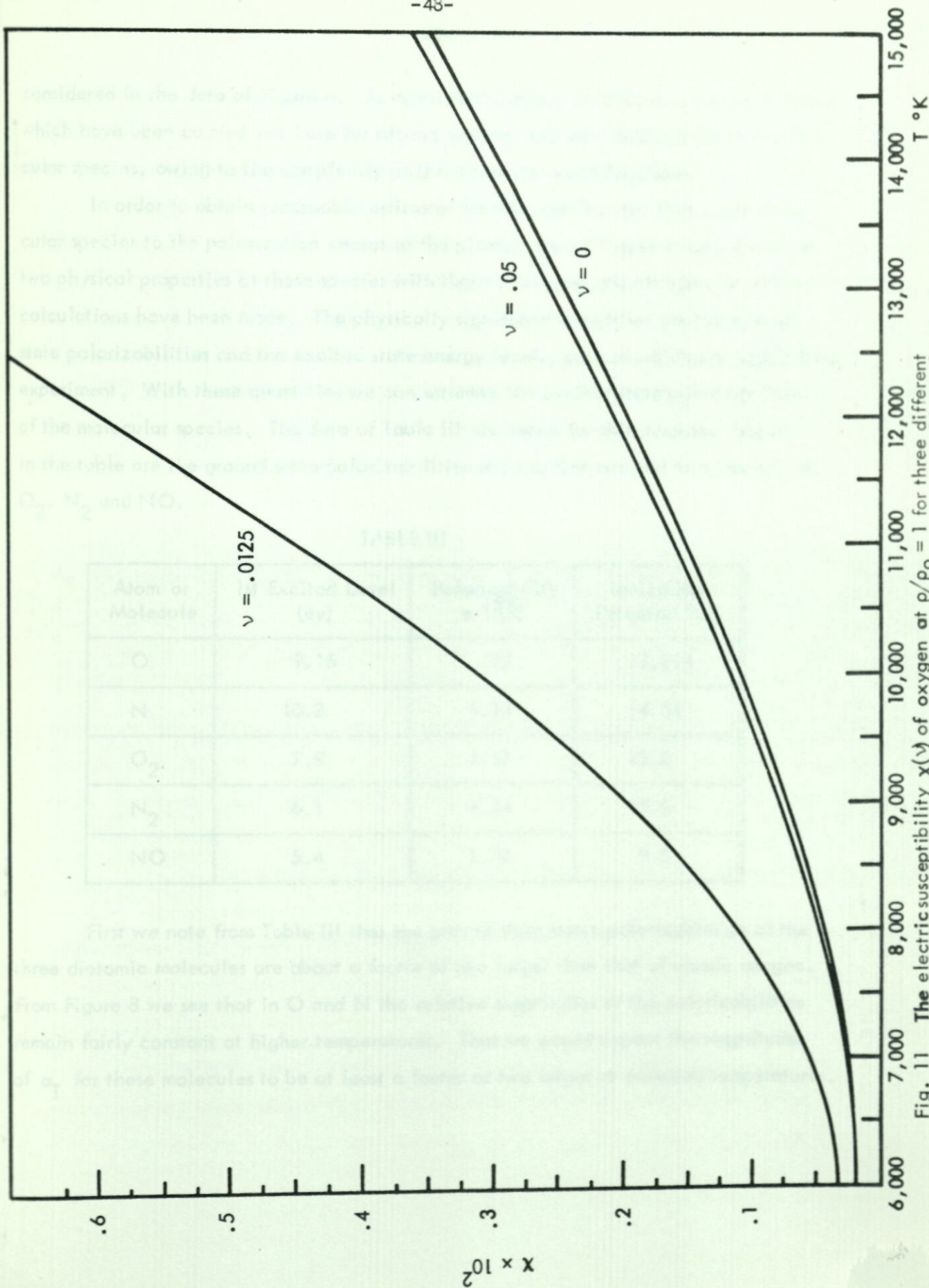


Fig. 11 The electric susceptibility  $\chi(\nu)$  of oxygen at  $p/p_0 = 1$  for three different frequencies (atomic units of frequency; units of  $\nu = 2.067 \times 10^{16} \text{ sec}^{-1}$ )

TABLE III

Atom or Molecule	Excited state (eV)	Polarizability (Å <sup>3</sup> )	Ionization potential (eV)
O	9.5	1.5	13.6
N	15.2	1.5	14.5
O <sub>2</sub>	7.4	2.0	12.1
N <sub>2</sub>	8.1	2.2	15.5
NO	5.4	1.5	9.0

First we note from Table III that the energy of the excited states for the three diatomic molecules are about a factor of two larger than that of atomic oxygen. From Figure 8 we see that in O and N the relative magnitudes of the polarizabilities remain fairly constant at higher temperatures. Thus we would expect the magnitudes of  $\alpha_1$  for these molecules to be at least a factor of two larger at higher temperatures.

considered in the data of Figure 6. As mentioned earlier, calculations similar to those which have been carried out here for atomic systems, are very difficult for the molecular species, owing to the complexity of the molecular wave functions.

In order to obtain reasonable estimates for the contributions from these molecular species to the polarization vector of the plasma, we will make a comparison of two physical properties of these species with those of oxygen and nitrogen for which calculations have been made. The physically significant quantities are the ground state polarizabilities and the excited state energy levels, both of which are known from experiment. With these quantities we can estimate the excited state polarizabilities of the molecular species. The data of Table III are useful for this purpose. Shown in the table are the ground state polarizabilities and the first excited level for O, N, O<sub>2</sub>, N<sub>2</sub> and NO.

TABLE III

Atom or Molecule	1st Excited Level (ev)	Polarizability $\alpha$ ( $\text{\AA}^3$ )	Ionization Potential (ev)
O	9.15	.77	13.614
N	10.3	1.13	14.54
O <sub>2</sub>	7.9	1.57	12.5
N <sub>2</sub>	6.1	1.74	15.5
NO	5.4	1.70	9.5

First we note from Table III that the ground state static polarizabilities of the three diatomic molecules are about a factor of two larger than that of atomic oxygen. From Figure 8 we see that in O and N the relative magnitudes of the polarizabilities remain fairly constant at higher temperatures. Thus we would expect the magnitudes of  $\alpha_T$  for these molecules to be at least a factor or two larger at elevated temperatures.



The second and more important point to note about the molecular species is the close spacing of energy levels in molecular states, and the lowering of the ionization potential in  $O_2$  and NO from that of O or N. Both of these characteristics tend to increase the polarizability of the excited states for these species to values which are greater than those of the atomic states of O and N. Taking NO for example, the first electronically excited state lies at 5.4 eV above the ground state compared to 9.15 and 10.3 eV for O and N respectively. Thus a very large polarization vector at a much lower temperature would result for these molecular states. Looking at the values in Table I and II for O and N, we could expect an increase in the polarizability of perhaps a factor of ten over that of N in the first excited state and even more for the closely lying levels of higher excitation.

One other factor which is even more significant in this comparison comes from an inspection of Eq. (3.31) for the frequency dependent polarizability  $\alpha(\nu)$ . We note that the frequency dependence is determined by the energy denominators  $(\epsilon_q - \epsilon_i)^2 - h^2 \nu^2$ . Thus the appearance of a resonance in the denominator depends on the energy spacing  $\epsilon_q - \epsilon_i$  which, in the case of molecular states, is much closer together. The result of this is that such resonances as shown in Figure 9 appear at much lower frequencies in molecular gases. Whereas, in the case of the atomic species O and N such effects were seen to lie in the infrared region of the spectrum, for excited molecular states the quantities  $\epsilon_q - \epsilon_i$  are much smaller and correspond in frequency to very long infrared and radio frequency regions.

Thus, we can expect that the value of  $\chi(\nu)$  from Eq. (3.30) in the temperature range of a typical atmospheric plasma will become significant for very high frequency propagation. The strong frequency dependence of  $\alpha(\nu)$  in a near resonance situation would make a tremendous difference in the dispersive properties of the plasma medium.

#### F. Conclusions

The results obtained in the present application are intended only to yield an estimate of the importance of the bound state electrons of a plasma medium to the electrical properties for high frequency propagation. The results for molecular species

are merely qualitative in nature as was our intention since a detailed calculation for very complicated systems would require a great deal of effort. The results obtained here do indicate, however, that such an effort would indeed be justified in view of the apparent magnitude of these effects on the problem of microwave propagation in plasma. Thus, it would seem that this problem which has received little attention from a theoretical or an experimental standpoint well deserves further investigation.

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