Chandra measurements of a complete sample of x-ray luminous clusters: the gas mass fraction

David Landry

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CHANDRA MEASUREMENTS OF A COMPLETE
SAMPLE OF X-RAY LUMINOUS CLUSTERS: THE GAS
MASS FRACTION

by

DAVID LANDRY

A DISSERTATION

Submitted in partial fulfillment of the requirements
for the degree of Doctor of Philosophy
in
The Department of Physics
to
The School of Graduate Studies
of
The University of Alabama in Huntsville

HUNTSVILLE, ALABAMA

2013
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David Landry

1/24/13
Dissertation Approval Form

Submitted by David Landry in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Physics and accepted on behalf of the Faculty of the School of Graduate Studies by the dissertation committee.

We, the undersigned members of the Graduate Faculty of The University of Alabama in Huntsville, certify that we have advised and/or supervised the candidate of the work described in this dissertation. We further certify that we have reviewed the dissertation manuscript and approve it in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Physics.

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Clusters of galaxies are the largest known bound systems in the Universe and are formed from the collapse of primordial density fluctuations. Since clusters are extremely large and massive, the cluster’s baryons and dark matter originated from approximately the same comoving volume, and thus it is believed that their ratio should be representative of the Universe. Current studies indicate that the cluster baryon fraction is lower than the cosmic baryon fraction as measured by the $\Omega_b/\Omega_M$ parameter.

To address this issue, the gas mass fraction, $f_{\text{gas}}$, is measured out to large radii for a complete sample of the 35 most luminous clusters from the Brightest Cluster Sample and its extension at redshifts $z = 0.15 - 0.30$ using Chandra X-ray data. This sample includes relaxed and unrelaxed clusters, and the data were analyzed using two different models for the gas density and temperature of the cluster. In accord with earlier studies, the gas mass fraction is shown to increase with radius, and thus the value of $f_{\text{gas}}$ depends on the radius used. However, the background of the surface brightness and temperature profiles often limits the radius out to which masses can be measured accurately. Therefore, measurements for the entire sample
were limited to $r_{500}$, the radius within which the mass density is 500 times the critical density. The average gas mass fraction for this sample of clusters at $r_{500}$ is measured to be $f_{\text{gas}} = 0.163 \pm 0.032$, which is in agreement with the cosmic baryon fraction ($\Omega_b/\Omega_M = 0.167 \pm 0.006$) at the 1σ level, after adding the stellar baryon fraction. In this analysis, it is shown that the most X-ray luminous clusters in the redshift range $z = 0.15 - 0.30$ have a gas mass fraction that is consistent with the cosmic value at $r_{500}$.
I would first and foremost like to thank God; without Him none of this would be possible. He has blessed me abundantly, and I hope that in everything I do, He will get the glory.

Next, I would like to thank my advisor, Dr. Bonamente. I simply cannot say enough great things about him. His constant patience and kindness while working with me is always refreshing and has always been appreciated. I could not pick a better person to work with, and will greatly miss every second I've spent with him. My only regret is that I wish I could have done a better job for him. He deserves more than a Nobel prize. It has truly been an honor and pleasure to work with Dr. Bonamente, and I only wish I could continue my career as his research assistant.

Also, I'm eternally grateful to my NASA advisor, Dr. Joy. He spent countless hours trying to help me write and was always more than willing to help in any way possible, even checking tedious calculations with me. I've tried to learn how to ask the right questions, and he's always tried to praise me, while still keeping me humble!

I would also like to thank Dr. Weisskopf, who has been my NASA advisor after Dr. Joy retired. He is always encouraging and has been completely supportive throughout my graduate career. I'm also grateful to Dr. Fix and Dr. Miller. Both have been extremely kind and helpful to me, even in the classroom.

I would not be where I am today if it wasn't for Dr. Golben. He always believed in me and has positively affected every student in his class. A better teacher
does not exist—he has changed my life and I will never forget him. I took every class available from him, and still wish I could take more.

Apart from all the wonderful teachers I have been blessed with, I would personally like to thank a few friends who have helped me tremendously in my schooling: Aaron Arthur, Jon Baker, Rob Freiderich, and Brian Fayock. I would not have survived graduate school without their help and support. They were always a joy to be around and I have many fond memories with them over the last four years. I will always miss being in class with them and the time we spent in the TA office.

I’m thankful for the opportunity to collaborate with Dr. Maughan and Paul Giles. They have both been extremely helpful since I began my research. Although it has been a long and time consuming process, it was always a pleasure working with them. I will miss our telecons. Also, I deeply appreciate all the help and guidance I received from Dr. Hasler when I started my research, and from Dr. Bulbul.

There are so many people who have helped me along the way, and I cannot name them all here, but I am grateful for their contributions. Lastly, I would like to thank all my friends and family for their love and support, especially my parents. They have always helped me and stuck by my side. I hope everything I accomplish in my life will make them proud.
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<tr>
<td>$\alpha$</td>
<td>index in density profile / cuspiness</td>
</tr>
<tr>
<td>$\beta$</td>
<td>index of core radius in density profile / slope of total mass profile / relativistic beta</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>index of scale radius in density profile / Lorentz factor</td>
</tr>
<tr>
<td>$\Gamma^\lambda_{\mu\nu}$</td>
<td>Christoffel symbol</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Kronecker delta</td>
</tr>
<tr>
<td>$\Delta$</td>
<td>overdensity / change in</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>X-ray emissivity / index of scale radius in density profile</td>
</tr>
<tr>
<td>$\varepsilon_0$</td>
<td>permittivity of free space</td>
</tr>
<tr>
<td>$\theta$</td>
<td>angular size</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>wavelength</td>
</tr>
<tr>
<td>$\Lambda$</td>
<td>cosmological constant</td>
</tr>
<tr>
<td>$\Lambda_{ee}$</td>
<td>band-averaged emissivity</td>
</tr>
<tr>
<td>$\mu$</td>
<td>total mean molecular weight</td>
</tr>
<tr>
<td>$\mu_e$</td>
<td>mean molecular weight of electrons</td>
</tr>
<tr>
<td>$\mu_i$</td>
<td>mean molecular weight of ions</td>
</tr>
<tr>
<td>$\nu$</td>
<td>frequency</td>
</tr>
<tr>
<td>Symbol</td>
<td>Meaning</td>
</tr>
<tr>
<td>--------</td>
<td>---------</td>
</tr>
<tr>
<td>$\tilde{\nu}$</td>
<td>wavenumber</td>
</tr>
<tr>
<td>$\xi$</td>
<td>cooling parameter</td>
</tr>
<tr>
<td>$\rho$</td>
<td>density</td>
</tr>
<tr>
<td>$\rho_{\text{crit}}$</td>
<td>critical density of the Universe</td>
</tr>
<tr>
<td>$\rho_i$</td>
<td>normalization of the total density</td>
</tr>
<tr>
<td>$\rho_{\text{tot}}$</td>
<td>total density</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>standard deviation</td>
</tr>
<tr>
<td>$\sigma_8$</td>
<td>normalization of power spectrum</td>
</tr>
<tr>
<td>$\tau_{\text{cool}}$</td>
<td>core cooling function</td>
</tr>
<tr>
<td>$\phi$</td>
<td>gravitational potential</td>
</tr>
<tr>
<td>$\chi$</td>
<td>comoving coordinate</td>
</tr>
<tr>
<td>$\psi_{n\ell m}$</td>
<td>hydrogen wave function</td>
</tr>
<tr>
<td>$\omega$</td>
<td>angular frequency</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>solid angle</td>
</tr>
<tr>
<td>$\Omega_\Lambda$</td>
<td>cosmological density parameter of dark energy</td>
</tr>
<tr>
<td>$\Omega_b$</td>
<td>cosmological density parameter of baryons</td>
</tr>
<tr>
<td>$\Omega_k$</td>
<td>cosmological density parameter of curvature</td>
</tr>
<tr>
<td>$\Omega_M$</td>
<td>cosmological density parameter of matter</td>
</tr>
<tr>
<td>$a$</td>
<td>index in temperature profile / acceleration</td>
</tr>
</tbody>
</table>
\( a_{\text{cool}} \)  cooling function index

\( \text{Å} \)  angström

\( A \)  element abundance (metallicity)

ACIS  AXAF CCD Imaging Spectrometer

APEC  Astrophysical Plasma Emission Code (XSPEC)

AXAF  Advanced X-ray Astrophysics Facility

\( b \)  index in temperature profile / impact parameter

\( B \)  magnetic field

BBN  Big Bang Nucleosynthesis

BCS  Brightest Cluster Sample

\( c \)  speed of light / index in temperature profile

CALDB  \textit{Chandra} Calibration Database

CCD  charge coupled device

CDM  Cold Dark Matter

CIAO  \textit{Chandra} Interactive Analysis of Observations

CMB  Cosmic Microwave Background

CXC  \textit{Chandra} X-ray Center

CXO  \textit{Chandra} X-ray Observatory

\( d \)  dipole moment
<table>
<thead>
<tr>
<th>Symbol</th>
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<tr>
<td>$dl$</td>
<td>line element</td>
</tr>
<tr>
<td>$ds$</td>
<td>metric</td>
</tr>
<tr>
<td>$D$</td>
<td>deuterium</td>
</tr>
<tr>
<td>$D$</td>
<td>distance</td>
</tr>
<tr>
<td>$D_A$</td>
<td>angular diameter distance</td>
</tr>
<tr>
<td>$D_M$</td>
<td>proper motion distance</td>
</tr>
<tr>
<td>d.o.f.</td>
<td>degrees of freedom</td>
</tr>
<tr>
<td>$e$</td>
<td>electron</td>
</tr>
<tr>
<td>$e$</td>
<td>electric charge</td>
</tr>
<tr>
<td>eBCS</td>
<td>extended Brightest Cluster Sample</td>
</tr>
<tr>
<td>eV</td>
<td>electron volt</td>
</tr>
<tr>
<td>$E$</td>
<td>energy / evolution function / electric field</td>
</tr>
<tr>
<td>$f_b$</td>
<td>total baryon fraction</td>
</tr>
<tr>
<td>$f_{gas}$</td>
<td>gas mass fraction</td>
</tr>
<tr>
<td>$f_{stars}$</td>
<td>stellar mass fraction</td>
</tr>
<tr>
<td>$F$</td>
<td>force / flux</td>
</tr>
<tr>
<td>Fe</td>
<td>iron</td>
</tr>
<tr>
<td>$g_{\mu\nu}$</td>
<td>metric tensor</td>
</tr>
<tr>
<td>$g_{\text{ff}}$</td>
<td>free-free Gaunt factor</td>
</tr>
</tbody>
</table>
\( G \) gravitational constant

\( G_{\mu\nu} \) Einstein tensor

\( h \) Planck’s constant / dimensionless Hubble constant

H hydrogen

\( H \) Hubble parameter

He helium

HETG High Energy Transmission Grating

HRC High Resolution Camera

HRMA High Resolution Mirror Assembly

i ion

ICL intracluster light

ICM intracluster medium

\( k \) curvature parameter / Boltzmann’s constant

\( k_B \) Boltzmann’s constant

K degrees Kelvin

keV kiloelectron volt

km kilometer

l line of sight

\( \ell \) azimuthal quantum number
\( L_X \) X-ray luminosity

LETG Low Energy Transmission Grating

Li lithium

\( m_\ell \) magnetic quantum number

\( m_p \) proton mass

\( M_\odot \) solar mass

\( M_{\text{gas}} \) gass mass

\( M_{\text{tot}} \) total mass

MCMC Monte Carlo Markov chain

Mpc megaparsec

\( n \) number density / polytropic index / principal quantum number

\( n_e \) number density of electrons

\( n_{e0} \) normalized density

\( n_i \) number density of ions

\( N \) number of particles

\( N_{H} \) neutral hydrogen

NFW Navarro Frenk White

p proton

\( P \) gas pressure / power
$P_e$  electron pressure

$P_{e0}$  normalized pressure

$q$  charge

$r$  radius

$r_\Delta$  overdensity radius

$r_c$  core radius

$r_{cool}$  cooling radius

$r_s$  scale radius

$R$  Rydberg constant

$R_{\mu\nu}$  Ricci tensor

$\mathcal{R}$  Ricci scalar

$ROSAT$  $Röntgensatellit$

$s$  seconds

$S_X$  X-ray surface brightness

$\text{Stdev}$  standard deviation

$SZ$  Sunyaev-Zel’dovich

$t$  time

$t_{cool}$  cooling time

$T$  temperature
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tr>
<td>$T_{\mu \nu}$</td>
<td>energy-momentum tensor</td>
</tr>
<tr>
<td>$T_0$</td>
<td>normalized temperature</td>
</tr>
<tr>
<td>$U_\mu$</td>
<td>four-velocity</td>
</tr>
<tr>
<td>$v$</td>
<td>velocity</td>
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<tr>
<td>$V$</td>
<td>volume</td>
</tr>
<tr>
<td>Var</td>
<td>variance</td>
</tr>
<tr>
<td>$w$</td>
<td>centroid shift / equation of state of dark energy</td>
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<tr>
<td>$W$</td>
<td>total energy</td>
</tr>
<tr>
<td>WMAP</td>
<td>Wilkinson Microwave Anisotropy Probe</td>
</tr>
<tr>
<td>XSPEC</td>
<td>XANADU spectral software package</td>
</tr>
<tr>
<td>$z$</td>
<td>redshift</td>
</tr>
<tr>
<td>$Z$</td>
<td>abundance / charge number / atomic number</td>
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</table>
For My loving parents
CHAPTER 1

INTRODUCTION

1.1 Basic Properties of Clusters of Galaxies

Galaxy clusters contain hundreds of galaxies and are a few Mpc in diameter. These massive systems of galaxies are formed from merger events due to the collapse of primordial density fluctuations in the hierarchical model of structure formation (Press & Schechter, 1974; White & Rees, 1978). Clusters are composed of diffuse intracluster light (ICL), galaxies, a hot intracluster medium (ICM) in the form of ionized gas at \( \sim 10^8 \) K, and dark matter, for a total mass of \( 10^{14} - 10^{15} M_\odot \). Galaxies and stars comprise \( \sim 2\% \) of the total mass, the hot gas makes up \( \sim 10\% \), and dark matter accounts for the remaining mass (Grego et al., 2001; Allen et al., 2002; Lin et al., 2003; Allen et al., 2004; LaRoque et al., 2006; Gonzalez et al., 2007; Giodini et al., 2009). Virgo is the closest rich cluster with Coma being the closest regular cluster. Groups of galaxies have fewer galaxies than clusters, and are also less massive than clusters of galaxies. The Milky Way is part of the Local Group, which hosts the Andromeda Galaxy and a number of smaller galaxies, and the Local Group is part of the Virgo Supercluster.
1.2 Observations of Clusters

The most extensive catalogs of clusters of galaxies were developed by Abell (1958) and Zwicky et al. (1961–1968). Both of these catalogs used the National Geographic Society-Palomar Observatory Sky Survey to distinguish clusters as enhancements in the number density of galaxies (Minkowski & Abell, 1963). The Abell catalog contains 2712 clusters based on the number of galaxies within a given magnitude range, radius, and redshift. The Zwicky catalog using fewer constraints comprises of over 30,000 clusters.

Currently, galaxy clusters are observed in optical, X-ray, and microwave wavelengths, and are often used in combination to better understand the physics in clusters. Optical observations measure the luminosity, velocity dispersion, and gravitational lensing of the galaxies. Gravitational lensing is produced by light from the galaxies being distorted by the gravitational potential of the cluster. These optical observations can provide measurements of the baryon fraction in the stars and galaxies (e.g., Lin et al., 2003; Gonzalez et al., 2007; Giodini et al., 2009) and the total cluster mass (Kaiser & Squires, 1993; Hoekstra et al., 1998; Umetsu et al., 1999; Bartelmann & Schneider, 2001). Sunyaev and Zel’dovich predicted that the hot gas in the ICM would cause distortions in the cosmic microwave background (CMB) (Sunyaev & Zeldovich, 1970, 1972). The Sunyaev-Zel’dovich (SZ) effect is caused by electrons distorting the CMB via inverse Compton scattering (see Birkinshaw 1999; Carlstrom et al. 2002 and references therein for a review). Recently, SZ measurements
and joint X-ray/SZ analyses are also being used for cosmology (e.g., Benson et al., 2011; Marrone et al., 2011; Hasler et al., 2012).

The first source of X-ray emission outside of our galaxy to be detected was M87 in the Virgo cluster (Byram et al., 1966). Sources in the directions of the Coma and Perseus clusters were detected after that (e.g., Fritz et al., 1971; Gursky et al., 1971a,b), and the launch of the Uhuru X-ray satellite in 1972 verified that clusters of galaxies were X-ray sources (Giacconi et al., 1972). The emission mechanism in clusters of galaxies was confirmed to be of thermal origin with the detection of Fe emission lines from the Perseus, Coma, and Virgo clusters (Mitchell et al., 1976; Serlemitsos et al., 1977). Clusters of galaxies are extremely luminous X-ray sources, with luminosities ranging from $L_X \sim 10^{43} - 10^{45}$ erg s$^{-1}$. The hot gas in the ICM is a powerful emitter of X-ray radiation via thermal bremsstrahlung emission, and the Chandra X-ray Observatory is perfectly designed to measure the X-ray surface brightness and plasma temperature. A more detailed discussion on X-ray emission is given in Section 3.1. Using the hot gas as tracer of the gravitational potential, the mass of the gas and the total mass of the cluster can be determined from the temperature and density profiles, assuming hydrostatic equilibrium. Derivations of the equations used for measuring cluster masses are provided in Chapter 3.

1.3 Chandra X-ray Observatory

The data used in this analysis was taken from the Chandra X-ray Observatory (CXO), one of NASA’s Great Observatories. The Advanced X-ray Astrophysics Facility (AXAF) was the original name, but NASA changed it to Chandra in 1998 after
the late Dr. Chandrasekhar, who was awarded the Nobel prize in 1983. *Chandra*, the most sophisticated X-ray observatory, was launched on July 23, 1999 by the Space Shuttle Columbia. The telescope was partly built at NASA’s Marshall Space Flight Center, where *Chandra* is managed, and home of the CXO project scientist, Dr. Weissskopf. The *Chandra* X-ray Center (CXC) is located at the Smithsonian Astrophysical Observatory (SAO) in Cambridge, Massachusetts, which operates the satellite and processes the data. This spacecraft is designed for high resolution imaging and spectroscopy, and consists of a High Resolution Mirror Assembly (HRMA), two focal plane instruments, and two transmission gratings. The mirror assembly is made up of four nested reflecting surfaces which have small reflection angles and is coated with iridium for high X-ray reflectivity and chemical stability (Weisskopf et al., 2000, 2002). The HRMA is used to focus X-rays onto the instruments. The two transmission grating spectrometers are made of two sets of gold gratings—one for low energy and one for high energy. The Low Energy Transmission Grating (LETG) is designed for energies below 2 keV, and has the highest spectral resolving power ($E/\Delta E > 1000$) for very low energies. The High Energy Transmission Grating (HETG) is designed for energies in the range $0.4 – 10$ keV. The spectral resolving power of HETG varies from $E/\Delta E \sim 800$ at 1.5 keV to $E/\Delta E \sim 200$ at 6 keV. The two focal plane instruments are the High Resolution Camera (HRC) and the Advanced CCD Imaging Spectrometer (ACIS). The HRC uses microchannel plate imaging detectors offering high spatial ($< 0.5''$) and temporal (16 ms) resolution. The ACIS instrument uses $1024 \times 1024$ pixel CCDs (charge coupled device) to convert the X-rays into number of electrons proportional to the photon energy, which allows the position of the X-ray to
be determined (Schwartz, 2004). ACIS-I is arranged in a $2 \times 2$ array for imaging wide fields ($16' \times 16'$) and ACIS-S is arranged in a $1 \times 6$ array primarily used as a readout of the HETG (see Figure 1.1).

1.4 Cluster Masses

Masses of galaxy clusters are important quantities because they can constrain cosmological parameters such as the normalization of the power spectrum, $\sigma_8$, the equation of state of dark energy, $w(z)$, the cosmological density of matter, $\Omega_M$, and provide information about the structure formation in the Universe (Allen et al., 2004; Kravtsov et al., 2006; Vikhlinin et al., 2009; Mantz et al., 2010). The cluster mass
function which is the number of clusters at a given mass and redshift can also be used in cosmology. Because cluster masses obtained through X-ray measurements depend on cosmology, incorrect cosmologies can be ruled out as seen at high redshift in Figure 1.2. The calculation of cluster masses is provided in greater detail in Section 3.3.

1.5 Measurements of Cosmological Parameters

Recent studies in cosmology not only predict a set of parameters which describe the Universe, but can precisely measure these parameters. Two cosmological parameters which are important for this analysis are briefly discussed: $\Omega_b$ and $\Omega_M$,
the cosmological density of baryons (e.g., protons and neutrons) and matter (baryonic and dark matter), respectively.

In Big Bang cosmology, the Universe was once very hot and dense. In this high density and temperature scenario, baryons and photons were coupled together due to Compton scattering (Dodelson, 2003). This is often referred to as a photon-baryon fluid. When this photon-baryon fluid existed, atoms could not form due to the high temperature of the photons. Any atom that tried to form would be ionized by energetic photons. Small mass density inhomogeneities would cause this fluid to oscillate, thereby creating cooler and hotter regions. Eventually the Universe expanded and cooled enough for neutral atoms to form. This is known as recombination (of electrons and protons), or decoupling, where photons decoupled from baryons. At this point, photons could travel freely. Observed today, these photons from the early Universe make up the Cosmic Microwave Background (CMB).

Once photons decoupled from matter, the oscillations were frozen onto the CMB. Therefore, the CMB should contain regions of higher and lower than the mean CMB temperature corresponding to the oscillation peaks, giving rise to anisotropies in the CMB. Measuring the angular scales of the anisotropies in the power spectrum allows cosmological parameters to be determined, for example, $\Omega_M$ (Dodelson, 2003; Maoz, 2007; Weinberg, 2008; Komatsu et al., 2011). This is exactly how the Wilkinson Microwave Anisotropy Probe (WMAP) determines $\Omega_b/\Omega_M$ (Komatsu et al., 2011).

Another way to measure the cosmological density of baryons is from Big Bang Nucleosynthesis (BBN), or the formation of elements. Light elements could begin to form once the Universe cooled sufficiently. The primordial abundance of these
elements depends on the baryon density, specifically, the baryon-to-photon density (Burles & Tytler, 1998; Tytler et al., 2000; Burles et al., 2001; Dodelson, 2003). Since the ratio is conserved and the photon density is known from CMB measurements (e.g., Fixsen et al., 1996; Bennett et al., 2003), an accurate determination of the abundance of light elements ($D$, $^3He$, $^4He$, and $^7Li$), can provide estimates of the baryon density, and thus $\Omega_b$ (Burles & Tytler, 1998; Tytler et al., 2000; Burles et al., 2001). The abundance of deuterium, $D$, gives an extremely accurate measurement of the baryon density (Burles & Tytler, 1998; Dodelson, 2003). Since D is easily destroyed by stellar processing, BBN is the only known source of D (e.g., Burles & Tytler, 1998). A common way of measuring the abundance of deuterium is from the absorption spectra of high redshift quasars (Burles et al., 2001; Dodelson, 2003).

1.6 Baryons in Clusters

The gas mass fraction, $f_{\text{gas}}$, is defined as the ratio of the gas mass, $M_{\text{gas}}$, to total mass (baryonic and dark matter), $M_{\text{tot}}$, i.e.,

$$f_{\text{gas}} = \frac{M_{\text{gas}}}{M_{\text{tot}}}. \quad (1.1)$$

The gas mass fraction is believed to be constant as a function of redshift, as seen in observations and simulations (Eke et al., 1998; Ettori et al., 2003; Allen et al., 2004; Crain et al., 2007; Allen et al., 2008; Ettori et al., 2009). Figure 1.3 shows $f_{\text{gas}}(z)$ as reported in Allen et al. (2008) for two Cold Dark Matter (CDM) cosmologies. A $\Lambda$CDM cosmology (shown in blue in Figure 1.3), currently the favored model of the
Figure 1.3: Plot of the gas mass fraction as a function of redshift. The left panel, shown in blue, corresponds to $f_{\text{gas}}$ measurements for a $\Lambda$CDM cosmology ($h = 0.7$, $\Omega_M = 0.3$, and $\Omega_\Lambda = 0.7$). The right panel (red) is the gas mass fraction assuming a SCDM cosmology, where $h = 0.5$, $\Omega_M = 1.0$, and $\Omega_\Lambda = 0.0$. The blue dashed line is the best-fit line consistent with a constant $f_{\text{gas}}(z)$. Data reproduced from Allen et al. (2008).

Figure 1.4: Constraints on cosmological parameters $\Omega_M$ and $\Omega_\Lambda$ found by Allen et al. (2008). This shows the 1 and 2$\sigma$ contours from measuring $f_{\text{gas}}$ with Chandra data. CMB and SNIa data were obtained from Davis et al. (2007). Figure reproduced from Allen et al. (2008).
Universe, corresponds to $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$, $\Omega_M = 0.3$, and $\Omega_\Lambda = 0.7$, and is consistent with a constant $f_{\text{gas}}(z)$. The Standard CDM (SCDM) cosmology, shown in red in Figure 1.3, is defined by $H_0 = 50 \text{ km s}^{-1} \text{ Mpc}^{-1}$, $\Omega_M = 1.0$, and $\Omega_\Lambda = 0.0$, and is clearly not consistent with the expected gas mass fraction being constant with redshift.

The gas mass fraction can be used to constrain the cosmological density of matter, $\Omega_M$, and the dark energy equation of state, $w(z)$ (e.g., Ettori et al., 2003; Allen et al., 2004, 2008; Ettori et al., 2009, 2011) by assuming that $f_{\text{gas}}(z)$ is constant (as seen in Figure 1.3). Most of the cluster’s baryons, i.e., regular matter, is in the form of hot gas in the ICM. However, a small amount $1 - 2\%$ of baryons are in stars and galaxies (Lin et al., 2003; Gonzalez et al., 2007; Giodini et al., 2009). Therefore, the total cluster baryon fraction, $f_b$, is a combination of $f_{\text{gas}}$ and $f_{\text{stars}}$, and can be written as

$$f_b = f_{\text{gas}} + f_{\text{stars}},$$  \hspace{1cm} \text{(1.2)}

where $f_{\text{stars}}$ is the baryonic fraction of stars and galaxies. Since clusters are the largest gravitationally bound structures in the Universe, it is believed that the cluster baryon fraction should be representative of the cosmic baryon fraction. Current studies indicate that the cluster baryon fraction is lower than the cosmic baryon fraction of the Universe as measured by the $\Omega_b/\Omega_M$ parameter, where $\Omega_b$ is the cosmological density parameter of baryons and $\Omega_M$ is the cosmological density of all matter (Vikhlinin et al., 2006; Afshordi et al., 2007; Arnaud et al., 2007; Giodini et al., 2009; Sun et al., 2009; Umetsu et al., 2009; Rasheed et al., 2010; Ettori et al., 2011; Komatsu et al.,
This has raised the question about these “missing baryons” (Rasheed et al., 2010). Therefore, the main goal of this analysis is to address this issue by measuring the gas mass fraction out to large radii to determine whether the cluster baryon fraction agrees with the cosmic baryon fraction, $\Omega_b/\Omega_M = 0.167 \pm 0.006$ as measured by the WMAP 7 year data (Komatsu et al., 2011).

1.7 Goals and Structure of Dissertation

The purpose of this analysis is to measure the gas mass fraction for a complete sample of 35 luminous clusters using Chandra X-ray data. The baryon fraction in clusters falls short of the cosmic value which is referred to as the “missing baryon” problem (Ettori et al. 2003, 2009; Rasheed et al. 2010 and references therein). The aim of this work is to measure the total baryon fraction to resolve this “missing baryon” problem in clusters of galaxies.

The structure of this dissertation is outlined below. In Chapter 2 the sample of clusters and selection function is discussed. The Chandra X-ray data reduction method is also presented. Chapter 3 discusses the X-ray emission processes in clusters of galaxies. The modeling of the Chandra data using the Vikhlinin et al. (2006) and Bulbul et al. (2010) models is described with the appropriate mass calculations shown in detail. The results of the measurement of the cluster gas mass fraction for this study are presented in Chapter 4. The criteria for classification of relaxed and unrelaxed clusters is presented, and the measurement of both subsamples is reported. Systematic effects on the measurement of $f_{\text{gas}}$ is then discussed in Chapter 5. The summary of results and conclusion is in Chapter 6.
CHAPTER 2

CLUSTER SAMPLE AND DATA REDUCTION METHODS

2.1 Cluster Sample

In order to accurately use cluster masses which represent the population of galaxy clusters, a large sample with a well defined selection function must be used. For this purpose, the sample used in this study consists of Chandra X-ray observations of all clusters from the Brightest Cluster Sample (BCS) and the low flux extension (eBCS) with X-ray luminosities $L_{X,0.1-2.4\text{ keV}} \geq 5 \times 10^{44} \text{ erg s}^{-1}$ in the redshift range $0.15 - 0.30$ (see Figure 2.1 and Table 2.1) (Ebeling et al., 1998, 2000; Dahle, 2006). These samples were selected from the Röntgensatellit (ROSAT) All-Sky Survey (RASS: Voges et al. 1992; Trümper 1993) and together make up the largest statistically defined sample in the northern hemisphere consisting of 203 BCS clusters and 107 eBCS clusters (Ebeling et al., 1998, 2000). There are several clusters that meet the luminosity and redshift requirements for the sample used in this study; however, Abell 689, one of the brightest clusters in the sample, is subject to point source contamination due to a BL Lacertae object at its center and is therefore not included in this sample (Giles et al., 2012). Thus, there are 35 clusters in this sample (see Table 2.1). These clusters are estimated to have masses of $M_{180} \sim 5 \times 10^{14}$
Figure 2.1: Sample of clusters taken from the BCS and eBCS with X-ray luminosities $L_{X,0.1−2.4\,\text{keV}} \geq 5 \times 10^{44}\,\text{erg s}^{-1}$ in the redshift range $0.15−0.30$. The red circles correspond to clusters from the original BCS and the blue circles are clusters from the low flux extension, eBCS.

$h^{-1}\,M_{\odot}$, making them the most massive in the BCS sample (Dahle, 2006). It is about 90% complete, whereas previous studies often used incomplete, ad hoc samples (Ebeling et al., 1998, 2000; Grego et al., 2001; Dahle, 2006; LaRoque et al., 2006). Also, this sample contains both relaxed and unrelaxed clusters, and is unbiased with respect to cluster morphology; making it possible to study the systematic effects in the measurement of masses in unrelaxed clusters. In addition to over 2 Ms of archival Chandra data, all of the clusters in this sample have SZ effect data and weak lensing data available making this sample suitable for additional studies (Dahle, 2006).
### Table 2.1: Sample of Galaxy Clusters

<table>
<thead>
<tr>
<th>Cluster</th>
<th>$z$</th>
<th>$D_A$ (Mpc)</th>
<th>$N_H$ (10^{20} \text{ cm}^{-2})</th>
<th>obsID</th>
<th>Exposure (ks)</th>
<th>Dynamical State</th>
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<td>0.1971</td>
<td>673.9</td>
<td>5.36</td>
<td>3233</td>
<td>43.6</td>
<td>Unrelaxed</td>
</tr>
<tr>
<td>Abell 1423</td>
<td>0.2130</td>
<td>716.1</td>
<td>1.81</td>
<td>538</td>
<td>35.1</td>
<td>Unrelaxed</td>
</tr>
<tr>
<td>Abell 1576</td>
<td>0.2790</td>
<td>876.1</td>
<td>1.08</td>
<td>7938</td>
<td>15.0</td>
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<tr>
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<td>749.5</td>
<td>1.04</td>
<td>11725</td>
<td>19.6</td>
<td>Unrelaxed</td>
</tr>
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<td>1.03</td>
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<td>10463</td>
<td>39.4</td>
<td>Relaxed</td>
</tr>
<tr>
<td>Zwicky 3146</td>
<td>0.2906</td>
<td>901.8</td>
<td>2.46</td>
<td>909</td>
<td>81.3</td>
<td>Relaxed</td>
</tr>
<tr>
<td>Zwicky 5247</td>
<td>0.2300</td>
<td>759.6</td>
<td>1.61</td>
<td>11727</td>
<td>19.2</td>
<td>Unrelaxed</td>
</tr>
<tr>
<td>Zwicky 5768</td>
<td>0.2660</td>
<td>846.4</td>
<td>1.49</td>
<td>7898</td>
<td>10.4</td>
<td>Unrelaxed</td>
</tr>
<tr>
<td>Zwicky 7215</td>
<td>0.2897</td>
<td>899.9</td>
<td>1.40</td>
<td>7899</td>
<td>13.0</td>
<td>Unrelaxed</td>
</tr>
</tbody>
</table>
2.2 *Chandra* X-ray Data Reduction

All of the data reduction was done using software versions of Chandra Interactive Analysis of Observations (CIAO) 4.2 and Chandra Calibration Database (CALDB) 4.3.1. As part of the data reduction, corrections were made for afterglows, charge transfer inefficiencies, bad pixels, and solar flares. Afterglows are caused from cosmic rays building up charge on the CCD. Bad pixels take into account hot pixels and afterglow events. Charge transfer ineffiency is due to proton damage to the ACIS chips and reduces the energy resolution. The variability of solar flares can cause periods of high background which need to be filtered out. A common way of removing these periods of solar flares is to follow the lightcurve filtering method of Markevitch et al. (2003). Before the lightcurve can be created, point sources of high and variable emission need to be excluded. The lightcurve is then created over the energy range 0.3 – 12.0 keV for ACIS-I observations and 2.5 – 6.0 keV for ACIS-S observations on a selected region of the *Chandra* CCD’s used as the local background. An example of a lightcurve can be seen in Figure 2.2. This lightcurve was filtered with an iterative algorithm (*deflare* command in CIAO) that was used to remove time intervals outside the $3\sigma$ range of the mean. The solid line shows the mean count rate for the observation and the red boxes show the regions that are filtered out. Most observations were taken in VFAINT (very faint) mode which uses a larger pixel event island to identify good events from bad events (e.g., cosmic rays). VFAINT cleaning was applied to both the cluster and blank-sky observations, when possible.
Figure 2.2: An example lightcurve for observation 9371 of Zwicky 3146 showing excluded time intervals due to solar flares (red boxes). Lightcurves are created over the whole energy range, 0.3 – 12.0 keV, using the local background region, 450 – 900″ in this case. The mean count rate is 0.596 s⁻¹ (dashed line) and the filtered exposure time is 36.3 ks.

Images were created in the 0.7 – 7.0 keV band to obtain the surface brightness as a function of radius. This energy band was chosen since the effective area is highest in this region (see Figure 2.3). The most crucial part of accurately analyzing the X-ray data of diffuse sources such as galaxy clusters is the background subtraction. For the purpose of background subtraction, ACIS blank-sky composite event files and measurements of the local background taken from source-free regions of the cluster observation were used (Markevitch et al., 2003). Since all the clusters in the sample have redshifts between 0.15 – 0.30, the Chandra ACIS-I detector leaves a large region to obtain a local background (see Figure 2.5). In Figure 2.5, $r_{500}$, the radius within which the matter density is 500 times the critical density, is about 270″ and the region used as the local background is from 450 – 900″. In nearly every cluster the
local background was chosen as an outer annulus beyond $r_{200}$ in which the surface brightness was approximately constant.

The local background of the cluster observation may differ from the ACIS blank-sky composites, since these observations were done at different times and positions in the sky. The blank-sky spectra has both spatial dependance (e.g., Snowden et al., 1997) and time-variability (e.g., Takei et al., 2008). Although the background flux may vary with time, the ratio of the flux within the $2 - 7$ keV and $9.5 - 12$ keV bands is constant in time, according to Hickox & Markevitch (2006). Therefore, the blank-sky spectrum can be rescaled by the ratio of the count rates in the $9.5 - 12$ keV band of cluster and blank-sky observations to obtain a clean subtraction of the background in the spectral region of interest, which is $0.7 - 7.0$ keV (Hickox & Markevitch, 2006). After subtracting the blank-sky background, residuals may still be present in the soft $0.7 - 2$ keV band. These soft residuals may be due to Galactic and extragalactic emission, as well as residual solar flares that were not removed by
Figure 2.4: Surface brightness profile of Zwicky 3146 in units of counts cm$^{-2}$ arcmin$^{-2}$ showing cluster emission consistent with a constant value beyond 450\arcsec.

Figure 2.5: Left: local background region used for ACIS-I observation 9371 of Zwicky 3146. The annulus used for the local background is from 450 – 900\arcsec. The dashed line indicates $r_{500}$ and is approximately 270\arcsec. Right: the blank-sky image used showing the same local background region selected.
lightcurve filtering and have spatial dependence (Snowden et al., 1997). This would also give rise to differences between the local background and the blank-sky observations. Solar flares may cause time variability on a timescale of hours, and therefore, greatly affect the observations (Takei et al., 2008). When present, the soft residual spectrum is then fit with a power law and a plasma emission model, and this model of the soft residuals is taken into account in the cluster spectra, as done in Bulbul et al. (2010) and Hasler et al. (2012). The top panel of Figure 2.6 shows an example of soft residuals below 2 keV for observation 6880 of Abell 1835. These soft residuals are fit with an APEC plasma emission model, and then this model is applied to the inner regions of the cluster using XSPEC software to obtain a temperature profile (Smith et al., 2001). Multiple observations were reduced individually to apply the correct calibration to each dataset. In these cases, the cluster surface brightness profile is obtained from merged images, and the temperature profile from fitting spectra from different observations simultaneously. The resultant temperature profile from analyzing observations 6880, 6881, and 7370 of Abell 1835 is shown in the bottom panel of Figure 2.6.
Figure 2.6: Top: spectrum of the local background (beyond $\sim 750''$) for observation 6880 of Abell 1835. This shows the soft residuals (below 2 keV) fit with an unabsorbed thermal plasma model with $kT \sim 0.25$ keV. Bottom: temperature profile of Abell 1835 obtained from analyzing multiple observations and fitting the spectra simultaneously.
CHAPTER 3

MODELS OF THE CHANDRA DATA

This research depends heavily on accurate mass measurements of galaxy clusters using data from Chandra. One method used to determine the mass of a cluster is by using the hot gas as tracer of gravitational potential, assuming hydrostatic equilibrium between the gas and dark matter (Sarazin, 1988). The derivation of the equation for hydrostatic equilibrium, mass equations, and modeling of the X-ray data are given below.

3.1 Radiative Processes

There are several processes responsible for radiative processes in galaxy clusters. The X-ray emitting plasma is composed of \( \sim 90\% \) H, \( \sim 10\% \) He, and small amounts of higher-Z elements (\( \approx 0.3 Z_\odot \)). The main emission process for a gas at temperature \( T \sim 10^8 \) K and density \( n \sim 10^{-3} \) cm\(^{-3}\) is thermal bremsstrahlung, or free-free emission. Bremsstrahlung is German for “braking radiation,” and is caused when a charged particle is accelerated by the presence of another charged particle, thereby emitting a photon. The emissivity, defined as the emitted energy per unit
time per unit frequency per unit volume, is defined as

$$\varepsilon_\nu = \frac{dE}{dt \, d\nu \, dV}. \quad (3.1)$$

For thermal bremsstrahlung emission the emissivity at frequency $\nu$ is given by the following:

$$\varepsilon^f_\nu = \frac{32\pi Z^2 e^6}{3m_e^2 c^3} \left(\frac{2\pi m_e}{3k_B T}\right)^{1/2} n_e n_i e^{-h\nu/k_B T} g^f(\nu). \quad (3.2)$$

where $Z$ is the charge number, $e$ is the charge, $n_e$ is the number density of electrons, $n_i$ is the number density of ions, $T$ is the temperature, and $g^f(\nu)$ is the Gaunt factor which corrects for quantum mechanical effects (see Appendix A for the derivation of Equation 3.2). Because of the dependence of mass in the above equation, electron-ion interactions are the most efficient and also the reason ion-ion interactions can be ignored. The emissivity can also be related to the band-averaged emissivity $\Lambda_{ee}(T, r)$ (counts cm$^3$ s$^{-1}$) by integrating the emissivity, $\varepsilon_\nu$, over the energy band 0.7 – 7.0 keV

$$\Lambda_{ee}(T, r) = \frac{\int_\nu \varepsilon_\nu \, d\nu}{n_e^2} \quad (3.3)$$

where $n_e$ is the number density of electrons (Bulbul et al., 2011). The band-averaged emissivity is calculated using the APEC code (version 1.3.1) from Smith et al. (2001) (Bulbul et al., 2010, 2011). The second process is radiative recombination, or free-bound emission. This is due to an electron being captured by an ion into a bound state while emitting a photon and gives rise to an additional contribution to the cluster’s continuum.
Line emission is caused when an electron makes a transition to a lower energy level in an atom which in turn emits a photon. Transitions between levels can be upward and downward. A transition to a higher energy level is caused from an electron absorbing a photon. For example, the energy levels for hydrogen are given by the following equation

\[ E_n = -\frac{13.6 \text{ eV}}{n^2}, \quad (3.4) \]

where \( n \) is the principal quantum number. An approximate formula to estimate the energy for a transition from an initial state, \( n_i \), to a final state, \( n_f \), of an X-ray emitting atom is given by Moseley’s Law:

\[ \frac{\tilde{\nu}}{R} = (Z - 1)^2 \left( \frac{1}{n_{f}^2} - \frac{1}{n_{i}^2} \right), \quad (3.5) \]

where \( \tilde{\nu} \) is the wavenumber, \( Z \) is the atomic number, and \( R \) is the Rydberg constant.

Emission lines are seen in the X-ray spectra of galaxy clusters and the strong detected lines are from highly ionized iron (\( \text{Fe}^{24+} \) and \( \text{Fe}^{25+} \)). Iron emission lines are seen at energies of 6.4 keV for the K\( \alpha \) line, 7.1 keV for the K\( \beta \) line, and 0.7 keV for the L\( \alpha \) line. K\( \alpha \) line emission is caused from electrons transitioning from the \( n = 2 \) shell to the \( n = 1 \) shell and K\( \beta \) lines are from the transition of the \( n = 3 \) to the \( n = 1 \) shell. Similarly, L\( \alpha \) lines are due to transitions from the \( n = 3 \) to \( n = 2 \) shell.

Spontaneous emission cannot account for all transitions to lower energy levels, since selection rules forbid certain transitions. For example, no transitions occur
unless the following two conditions hold:

\[ \Delta m_\ell = \pm 1, 0 \]
\[ \Delta \ell = \pm 1 \]  \hspace{1cm} (3.6)

by conservation of angular momentum for \( \Delta m_\ell \), where \( m_\ell \) is the magnetic quantum number and electric dipole radiation does not allow \( \Delta \ell = 0 \), where \( \ell \) is the azimuthal quantum number. Consider the 2s state of hydrogen (\( \psi_{200} \) for \( \psi_{n\ell m} \)): this is called a metastable state because there is no lower energy state with sublevel \( \ell = 1 \). Metastable states eventually decay by collisions (also called “forbidden transitions”). As an example, the transition from the 2s state to the 1s state for hydrogen is not possible since that would imply \( \Delta \ell = 0 \). However, the transition from the 2s state to the 2p state is possible as is the transition from the 2p state to the 1s state. If two photons with opposite polarizations are emitted, then angular momentum will be conserved, and a transition will be possible. This is an example of the two-photon process which is another method by which galaxy clusters emit X-rays. This is due to spontaneous emission of two photons that have a combined frequency equal to that of a corresponding Lyman-\( \alpha \) photon. The Lyman series corresponds to \( n_f = 1 \) and \( n_i = 2 \) and higher in the Rydberg formula given as

\[ \frac{1}{\lambda} = \frac{m_e e^4}{8\varepsilon_0^2 h^3 c} \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right) \]  \hspace{1cm} (3.7)
and the Lyman-α wavelength is 1216 Å. All of these emission processes are accounted for in this analysis of X-ray clusters, by use of the APEC code developed by Smith et al. (2001).

### 3.2 Hydrostatic Equilibrium

A critical assumption for calculating cluster masses using X-ray observations is that the gas is in hydrostatic equilibrium. Here, the derivation of the hydrostatic equilibrium equation is presented. Consider a cylindrical differential mass element of length $dr$ and mass $dm$ located a distance $r$ from the center of the cluster. Let the density be given by $\rho(r)$, and the area of the top and bottom faces of the cylinder be $dA$. Then, $dm = \rho(r) dr dA$. The cylinder feels a gravitational force towards the center of the cluster, $F_g$, pressure towards the center of the cluster, $P(r + dr)$, and pressure pushing the cylinder away from the cluster center, $P(r)$. The force due to gravity is given by

$$F_g = -\frac{GM(r)dm}{r^2}. \quad (3.8)$$

Substituting in for $dm$ gives the following equation:

$$F_g = -\frac{GM(r)\rho(r)dr}{r^2} \quad dA. \quad (3.9)$$

The net pressure on the cylinder, $dP$, is the difference between the two pressures, i.e.,

$$dP = P(r + dr) - P(r). \quad (3.10)$$
Since pressure is simply force per area, the force due to the pressure is given by

\[ F_P = [P(r + dr) - P(r)] dA = dP dA. \tag{3.11} \]

The condition for hydrostatic equilibrium is that the force due to pressure balances the force due to gravity, i.e.,

\[ F_P = F_g. \tag{3.12} \]

Setting the two equations equal yields the following:

\[ dP dA = -\frac{GM(r)\rho(r)dr}{r^2} dA. \tag{3.13} \]

Cancelling the \( dA \)'s and bringing \( dr \) to the left-hand side gives the desired equation for hydrostatic equilibrium,

\[ \frac{dP}{dr} = -\frac{GM(r)\rho(r)}{r^2}, \tag{3.14} \]

where \( P \) is the pressure of the gas, \( G \) is the gravitational constant, \( M \) is the total mass, \( \rho \) is the density of the gas, and \( r \) is the distance from the center of the cluster.
3.3 Gas Mass and Total Mass of Clusters

In this section, the equations used for calculating \( M_{\text{gas}} \) and \( M_{\text{tot}} \) are given. The mass of the gas is simply given by

\[
M_{\text{gas}}(r) = \int \rho(r) \, dV. \tag{3.15}
\]

Assuming spherical symmetry, this simplifies to

\[
M_{\text{gas}}(r) = 4\pi \int \rho(r) r^2 \, dr. \tag{3.16}
\]

Substituting \( \rho(r) = \mu_e n_e(r) m_p \) in the above equation yields the final equation for \( M_{\text{gas}} \):

\[
M_{\text{gas}}(r) = 4\pi \mu_e m_p \int n_e(r) r^2 \, dr, \tag{3.17}
\]

where \( \mu_e \) is the mean molecular weight of electrons, \( n_e \) is the number density of electrons, and \( m_p \) is the mass of a proton. The mean molecular weight is given by the following

\[
\frac{1}{\mu m_H} = \frac{\text{total number of particles}}{\text{total mass of gas}} \tag{3.18}
\]

where \( m_H \) is the mass of hydrogen. More formally, this can be written as

\[
\frac{1}{\mu} = \sum_i \left( \frac{X_i}{A_i} \right) + \sum_i \left( \frac{Z_i}{A_i} \right) X_i = \frac{1}{\mu_{\text{ions}}} + \frac{1}{\mu_{\text{electrons}}}, \tag{3.19}
\]
where $X_i$ is the fraction of the total mass contributed by the $i$th species, $A_i$ is the atomic weight of the $i$th species, and $Z_i$ is the effective number of electrons supplied by each atom of the $i$th species (Padmanabhan, 2000). As an example, consider a gas composed of 90% H by number, and 10% He. Then, $n_{\text{He}}/n_{\text{H}} = 0.1$ and the total mass is given as $m_{\text{He}}n_{\text{He}} + m_{\text{H}}n_{\text{H}} = 1.2802\ m_{\text{H}}$. This results in $\mu_{\text{H}} = 1.4082$, $\mu_e = 1.1735$ and $\mu_{\text{tot}} = 0.6401$.

To derive the equation for total mass, the gas in the ICM is assumed to be ideal. This is a reasonably safe assumption since clusters have high temperatures and low densities. The ideal gas law is given by

$$PV = N k_B T,$$  (3.20)

where $P$ is the pressure, $V$ is the volume, $N$ is the number of particles, $k_B$ is the Boltzmann constant, and $T$ is temperature. Dividing both sides by $V$, the pressure can be written simply as

$$P = n k_B T,$$  (3.21)

where $n = N/V$ is the number density. The density is given by $\rho = \mu n m_p$, where $\mu$ is the mean molecular weight, $n$ is the number density, and $m_p$ is the mass of a proton. Solving for $n$ and plugging this into the above equation, the pressure can be written as

$$P = \frac{\rho k_B T}{\mu m_p}.$$  (3.22)
Now, the total mass is given in the hydrostatic equilibrium equation and is given by

\[ M_{\text{tot}}(r) = -\frac{r^2}{G \rho(r)} \frac{dP}{dr}. \]  \hspace{1cm} (3.23)

Therefore, to derive the equation for the total mass of a cluster, take the derivative of this equation with respect to \( r \).

\[ \frac{dP}{dr} = \frac{d}{dr} \left( \frac{\rho k_B T}{\mu m_p} \right) = \frac{k_B}{\mu m_p} \frac{d}{dr} (\rho T) = \frac{k_B}{\mu m_p} \left( T \frac{d\rho}{dr} + \rho \frac{dT}{dr} \right) \]  \hspace{1cm} (3.24)

Substituting this equation into the equation for total mass above results in

\[ M_{\text{tot}}(r) = -\frac{r^2}{G \rho} \frac{k_B}{\mu m_p} \left( T \frac{d\rho}{dr} + \rho \frac{dT}{dr} \right). \]  \hspace{1cm} (3.25)

Now, multiplying through by the \( 1/\rho \) term gives the following:

\[ M_{\text{tot}}(r) = -\frac{k_B r^2}{\mu m_p G} \left( T \frac{d\rho}{dr} + \rho \frac{dT}{dr} \right), \]  \hspace{1cm} (3.26)

after some rearranging. Factoring out a \( T \) from the terms in parentheses yields

\[ M_{\text{tot}}(r) = -\frac{k_B T r^2}{\mu m_p G} \left( \frac{1}{\rho} \frac{d\rho}{dr} + \frac{1}{T} \frac{dT}{dr} \right). \]  \hspace{1cm} (3.27)

Substituting in \( \rho = \mu n_e m_p \), this can be rewritten as

\[ M_{\text{tot}}(r) = -\frac{k_B T r^2}{\mu m_p G} \left( \frac{1}{n_e} \frac{dn_e}{dr} + \frac{1}{T} \frac{dT}{dr} \right). \]  \hspace{1cm} (3.28)

The terms in parentheses can be written in the form of logarithmic derivatives which actually turns out to be more useful in terms of computing (see Equation 3.55 and Equation 3.60). Consider the two functions \( u = \ln x \) and \( v = \ln r \). Then,

\[
du = \frac{1}{x} \, dx \quad \text{and} \quad dv = \frac{1}{r} \, dr,
\]

(3.29)

leads to

\[
\frac{du}{dv} = \frac{d \ln x}{d \ln r} = \frac{r \, dx}{x \, dr}.
\]

(3.30)

Using this result for the equation for total mass and multiplying through by \( r \) gives

\[
M_{\text{tot}}(r) = -\frac{k_B T}{\mu m_p G} \left( \frac{r}{n_e} \frac{dn_e}{dr} + \frac{r}{T} \frac{dT}{dr} \right).
\]

(3.31)

Therefore, the total mass can be written in the following way:

\[
M_{\text{tot}}(r) = -\frac{k_B T}{\mu m_p G} \left( \frac{d \ln n_e}{d \ln r} + \frac{d \ln T}{d \ln r} \right).
\]

(3.32)

### 3.4 Calculation of an Overdensity Radius

The mass of the cluster should be measured as far out radially as possible. However, the background and low-quality data often limit the radius to which the mass measurement can be made. Ideally, the mass would be measured at the virial radius. Virial equilibrium is obtained when an overdense region collapses under the effect of its own gravity, and the virial radius is defined as the radius which is half of that at maximum expansion (Peebles, 1980; Lacey & Cole, 1993). The virial radius
can be defined as the radius within which the average cluster density is $\Delta_v(z)$ times the critical density, where the critical density is given by

$$\rho_c(z) = \frac{3H_0^2 E^2(z)}{8\pi G}. \quad (3.33)$$

Bryan & Norman (1998) showed through numerical simulations that the ratio of mean density to the background density at the time of virialization for a universe with $\Omega_k = 0$ is given by $\Delta_v(z) \simeq 18\pi^2 + 82x - 39x^2$, where $\Omega(z) = \Omega_M (1 + z)^3 / E^2(z)$, $E^2(z) = \Omega_M (1 + z)^3 + \Omega_k (1 + z)^2 + \Omega_\Lambda$, and $x = \Omega(z) - 1$. For a universe with $\Omega_M = 0.27$, $\Omega_\Lambda = 0.73$ and $\Omega_k = 1 - \Omega_M - \Omega_\Lambda$, the overdensity, $\Delta_v = 97$ for $z = 0$.

Therefore, the mass out to the virial radius may be written as the following

$$M(r_{\Delta_v(z)}) = \frac{4}{3} \pi \rho_c(z) \Delta_v(z) r_{\text{vir}}^3 \quad (3.34)$$

(Bonamente et al., 2008). Since current measurements cannot always be made out to this radius, a smaller radius is used. Therefore, all mass measurements were made at $r_{2500}$ and $r_{500}$, where the average density is $\Delta = 2500$ and 500 times the critical density, respectively ($r_{2500} \approx 0.5 r_{500}$, $r_{500} \approx 0.7 r_{200}$, and $r_{200} \approx 0.8 r_{100}$).

### 3.5 Modeling of the Chandra X-ray Data

The total mass of clusters can be inferred from the density and temperature of the X-ray emitting ionized plasma, assuming that the hot gas is in hydrostatic equilibrium within the gravitational cluster potential, as shown in Section 3.3. This
study uses high resolution X-ray imaging and spectroscopy from the \textit{Chandra} X-ray Observatory to determine $n_e(r)$ and $T(r)$. The observed X-ray surface brightness, $S_X$, is related to the electron number density, $n_e$, and the relationship is shown below.

### 3.5.1 The X-ray Surface Brightness

The cluster surface brightness is one of the two X-ray observables, however, the density of the gas is needed to calculate cluster masses. Therefore, the relation between the gas density and surface brightness is described below. Consider the emissivity, $\varepsilon(E)$, of a plasma with temperature $T$, metal abundance $A$, and photon energy, $E$. The cooling function is defined as

$$\Lambda_{ee} = \frac{\varepsilon(E)}{n_e^2},$$  \hspace{1cm} (3.35)

where $n_e$ is the electron number density. The luminosity at photon energy $E$ is given by

$$L(E) = \int \varepsilon(E) \, dV = \int n_e^2 \Lambda_{ee} \, dV. \hspace{1cm} (3.36)$$

Therefore, the flux received at a distance $D$ is given by

$$F(E) = \frac{L(E)}{4\pi D^2} = \frac{1}{4\pi D^2} \int n_e^2 \Lambda_{ee} \, dV. \hspace{1cm} (3.37)$$

For an expanding universe, the frequency observed $\nu'$ of a photon with frequency $\nu$ is given by the redshift relation

$$\nu' = \frac{\nu}{(1 + z)} \hspace{1cm} (3.38)$$
Since photon energy depends on frequency, the flux received at energy $E'$ from a source of energy $E$ at redshift $z$ is given by

$$E' = \frac{E}{(1 + z)}. \quad (3.39)$$

Since the arrival time between photons increases by $(1 + z)$ due to time dilation, the energy flux can be written as

$$F(E') = \frac{1}{4\pi D^2 (1 + z)^2} \int n_e^2 \Lambda_{ee} \, dV. \quad (3.40)$$

Making use of Equation 3.39, the flux measured in number of photons, $F_\gamma(E')$, is related to the energy flux as

$$F_\gamma(E') = (1 + z) F(E'), \quad (3.41)$$

with the cooling function, $\Lambda_{ee}$, also measured in photons. The surface brightness, $S_X(E)$, is defined as

$$S_X(E) = \frac{F}{\Omega}, \quad (3.42)$$

where $\Omega = A/D^2$ is the solid angle of the source with physical size $A$. In the case of an expanding universe, this is written as

$$\Omega = \frac{(1 + z)^2 A}{D^2} = \frac{A}{D_\Lambda^2}, \quad (3.43)$$
where $D_A$ is the angular diameter distance. The surface brightness measured at energy $E'$ is then given by

$$S_X(E') = \frac{1}{4\pi D^2 (1+z)^2} \frac{D^2}{(1+z)^2 A} \int n_e^2 \Lambda_{ee} dV. \quad (3.44)$$

Assuming constant density across the area of interest gives

$$\int n_e^2 dV = A \int n_e^2 dl, \quad (3.45)$$

where $dl$ is along the line of sight. The surface brightness can now be written as

$$S_X(E') = \frac{1}{4\pi (1+z)^4} \int n_e^2 \Lambda_{ee} dl, \quad (3.46)$$

measured in ergs arcmin$^{-2}$ cm$^{-2}$ s$^{-1}$. Therefore, the surface brightness as measured in number of photons and used in this analysis is given by

$$S_X = \frac{1}{4\pi (1+z)^3} \int n_e^2 \Lambda_{ee} dl, \quad (3.47)$$

where $\Lambda_{ee}$ is in counts cm$^3$ s$^{-1}$. The surface brightness in Equation 3.47 is measured in detector units, counts arcmin$^{-2}$ cm$^{-2}$ s$^{-1}$. Sometimes the surface brightness is written where $S_X \propto (1+z)^{-4}$ as in Equation 3.46, instead of $(1+z)^{-3}$ as in Equation 3.47, and this is due to the use of different units. If $S_X$ is measured in ergs instead of counts, then another factor of $(1+z)$ appears since the photon energy is proportional to frequency. The linear distance, $r$, is given by $r = \theta D_A$, where $\theta$ is the apparent
angular size and \( D_A \) is the angular diameter (Carroll et al., 1992). To measure the density from the surface brightness, the angular diameter must be known. The value of \( D_A \) can be calculated by the following equation (valid for \( \Omega_k = 0 \)):

\[
D_A = \frac{c}{(1+z)H_0} \int_0^z \frac{1}{E(z)} \, dz,
\]

(3.48)

where

\[
E^2(z) = \Omega_M (1+z)^3 + \Omega_k (1+z)^2 + \Omega_\Lambda,
\]

(3.49)

and \( H_0 \) is the present value of the Hubble constant. The angular diameter is derived for the general case allowing curvature in Appendix C. The temperature profile is determined from radially-averaged X-ray spectra fit to an APEC optically-thin emission model (Smith et al., 2001), with Galactic HI column density measured from the Leiden-Argentine-Bonn survey (Kalberla et al., 2005). XSPEC software version 12.6.0s was used for all spectral fitting (Arnaud, 1996).

### 3.5.2 The Vikhlinin et al. (2006) Model

The \( \beta \)-model has been commonly used to describe the gas density in clusters and assumes that the temperature is isothermal. (Cavaliere & Fusco-Femiano, 1976, 1978). However, observations show that the temperature profile is not isothermal, the surface brightness profile can steepen at large radii, and thus a more sophisticated model of the ICM is needed to accurately describe the data (Vikhlinin et al., 1999, 2006; Bulbul et al., 2010). This research project used the Vikhlinin et al. (2006) model to describe the density and temperature of the cluster to measure masses. However, a
second and completely different model which describes the density and temperature of the cluster was used to assess any bias from the use of a particular model. The two models used for the X-ray data are presented by Vikhlinin et al. (2006) and Bulbul et al. (2010) to describe the density and temperature profiles of the hot plasma in galaxy clusters. The first model discussed is one developed by Vikhlinin et al. (2006). The three-dimensional gas density is modeled as a generalization of the $\beta$-model and is given by,

$$n_p n_e = n_0^2 \frac{(r/r_c)^{-\alpha}}{(1 + r^2/r_c^2)^{3\beta - \alpha/2}} \frac{1}{(1 + r^\gamma/r_s^\gamma)^{\varepsilon/\gamma}} + \frac{n_{02}^2}{(1 + r^2/r_c^2)^{3\beta_2}}$$

which uses a total of ten free parameters. The $\alpha$ component in Equation 3.50 is used to model a power law type cusp at the center of clusters. The second term allows for steepening of the surface brightness at large radii. The model used for the temperature profile is given by the phenomenological function

$$T(r) = T_0 \frac{(r/r_{cool})^{a_{cool}} + (T_{\text{min}}/T_0) (r/r_t)^{-a}}{(r/r_{cool})^{a_{cool}} + 1} \frac{(r/r_t)^{-a}}{[1 + (r/r_t)^b]^{\varepsilon/b}};$$

which has eight free parameters and can model nearly any smooth temperature distribution. The second term in the temperature profile describes the region outside of the cool-core as a broken power law with a transition region. Therefore, the Vikhlinin et al. (2006) model has a total of 18 free parameters. In fitting the X-ray data with the Vikhlinin et al. (2006) model, $\alpha$ and $n_{02}$ were set to 0, and as suggested by Vikhlinin et al. (2006) $\gamma = 3$ and $\varepsilon < 5$ for all clusters (see Table 4.4). The derivatives of the
density and temperature profiles are needed to obtain the total cluster mass using the Vikhlinin et al. (2006) model. This is shown below as a reference. Neglect the second $\beta$-model in the density profile for simplicity and since it wasn’t used. First, take the square root of the density and rewrite the fractions as products to obtain

$$\frac{n_0(r)}{r_c} = \frac{n_0}{r_c} \left( \frac{r}{r_c} \right)^{-\alpha/2} \left( 1 + \frac{r^2}{r_c^2} \right)^{-(3\beta-\alpha/2)/2} \left( 1 + \frac{r^\gamma}{r_s^\gamma} \right)^{-\varepsilon/2\gamma}. \quad (3.52)$$

Taking the derivative of this equation gives the following:

$$\frac{d}{dr} \left[ \frac{n_0(r)}{r_c} \right] = n_0 \left[ \left( -\frac{\alpha}{2} \right) \left( \frac{r}{r_c} \right)^{-\alpha/2-1} \left( 1 + \frac{r^2}{r_c^2} \right)^{-(3\beta-\alpha/2)/2} \left( 1 + \frac{r^\gamma}{r_s^\gamma} \right)^{-\varepsilon/2\gamma} 
+ \left( \frac{r}{r_c} \right)^{-\alpha/2} \left( -3\beta - \alpha/2 \right) \left( 1 + \frac{r^2}{r_c^2} \right)^{-(3\beta-\alpha/2)/2-1} \left( 2\frac{r}{r_c^2} \right) \left( 1 + \frac{r^\gamma}{r_s^\gamma} \right)^{-\varepsilon/2\gamma} 
+ \left( -\frac{\alpha}{2} \right) \left( \frac{r}{r_c} \right)^{-\alpha/2-1} \left( 1 + \frac{r^2}{r_c^2} \right)^{-(3\beta-\alpha/2)/2} \left( \frac{-\varepsilon}{2\gamma} \right) \left( 1 + \frac{r^\gamma}{r_s^\gamma} \right)^{-\varepsilon/2\gamma-1} \left( \frac{r^\gamma}{r_s^\gamma} \right) \right]. \quad (3.53)$$

Divide both sides by $1/n_0(r)$ to get

$$\frac{1}{n_0(r)} \frac{d}{dr} \left[ \frac{n_0(r)}{r_c} \right] = -\frac{\alpha}{2} \frac{1}{r_c} \left( \frac{r}{r_c} \right)^{-1} \left( -3\beta - \alpha/2 \right) \left( 1 + \frac{r^2}{r_c^2} \right)^{-1} 2\frac{r}{r_c^2} 
+ \frac{-\varepsilon}{2\gamma} \left( 1 + \frac{r^\gamma}{r_s^\gamma} \right)^{-1} \frac{\gamma r^\gamma-1}{r_s^\gamma}. \quad (3.54)$$

Finally, multiplying through by $r$ gives the desired logarithmic derivative.
\[
\frac{d \ln n_0(r)}{d \ln r} = \frac{r}{n_0(r)} \frac{d n_0(r)}{dr} = -\frac{\alpha}{2} - \left(3\beta - \frac{\alpha}{2}\right) \left(1 + \frac{r^2}{r_c^2}\right) \frac{r^2}{r_c^2}
- \frac{\varepsilon}{2} \left(1 + \frac{r^\gamma}{r_s^\gamma}\right) \frac{r^\gamma}{r_s^\gamma}.
\] (3.55)

Similarly, the logarithmic derivative of the temperature profile is needed. Start by writing all the fractions as products:

\[
T(r) = T_0 \left(\frac{r}{r_t}\right)^{-a} \left(1 + \frac{r^b}{r_t^b}\right)^{-c/b} \left[\left(\frac{r}{r_{cool}}\right)^{a_{cool}} + \frac{T_{min}}{T_0}\right] \left[\left(\frac{r}{r_{cool}}\right)^{a_{cool}} + 1\right]^{-1}. \tag{3.56}
\]

The derivative with respect to \(r\) is given by

\[
\frac{dT(r)}{dr} \implies \quad T_0 \left\{ -a \left(\frac{r}{r_t}\right)^{-a-1} \left(\frac{1}{r_t}\right) \left(1 + \frac{r^b}{r_t^b}\right)^{-c/b} \left[\left(\frac{r}{r_{cool}}\right)^{a_{cool}} + \frac{T_{min}}{T_0}\right] \left[\left(\frac{r}{r_{cool}}\right)^{a_{cool}} + 1\right]^{-1}
+ \left(\frac{r}{r_t}\right)^{-a} \left(\frac{-c/b}{b}\right) \left(1 + \frac{r^b}{r_t^b}\right)^{-c/b-1} \left(\frac{b r^{b-1}}{r_t^{b-1}}\right) \left[\left(\frac{r}{r_{cool}}\right)^{a_{cool}} + \frac{T_{min}}{T_0}\right] \left[\left(\frac{r}{r_{cool}}\right)^{a_{cool}} + 1\right]^{-1}
+ \left(\frac{r}{r_t}\right)^{-a} \left(1 + \frac{r^b}{r_t^b}\right)^{-c/b} a_{cool} \left(\frac{r}{r_{cool}}\right)^{a_{cool}-1} \left(\frac{1}{r_{cool}}\right) \left[\left(\frac{r}{r_{cool}}\right)^{a_{cool}} + 1\right]^{-1}
+ \left(\frac{r}{r_t}\right)^{-a} \left(1 + \frac{r^b}{r_t^b}\right)^{-c/b} \left[\left(\frac{r}{r_{cool}}\right)^{a_{cool}} + \frac{T_{min}}{T_0}\right] (-1) \left[\left(\frac{r}{r_{cool}}\right)^{a_{cool}} + 1\right]^{-2} a_{cool}
\times \left(\frac{r}{r_{cool}}\right)^{a_{cool}-1} \left(\frac{1}{r_{cool}}\right) \right\}. \tag{3.57}
\]
Dividing through by $T(r)$ gives the following

$$\frac{1}{T(r)} \frac{dT(r)}{dr} = -a \left( \frac{r}{r_t} \right)^{-1} \left( \frac{1}{r_t} \right) + \left( \frac{-c}{b} \right) \left( 1 + \frac{r^b}{r_t^b} \right)^{-1} \left( \frac{b r^b}{r_t^b} \right)$$
$$+ \left( \frac{a_{cool}}{r_{cool}} \right) \left( \frac{r}{r_{cool}} \right)^{a_{cool}^{-1}} \left[ \left( \frac{r}{r_{cool}} \right)^{a_{cool}} + 1 \right]^{-1}$$
$$+ (-1) \left[ \left( \frac{r}{r_{cool}} \right)^{a_{cool}} + 1 \right]^{-1} \left( \frac{a_{cool}}{r_{cool}} \right) \left( \frac{r}{r_{cool}} \right)^{a_{cool}^{-1}}. \quad (3.58)$$

Finally, multiplying by $r$ gives the logarithmic derivative with

$$\frac{r}{T(r)} \frac{dT(r)}{dr} = -a \left( \frac{r}{r_t} \right)^{-1} \left( \frac{r}{r_t} \right) + \left( \frac{-c}{b} \right) \left( 1 + \frac{r^b}{r_t^b} \right)^{-1} \left( \frac{b r^b}{r_t^b} \right)$$
$$+ a_{cool} \left( \frac{r}{r_{cool}} \right) \left( \frac{r}{r_{cool}} \right)^{a_{cool}^{-1}} \left[ \left( \frac{r}{r_{cool}} \right)^{a_{cool}} + 1 \right]^{-1}$$
$$+ (-1) \left[ \left( \frac{r}{r_{cool}} \right)^{a_{cool}} + 1 \right]^{-1} a_{cool} \left( \frac{r}{r_{cool}} \right) \left( \frac{r}{r_{cool}} \right)^{a_{cool}^{-1}}. \quad (3.59)$$

Simplifying this equation yields the desired result

$$\frac{d \ln T(r)}{d \ln r} = \frac{r}{T(r)} \frac{dT(r)}{dr} = -a - c \left( 1 + \frac{r^b}{r_t^b} \right)^{-1} \left( \frac{r^b}{r_t^b} \right) + a_{cool} \left( \frac{r}{r_{cool}} \right)^{a_{cool}}$$
$$\times \left[ \left( \frac{r}{r_{cool}} \right)^{a_{cool}} + \frac{T_{\text{min}}}{T_0} \right]^{-1} - \left[ \left( \frac{r}{r_{cool}} \right)^{a_{cool}} + 1 \right]^{-1}. \quad (3.60)$$

### 3.5.3 The Bulbul et al. (2010) Model

The second model used is one by Bulbul et al. (2010), and assumes a polytropic equation of state and hydrostatic equilibrium. The full derivation of the necessary
equations for this model, $M_{\text{tot}}(r)$, $n_e(r)$, and $T(r)$ are provided for completeness. The total density profile is based on the distribution given in Navarro, Frenk, & White (1996, 1997), called an NFW profile, and generalized to allow a variable asymptotic slope at large radii following Suto et al. (1998); Ascasibar et al. (2003); Ascasibar & Diego (2008),

$$\rho_{\text{tot}}(r) = \frac{\rho_i}{r/r_s (1 + r/r_s)^\beta}, \quad (3.61)$$

The total mass profile is given by performing a volume integral of the total density distribution,

$$M_{\text{tot}}(r) = \int \rho_{\text{tot}}(r) \, dV = \int_0^{2\pi} d\phi \int_0^\pi \sin \theta \, d\theta \int_0^r \rho_{\text{tot}}(r) \, r^2 \, dr. \quad (3.62)$$

The first two integrals evaluate to $4\pi$ and thus,

$$M_{\text{tot}}(r) = 4\pi \rho_i \int_0^r \frac{r^2}{r/r_s (1 + r/r_s)^\beta} \, dr. \quad (3.63)$$

Let $x = r/r_s$, and therefore, $dx = dr/r_s$. Rewriting the above equation gives the following:

$$4\pi \rho_i \int_0^{r/r_s} \frac{x^2 \, r_s^2}{x (1 + x)^\beta} \, dx = 4\pi \rho_i r_s^3 \int_0^{r/r_s} \frac{x}{(1 + x)^\beta} \, dx. \quad (3.64)$$

Evaluating the integral results in

$$4\pi \rho_i r_s^3 \left[ \frac{(1 + x)^{1-\beta}}{(1 - \beta)(2 - \beta)} \left(x(1 - \beta) - 1\right) \right]_0^{r/r_s}$$
\[ \Rightarrow 4\pi \rho r_s^3 \left[ \frac{(1 + r/r_s)^{1-\beta}}{(1-\beta)(2-\beta)} \left( \frac{r/r_s(1-\beta)}{(1-\beta)} - 1 \right) - \frac{1}{(1-\beta)(2-\beta)(-1)} \right] \]

\[ \Rightarrow \frac{4\pi \rho r_s^3}{(1-\beta)(2-\beta)} \left[ (1 + r/r_s)^{1-\beta} \left( \frac{r/r_s(1-\beta)}{(1-\beta)} - 1 \right) + 1 \right] \]

\[ \Rightarrow \frac{4\pi \rho r_s^3}{(1-\beta)(2-\beta)} \left[ \frac{r/r_s(1-\beta)}{(1 + r/r_s)^{\beta-1}} + 1 \right] \]

\[ \Rightarrow M_{\text{tot}}(r) = \frac{4\pi \rho r_s^3}{(\beta - 1)(\beta - 2)} \left[ 1 + \frac{r/r_s(1-\beta)}{(1 + r/r_s)^{\beta-1}} \right]. \quad (3.65) \]

Now the gravitational potential, \( \phi(r) \), can be calculated from integrating the total mass.

\[ d\phi(r) = \frac{GM_{\text{tot}}(r)}{r^2} \, dr \quad \Rightarrow \quad \phi(r) = G \int_{r}^{r_s} \frac{M_{\text{tot}}(r)}{r^2} \, dr. \quad (3.66) \]

Again make the substitution \( x = r/r_s \) to perform the integration.

\[ \phi(x) = G \int_{\infty}^{r/r_s} \frac{M_{\text{tot}}(x)}{r_s^2 x^2} r_s \, dx = \frac{G}{r_s} \int_{\infty}^{r/r_s} \frac{M_{\text{tot}}(x)}{x^2} \, dx. \quad (3.67) \]

The total mass is given in Equation 3.65 and in terms of \( x \) is given as

\[ M_{\text{tot}}(x) = \frac{4\pi \rho r_s^3}{(\beta - 1)(\beta - 2)} \left[ 1 + \frac{x(1-\beta)}{(1 + x)^{\beta-1}} \right]. \quad (3.68) \]

Rewriting the total mass by multiplying through by \( 1/(\beta - 1) \)

\[ M_{\text{tot}}(x) = \frac{4\pi \rho r_s^3}{(\beta - 2)} \left[ \frac{1}{(\beta - 1)} + \frac{-x - \frac{1}{\beta - 1}}{(1 + x)^{\beta-1}} \right] \]

\[ = \frac{4\pi \rho r_s^3}{(\beta - 2)} \left[ \frac{1}{(\beta - 1)} + \frac{1}{1-\beta - x} \right]. \quad (3.69) \]
will help simplify the integration. Therefore, the gravitational potential may now be written as

\[
\phi(x) = \frac{4\pi G \rho r_s^2}{(\beta - 2)} \left[ \int_{\infty}^{r/r_s} \frac{dx}{(\beta - 1) x^2} + \int_{\infty}^{r/r_s} \frac{dx}{(1 - \beta) x^2 (1 + x)^{\beta - 1}} - \int_{\infty}^{r/r_s} \frac{dx}{x (1 + x)^{\beta - 1}} \right]
\]

\[
\Rightarrow \phi(x) = \frac{4\pi G \rho r_s^2}{(\beta - 2)} \left[ \varphi_A + \varphi_B - \varphi_C \right]. \quad (3.70)
\]

\[
\varphi_A = \int_{\infty}^{r/r_s} \frac{dx}{(\beta - 1) x^2} = \frac{1}{(\beta - 1) \frac{r}{r_s}}. \quad (3.71)
\]

\[
\varphi_B = \int_{\infty}^{r/r_s} \frac{dx}{(1 - \beta) x^2 (1 + x)^{\beta - 1}} = \frac{1}{(1 - \beta)} \left[ -\frac{1}{r/r_s (1 + r/r_s)^{\beta - 2}} - (\beta - 1) \int_{\infty}^{r/r_s} \frac{dx}{x (1 + x)^{\beta - 1}} \right]
\]

\[
\Rightarrow \varphi_B = \frac{1}{(\beta - 1) \frac{r}{r_s}} + \frac{1}{(1 - \beta) (1 + r/r_s)^{\beta - 2}} \int_{\infty}^{r/r_s} \frac{dx}{x (1 + x)^{\beta - 1}}. \quad (3.72)
\]

\[
\varphi_C = \int_{\infty}^{r/r_s} \frac{dx}{x (1 + x)^{\beta - 1}}. \quad (3.73)
\]

Putting this all together gives

\[
\phi(r) = \frac{4\pi G \rho r_s^2}{(\beta - 2)} \left[ \frac{1}{(\beta - 1) \frac{r}{r_s}} - \frac{1}{(1 - \beta) \frac{r}{r_s} (1 + r/r_s)^{\beta - 2}} + \int_{\infty}^{r/r_s} \frac{dx}{x (1 + x)^{\beta - 1}} \right] - \int_{\infty}^{r/r_s} \frac{dx}{x (1 + x)^{\beta - 1}} \right]
\]

\[
(3.74)
\]
which results in the last two integrals cancelling each other. After factoring out the $1/(\beta - 1)$ term, the gravitational potential is simply

$$
\phi(r) = \frac{4\pi G \rho r_s^2}{(\beta - 1)(\beta - 2)} \left[ -\frac{1}{r/r_s} + \frac{1}{r/r_s (1 + r/r_s)^{\beta - 2}} \right] 
$$

$$
= -\frac{4\pi G \rho r_s^2}{(\beta - 1)(\beta - 2)} \left[ \frac{(1 + r/r_s)^{\beta - 2} - 1}{r/r_s (1 + r/r_s)^{\beta - 2}} \right] 
$$

$$
= -\frac{4\pi G \rho r_s^2}{(\beta - 1)} \left[ \frac{1}{(\beta - 2)} \frac{(1 + r/r_s)^{\beta - 2} - 1}{r/r_s (1 + r/r_s)^{\beta - 2}} \right]. \quad (3.75)
$$

The gravitational potential can then be written as

$$
\phi(r) = \phi_0 \left[ \frac{1}{(\beta - 2)} \frac{(1 + r/r_s)^{\beta - 2} - 1}{r/r_s (1 + r/r_s)^{\beta - 2}} \right], \quad (3.76)
$$

where

$$
\phi_0 = -\frac{4\pi G \rho r_s^2}{(\beta - 1)}. \quad (3.77)
$$

Letting $\beta = 2$ in Equation 3.61 corresponds to the NFW profile (Navarro et al., 1996, 1997). Thus, for the case $\beta = 2$, the total mass and gravitational potential should match that of the NFW profile. The total mass and gravitational potential are shown again for convenience

$$
M_{\text{tot}}(x) = \frac{4\pi \rho \mu r_s^3}{(\beta - 1)(\beta - 2)} \left[ 1 + \frac{1 - x(\beta - 1)}{(1 + x)^{\beta - 1}} \right] 
$$

$$
\phi(x) = -\frac{4\pi G \rho r_s^2}{(\beta - 1)} \left[ \frac{1}{(\beta - 2)} \frac{(1 + x)^{\beta - 2} - 1}{x (1 + x)^{\beta - 2}} \right],
$$
where $x = r/r_s$. Both functions can be evaluated at $\beta = 2$ using l’Hôpital’s rule, given by

$$\lim_{x \to a} \frac{f'(x)}{g'(x)} = \frac{f''(a)}{g''(a)}.$$ \hspace{1cm} (3.78)

First, consider the total mass and let

$$f(\beta) = 4\pi \rho r_s^3 [(1 + x)^{\beta-1} + 1 - x(\beta - 1)]$$ \hspace{1cm} (3.79)

and

$$g(\beta) = (\beta - 1)(\beta - 2)(1 + x)^{\beta-1}. \hspace{1cm} (3.80)$$

Taking the derivatives with respect to $\beta$ gives

$$f'(\beta) = 4\pi \rho r_s^3 [ (1 + x)^{\beta-1} \ln(1 + x) - x ]$$ \hspace{1cm} (3.81)

and

$$g'(\beta) = (\beta - 2)(1 + x)^{\beta-1} + (\beta - 1)(1 + x)^{\beta-1} + (\beta - 1)(\beta - 2)(1 + x)^{\beta-1} \ln(1 + x). \hspace{1cm} (3.82)$$

Therefore,

$$\lim_{\beta \to 2} \frac{f'(\beta)}{g'(\beta)} = \frac{4\pi \rho r_s^3 (1 + x) \ln(1 + x) - x}{(1 + x) (1 + x)} = 4\pi \rho r_s^3 \left[ \ln(1 + x) - \frac{x}{(1 + x)} \right]. \hspace{1cm} (3.83)$$
\[ M_{\text{tot}}(r) = 4\pi \rho_i r_s^3 \left[ \ln(1 + r/r_s) - \frac{r/r_s}{1 + r/r_s} \right] \quad [\beta = 2] \quad (3.84) \]

which matches the functional form of the NFW total mass profile. Similarly, the gravitational potential can be determined for \( \beta = 2 \) using l’Hôpital’s rule. Let

\[ f(\beta) = -4\pi G \rho_i r_s^2 \left[ (1 + x)^{\beta-2} - 1 \right] \quad (3.85) \]

and

\[ g(\beta) = (\beta - 1)(\beta - 2)x(1 + x)^{\beta-2}. \quad (3.86) \]

The derivatives with respect to \( \beta \) are

\[ f'(\beta) = -4\pi G \rho_i r_s^2 (1 + x)^{\beta-2} \ln(1 + x) \quad (3.87) \]

and

\[ g'(\beta) = (\beta - 2) x (1 + x)^{\beta-2} + (\beta - 1) x (1 + x)^{\beta-2} \]
\[ + (\beta - 1)(\beta - 2) x (1 + x)^{\beta-2} \ln(1 + x). \quad (3.88) \]

Taking the limit as \( \beta \to 2 \) gives

\[ \lim_{\beta \to 2} \frac{f'(\beta)}{g'(\beta)} = -4\pi G \rho_i r_s^2 \frac{\ln(1 + x)}{x}, \quad (3.89) \]
Figure 3.1: Plot of the Bulbul et al. (2010) gravitational potential for various values of $\beta$.

\[
\phi(r) = -4\pi G \rho_i r_s^2 \left[ \frac{\ln(1 + r/r_s)}{r/r_s} \right] \quad [\beta = 2].
\] (3.90)

Equation 3.90 has the same functional form as the NFW gravitational potential. A plot of the gravitational potential as a function of radius $(r/r_s)$ for various values of $\beta$ is shown in Figure 3.1. The limiting case $\beta = 1$ corresponds to a constant potential.

With these equations, the density, temperature, and pressure can be derived. Assume hydrostatic equilibrium, which can be written as

\[
\frac{1}{\rho(r)} \frac{dP(r)}{dr} = -\frac{GM(r)}{r^2} = \frac{1}{\mu m_p n_e(r)} \frac{dP(r)}{dr} = -\frac{d\phi(r)}{dr}
\] (3.91)
and the ideal gas law, \( P(r) = n_e(r) k_B T(r) \). To find the density and temperature, assume the gas follows a polytropic equation of state, i.e.,

\[
\frac{n_e(r)}{n_{e0}} = \left[ \frac{T(r)}{T_0} \right]^n,
\]

(3.92)

where \( n \) is the polytropic index and \( n_{e0} \) and \( T_0 \) is the central density and temperature at \( r = 0 \), respectively. First solve for the temperature and note that

\[
n_e(r) = n_{e0} \frac{T^n(r)}{T_0^n}.
\]

(3.93)

Taking the derivative of the pressure and substituting it into Equation 3.91 implies

\[
\frac{k_B}{\mu m_p} \left[ T(r) \frac{dn_e(r)}{dr} + n_e(r) \frac{dT(r)}{dr} \right] = -n_e(r) \frac{d\phi(r)}{dr}
\]

\[
\Rightarrow \frac{k_B}{\mu m_p} \left[ T(r) \frac{n_{e0}}{T_0^n} \frac{dT^n(r)}{dr} + \frac{n_{e0}}{T_0^n} T^n(r) \frac{dT(r)}{dr} \right] = -n_e(r) \frac{d\phi(r)}{dr}
\]

\[
\Rightarrow \frac{k_B}{\mu m_p} n_e(r) \frac{T^n(r)}{T_0^n} \left[ n \frac{dT(r)}{dr} + \frac{dT(r)}{dr} \right] = -n_e(r) \frac{d\phi(r)}{dr}
\]

\[
\Rightarrow \frac{k_B}{\mu m_p} n_{e0} T^n(r) \frac{dT(r)}{dr} (n + 1) = -n_e(r) \frac{d\phi(r)}{dr}
\]

\[
\Rightarrow \frac{k_B}{\mu m_p} \frac{dT(r)}{dr} (n + 1) = -\frac{d\phi(r)}{dr}
\]

\[
\Rightarrow \frac{dT(r)}{dr} = -\frac{1}{(n + 1)} \frac{\mu m_p}{k_B} \frac{d\phi(r)}{dr}
\]

\[
\Rightarrow T(r) = -\frac{1}{(n + 1)} \frac{\mu m_p}{k_B} \phi(r)
\]

(3.94)
\[ T(r) = T_0 \left( \frac{1}{(\beta - 2)} \frac{(1 + r/r_s)^{\beta-2} - 1}{r/r_s (1 + r/r_s)^{\beta-2}} \right), \]  
\[ (3.95) \]

where
\[ T_0 = \phi_0 \frac{-1}{(n + 1)} \frac{\mu m_p}{k_B} = \frac{4\pi G \mu m_p \rho_i r_s^2}{k_B(n + 1)(\beta - 1)} \]  
\[ (3.96) \]
is obtained from Equations 3.77 and 3.94. Knowing the temperature profile allows the density to be determined from the polytropic equation of state (Equation 3.92),
\[ n_e(r) = n_{e0} \left( \frac{1}{(\beta - 2)} \frac{(1 + r/r_s)^{\beta-2} - 1}{r/r_s (1 + r/r_s)^{\beta-2}} \right)^n. \]  
\[ (3.97) \]
The electron gas pressure can now be determined using the ideal gas law and Equations 3.95 and 3.97
\[ P_e(r) = P_{e0} \left( \frac{1}{(\beta - 2)} \frac{(1 + r/r_s)^{\beta-2} - 1}{r/r_s (1 + r/r_s)^{\beta-2}} \right)^{n+1}. \]  
\[ (3.98) \]
As with the other derived quantities, the pressure is also analytic as \( \beta \to 2 \) with
\[ P_e(r) = P_{e0} \left[ \frac{\ln(1 + r/r_s)}{r/r_s} \right]^{n+1} \quad [\beta = 2]. \]  
\[ (3.99) \]

An appealing feature of this model is the self-consistent thermodynamic relationship between the electron number density, \( n_e(r) \), temperature, \( T(r) \), and the gas pressure, \( P_e(r) \). In addition to its use for analyzing X-ray data, the Bulbul et al. (2010) model is also well suited for Sunyaev-Zel’dovich Effect data and joint X-ray/SZ analyses (e.g., Hasler et al., 2012; Bonamente et al., 2012). While this polytropic model is
accurate at large radii, cool-core clusters have a significant drop in temperature at small radii which cannot be described by the polytropic equation of state (Vikhlinin et al., 2005; Sanderson et al., 2006; Vikhlinin et al., 2006; Baldi et al., 2007; Snowden et al., 2008). To account for cooling in the cluster core, a modified temperature profile is introduced with

\[
\tau_{\text{cool}} = \frac{(r/r_{\text{cool}})^{a_{\text{cool}}} + \xi}{(r/r_{\text{cool}})^{a_{\text{cool}}} + 1}
\]  

(3.100)

which has been adopted from Vikhlinin et al. (2006). In Equation 3.100, \( \xi \) is a free parameter \( (0 < \xi < 1) \) that measures the amount of central cooling and \( r_{\text{cool}} \) is the characteristic cooling radius. The terms \( \xi \) and \( a_{\text{cool}} \) in Equation 3.100 are equivalent to \( T_{\text{min}}/T_0 \) and \( a_{\text{cool}} \) in Equation 3.51, respectively, in the Vikhlinin et al. (2006) model. Therefore, the temperature and density profile of the Bulbul et al. (2010) model is given as

\[
T(r) = T_0 \left( \frac{1}{(\beta - 2)} \frac{1 + r/r_s}{r/r_s (1 + r/r_s)^{\beta - 2} - 1} \right)^{\tau_{\text{cool}}}
\]  

(3.101)
\[ n_e(r) = n_{e0} \left( \frac{1}{(\beta - 2)} \frac{(1 + r/r_s)^{\beta - 2} - 1}{r/r_s (1 + r/r_s)^{\beta - 2}} \right)^n \tau_{\text{cool}}^{-1} \] (3.102)

which has a total of 8 free parameters and makes this an analytic model appropriate for analyzing X-ray observations of galaxy clusters (Bulbul et al., 2010). Plots of the temperature and density from the Bulbul et al. (2010) model with \( \xi = 0.3 \), \( n = 3.0 \), \( a_{\text{cool}} = 2.0 \), and \( r_{\text{cool}} = r_s \) are shown for various values of \( \beta \) in Figure 3.2. The comparison of results from the two models can be found in Chapter 5.
CHAPTER 4

MEASUREMENT OF THE GAS MASS FRACTION

4.1 Results for the Complete Sample

This section presents the results from this analysis. The gas mass, total mass, and gas mass fraction for each cluster are reported in Table 4.2 The average values were calculated as the median and 68.3% confidence interval from the combined Monte Carlo Markov chains (MCMC) of all clusters to find

\[
\begin{align*}
  f_{\text{gas}}(r_{2500}) &= 0.110 \pm 0.017 \\
  f_{\text{gas}}(r_{500}) &= 0.163 \pm 0.032.
\end{align*}
\]  

(4.1)

The combined radial profile of the gas mass fraction for all clusters is shown in Figure 4.1 and shows an increase with radius as also found in Vikhlinin et al. (2006) and Rasheed et al. (2010). Since clusters are the largest gravitationally bound structures in the Universe, it is believed that the baryon fraction should represent the cosmic baryon fraction. Current measurements from WMAP indicate that the cosmic baryonic fraction is

\[
\Omega_b/\Omega_M = 0.167 \pm 0.006
\]  

(4.2)
(Komatsu et al., 2011). In order to accurately compare the gas mass fraction with the cosmic baryon fraction, the baryons in stars and galaxies need to be taken into account. Giodini et al. (2009) measured the baryon fraction of stars and galaxies and determined the baryonic mass fraction as a function of $M_{500}$ finding

$$f_{\text{stars},500} = 0.019 \pm 0.002$$

(4.3)

for clusters with mass $M(r_{500}) = 7.1 \times 10^{14} \, M_\odot$ (Giodini et al., 2009). This result is slightly higher than the value reported by Gonzalez et al. (2007) who found a value of $\simeq 0.012$, using the same total mass. Therefore, the total baryon fraction in clusters is given by

$$f_b = f_{\text{stars}} + f_{\text{gas}},$$

(4.4)

and this baryon fraction can be accurately compared to the cosmic baryon fraction, $\Omega_b/\Omega_M$. The total cluster baryon fraction as measured with the Vikhlinin et al. (2006) model and using the results from Giodini et al. (2009) is

$$f_b(r_{500}) = 0.182 \pm 0.032.$$  

(4.5)

The difference between the total baryon fraction measured at $r_{500}$ and the cosmic baryon fraction from WMAP is

$$f_b(r_{500}) - \Omega_b/\Omega_M = +0.015 \pm 0.033.$$  

(4.6)
Therefore, the measurement of $f_{\text{gas}}(r_{500})$ using Vikhlinin et al. (2006) model after accounting for the baryons in stars and galaxies according to the results from either Giodini et al. (2009) or Gonzalez et al. (2007) matches the known cosmic baryon fraction as shown in Figure 4.1. The measurement of the baryon fraction using the Bulbul et al. (2010) model also agrees with $\Omega_b/\Omega_M$ (see Chapter 5 for discussion).

4.2 Classification of Relaxed and Unrelaxed Clusters

This section describes the classification of relaxed and unrelaxed clusters. Relaxed clusters which also feature a cool-core are of interest since these clusters are best believed to be in hydrostatic equilibrium. Three parameters were used to determine if a cluster can be classified as dynamically relaxed or not: the centroid shift, central cooling time, and cuspiness. For clusters with high quality data, the central cooling time is the best method for determining if it has a cool-core or not (Hudson et al., 2010). The central cooling time is defined as

$$t_{\text{cool}} \equiv \frac{T}{\frac{dT}{dt}} = \left(\frac{d\ln T}{dt}\right)^{-1}$$

(4.7)

(e.g., Sarazin, 1988), where $T$ is the temperature of the hot gas at the center of the cluster. Consider radiative cooling only and emission from thermal bremsstrahlung radiation. The emissivity due to thermal bremsstrahlung emission is

$$\varepsilon_{\text{ff}} = 1.62 \times 10^{-27} \ T^{-1/2} \ n^2 \ \text{erg cm}^{-3} \ \text{s}^{-1},$$

(4.8)
where \( n \) is the number density and \( T \) is the temperature (Rybicki & Lightman, 1979).

The average energy is given by

\[
E \approx n k_B T,
\]

(4.9)

and

\[
\varepsilon_{\text{ff}} = \frac{dE}{dt} = n k_B \frac{dT}{dt}.
\]

(4.10)

The equation for \( t_{\text{cool}} \) becomes

\[
t_{\text{cool}} = \frac{T}{\varepsilon_{\text{ff}}/n k_B} = \frac{n k_B T}{1.62 \times 10^{-27} T^{-1/2} n^2 \text{ erg cm}^{-3} \text{ s}^{-1}}
\]

\[
= 8.5 \times 10^{10} \text{ s}.
\]

(4.11)

Therefore, the central cooling time was calculated by using

\[
t_{\text{cool}} = 8.5 \times 10^{10} \text{ yr} \left( \frac{n_p}{10^{-3} \text{ cm}^{-3}} \right)^{-1} \left( \frac{T}{10^8 \text{ K}} \right)^{1/2},
\]

(4.12)

where \( n_p \) is the number density of protons and \( T \) is the central temperature of the gas at 0.048 \( r_{500} \) (Sarazin, 1988; Hudson et al., 2010). Cuspinness is defined as the logarithmic derivative of the density profile,

\[
\alpha = \frac{d \log n_e}{d \log r},
\]

(4.13)

evaluated at \( r = 0.04 r_{500} \) (Vikhlinin et al., 2007). Hudson et al. (2010) indicate that this parameter is the preferred choice to mark the presence of a cool-core. The
centroid shift is defined as the standard deviation of the distance between the peak X-ray emission and the centroid (e.g., Poole et al., 2006), given by

$$\langle w \rangle = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} (w_i - \overline{w})}, \quad (4.14)$$

where $\overline{w}$ is the average centroid shift and $w_i$ is the $i$th centroid shift at a given radius. This is calculated as

$$w_i = \sqrt{(x_i - x_{\text{peak}})^2 + (y_i - y_{\text{peak}})^2}, \quad (4.15)$$

where $x_i$ and $y_i$ are the centroid coordinates for the $i$th iteration and $x_{\text{peak}}$ and $y_{\text{peak}}$ are the coordinates of the X-ray peak. The centroid shifts were measured in annular bins centered on the X-ray peak decreasing by 0.05 $r_{500}$ as suggested by Poole et al. (2006). Since the centroid shift is defined as the square root of the variance, i.e., the standard deviation, error propagation must be used to determine the uncertainty in the standard deviation. The variance, $\langle w \rangle^2$, is the actual measurement, so the uncertainty in the centroid shift is given by the variance of this sample variance. Since $w = \sqrt{v}$, where $v$ is the sample variance of $w_i$, consider the equation $f(v) = \sqrt{v}$. The variance of this function, $f(v)$, is given by

$$\sigma_f^2 = \left( \frac{\partial f}{\partial v} \right)^2 \sigma_v^2 = \left( \frac{1}{2\sqrt{v}} \right)^2 \sigma_v^2. \quad (4.16)$$
Therefore, the standard deviation of $f$ is given as

$$\sigma_f = \left( \frac{1}{2\sqrt{v}} \right) \sigma_v = \left( \frac{1}{2w} \right) \sigma_{w^2}, \quad (4.17)$$

since $w^2 = v$. However, the standard deviation of $w^2$ ($\sigma_{w^2}$ in Equation 4.17) is still unknown. Assume that the measured centroid shift is the population standard deviation, i.e., $w = \sigma$. Then, the variance, $\text{Var}(S^2)$, of the sample variance $\langle w^2 \rangle = \sigma^2$, can be determined by the following argument. It is known that the sample variance is related to the chi-squared distribution with $N - 1$ degrees of freedom by

$$(N - 1) \frac{S^2}{\sigma^2} \sim \chi^2(N - 1). \quad (4.18)$$

Making the substitution $w^2 = \sigma^2$, this can be rewritten as

$$(N - 1) \frac{S^2}{w^2} \sim \chi^2(N - 1). \quad (4.19)$$

The variance of $S^2$ is given as

$$\text{Var} \left[ (N - 1) \frac{S^2}{w^2} \right] = 2(N - 1). \quad (4.20)$$

Since $(N - 1)/w^2$ is a constant, then

$$\text{Var} \left[ (N - 1) \frac{S^2}{w^2} \right] = \frac{(N - 1)^2}{w^4} \text{Var}(S^2), \quad (4.21)$$
and now Equation 4.20 can be written as

\[
\frac{(N - 1)^2}{w^4} \text{Var}(S^2) = 2(N - 1). \quad (4.22)
\]

Therefore, the variance of the sample variance is given as

\[
\text{Var}(S^2) = \frac{2w^4}{(N - 1)}, \quad (4.23)
\]

and the standard deviation of the sample variance is

\[
\text{Stdev}(S^2) = \sqrt{\frac{2}{(N - 1)} w^2}. \quad (4.24)
\]

The centroid shift, \( \langle w \rangle \), has units of [distance], and both Equations 4.23 and 4.24 have the correct units. The variance has units of [distance]², and the standard deviation has units of [distance]. In the cases of Equations 4.23 and 4.24, \( S^2 \propto \text{[distance]}^2 \), so \( \text{Var}(S^2) \propto \text{(distance)}^4 \), and \( \text{Stdev}(S^2) \propto \text{[distance]}^2 \). Therefore,

\[
\sigma_{w^2} = \sqrt{\frac{2}{(N - 1)} w^2}, \quad (4.25)
\]

but this is applicable for \( w^2 \). Instead, to get \( \sigma_w \), the error propagation equation in Equation 4.17 must be used to related \( \sigma_{w^2} \) to \( \sigma_w \). The final uncertainty in \( w \) is given as

\[
\sigma_w = \left( \frac{1}{2w} \right) \sqrt{\frac{2}{(N - 1)} w^2} = \sqrt{\frac{1}{2} \frac{1}{(N - 1)} w}. \quad (4.26)
\]
Table 4.1: Measurements of $f_{\text{gas}}$ using the Vikhlinin et al. (2006) Model

<table>
<thead>
<tr>
<th>Sample</th>
<th>$f_{\text{gas}}$ at $r_{2500}$</th>
<th>$f_{\text{gas}}$ at $r_{500}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relaxed</td>
<td>0.111 ± 0.017</td>
<td>0.155 ± 0.026</td>
</tr>
<tr>
<td>Unrelaxed</td>
<td>0.108 ± 0.017</td>
<td>0.173 ± 0.036</td>
</tr>
<tr>
<td>All Clusters</td>
<td>0.110 ± 0.017</td>
<td>0.163 ± 0.032</td>
</tr>
</tbody>
</table>

The central cooling times, cuspiness, and centroid shifts were calculated for all the clusters in the sample, and relaxed clusters were classified as those which satisfied the following three conditions: $\langle w \rangle < 0.0125 r_{500}$, $\alpha > 0.6$, and $t_{\text{cool}} < 8$ Gyr (see also Giles et al. 2013). This classification results in 13 relaxed clusters and 22 unrelaxed clusters (see Table 2.1 and Table 4.3). The results for each subsample are shown in Table 4.1. The measurements of $f_{\text{gas}}$ for the relaxed and unrelaxed clusters are consistent with one another, however the gas mass fraction for the unrelaxed sample is slightly higher than both the relaxed and entire sample. This could be due to the presence of non-thermal pressure support in unrelaxed clusters (see Chapter 5 for further details).

4.3 Dependence of Cluster Properties on Cosmology

The entire sample of clusters was analyzed by running Markov chains and calculating masses assuming a cosmology of $h = 0.73$, $\Omega_m = 0.27$, and $\Omega_\Lambda = 0.73$. However, to convert to a cosmology determined from WMAP7 ($h = 0.702$ instead of $h = 0.73$) the dependence of $f_{\text{gas}}$ on $H_0$ must be known. Assume a cosmology with
\[ \Omega_M = 0.27 \text{ and } \Omega_A = 0.73. \] Hubble’s Law is given by

\[ H_0 D_A = c z, \quad (4.27) \]

and therefore

\[ H_0 \propto D_A^{-1}. \quad (4.28) \]

This is true for any redshift, provided \( \Omega_M \) and \( \Omega_A \) do not change, which is the approach in this analysis. The relation between the physical radius of an object and its angular size is given by

\[ r = \theta D_A, \quad (4.29) \]

where \( r \) is the physical radius (measured in cm) and \( \theta \) is the angular size (measured in arcseconds), and \( D_A \) is the angular diameter distance. Since \( r \) depends on \( D_A \), then

\[ r \propto H_0^{-1}, \quad (4.30) \]

when measured in physical units. However, \( \theta \) is independent of \( H_0 \), so

\[ \theta \not\propto H_0. \quad (4.31) \]

This can easily be seen from the equation used to calculate the overdensity radius, \( r_\Delta \):

\[ \frac{4}{3} \pi r_\Delta^3 \Delta \rho_{\text{crit}}(z) = \frac{k_B T(r_\Delta)}{\mu m_p G} \left( \frac{d \ln n_e(r_\Delta)}{d \ln r_\Delta} + \frac{d \ln T(r_\Delta)}{d \ln r_\Delta} \right), \quad (4.32) \]
where

\[ \rho_{\text{crit}}(z) = \frac{3H_0^2 E^2(z)}{8\pi G}. \]  \hfill (4.33)

Using Equation 4.29, the overdensity angular size, \( \theta_\Delta \), is given by

\[ \frac{4}{3} \pi (\theta_\Delta D_A)^3 \Delta \rho_{\text{crit}}(z) = \frac{-k_B T(\theta_\Delta D_A) \theta_\Delta D_A}{\mu m_p G} \left( \frac{d \ln n_e(\theta_\Delta D_A)}{d \ln (\theta_\Delta D_A)} + \frac{d \ln T(\theta_\Delta D_A)}{d \ln (\theta_\Delta D_A)} \right). \]  \hfill (4.34)

Since the dependence on cosmology is of interest, i.e., \( D_A \), the left-hand side of Equation 4.34 goes as

\[ \frac{4}{3} \pi (\theta_\Delta D_A)^3 \Delta \rho_{\text{crit}}(z) \propto (\theta_\Delta D_A)^3 D_A^{-2}, \]  \hfill (4.35)

while the right-hand side of Equation 4.34 is simply

\[ \frac{-k_B T(\theta_\Delta D_A) \theta_\Delta D_A}{\mu m_p G} \left( \frac{d \ln n_e(\theta_\Delta D_A)}{d \ln (\theta_\Delta D_A)} + \frac{d \ln T(\theta_\Delta D_A)}{d \ln (\theta_\Delta D_A)} \right) \propto \theta_\Delta D_A. \]  \hfill (4.36)

The dependence on \( D_A \) cancels out as seen from Equations 4.35 and 4.36:

\[ \theta_\Delta^3 D_A^3 D_A^{-2} = \theta_\Delta^3 D_A \propto \theta_\Delta D_A \implies \theta_\Delta \not\propto D_A. \]  \hfill (4.37)

Therefore,

\[ \theta_\Delta \not\propto H_0, \]  \hfill (4.38)

which was tested for different values of \( H_0 \) and confirmed in the analysis.

The cosmological dependence of the thermodynamic quantities, \( n_e(r) \) and \( T(r) \), must also be known in order to understand the mass dependencies on cos-
mology. The X-ray observables, surface brightness and temperature, allow for the cluster density and temperature to be determined. Since the observables are not dependent on cosmology, then $T(r)$ must not depend on $D_A$. To determine how $n_e(r)$ depends on $D_A$, consider the equation relating the surface brightness and density:

$$S_X = \frac{1}{4\pi (1 + z)^3} \int n_e^2 \Lambda_{ee} \, dl. \tag{4.39}$$

Since neither $S_X$ nor $\Lambda_{ee}$ depend on cosmology, and $dl = D_A \, d\theta$, then

$$n_e(r) \propto D_A^{-1/2}, \tag{4.40}$$

when measured from X-ray observations. The dependence on $D_A$ of $M_{\text{gas}}$, $M_{\text{tot}}$, and $f_{\text{gas}}$ can now be determined.

### 4.3.1 Dependence of Cluster Masses on Cosmology

How cluster mass measurements of $M_{\text{gas}}$, $M_{\text{tot}}$, and $f_{\text{gas}}$ depend on $D_A$ are necessary in order to convert these measurements to a cosmology assuming a different Hubble constant. The gas mass is given in Equation 3.17 as

$$M_{\text{gas}}(r) = 4\pi \mu_e m_p \int n_e(r) r^2 \, dr. \tag{4.41}$$

Making use of Equation 4.29, $M_{\text{gas}}$ can be written as

$$M_{\text{gas}}(\theta D_A) = 4\pi \mu_e m_p \int n_e(\theta D_A) (\theta D_A)^2 D_A \, d\theta, \tag{4.42}$$
and thus

\[ M_{\text{gas}} \propto n_e D_A^3. \]  \hspace{1cm} (4.43)

From Equation 4.40 the dependence on \( D_A \) can be seen as

\[ M_{\text{gas}} \propto D_A^{5/2}, \]  \hspace{1cm} (4.44)

when measured from X-ray observations. The left-hand side of Equation 4.36 is simply the total mass and is shown to give

\[ M_{\text{tot}} \propto D_A. \]  \hspace{1cm} (4.45)

Therefore, the gas mass fraction is

\[ f_{\text{gas}} = \frac{M_{\text{gas}}}{M_{\text{tot}}} \propto \frac{D_A^{5/2}}{D_A} \propto D_A^{3/2}, \]  \hspace{1cm} (4.46)

as measured from X-ray observations.

### 4.3.2 Rescaling Parameters and Mass Measurements for Different Hubble Constants

Since the main goal of this section is to know the value of cluster masses for \( h = 0.702 \), instead of \( h = 0.73 \), a rescaling of the density must be performed. The density using either the Vikhlinin et al. (2006) or the Bulbul et al. (2010) model can
be written as

\[ n_e(r) = n_{e0} f(x), \]  

(4.47)

where \( n_{e0} \) is the normalization, \( f(x) \) is functional form of the density model, and \( x \) is a scaled radius (e.g., \( r/r_c \) or \( r/r_s \)). Since the dependence of \( n_e(r) \) on \( D_A \) from X-ray observations is known from Equation 4.40, the following relation holds for different values of \( D_A \) (and thus \( H_0 \)):

\[ n_{e0} \sqrt{D_A} = n_{e0, \text{ref}} \sqrt{D_{A, \text{ref}}}, \]  

(4.48)

where \( n_{e0, \text{ref}} \) and \( D_{A, \text{ref}} \) are the reference values obtained from the assumed Hubble constant in the Markov chain. Therefore, the new normalization, \( n_{e0} \), corresponding to \( h = 0.702 \) is given by

\[ n_{e0} = n_{e0, \text{ref}} \sqrt{\frac{D_{A, \text{ref}}}{D_A}}, \]  

(4.49)

where “ref” indicates values obtained assuming \( h = 0.73 \). Cluster masses were calculated with this new normalization corresponding to a cosmology for \( h = 0.702 \). Similarly, the cluster masses can be rescaled according to their dependence on \( D_A \) from Equations 4.44, 4.45 and 4.46:

\[ M_{\text{gas}} = M_{\text{gas, ref}} \left( \frac{D_A}{D_{A, \text{ref}}} \right)^{5/2} \]  

(4.50)

\[ M_{\text{tot}} = M_{\text{tot, ref}} \left( \frac{D_A}{D_{A, \text{ref}}} \right) \]  

(4.51)
and

\[ f_{\text{gas}} = f_{\text{gas, ref}} \left( \frac{D_A}{D_{A, \text{ref}}} \right)^{3/2}, \]  

(4.52)

where again the “ref” indicates values obtained assuming \( h = 0.73 \).
Figure 4.1: Top: average gas mass fraction profile using the Vikhlinin et al. (2006) model for the entire sample with $f_{\text{gas}}(r_{500}) = 0.163 \pm 0.032$. The red line is the median and the hatched region is the 68.3% confidence interval. The blue region is the average $f_{\text{gas}}$ profile for all clusters. The gray envelope is the difference of the cosmic baryon fraction and the fraction of baryons in stars and galaxies, $\Omega_b/\Omega_M - f_{\text{stars}} = 0.148 \pm 0.006$ (Giodini et al., 2009; Komatsu et al., 2011). Bottom: same as above, except for the Bulbul et al. (2010) model and $f_{\text{gas}}(r_{500}) = 0.161 \pm 0.029$. 
Figure 4.2: Top: average gas mass fraction profile using the Vikhlinin et al. (2006) model for the relaxed clusters with $f_{\text{gas}}(r_{500}) = 0.155 \pm 0.026$. The red line is the median and the hatched region is the 68.3% confidence interval. The blue region is the average $f_{\text{gas}}$ profile for all clusters. The gray envelope is the difference of the cosmic baryon fraction and the fraction of baryons in stars and galaxies, $\Omega_b/\Omega_M - f_{\text{stars}} = 0.148 \pm 0.006$ (Giodini et al., 2009; Komatsu et al., 2011). Bottom: same as above, except for the Bulbul et al. (2010) model and $f_{\text{gas}}(r_{500}) = 0.155 \pm 0.027$. 
Figure 4.3: Top: average gas mass fraction profile using the Vikhlinin et al. (2006) model for the unrelaxed clusters with $f_{\text{gas}}(r_{500}) = 0.173 \pm 0.036$. The red line is the median and the hatched region is the 68.3% confidence interval. The blue region is the average $f_{\text{gas}}$ profile for all clusters. The gray envelope is the difference of the cosmic baryon fraction and the fraction of baryons in stars and galaxies, $\Omega_b/\Omega_M - f_{\text{stars}} = 0.148 \pm 0.006$ (Giodini et al., 2009; Komatsu et al., 2011). Bottom: same as above, except for the Bulbul et al. (2010) model and $f_{\text{gas}}(r_{500}) = 0.166 \pm 0.030$. 
Table 4.2: Cluster Masses

<table>
<thead>
<tr>
<th>Cluster</th>
<th>Model</th>
<th>$r_\Delta$ (arcsec)</th>
<th>$M_{\text{gas}}$ (10$^{13}$ M$_{\odot}$)</th>
<th>$M_{\text{tot}}$ (10$^{14}$ M$_{\odot}$)</th>
<th>$f_{\text{gas}}$</th>
<th>$r_\Delta$ (arcsec)</th>
<th>$M_{\text{gas}}$ (10$^{13}$ M$_{\odot}$)</th>
<th>$M_{\text{tot}}$ (10$^{14}$ M$_{\odot}$)</th>
<th>$f_{\text{gas}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A115</td>
<td>Vick</td>
<td>134.5 ±13.7</td>
<td>1.57 ±0.28</td>
<td>1.45 ±0.49</td>
<td>0.108 ±0.015</td>
<td>360.4 ±21.4</td>
<td>8.65 ±0.84</td>
<td>5.58 ±1.06</td>
<td>0.155 ±0.013</td>
</tr>
<tr>
<td>A1423</td>
<td>Poly</td>
<td>139.7 ±4.0</td>
<td>1.68 ±0.09</td>
<td>1.62 ±0.15</td>
<td>0.103 ±0.006</td>
<td>318.7 ±11.3</td>
<td>7.08 ±0.42</td>
<td>3.86 ±0.42</td>
<td>0.184 ±0.010</td>
</tr>
<tr>
<td>A1576</td>
<td>Vick</td>
<td>126.7 ±7.0</td>
<td>1.59 ±0.15</td>
<td>1.48 ±0.26</td>
<td>0.108 ±0.008</td>
<td>274.0 ±18.8</td>
<td>5.73 ±0.58</td>
<td>2.99 ±0.66</td>
<td>0.192 ±0.020</td>
</tr>
<tr>
<td>A1682</td>
<td>Poly</td>
<td>130.8 ±4.3</td>
<td>1.69 ±0.09</td>
<td>1.63 ±0.16</td>
<td>0.104 ±0.005</td>
<td>267.3 ±10.5</td>
<td>5.49 ±0.61</td>
<td>2.78 ±0.31</td>
<td>0.198 ±0.012</td>
</tr>
<tr>
<td>A1758</td>
<td>Vick</td>
<td>114.8 ±8.5</td>
<td>2.23 ±0.28</td>
<td>2.15 ±0.52</td>
<td>0.104 ±0.014</td>
<td>239.6 ±20.2</td>
<td>7.01 ±0.78</td>
<td>3.91 ±1.07</td>
<td>0.179 ±0.033</td>
</tr>
<tr>
<td>A1763</td>
<td>Poly</td>
<td>114.5 ±6.5</td>
<td>2.28 ±0.22</td>
<td>2.14 ±0.38</td>
<td>0.107 ±0.010</td>
<td>242.5 ±15.5</td>
<td>7.26 ±0.56</td>
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### Cluster Masses – Continued

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Cluster Masses – Continued

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Cluster Masses – Continued
Cluster Masses – Continued

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## Cluster Masses – Continued

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Figure 4.4: Cluster classification showing the centroid shift, cuspiness, and cooling time. Clusters with $\langle w \rangle < 0.0125 \, r_{500}$, $\alpha > 0.6$, and $t_{\text{cool}} < 8 \, \text{Gyr}$ were classified as relaxed and hosting a cool-core.
4.4 Model Fits to the Chandra X-ray Data

Both the Vikhlinin et al. (2006) and Bulbul et al. (2010) model are able to fit the surface brightness and temperature profiles of the Chandra X-ray data. In order to fit the data to the two models, a Monte Carlo Markov chain with a Metropolis-Hastings algorithm as described by Bonamente et al. (2004) was implemented. The analysis of Bulbul et al. (2010); Hasler et al. (2012) for the sources of uncertainty for the X-ray data was used. This comes out to a 1% systematic uncertainty for each bin of the surface brightness profile and a 10% systematic uncertainty for each bin of the temperature profile (Bulbul et al., 2010; Hasler et al., 2012). In Chapter 5 additional sources of systematic errors that can affect the gas mass fraction are discussed. To measure masses at $r_{500}$, the data must be fit beyond this radius to constrain the slope of the temperature profile. Out of the entire sample of 35 clusters, 28 have temperature and surface brightness measurements out to or beyond $r_{500}$, and only 7 required slight extrapolation out to $r_{500}$ (see Table 4.7 for additional details). Model parameters using the Vikhlinin et al. (2006) and Bulbul et al. (2010) model are reported in Tables 4.4, 4.5 and 4.6 (fixed parameters are shown without error bars). Fits to the surface brightness and temperature profiles for the entire sample of clusters are shown in Figures 4.5–4.39 where blue and red represent the Vikhlinin et al. (2006) and Bulbul et al. (2010) model, respectively.
Table 4.4: Vikhlinin et al. (2006) Model Density Parameters

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<tr>
<th>Cluster</th>
<th>$n_{e0}$ ($10^{-2}$ cm$^{-3}$)</th>
<th>$r_c$ (arcsec)</th>
<th>$\beta$</th>
<th>$r_s$ (arcsec)</th>
<th>$\varepsilon$</th>
<th>$n_{e02}$</th>
<th>$\gamma$</th>
<th>$\alpha$</th>
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<td>4.89 ± 0.72</td>
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<td>751.40 ± 30.53</td>
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<td>0.44 ± 0.01</td>
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<td>0.33 ± 0.04</td>
<td>190.80 ± 17.41</td>
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<td>70.00 ± 44.92</td>
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<td>233.40 ± 110.30</td>
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### Vikhlinin et al. (2006) Model Density Parameters - Continued

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## Table 4.5: Vikhlinin et al. (2006) Model Temperature Parameters

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<th>$a_{\text{cool}}$</th>
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Bulbul et al. (2010) Model Parameters – Continued

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<th>$n$</th>
<th>$\beta + 1$</th>
<th>$T_0$ (keV)</th>
<th>$r_{\text{cool}}$ (arcsec)</th>
<th>$\xi$</th>
<th>$a_{\text{cool}}$</th>
<th>$\chi^2_{\text{tot}}$ (d.o.f.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A697</td>
<td>$0.99 \pm 0.19$</td>
<td>$68.92 \pm 18.88$</td>
<td>$10.27 \pm 4.50$</td>
<td>$2.36 \pm 0.28$</td>
<td>$13.14 \pm 2.52$</td>
<td>$100.0$</td>
<td>$0.79 \pm 0.13$</td>
<td>2.0</td>
<td>31.15 (119)</td>
</tr>
<tr>
<td>A773</td>
<td>$1.33 \pm 0.03$</td>
<td>$56.17 \pm 3.71$</td>
<td>$7.11 \pm 0.24$</td>
<td>2.5</td>
<td>$9.58 \pm 0.39$</td>
<td>10.0</td>
<td>1.0</td>
<td>2.0</td>
<td>139.93 (94)</td>
</tr>
<tr>
<td>A781</td>
<td>$0.36 \pm 0.02$</td>
<td>$1442.00 \pm 375.60$</td>
<td>$33.81 \pm 10.97$</td>
<td>3.0</td>
<td>$6.18 \pm 0.89$</td>
<td>10.0</td>
<td>1.0</td>
<td>2.0</td>
<td>38.88 (74)</td>
</tr>
<tr>
<td>A963</td>
<td>$1.54 \pm 0.14$</td>
<td>$65.20 \pm 13.01$</td>
<td>$3.78 \pm 0.58$</td>
<td>$3.22 \pm 0.35$</td>
<td>$11.54 \pm 1.04$</td>
<td>$21.27 \pm 3.08$</td>
<td>$0.49 \pm 0.04$</td>
<td>2.0</td>
<td>82.70 (77)</td>
</tr>
<tr>
<td>M1455</td>
<td>$3.27 \pm 0.69$</td>
<td>$7.45 \pm 1.27$</td>
<td>$3.19 \pm 0.60$</td>
<td>$2.68 \pm 0.12$</td>
<td>$15.79 \pm 1.90$</td>
<td>$40.92 \pm 3.15$</td>
<td>$0.31 \pm 0.06$</td>
<td>2.10</td>
<td>145.54 (100)</td>
</tr>
<tr>
<td>R0437</td>
<td>$2.61 \pm 0.31$</td>
<td>$17.57 \pm 2.43$</td>
<td>$48.98 \pm 14.49$</td>
<td>$2.05 \pm 0.01$</td>
<td>$7.78 \pm 0.56$</td>
<td>$16.76 \pm 2.61$</td>
<td>$0.57 \pm 0.06$</td>
<td>2.0</td>
<td>57.38 (58)</td>
</tr>
<tr>
<td>R0439</td>
<td>$0.72 \pm 0.22$</td>
<td>$288.90 \pm 148.90$</td>
<td>$2.45 \pm 0.33$</td>
<td>$7.47 \pm 2.79$</td>
<td>$17.08 \pm 3.82$</td>
<td>$41.93 \pm 6.97$</td>
<td>$0.16 \pm 0.11$</td>
<td>1.04</td>
<td>14.04 (89)</td>
</tr>
<tr>
<td>R1720</td>
<td>$3.78 \pm 0.29$</td>
<td>$18.30 \pm 1.66$</td>
<td>$14.53 \pm 2.39$</td>
<td>$2.17 \pm 0.02$</td>
<td>$8.81 \pm 0.49$</td>
<td>$25.12 \pm 1.46$</td>
<td>$0.42 \pm 0.03$</td>
<td>2.07</td>
<td>172.51 (102)</td>
</tr>
<tr>
<td>R2129</td>
<td>$1.99 \pm 0.35$</td>
<td>$25.84 \pm 7.17$</td>
<td>$4.39 \pm 1.15$</td>
<td>$2.66 \pm 0.19$</td>
<td>$13.81 \pm 1.64$</td>
<td>$23.23 \pm 3.01$</td>
<td>$0.25 \pm 0.04$</td>
<td>1.48</td>
<td>46.52 (75)</td>
</tr>
<tr>
<td>Z2089</td>
<td>$5.13 \pm 1.20$</td>
<td>$6.20 \pm 3.94$</td>
<td>$4.53 \pm 1.16$</td>
<td>$2.49 \pm 0.22$</td>
<td>$12.05 \pm 2.75$</td>
<td>$24.99 \pm 10.42$</td>
<td>$0.2$</td>
<td>1.06</td>
<td>6.72 (43)</td>
</tr>
<tr>
<td>Z3146</td>
<td>$5.22 \pm 0.32$</td>
<td>$19.25 \pm 1.15$</td>
<td>$13.31 \pm 5.10$</td>
<td>$2.23 \pm 0.04$</td>
<td>$8.88 \pm 0.46$</td>
<td>$15.88 \pm 3.56$</td>
<td>$0.47 \pm 0.03$</td>
<td>3.30</td>
<td>90.40 (90)</td>
</tr>
<tr>
<td>Z5247</td>
<td>$0.24 \pm 0.01$</td>
<td>$536.40 \pm 26.68$</td>
<td>$10.0$</td>
<td>3.0</td>
<td>$5.17 \pm 0.43$</td>
<td>10.0</td>
<td>1.0</td>
<td>2.0</td>
<td>35.03 (82)</td>
</tr>
<tr>
<td>Z5768</td>
<td>$0.49 \pm 0.09$</td>
<td>$82.87 \pm 13.23$</td>
<td>$5.0$</td>
<td>3.0</td>
<td>$4.82 \pm 1.04$</td>
<td>10.0</td>
<td>1.0</td>
<td>2.0</td>
<td>26.63 (60)</td>
</tr>
<tr>
<td>Z7215</td>
<td>$0.59 \pm 0.15$</td>
<td>$319.80 \pm 119.70$</td>
<td>$9.07 \pm 3.42$</td>
<td>$3.45 \pm 0.96$</td>
<td>$9.25 \pm 1.70$</td>
<td>$64.41 \pm 35.54$</td>
<td>$0.64 \pm 0.18$</td>
<td>2.0</td>
<td>17.64 (57)</td>
</tr>
</tbody>
</table>
Table 4.7: Extrapolation of Temperature Profiles Out to $r_{500}$

<table>
<thead>
<tr>
<th>Cluster</th>
<th>Max Radius of $T(r)$ (arcsec)</th>
<th>$r_{500}$ (arcsec)</th>
<th>Percent of $r_{500}$ (%)</th>
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</thead>
<tbody>
<tr>
<td>A1758</td>
<td>240</td>
<td>330</td>
<td>73</td>
</tr>
<tr>
<td>A1914</td>
<td>420</td>
<td>440</td>
<td>95</td>
</tr>
<tr>
<td>A611</td>
<td>180</td>
<td>285</td>
<td>63</td>
</tr>
<tr>
<td>R0437</td>
<td>240</td>
<td>260</td>
<td>92</td>
</tr>
<tr>
<td>R1720</td>
<td>400</td>
<td>440</td>
<td>91</td>
</tr>
<tr>
<td>Z2089</td>
<td>180</td>
<td>210</td>
<td>86</td>
</tr>
<tr>
<td>Z7215</td>
<td>240</td>
<td>275</td>
<td>87</td>
</tr>
</tbody>
</table>
Figure 4.5: Top: Abell 115 temperature profile for the Vikhlinin et al. (2006) model (blue) and the Bulbul et al. (2010) model (red). The solid line is the best-fit line and the hatched region is the 68.3% confidence envelope. Bottom: Abell 115 surface brightness profile for the Vikhlinin et al. (2006) model (blue) and the Bulbul et al. (2010) model (red). The black data were obtained from the X-ray images and the green line is the background obtained from blank-sky observations.
Figure 4.6: Top: Abell 1423 temperature profile for the Vikhlinin et al. (2006) model (blue) and the Bulbul et al. (2010) model (red). The solid line is the best-fit line and the hatched region is the 68.3% confidence envelope. Bottom: Abell 1423 surface brightness profile for the Vikhlinin et al. (2006) model (blue) and the Bulbul et al. (2010) model (red). The black data were obtained from the X-ray images and the green line is the background obtained from blank-sky observations.
Figure 4.7: Top: Abell 1576 temperature profile for the Vikhlinin et al. (2006) model (blue) and the Bulbul et al. (2010) model (red). The solid line is the best-fit line and the hatched region is the 68.3% confidence envelope. Bottom: Abell 1576 surface brightness profile for the Vikhlinin et al. (2006) model (blue) and the Bulbul et al. (2010) model (red). The black data were obtained from the X-ray images and the green line is the background obtained from blank-sky observations.
Figure 4.8: Top: Abell 1682 temperature profile for the Vikhlinin et al. (2006) model (blue) and the Bulbul et al. (2010) model (red). The solid line is the best-fit line and the hatched region is the 68.3% confidence envelope. Bottom: Abell 1682 surface brightness profile for the Vikhlinin et al. (2006) model (blue) and the Bulbul et al. (2010) model (red). The black data were obtained from the X-ray images and the green line is the background obtained from blank-sky observations.
Figure 4.9: Top: Abell 1758 temperature profile for the Vikhlinin et al. (2006) model (blue) and the Bulbul et al. (2010) model (red). The solid line is the best-fit line and the hatched region is the 68.3% confidence envelope. Bottom: Abell 1758 surface brightness profile for the Vikhlinin et al. (2006) model (blue) and the Bulbul et al. (2010) model (red). The black data were obtained from the X-ray images and the green line is the background obtained from blank-sky observations.
Figure 4.10: Top: Abell 1763 temperature profile for the Vikhlinin et al. (2006) model (blue) and the Bulbul et al. (2010) model (red). The solid line is the best-fit line and the hatched region is the 68.3% confidence envelope. Bottom: Abell 1763 surface brightness profile for the Vikhlinin et al. (2006) model (blue) and the Bulbul et al. (2010) model (red). The black data were obtained from the X-ray images and the green line is the background obtained from blank-sky observations.
Figure 4.11: Top: Abell 1835 temperature profile for the Vikhlinin et al. (2006) model (blue) and the Bulbul et al. (2010) model (red). The solid line is the best-fit line and the hatched region is the 68.3% confidence envelope. Bottom: Abell 1835 surface brightness profile for the Vikhlinin et al. (2006) model (blue) and the Bulbul et al. (2010) model (red). The black data were obtained from the X-ray images and the green line is the background obtained from blank-sky observations.
Figure 4.12: Top: *Abell 1914* temperature profile for the Vikhlinin et al. (2006) model (blue) and the Bulbul et al. (2010) model (red). The solid line is the best-fit line and the hatched region is the 68.3% confidence envelope. Bottom: *Abell 1914* surface brightness profile for the Vikhlinin et al. (2006) model (blue) and the Bulbul et al. (2010) model (red). The black data were obtained from the X-ray images and the green line is the background obtained from blank-sky observations.
Figure 4.13: Top: *Abell 2111* temperature profile for the Vikhlinin et al. (2006) model (blue) and the Bulbul et al. (2010) model (red). The solid line is the best-fit line and the hatched region is the 68.3% confidence envelope. Bottom: *Abell 2111* surface brightness profile for the Vikhlinin et al. (2006) model (blue) and the Bulbul et al. (2010) model (red). The black data were obtained from the X-ray images and the green line is the background obtained from blank-sky observations.
Figure 4.14: Top: *Abell 2204* temperature profile for the Vikhlinin et al. (2006) model (blue) and the Bulbul et al. (2010) model (red). The solid line is the best-fit line and the hatched region is the 68.3% confidence envelope. Bottom: *Abell 2204* surface brightness profile for the Vikhlinin et al. (2006) model (blue) and the Bulbul et al. (2010) model (red). The black data were obtained from the X-ray images and the green line is the background obtained from blank-sky observations.
Figure 4.15: Top: *Abell 2219* temperature profile for the Vikhlinin et al. (2006) model (blue) and the Bulbul et al. (2010) model (red). The solid line is the best-fit line and the hatched region is the 68.3% confidence envelope. Bottom: *Abell 2219* surface brightness profile for the Vikhlinin et al. (2006) model (blue) and the Bulbul et al. (2010) model (red). The black data were obtained from the X-ray images and the green line is the background obtained from blank-sky observations.
Figure 4.16: Top: Abell 2261 temperature profile for the Vikhlinin et al. (2006) model (blue) and the Bulbul et al. (2010) model (red). The solid line is the best-fit line and the hatched region is the 68.3% confidence envelope. Bottom: Abell 2261 surface brightness profile for the Vikhlinin et al. (2006) model (blue) and the Bulbul et al. (2010) model (red). The black data were obtained from the X-ray images and the green line is the background obtained from blank-sky observations.
Figure 4.17: Top: Abell 2390 temperature profile for the Vikhlinin et al. (2006) model (blue) and the Bulbul et al. (2010) model (red). The solid line is the best-fit line and the hatched region is the 68.3% confidence envelope. Bottom: Abell 2390 surface brightness profile for the Vikhlinin et al. (2006) model (blue) and the Bulbul et al. (2010) model (red). The black data were obtained from the X-ray images and the green line is the background obtained from blank-sky observations.
Figure 4.18: Top: Abell 2552 temperature profile for the Vikhlinin et al. (2006) model (blue) and the Bulbul et al. (2010) model (red). The solid line is the best-fit line and the hatched region is the 68.3% confidence envelope. Bottom: Abell 2552 surface brightness profile for the Vikhlinin et al. (2006) model (blue) and the Bulbul et al. (2010) model (red). The black data were obtained from the X-ray images and the green line is the background obtained from blank-sky observations.
Figure 4.19: Top: *Abell 2631* temperature profile for the Vikhlinin et al. (2006) model (blue) and the Bulbul et al. (2010) model (red). The solid line is the best-fit line and the hatched region is the 68.3% confidence envelope. Bottom: *Abell 2631* surface brightness profile for the Vikhlinin et al. (2006) model (blue) and the Bulbul et al. (2010) model (red). The black data were obtained from the X-ray images and the green line is the background obtained from blank-sky observations.
Figure 4.20: Top: *Abell 267* temperature profile for the Vikhlinin et al. (2006) model (blue) and the Bulbul et al. (2010) model (red). The solid line is the best-fit line and the hatched region is the 68.3% confidence envelope. Bottom: *Abell 267* surface brightness profile for the Vikhlinin et al. (2006) model (blue) and the Bulbul et al. (2010) model (red). The black data were obtained from the X-ray images and the green line is the background obtained from blank-sky observations.
Figure 4.21: Top: Abell 520 temperature profile for the Vikhlinin et al. (2006) model (blue) and the Bulbul et al. (2010) model (red). The solid line is the best-fit line and the hatched region is the 68.3% confidence envelope. Bottom: Abell 520 surface brightness profile for the Vikhlinin et al. (2006) model (blue) and the Bulbul et al. (2010) model (red). The black data were obtained from the X-ray images and the green line is the background obtained from blank-sky observations.
Figure 4.22: Top: *Abell 586* temperature profile for the Vikhlinin et al. (2006) model (blue) and the Bulbul et al. (2010) model (red). The solid line is the best-fit line and the hatched region is the 68.3% confidence envelope. Bottom: *Abell 586* surface brightness profile for the Vikhlinin et al. (2006) model (blue) and the Bulbul et al. (2010) model (red). The black data were obtained from the X-ray images and the green line is the background obtained from blank-sky observations.
Figure 4.23: Top: Abell 611 temperature profile for the Vikhlinin et al. (2006) model (blue) and the Bulbul et al. (2010) model (red). The solid line is the best-fit line and the hatched region is the 68.3% confidence envelope. Bottom: Abell 611 surface brightness profile for the Vikhlinin et al. (2006) model (blue) and the Bulbul et al. (2010) model (red). The black data were obtained from the X-ray images and the green line is the background obtained from blank-sky observations.
Figure 4.24: Top: Abell 665 temperature profile for the Vikhlinin et al. (2006) model (blue) and the Bulbul et al. (2010) model (red). The solid line is the best-fit line and the hatched region is the 68.3% confidence envelope. Bottom: Abell 665 surface brightness profile for the Vikhlinin et al. (2006) model (blue) and the Bulbul et al. (2010) model (red). The black data were obtained from the X-ray images and the green line is the background obtained from blank-sky observations.
Figure 4.25: Top: Abell 68 temperature profile for the Vikhlinin et al. (2006) model (blue) and the Bulbul et al. (2010) model (red). The solid line is the best-fit line and the hatched region is the 68.3% confidence envelope. Bottom: Abell 68 surface brightness profile for the Vikhlinin et al. (2006) model (blue) and the Bulbul et al. (2010) model (red). The black data were obtained from the X-ray images and the green line is the background obtained from blank-sky observations.
Figure 4.26: Top: Abell 697 temperature profile for the Vikhlinin et al. (2006) model (blue) and the Bulbul et al. (2010) model (red). The solid line is the best-fit line and the hatched region is the 68.3% confidence envelope. Bottom: Abell 697 surface brightness profile for the Vikhlinin et al. (2006) model (blue) and the Bulbul et al. (2010) model (red). The black data were obtained from the X-ray images and the green line is the background obtained from blank-sky observations.
Figure 4.27: Top: Abell 773 temperature profile for the Vikhlinin et al. (2006) model (blue) and the Bulbul et al. (2010) model (red). The solid line is the best-fit line and the hatched region is the 68.3% confidence envelope. Bottom: Abell 773 surface brightness profile for the Vikhlinin et al. (2006) model (blue) and the Bulbul et al. (2010) model (red). The black data were obtained from the X-ray images and the green line is the background obtained from blank-sky observations.
Figure 4.28: Top: *Abell 781* temperature profile for the Vikhlinin et al. (2006) model (blue) and the Bulbul et al. (2010) model (red). The solid line is the best-fit line and the hatched region is the 68.3% confidence envelope. Bottom: *Abell 781* surface brightness profile for the Vikhlinin et al. (2006) model (blue) and the Bulbul et al. (2010) model (red). The black data were obtained from the X-ray images and the green line is the background obtained from blank-sky observations.
Figure 4.29: Top: Abell 963 temperature profile for the Vikhlinin et al. (2006) model (blue) and the Bulbul et al. (2010) model (red). The solid line is the best-fit line and the hatched region is the 68.3% confidence envelope. Bottom: Abell 963 surface brightness profile for the Vikhlinin et al. (2006) model (blue) and the Bulbul et al. (2010) model (red). The black data were obtained from the X-ray images and the green line is the background obtained from blank-sky observations.
Figure 4.30: Top: MS 1455.0+2232 temperature profile for the Vikhlinin et al. (2006) model (blue) and the Bulbul et al. (2010) model (red). The solid line is the best-fit line and the hatched region is the 68.3% confidence envelope. Bottom: MS 1455.0+2232 surface brightness profile for the Vikhlinin et al. (2006) model (blue) and the Bulbul et al. (2010) model (red). The black data were obtained from the X-ray images and the green line is the background obtained from blank-sky observations.
Figure 4.31: Top: RX J0437.1+0043 temperature profile for the Vikhlinin et al. (2006) model (blue) and the Bulbul et al. (2010) model (red). The solid line is the best-fit line and the hatched region is the 68.3% confidence envelope. Bottom: RX J0437.1+0043 surface brightness profile for the Vikhlinin et al. (2006) model (blue) and the Bulbul et al. (2010) model (red). The black data were obtained from the X-ray images and the green line is the background obtained from blank-sky observations.
Figure 4.32: Top: RX J0439.0+0715 temperature profile for the Vikhlinin et al. (2006) model (blue) and the Bulbul et al. (2010) model (red). The solid line is the best-fit line and the hatched region is the 68.3% confidence envelope. Bottom: RX J0439.0+0715 surface brightness profile for the Vikhlinin et al. (2006) model (blue) and the Bulbul et al. (2010) model (red). The black data were obtained from the X-ray images and the green line is the background obtained from blank-sky observations.
Figure 4.33: Top: RX J1720.1+2638 temperature profile for the Vikhlinin et al. (2006) model (blue) and the Bulbul et al. (2010) model (red). The solid line is the best-fit line and the hatched region is the 68.3% confidence envelope. Bottom: RX J1720.1+2638 surface brightness profile for the Vikhlinin et al. (2006) model (blue) and the Bulbul et al. (2010) model (red). The black data were obtained from the X-ray images and the green line is the background obtained from blank-sky observations.
Figure 4.34: Top: RX J2129.6+0005 temperature profile for the Vikhlinin et al. (2006) model (blue) and the Bulbul et al. (2010) model (red). The solid line is the best-fit line and the hatched region is the 68.3% confidence envelope. Bottom: RX J2129.6+0005 surface brightness profile for the Vikhlinin et al. (2006) model (blue) and the Bulbul et al. (2010) model (red). The black data were obtained from the X-ray images and the green line is the background obtained from blank-sky observations.
Figure 4.35: Top: Zwicky 2089 temperature profile for the Vikhlinin et al. (2006) model (blue) and the Bulbul et al. (2010) model (red). The solid line is the best-fit line and the hatched region is the 68.3% confidence envelope. Bottom: Zwicky 2089 surface brightness profile for the Vikhlinin et al. (2006) model (blue) and the Bulbul et al. (2010) model (red). The black data were obtained from the X-ray images and the green line is the background obtained from blank-sky observations.
Figure 4.36: Top: *Zwicky 3146* temperature profile for the Vikhlinin et al. (2006) model (blue) and the Bulbul et al. (2010) model (red). The solid line is the best-fit line and the hatched region is the 68.3% confidence envelope. Bottom: *Zwicky 3146* surface brightness profile for the Vikhlinin et al. (2006) model (blue) and the Bulbul et al. (2010) model (red). The black data were obtained from the X-ray images and the green line is the background obtained from blank-sky observations.
Figure 4.37: Top: Zwicky 5247 temperature profile for the Vikhlinin et al. (2006) model (blue) and the Bulbul et al. (2010) model (red). The solid line is the best-fit line and the hatched region is the 68.3% confidence envelope. Bottom: Zwicky 5247 surface brightness profile for the Vikhlinin et al. (2006) model (blue) and the Bulbul et al. (2010) model (red). The black data were obtained from the X-ray images and the green line is the background obtained from blank-sky observations.
Figure 4.38: Top: Zwicky 5768 temperature profile for the Vikhlinin et al. (2006) model (blue) and the Bulbul et al. (2010) model (red). The solid line is the best-fit line and the hatched region is the 68.3% confidence envelope. Bottom: Zwicky 5768 surface brightness profile for the Vikhlinin et al. (2006) model (blue) and the Bulbul et al. (2010) model (red). The black data were obtained from the X-ray images and the green line is the background obtained from blank-sky observations.
Figure 4.39: Top: Zwicky 7215 temperature profile for the Vikhlinin et al. (2006) model (blue) and the Bulbul et al. (2010) model (red). The solid line is the best-fit line and the hatched region is the 68.3% confidence envelope. Bottom: Zwicky 7215 surface brightness profile for the Vikhlinin et al. (2006) model (blue) and the Bulbul et al. (2010) model (red). The black data were obtained from the X-ray images and the green line is the background obtained from blank-sky observations.
4.5 Comparison with Previous Results

In this section, the results obtained from this analysis are compared to previous measurements of the gas mass fraction using hydrostatic mass estimates. The result from this research shows that clusters agree with the cosmic baryon fraction, and thus there are no “missing baryons” in X-ray luminous, massive clusters. An obvious question is then, why are there “missing baryons” in previous analyses of $f_{\text{gas}}$? This section aims to answer that question, by giving insight to the comparison from recent literature.

4.5.1 Description of Previous Samples

There are four recent measurements of the gas mass fraction from hydrostatic mass estimates that are compared to the result from this analysis: Vikhlinin et al. 2006, hereafter, V06, Arnaud et al. 2007, hereafter, A07, Ettori et al. 2009, hereafter, E09, and Sun et al. 2009, hereafter, S09. Studies that use scaling relations to measure cluster masses were not used as a comparison, since scaling relations were not used here. Comparisons were made at the same overdensity used here, $\Delta = 500$, i.e., mass measurements made out to $r_{500}$.

The sample used in this analysis consists of the brightest clusters from the BCS and the low flux extension, eBCS. There are 35 clusters with luminosities in the 0.1 – 2.4 keV band $L_X \geq 6 \times 10^{44}$ erg s$^{-1}$ (for a concordance-model universe with $h = 0.7$), and are therefore estimated to be the most massive. These clusters are all between $0.15 \leq z \leq 0.30$ and the sample is about 90% complete (Ebeling et al., 1998,
For consistency, only the relaxed subsample from this study is compared to previous studies, as most of the clusters selected in the previous analyses are relaxed. The weighted average of the subsample of 13 relaxed clusters is

\[ f_{\text{gas, relaxed}} = 0.150 \pm 0.004. \]

Notice that the value reported in Table 4.1 \((f_{\text{gas}} = 0.155 \pm 0.026)\) is the value of the gas mass fraction for the average cluster profile, i.e., obtained from the combination of all Markov chains. In this section the weighted average of the 13 measurements for the relaxed clusters is used, since this number can be compared directly with the averages obtained from the data published in previous papers.

1. V06: Vikhlinin et al. (2006) measured masses for 10 clusters out to \(r_{500}\). The data used to measure temperature and surface brightness profiles is from *Chandra* as well as *ROSAT*, for improved statistical accuracy of \(S_X\) for low-\(z\) clusters. The sample of clusters in V06 span the redshift range from 0.02 – 0.23. *Abell 2390* is a cluster in common with the sample used here. The weighted average of the gas mass fraction found by Vikhlinin et al. (2006) is

\[ f_{\text{gas, V06}} = 0.105 \pm 0.002. \]

2. A07: Another study done by Arnaud et al. (2007) also measure the gas mass fraction for 10 clusters out to \(r_{500}\). The temperature and surface brightness profiles obtained in this study use *XMM-Newton* data. These 10 clusters are all
low redshift with \( z \leq 0.15 \). The only cluster that overlaps with this sample is Abell 2204. Arnaud et al. (2007) find an average value of the gas mass fraction to be

\[
    f_{\text{gas}, A07} = 0.106 \pm 0.004.
\]

3. E09: Ettori et al. (2009) measured the gas mass fraction out to large radii for a sample of 52 clusters for constraining cosmological parameters. Chandra data was used to measure the surface brightness and temperature profile. All of the clusters in the E09 sample have redshifts \( z > 0.3 \), and therefore none overlap with the sample from this study. The average \( f_{\text{gas}} \) found by E09 is

\[
    f_{\text{gas}, E09} = 0.112 \pm 0.004.
\]

Due to the high redshift range of the sample in E09, further comparison is not made.

4. S09: A study on low mass clusters and groups is presented by Sun et al. (2009). Gas mass fractions were measured out to \( r_{500} \) for a total of 23 out of the 43 galaxy groups and low mass systems. As done in V06, Chandra and when applicable, ROSAT data were used to measure the temperature and surface brightness profiles. Since groups have less mass, and thus gravitational potential, the gas cannot stay within the inner regions of the system. Therefore, the gas fraction is expected to be lower than that for more massive systems. The average gas
mass fraction found by Sun et al. (2009) is

\[ f_{\text{gas},S09} = 0.076 \pm 0.002. \]

Since the sample of clusters in S09 consists of groups and low mass systems, comparing the gas mass fraction will obviously differ from the most X-ray luminous, massive clusters used here, and thus further comparison is not made.

The results of the two clusters that overlap with the samples of V06 and A07 are discussed here. A07 measured the gas mass fraction for Abell 2204 and found \( f_{\text{gas}}(r_{500}) = 0.126 \pm 0.013 \). The result reported here for this same cluster is \( f_{\text{gas}}(r_{500}) = 0.163 \pm 0.010 \). Obviously, there is a discrepancy between the two measurements. However, the temperature profile of Abell 2204 is given in Pointecouteau et al. (2005), and does not match the one obtained here. A significant drop in \( T(r) \) is observed at large radii from this analysis, but not from Pointecouteau et al. (2005); Arnaud et al. (2007). Since the temperature profile is the most dominant term in the mass equation, which also yields \( r_{500} \), any discrepancy would greatly affect the gas mass fraction too.

Abell 2390 is one of the 10 clusters for which V06 analyzed and obtained masses. The gas mass fraction found by V06 is \( f_{\text{gas}}(r_{500}) = 0.141 \pm 0.009 \). Comparing the temperature profile obtained here with that of V06, good agreement is found between the two studies, both showing a significant decrease at large radii. Therefore, the gas mass fraction for Abell 2390 found from this analysis, \( 0.131 \pm 0.024 \), is in excellent agreement with V06.
4.5.2 Comparison of Total Mass and Gas Fraction with Previous Samples

The masses from this cluster sample was also compared with those in the V06 and A07 samples. As seen in Figure 4.40, the clusters used in this analysis are generally more massive than those in the V06 or A07 samples, yet there is an overlap in the range of masses for all three samples. To compare clusters in a similar mass range, clusters were grouped in two bins: Bin 1 with $2 < M_{\text{tot}} < 5 \times 10^{14} M_\odot$, and Bin 2 with $M_{\text{tot}} > 5 \times 10^{14} M_\odot$. The weighted average of $f_{\text{gas}}$ for these bins are reported in Table 4.8, showing that there is a significant difference between the $f_{\text{gas}}$ measurements from this study and those of V06 and A07, especially for Bin 1. Also, contrary to V06 and A07, the results from this analysis do not show an increase in $f_{\text{gas}}$ with mass between the two mass bins.

To further investigate this disagreement, all of the V06 clusters that did not require ROSAT data for the measurement of the background, namely, Abell 133, Abell 1413, Abell 383, and Abell 907 were analyzed (Vikhlinin et al., 2006). Temperature profiles were extracted and surface brightness profiles were obtained for these four clusters, and then masses were measured, using the same reduction and analysis procedure as for all other clusters in this work. The weighted average for the four clusters analyzed here is $0.114 \pm 0.006$ (see Table 4.9). This is in agreement with the results from V06, who measure a value of $0.109 \pm 0.003$ for these clusters. Therefore, it is concluded that on average, the method of analysis used here yields statistically consistent results to those of V06.
Of the four clusters in this comparison sample, three are in agreement at the 1σ level. The only cluster which is not in statistical agreement with V06 is Abell 1413: this study measures $f_{\text{gas}}(r_{500}) = 0.161 \pm 0.011$, whereas V06 reports $0.107 \pm 0.007$. Since the temperature obtained for Abell 1413 at large radii is lower than that of V06, this is likely the reason for the disagreement, since a lower temperature would result in a lower total mass and thus a higher gas mass fraction. Differences in the Chandra calibration or other aspects of the data analysis are likely responsible for the disagreement between the results from this study and those of V06 for Abell 1413. Also, it should be noted that A07 also analyzed Abell 1413 and measured $f_{\text{gas}}(r_{500}) = 0.157 \pm 0.015$, which is in very good agreement with the value obtained here.

### 4.5.3 Possible Luminosity-Selection Bias for the Gas Fraction

The sample of 35 clusters used in this work was selected as the most X-ray luminous in the 0.15–0.30 redshift range. The X-ray luminosity was calculated in the 0.6–9.0 keV band using spectra within the $(0.15 - 1) r_{500}$ region (Giles et al., 2013). The luminosity for the five relaxed clusters in Bin 1 from the sample used here, and the three clusters (Abell 133, Abell 383, and Abell 907) from the V06 sample in the same mass range were compared. Abell 133, Abell 383, and Abell 907 were found to have luminosity of respectively $5.5 \times 10^{43}$, $2.8 \times 10^{44}$, and $3.8 \times 10^{44}$ erg s$^{-1}$, for an average of $2.4 \times 10^{44}$ erg s$^{-1}$, a factor of approximately three times lower than the values for the five clusters in the same mass range present in this sample (Table 4.8). This analysis of clusters in the same mass range indicates that the selection of clusters
Table 4.8: Cluster Properties for Mass Bin 1 ($2 < M_{\text{tot}} < 5 \times 10^{14} M_\odot$) and Mass Bin 2 ($M_{\text{tot}} > 5 \times 10^{14} M_\odot$)

<table>
<thead>
<tr>
<th>Sample</th>
<th>$f_{\text{gas}}(r_{500})$</th>
<th>Number of Clusters</th>
<th>$L_X$ ($10^{44}$ erg s$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Bin 1 Bin 2</td>
<td>Bin 1 Bin 2</td>
<td>Bin 1</td>
</tr>
<tr>
<td>Vikhlinin et al. (2006)</td>
<td>$0.109 \pm 0.004$ $0.116 \pm 0.003$</td>
<td>3 5</td>
<td>2.38</td>
</tr>
<tr>
<td>Arnaud et al. (2007)</td>
<td>$0.111 \pm 0.006$ $0.131 \pm 0.009$</td>
<td>3 3</td>
<td>$\ldots$</td>
</tr>
<tr>
<td>Relaxed Clusters</td>
<td>$0.158 \pm 0.009$ $0.147 \pm 0.005$</td>
<td>5 8</td>
<td>7.14</td>
</tr>
</tbody>
</table>
Table 4.9: Comparison of Clusters with Vikhlinin et al. (2006)

<table>
<thead>
<tr>
<th>Cluster</th>
<th>$r_{500}$ (kpc)</th>
<th>$M_{\text{tot}}(r_{2500})$ (10$^{14}$ M$_{\odot}$)</th>
<th>$f_{\text{gas}}(r_{2500})$</th>
<th>$M_{\text{tot}}(r_{500})$ (10$^{14}$ M$_{\odot}$)</th>
<th>$f_{\text{gas}}(r_{500})$</th>
<th>Weighted Avg. $f_{\text{gas}}(r_{500})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vikhlinin et al. (2006)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A133</td>
<td>1007 ± 41</td>
<td>1.13 ± 0.07</td>
<td>0.067 ± 0.002</td>
<td>3.17 ± 0.38</td>
<td>0.083 ± 0.006</td>
<td>0.109 ± 0.003</td>
</tr>
<tr>
<td>A1413</td>
<td>1299 ± 43</td>
<td>3.01 ± 0.18</td>
<td>0.094 ± 0.003</td>
<td>7.57 ± 0.76</td>
<td>0.107 ± 0.007</td>
<td></td>
</tr>
<tr>
<td>A383</td>
<td>944 ± 32</td>
<td>1.64 ± 0.14</td>
<td>0.092 ± 0.005</td>
<td>3.06 ± 0.31</td>
<td>0.124 ± 0.007</td>
<td>0.124 ± 0.006</td>
</tr>
<tr>
<td>A907</td>
<td>1096 ± 30</td>
<td>2.21 ± 0.14</td>
<td>0.091 ± 0.003</td>
<td>4.56 ± 0.37</td>
<td>0.124 ± 0.006</td>
<td></td>
</tr>
<tr>
<td>Landry et al. (2012)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A133</td>
<td>1027 ± 48</td>
<td>1.11 ± 0.07</td>
<td>0.066 ± 0.002</td>
<td>3.26 ± 0.46</td>
<td>0.080 ± 0.008</td>
<td></td>
</tr>
<tr>
<td>A1413</td>
<td>1160 ± 40</td>
<td>3.55 ± 0.45</td>
<td>0.097 ± 0.007</td>
<td>5.06 ± 0.52</td>
<td>0.161 ± 0.011</td>
<td>0.114 ± 0.006</td>
</tr>
<tr>
<td>A383</td>
<td>886 ± 98</td>
<td>1.40 ± 0.46</td>
<td>0.120 ± 0.022</td>
<td>2.36 ± 0.79</td>
<td>0.169 ± 0.045</td>
<td></td>
</tr>
<tr>
<td>A907</td>
<td>1142 ± 53</td>
<td>2.18 ± 0.12</td>
<td>0.104 ± 0.003</td>
<td>5.06 ± 0.63</td>
<td>0.132 ± 0.013</td>
<td></td>
</tr>
</tbody>
</table>
Figure 4.40: $f_{\text{gas}} - M$ plot for Vikhlinin et al. (2006), Arnaud et al. (2007), and the relaxed clusters from this work. The mass ranges show overlap between the samples.

Based on X-ray luminosity—as in the case of the sample used here—may result in the preferential selection of the high-$f_{\text{gas}}$ tail of the cluster $f_{\text{gas}}$ distribution for a given mass. This is not surprising, since both $f_{\text{gas}}$ and $L_X$ depend on the gas mass content of the cluster. This conclusion is also supported by the scaling relations measured by Giles et al. (2013), in which the same sample of high-$L_X$ clusters was used to find that there is an offset with respect to earlier studies that can be explained by a higher gas mass for a fixed total mass.

Using the same sample from this analysis, Giles et al. (2013) derived several scaling relations for $M - Y_X$, $M - T_X$, $M - M_{\text{gas}}$, $M - L_X$, and $M_{\text{gas}} - T_X$; also finding results that differ from previous analyses. For the $M - Y_X$ scaling relation, they find...
a slope consistent with previous results, but a different normalization. Looking at
the $M - T_X$ relation, they find that the temperature of the clusters in this sample
are about 10% higher than previous studies. The difference in the normalization of
the $M - M_{\text{gas}}$ relation increases with decreasing mass, which is opposite to that of
the $M - T_X$ relation, and is also less evident in the $M - Y_X$ relation. Therefore, $M_{\text{gas}}$
is proposed as the main reason behind the differences in the $M - Y_X$ relation, which
is seen from the $M - M_{\text{gas}}$ relation. The offset in the observed $M - L_X$ relation are
due to the underlying $M - Y_X$ assumption. From these scaling relations, Giles et al.
(2013) conclude that deviations from previous studies are due to the individual cluster
properties based on the redshift range and a larger gas mass. This aligns with the
above conclusion that the higher luminosity of this sample yields a larger gas mass
fraction.
CHAPTER 5

SYSTEMATIC EFFECTS ON THE GAS MASS FRACTION

There are several factors which have an effect on the gas mass fraction. Hydrostatic equilibrium, spherical symmetry, and a uniform gas distribution are just a few examples that may not hold at large radii (e.g., Nagai et al., 2007; Lau et al., 2009; Nagai & Lau, 2011). In this section the systematics and physical processes (e.g., clumping of the gas) that affect the measurement of $f_{\text{gas}}$ are discussed.

5.1 Hydrostatic Equilibrium Assumption

One way in which the gas mass fraction can be affected is from the assumption of hydrostatic equilibrium. If there is an additional component of pressure that is non-thermal, then this must be taken into account when applying the equation of hydrostatic equilibrium. The hydrostatic equilibrium equation is used to determine the mass of the cluster from X-ray observations:

$$M_{\text{tot}}(r) = \frac{-r^2}{\rho G} \frac{dP}{dr},$$  \hspace{1cm} (5.1)
Continued accretion of the gas onto clusters along filaments, mergers, and supersonic motions of galaxies through the intracluster medium are believed to cause gas motions which give rise to non-thermal pressure (Lau et al., 2009). Suppose that the pressure of the hot gas consists of a thermal and a non-thermal component, \( P = P_{\text{th}} + P_{\text{non-th}} \). Then, the equation for mass becomes

\[
M_{\text{tot}}(r) = -\frac{\rho r^2}{G} \left( \frac{dP_{\text{th}}}{dr} + \frac{dP_{\text{non-th}}}{dr} \right),
\]  

(5.2)

and yields a higher mass compared to the mass from thermal pressure only. Separating the mass into components which correspond to different pressure terms, the total mass can be written as

\[
M_{\text{tot}} = M_{\text{th}} + M_{\text{non-th}}.
\]  

(5.3)

Gas motions are expected to cause a non-thermal pressure in the amount of \( \gtrsim 5\% - 15\% \) of the thermal pressure, and this non-thermal pressure will cause an underestimate of mass at large radii of \( 8\% \pm 2\% \) for relaxed systems and \( 11\% \pm 6\% \) for unrelaxed systems at \( r_{500} \) (Lau et al., 2009). Lau et al. (2009) show that the hydrostatic mass underestimates the true mass for several simulated clusters, especially at large radii. In the presence of non-thermal pressure the true mass of the cluster is given by Equation 5.2 and the use of Equation 5.1 will lead to an underestimate of the mass, and therefore an overestimate of the gas mass fraction. Giles et al. (2013) compared X-ray hydrostatic masses with weak lensing masses for these clusters, and found that the total mass obtained through X-ray measurements is underestimated by
a factor of $1.21 \pm 0.23$ and $1.41 \pm 0.15$ for relaxed and unrelaxed clusters, respectively.

This comparison would indicate a departure from hydrostatic equilibrium and cause the gas mass fraction to be overestimated. Based on the Lau et al. (2009) results, a systematic uncertainty of $+10\%$ on $M_{\text{tot}}(r_{500})$ for all clusters was adopted. This results in a possible systematic error of $-10\%$ on $f_{\text{gas}}(r_{500})$. 

**Figure 5.1:** Plot showing the underestimate of total mass assuming hydrostatic equilibrium due to the presence of non-thermal pressure. Figure reproduced from Lau et al. (2009).
5.2 *Chandra* Calibration

Uncertainties in the calibration of the *Chandra* X-ray data can also affect the measurement of the gas mass fraction. The efficiency of the ACIS detector has a spatial dependency, and deviates from being uniform at the ±1% level (Bulbul et al., 2010). Therefore, a ±1% uncertainty to the surface brightness data was added. The temperature profile used in the analysis is also subject to various sources of systematic uncertainty. One source of uncertainty is from the subtraction of the local background. To subtract the background, the blank-sky spectrum must first be rescaled to match that of the high-energy (9.5 – 12 keV) flux of the cluster observation (see Chapter 2) (Hickox & Markevitch, 2006). The effect of the background subtraction was estimated by using the longest observation of *Abell 1835* (ObsID 6880) which has the highest signal-to-noise ratio. To obtain a clean background subtraction, the correction to the blank-sky spectrum was found to be −0.04 ± 0.01. The temperature was measured in an outer region from 240 – 330″ (∼ *r*<sub>500</sub>) using the best-fit correction factor, −0.04, and found to be *kT* = 5.77 keV. The correction to the blank-sky spectrum was changed to −0.03 and −0.05 to over- and under-subtract the background by ±1σ. With these values, the temperature was found to change to *kT* = 5.38 keV and 6.23 keV, respectively. Therefore, the uncertainty in temperature due to the background subtraction is approximately 7.5%. Another source of error is caused by contamination on the Optical Blocking Filter, which is known to affect cluster temperatures by up to 5% (Bulbul et al., 2010; Hasler et al., 2012). Adding these errors
in quadrature, a ±10% systematic uncertainty was used in fitting the temperature data.

### 5.3 Spherical Symmetry

Spherical symmetry is assumed when measuring cluster masses, even though many clusters have disturbed morphologies. Therefore, to estimate the uncertainty due to spherical symmetry, one of the most disturbed clusters, *Abell 520*, was considered as a test case. Observation ID 9426 was used since this observation had the best aimpoint for the task. After performing the *Chandra* data reduction as described in Chapter 2, this observation was left with 96.5 ks of filtered exposure time. Using the temperature map of *Abell 520* from Govoni et al. (2004), the cluster was divided into two sections (see Figure 5.2): the northern section (with respect to the azimuthal angle) was chosen because it encompass a gas with temperature \( \sim 10 \text{ keV} \), while the eastern section was selected for a cooler temperature of \( \lesssim 8 \text{ keV} \) (Govoni et al., 2004). The two sections chosen here are representative of an extreme example of the dynamical states that would be included in this analysis. A temperature profile and surface brightness profile was extracted for the two sectors shown in Figure 5.2 and masses were measured within these regions. Figure 5.3 shows the temperature fits to the two sections extracted from *Abell 520*. For the northern section, \( f_{\text{gas}}(r_{2500}) = 0.109 \pm 0.010 \) and \( f_{\text{gas}}(r_{500}) = 0.179 \pm 0.015 \), and for the eastern section, \( f_{\text{gas}}(r_{2500}) = 0.101 \pm 0.007 \) and \( f_{\text{gas}}(r_{500}) = 0.202 \pm 0.015 \) (see Table 5.1 for more information). Using these measurements there is an estimated ±6% and ±8% systematic uncertainty due to the assumption of spherical symmetry at \( r_{2500} \) and \( r_{500} \), respectively.
Figure 5.2: Image of Abell 520 showing the two sections analyzed. The regions were selected by using the temperature map from Govoni et al. (2004).

Figure 5.3: Temperature profile of Abell 520 for the eastern and northern sections analyzed in Figure 5.2. The blue lines correspond to the eastern section and red corresponds to the northern section. The black data is the temperature profile obtained from analyzing the whole cluster.
Table 5.1: Cluster Properties of Sections of Abell 520

<table>
<thead>
<tr>
<th>Cluster</th>
<th>$r_\Delta$ (arcsec)</th>
<th>$M_{\text{gas}}$ ($10^{13}$ M$_\odot$)</th>
<th>$M_{\text{tot}}$ ($10^{14}$ M$_\odot$)</th>
<th>$f_{\text{gas}}$</th>
<th>$r_\Delta$ (arcsec)</th>
<th>$M_{\text{gas}}$ ($10^{13}$ M$_\odot$)</th>
<th>$M_{\text{tot}}$ ($10^{14}$ M$_\odot$)</th>
<th>$f_{\text{gas}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A520</td>
<td>166.5 ± 6.0_7.2</td>
<td>3.16 ± 0.22_0.25</td>
<td>2.82 ± 0.31_0.35</td>
<td>0.112 ± 0.006_0.005</td>
<td>367.1 ± 9.3_9.2</td>
<td>10.71 ± 0.30_0.29</td>
<td>6.04 ± 0.47_0.44</td>
<td>0.177 ± 0.009_0.009</td>
</tr>
<tr>
<td>Hot Slice</td>
<td>168.1 ± 12.3_13.1</td>
<td>3.18 ± 0.44_0.44</td>
<td>2.90 ± 0.68_0.63</td>
<td>0.109 ± 0.011_0.009</td>
<td>375.5 ± 15.4_15.3</td>
<td>11.60 ± 0.59_0.57</td>
<td>6.47 ± 0.88_0.76</td>
<td>0.179 ± 0.015_0.015</td>
</tr>
<tr>
<td>Cool Slice</td>
<td>165.0 ± 8.7_10.9</td>
<td>2.76 ± 0.29_0.34</td>
<td>2.74 ± 0.46_0.51</td>
<td>0.101 ± 0.006_0.006</td>
<td>338.5 ± 14.5_14.9</td>
<td>9.58 ± 0.58_0.58</td>
<td>4.74 ± 0.63_0.60</td>
<td>0.202 ± 0.016_0.014</td>
</tr>
</tbody>
</table>
5.4 Modeling of the X-ray Data

Another source of error is due to the choice of model for the X-ray data. Two different models were used for measuring the gas mass fraction to assess this uncertainty—the Vikhlinin et al. (2006) model and the Bulbul et al. (2010) model (see Chapter 3 for more detail about the models). The masses for all the clusters using the two models can be found in Table 4.2 and the parameters used are reported in Tables 4.6, 4.4 and 4.5 (fixed parameters are shown without error bars). A plot of the average gas mass fraction profile for the entire sample using the Bulbul et al. (2010) model is given in Figure 4.1. While the two models give the same result of $f_{\text{gas}}$ at $r_{500}$, it is interesting to note that the slope tends to flatten around $\sim r_{500}$ when using the Bulbul et al. (2010) model. The average value for each measurement is compared below by using the median and the 68.3% confidence interval. The average values of $r_{\Delta}$, $M_{\text{gas}}$, $M_{\text{tot}}$, and $f_{\text{gas}}$ for the two models are reported in Table 5.2 and plots comparing each measurement can be found in Figures 5.4–5.7. In the model comparison plot of the gas mass fraction at large radii (bottom panel of Figure 5.7), the $y = x$ line (black) gives a $\chi^2 = 24.15$ for 35 degrees of freedom, so there is no intrinsic scatter. By using two different models to measure cluster masses there is an estimated $6\% \pm 2\%$ uncertainty on $f_{\text{gas}}$ at $r_{2500}$ and a $1\% \pm 2\%$ uncertainty on $f_{\text{gas}}$ at $r_{500}$ due to modeling the data, i.e., the two models give the same answer at large radii. Also, two independent pipelines were used to reduce and analyze the same Chandra data, one by Landry and Bonamente (LB) and one by Giles and Maughan (GM). Additionally, Giles and Maughan measured masses independently of Landry
and Bonamente, and the results at $r_{2500}$ and $r_{500}$ are shown below:

\[
\begin{align*}
\left\{ \begin{array}{l}
f_{\text{gas}}(r_{2500}) : & \frac{\text{GM} - \text{LB}}{\text{GM}} = -2\% \pm 3\% \\
f_{\text{gas}}(r_{500}) : & \frac{\text{GM} - \text{LB}}{\text{GM}} = -3\% \pm 3%,
\end{array} \right. 
\end{align*}
\]

which indicates that the two analyses are consistent with one another and cluster masses are robust to the various choices made in each individual analysis. The measurement of the gas mass fraction in this analysis is higher than previous studies, e.g., Ettori et al. (2003); Vikhlinin et al. (2006); Arnaud et al. (2007); Allen et al. (2008); Ettori et al. (2009); Sun et al. (2009); Rasheed et al. (2010), but the sample used in this analysis was restricted to only the brightest clusters and in the redshift range $0.15 \leq z \leq 0.30$. These previous studies often used a wide range of redshifts and usually only considered relaxed clusters, whereas this analysis uses both relaxed and unrelaxed clusters.

It should be noted that a recent study by Mantz & Allen (2011) shows how the use of a fully parametric model for the density and temperature of the ICM introduces an implicit prior due to the assumption of hydrostatic equilibrium. If the models are not flexible enough or too general, then the derived scaling relations will be biased towards self-similarity (Mantz & Allen, 2011).

### 5.5 Gas Clumping

Clumping of the gas due to accretion or mergers will also affect the gas mass fraction. The main process for X-ray emission in clusters is through bremsstrahlung
emission, which is proportional to $n_e^2$. If the gas is clumped, instead of being distributed uniformly, then the density of electrons, $n_e$, will be overestimated by making the assumption of a uniform distribution (Simionescu et al., 2011). Observations and simulations show that gas clumping is most evident in the outskirts of clusters ($r > r_{500}$) (Mathiesen et al., 1999; Nagai & Lau, 2011; Eckert et al., 2012), though even within $r_{500}$ the gas mass can be overestimated by $\sim 10\%$ due to clumping (Mathiesen et al., 1999). Therefore, the measurements from this analysis of $f_{\text{gas}}$ at $r_{500}$ would remain consistent with the cosmological value of $\Omega_b/\Omega_M$, even after accounting for a 10\% reduction due to possible clumping.
Table 5.2: Vikhlinin et al. (2006) and Bulbul et al. (2010) Model Comparison of Sample Average Cluster Properties

<table>
<thead>
<tr>
<th>Model</th>
<th>( r_{2500} ) (arcsec)</th>
<th>( M_{\text{gas}} ) (10(^{13}) M(_{\odot}))</th>
<th>( M_{\text{tot}} ) (10(^{14}) M(_{\odot}))</th>
<th>( f_{\text{gas}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vikhlinin et al. (2006)</td>
<td>137.4 ± 34.7</td>
<td>2.80 ± 1.50</td>
<td>2.53 ± 1.38</td>
<td>0.108 ± 0.017</td>
</tr>
<tr>
<td>Bulbul et al. (2010)</td>
<td>132.4 ± 27.7</td>
<td>2.73 ± 1.05</td>
<td>2.36 ± 0.96</td>
<td>0.115 ± 0.016</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Model</th>
<th>( r_{500} ) (arcsec)</th>
<th>( M_{\text{gas}} ) (10(^{13}) M(_{\odot}))</th>
<th>( M_{\text{tot}} ) (10(^{14}) M(_{\odot}))</th>
<th>( f_{\text{gas}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vikhlinin et al. (2006)</td>
<td>292.2 ± 71.1</td>
<td>8.07 ± 3.28</td>
<td>5.05 ± 2.43</td>
<td>0.163 ± 0.032</td>
</tr>
<tr>
<td>Bulbul et al. (2010)</td>
<td>301.2 ± 59.4</td>
<td>8.57 ± 2.78</td>
<td>5.58 ± 2.26</td>
<td>0.161 ± 0.029</td>
</tr>
</tbody>
</table>

Table 5.3: Systematic Uncertainties and Effects on \( f_{\text{gas}} \)

<table>
<thead>
<tr>
<th>Source</th>
<th>Effect on ( f_{\text{gas}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>\textit{Chandra} Instrument Calibration</td>
<td>( f_{\text{gas}}(r_{2500}) ) \hspace{1cm} ( f_{\text{gas}}(r_{500}) )</td>
</tr>
<tr>
<td>Surface Brightness</td>
<td>±1%</td>
</tr>
<tr>
<td>Temperature</td>
<td>±10%</td>
</tr>
<tr>
<td>Hydrostatic Equilibrium</td>
<td>−8%</td>
</tr>
<tr>
<td>Spherical Symmetry Assumption</td>
<td>±6%</td>
</tr>
<tr>
<td>Modeling of X-ray Data</td>
<td>±6%</td>
</tr>
<tr>
<td>Clumping of Gas</td>
<td>\cdots</td>
</tr>
</tbody>
</table>
Figure 5.4: Top: comparison plot between the Vikhlinin et al. (2006) and Bulbul et al. (2010) model for $r_{2500}$. The black line is $y = x$, unrelaxed clusters are shown in red, and the blue data points correspond to relaxed clusters. Bottom: same as above, except for $r_{500}$. 


Figure 5.5: Top: comparison plot between the Vikhlinin et al. (2006) and Bulbul et al. (2010) model for $M_{\text{gas}(r_{2500})}$ ($M_{\odot}$). The black line is $y = x$, unrelaxed clusters are shown in red, and the blue data points correspond to relaxed clusters. Bottom: same as above, except for $M_{\text{gas}(r_{500})}$. 

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Figure 5.6: Top: comparison plot between the Vikhlinin et al. (2006) and Bulbul et al. (2010) model for $M_{\text{tot}}(r_{2500})$. The black line is $y = x$, unrelaxed clusters are shown in red, and the blue data points correspond to relaxed clusters. Bottom: same as above, except for $M_{\text{tot}}(r_{500})$. 
Figure 5.7: Top: comparison plot between the Vikhlinin et al. (2006) and Bulbul et al. (2010) model for $f_{\text{gas}}(r_{2500})$. The black line is $y = x$, unrelaxed clusters are shown in red, and the blue data points correspond to relaxed clusters. Bottom: same as above, except for $f_{\text{gas}}(r_{500})$. 
CHAPTER 6

CONCLUSIONS

Clusters of galaxies are massive structures that have formed from smaller merger events (Press & Schechter, 1974). The composition of galaxy clusters are galaxies, the intracluster medium, and dark matter. The hot gas in the ICM accounts for $\sim 10\%$ of the total mass, while the majority of the cluster’s mass is in the form of dark matter (Grego et al., 2001; Allen et al., 2002, 2004; LaRoque et al., 2006; Allen et al., 2008; Vikhlinin et al., 2006; Arnaud et al., 2007; Ettori et al., 2009; Sun et al., 2009; Mantz et al., 2010; Hasler et al., 2012).

The mass of clusters plays an important role in cosmology and can be used to constrain cosmological parameters such as the equation of state of dark energy, normalization of the power spectrum, and density of matter (Ettori et al., 2003; Allen et al., 2004; Kravtsov et al., 2006; Allen et al., 2008; Ettori et al., 2009; Vikhlinin et al., 2009; Benson et al., 2011; Mantz et al., 2010). Since clusters of galaxies are extremely large and massive, the baryonic matter and dark matter originated from about the same comoving volume; it is believed their composition should be representative of the Universe (Metzler & Evrard, 1994). White et al. (1993) measured the baryon fraction of the Coma cluster to suggest a low matter density of the Universe. The
gas mass fraction is the ratio of the gas mass to total mass, i.e., \( f_{\text{gas}} = \frac{M_{\text{gas}}}{M_{\text{tot}}} \), and thus the total baryon fraction in clusters is given by \( f_b = f_{\text{stars}} + f_{\text{gas}} \). The gas mass fraction and its evolution with redshift have also been used for cosmology. Allen et al. (2002, 2004, 2008); Ettori et al. (2009) have used the gas mass fraction to place constraints on \( \Omega_M \) and \( w(z) \). Since clusters are the largest gravitationally bound structures in the Universe, it is difficult for baryons to escape the cluster gravitational potential, indicating that the baryon fraction should match the cosmic baryon fraction. However, recent studies of the cluster baryon fraction fall short of the cosmic baryon fraction (Vikhlinin et al., 2006; Afshordi et al., 2007; Arnaud et al., 2007; Sun et al., 2009; Umetsu et al., 2009; Rasheed et al., 2010). This is known as the “missing baryon” problem in clusters (Rasheed et al., 2010).

For an accurate representation of the population of clusters of galaxies, a statistically complete sample with a well defined selection function must be used. This study used Chandra X-ray observations of the brightest clusters taken from the Brightest Cluster Sample and its low flux extension. This sample consists of 35 clusters with luminosities \( L_{\text{X},0.1–2.4\text{ keV}} \geq 5 \times 10^{44} \text{ erg s}^{-1} \) in the redshift range 0.15 – 0.30 (Dahle, 2006). Since these clusters are the brightest, they are also estimated to be the most massive (Dahle, 2006). This sample also consists of both relaxed and unrelaxed clusters, and thus is unbiased with respect to cluster morphology; allowing for the study of systematic effects in measuring unrelaxed cluster masses. Out of the 35 clusters in this sample, 13 were determined to be relaxed, leaving 22 classified as unrelaxed (see Table 2.1).
For this analysis, the Vikhlinin et al. (2006) model was used to describe the plasma density and temperature of the cluster. In order to fit the data to the models, a Monte Carlo Markov chain with a Metropolis-Hastings algorithm was used in this analysis as described in Bonamente et al. (2004). The average $f_{\text{gas}}$ measurement for the sample of clusters was determined by the median and 68.3% confidence interval from the combined Monte Carlo Markov chains

$$
\begin{align*}
    f_{\text{gas}}(r_{2500}) &= 0.110 \pm 0.017 \\
    f_{\text{gas}}(r_{500}) &= 0.163 \pm 0.032.
\end{align*}
$$

The combined radial profile of the gas mass fraction for all clusters is shown in Figure 4.1 and shows an increase with radius as also found in Vikhlinin et al. (2006) and Rasheed et al. (2010). Current measurements from WMAP indicate that the cosmic baryonic fraction is $\Omega_b/\Omega_M = 0.167 \pm 0.006$ (Komatsu et al., 2011). The baryon fraction from the stars and galaxies as reported in Giodini et al. (2009) who find $f_{\text{stars,500}} = 0.019 \pm 0.002$ was used to calculate the total baryon fraction. The difference between the total baryon fraction measured at $r_{500}$ and the cosmic baryon fraction from WMAP is

$$
f_b(r_{500}) - \frac{\Omega_b}{\Omega_M} = +0.015 \pm 0.033.
$$

Therefore, the measurement of $f_{\text{gas}}(r_{500})$ using Vikhlinin et al. (2006) model after accounting for the baryons in stars and galaxies according to the results from either
Giodini et al. (2009) or Gonzalez et al. (2007) matches the known cosmic baryon fraction (see Figure 4.1). Therefore, it is found that in this sample of high-$L_X$ clusters, baryons at $r_{500}$ are consistent with the cosmological value.

Three parameters were used to determine if a cluster can be classified as dynamically relaxed or not: the centroid shift, central cooling time, and cuspiness. The gas mass fraction was then calculated for both subsamples of relaxed clusters and unrelaxed clusters. The results show that at $r_{2500}$ and $r_{500}$ the gas mass fractions for relaxed clusters are in excellent agreement with that of unrelaxed clusters. Also, after accounting for the baryons in stars and galaxies as done above, both subsamples are statistically consistent with the cosmic baryon fraction at $r_{500}$ (see Figures 4.2 and 4.3).

In accord with earlier studies, this study finds that the gas mass fraction increases with radius, and thus the value of $f_{\text{gas}}$ depends on the radius used. The mass of the cluster should be measured at the largest radii to include as much of the matter of the cluster as possible, i.e., ideally measurements should be made at the virial radius. However, the background and the cluster surface brightness profile limits the radius to which the mass measurement can be made, and therefore all of the measurements in this analysis were made at $r_{500}$. Comparing the total baryon fraction with the cosmic baryon fraction, this analysis shows that the two measurements agree at the $1\sigma$ level, i.e., the gas mass fraction at $r_{500}$ is in fact consistent with the cosmological value of $\Omega_b/\Omega_M$, and there are no “missing baryons” within $r_{500}$ in the most X-ray luminous, massive clusters. A recent study by Miller et al. (2012) also
find a gas mass fraction in agreement with the cosmic baryon fraction using data from
*Chandra*, *XMM-Newton*, and *Suzaku*.

One question that remains open is what happens to $f_{\text{gas}}$ beyond $r_{500}$? As seen in Figure 4.1, the gas mass fraction increases with radius, and naively extrapolating beyond $r_{500}$ will give an even higher value for the gas mass fraction. The total baryon fraction in this case will be greater than the cosmological value, $\Omega_b/\Omega_M$, as reported in recent studies based on *Suzaku* data (e.g., Simionescu et al., 2011) and *Chandra* data (e.g., Bonamente et al., 2013). However, there are two important reasons why simple extrapolations to large radii of the measured radial trend of $f_{\text{gas}}$ may not be valid. First, the gas beyond $r_{500}$ is not likely to be in hydrostatic equilibrium. Non-thermal pressure support will become more significant at radii $> r_{500}$ and will affect the measurement of the total mass (Nagai et al., 2007; Lau et al., 2009). An increase of non-thermal pressure support would cause an underestimate of total mass, and thus an overestimate of $f_{\text{gas}}$. Therefore, the true gas mass fraction would be lower than the measured value reported here in the presence of non-thermal pressure. The second reason is that clumping of the gas may also become more predominant at large radii; observations and simulations show that the gas clumping factor increases beyond $r_{500}$ and will considerably bias X-ray mass measurements (Mathiesen et al., 1999; Nagai & Lau, 2011; Simionescu et al., 2011; Eckert et al., 2012). As a result of clumping, the true gas mass fraction at large radii would be lower than the value measured assuming a smooth distribution of the gas.

Also, the sample selection criteria could have an effect on the measurement of $f_{\text{gas}}$. The sample used here is comprised of clusters selected based on their high
X-ray flux and luminosity, i.e., this sample preferentially selects clusters with a high $L_X$ for a given mass. Since $L_X$ depends on the gas mass, a luminosity-selected sample may have a larger fraction of clusters with high $f_{\text{gas}}$ than samples selected with other criteria. The extent of any such bias would depend on the correlation between the scatter in $f_{\text{gas}}$ and $L_X$ at fixed mass, and the magnitude of the intrinsic scatter in $f_{\text{gas}}$.

Full account of this effect requires modeling of the selection function and population scatter of the sample, which is presented in Giles et al. (2013). For the current study, the intrinsic scatter of $f_{\text{gas}}$ in this sample was estimated as $11\% \pm 4\%$. This may be underestimated if clusters from the low $f_{\text{gas}}$ tail of the distribution are excluded by the luminosity selection of the sample. If this bias exists, then it could pose problems for using the gas mass fraction for cosmology. In fact, clusters used for cosmological applications are generally selected on the basis of their high luminosity, allowing their study at high redshift. Such clusters would be biased towards high $f_{\text{gas}}$, potentially distorting the cosmological constraints derived from the $f_{\text{gas}}(z)$ tests. The implication is that complete (or at least statistically representative) samples should be used for the $f_{\text{gas}}(z)$ tests, so that the selection bias can be corrected. While these samples would necessarily contain morphologically disturbed clusters, the results from this analysis show that $f_{\text{gas}}$ measurements are not significantly affected by cluster morphology, at least out to $r_{500}$; Giles et al. (2013) also find an agreement between scaling relations for relaxed and unrelaxed hydrostatic masses. This suggests that complete samples of clusters could be used for $f_{\text{gas}}(z)$ tests. A complementary approach would be to study the baryon fraction of clusters selected independently of the ICM (e.g., red sequence or weak lensing selected clusters). This would give a measurement of the
range of baryon fractions that is free from selection biases associated with their X-ray emission.

The conclusion from this analysis is that the large value of $f_{\text{gas}}$ at $r_{500}$ measured for this sample is representative of the high-$f_{\text{gas}}$ tail of the cluster population, and that the most massive and X-ray luminous clusters in this redshift range have the cosmological ratio of baryons to dark matter even at $r_{500}$. 
APPENDICES
APPENDIX A

THERMAL BREMSSTRAHLUNG

This is the derivation of the emissivity, $\varepsilon_\nu$, for free-free, or thermal bremsstrahlung emission as outlined in Rybicki & Lightman (1979) and reproduced here for completeness. Bremsstrahlung is German for “braking radiation,” and occurs when an electron is accelerated by the presence of a ion. Only electron-ion collisions are considered here, since this is the dominant emission mechanism due to the mass ratio of an ion to an electron. It is assumed that the electron is traveling in a Coulomb field of the ion with impact parameter $b$.

Consider a pure $E$-field. For a moving electron, the Coulomb field of an ion is Lorentz transformed:

\[
\begin{align*}
E'_\parallel &= E_\parallel & B'_\parallel &= 0 \\
E'_\perp &= \gamma E_\perp & B'_\perp &= -\gamma \beta \times E_\perp,
\end{align*}
\]  

(A.1)  

(A.2)
where $\beta = v/c$ and $\gamma = (1 - \beta^2)^{-1/2}$. Assume the electron is moving in the $+x$-direction, so

\[
E_x = \frac{q x'}{r'^3} = \frac{q \gamma (x - vt)}{r^3} \quad B_x = 0
\]

\[
E_y = \frac{q y'}{r'^3} = \frac{q \gamma y}{r^3} \quad B_y = \frac{-q \gamma \beta z'}{r'^3} = \frac{-q \gamma \beta z}{r^3}
\]

\[
E_z = \frac{q z'}{r'^3} = \frac{q \gamma z}{r^3} \quad B_z = \frac{q \gamma \beta y'}{r'^3} = \frac{q \gamma \beta y}{r^3},
\]

(A.3)

where $r'^2 = x'^2 + y'^2 + z'^2$ and $r^2 = x^2 + y^2 + z^2$. Placing the electron at the position $x = 0, y = b, z = 0$ gives

\[
\mathbf{E} = \frac{q \gamma}{(\gamma^2 v^2 t^2 + b^2)^{3/2}} (vt, b, 0)
\]

(A.4)

and

\[
\mathbf{B} = (0, 0, \beta E_y).
\]

(A.5)

Now, consider an ion of charge $q = Ze$; the acceleration of the electron is given as

\[
a_x = \frac{-e E_x}{m_e} = \frac{-Ze^2 \gamma vt}{m_e (\gamma^2 v^2 t^2 + b^2)^{3/2}}
\]

(A.6)

\[
a_y = \frac{-e E_y}{m_e} = \frac{Ze^2 \gamma b}{m_e (\gamma^2 v^2 t^2 + b^2)^{3/2}}
\]

(A.7)

From here on, only the non-relativistic case will be considered, i.e., $\gamma = 1$. The dipole moment is given as $\mathbf{d} = q_i \mathbf{r}_i$. Instead of $d(t)$, the frequency dependence of $\mathbf{d}$ will be
more useful and is found with the Fourier transform,

\[ d(t) = \int_{-\infty}^{\infty} \hat{d}(\omega) e^{-i\omega t} \, d\omega \]  
(A.8)

and

\[ \ddot{d}(t) = \int_{-\infty}^{\infty} -\omega^2 \hat{d}(\omega) e^{-i\omega t} \, d\omega. \]  
(A.9)

The total energy, \( W \), per (angular) frequency is given as

\[ \frac{dW}{d\omega} = \frac{8\pi \omega^4}{3c^3} |\dot{d}(\omega)|^2, \]  
(A.10)

where \( \omega = 2\pi \nu \). Larmor’s formula with the dipole approximation is given by

\[ P = \frac{2}{3c^3} \ddot{d} \]  
(A.11)

and

\[ \ddot{d} = -e \mathbf{a}, \]  
(A.12)

where \( e \) is the charge and \( \mathbf{a} \) is the acceleration. Thus, Fourier transforming \( \ddot{d} \) gives

\[ -\omega^2 \hat{d}(\omega) = \frac{-e}{2\pi} \int_{-\infty}^{\infty} \mathbf{a} e^{i\omega t} \, dt. \]  
(A.13)
The time interval of the collision is of order $\Delta t \sim b/v$. For $\omega \gg v/b$, the integral is small, so only $\omega \ll v/b$ is considered. In this limit, $e^{i\omega \Delta t} \approx 1$, so

$$\hat{d}(\omega) = \frac{e}{2\pi \omega^2} \Delta v. \quad (A.14)$$

Substituting this into Equation A.10 gives

$$\frac{dW}{d\omega} = \frac{8\pi \omega^4}{3e^3} \frac{e^2}{4\pi^2 \omega^4} |\Delta v|^2 = \frac{2e^2}{3\pi c^3} |\Delta v|^2. \quad (A.15)$$

Considering the perpendicular component of the acceleration, $a_y$, $\Delta v$ can be estimated:

$$\Delta v = \int_{-\infty}^{\infty} a_y \, dt = \int_{-\infty}^{\infty} \frac{Ze^2 b}{m_e(v^2 t^2 + b^2)^{3/2}} \, dt = \frac{2Ze^2}{m_e vb}. \quad (A.16)$$

Substituting this result into Equation A.15 yields

$$\frac{dW}{d\omega} = \frac{8Z^2 e^6}{3\pi m_e^2 c^3 v^2 b^2}. \quad (A.17)$$

The total emission for a given electron number density, $n_e$, and ion number density, $n_i$ is given by

$$\frac{dW}{dt \, d\omega \, dV} = n_e n_i v \int \frac{dW}{d\omega} \, dA = n_e n_i v 2\pi \int_{b_{\text{min}}}^{b_{\text{max}}} \frac{8Z^2 e^6}{3\pi m_e^2 c^3 v^2 b^2} b \, db. \quad (A.18)$$
This is simply given by

\[
dW \over dt \, d\omega \, dV = n_e n_i \frac{16Z^2 e^6}{3m_e^2 c^3 v} \ln \left( \frac{b_{\text{max}}}{b_{\text{min}}} \right) = n_e n_i \frac{16\pi Z^2 e^6}{3\sqrt{3}m_e^2 c^3 v} g_{\text{ff}}(v, \omega), \tag{A.19}
\]

where \(g_{\text{ff}}(v, \omega)\) is the Gaunt factor that corrects for quantum mechanical effects and is defined here as

\[
g_{\text{ff}}(v, \omega) = \sqrt{3} \ln \left( \frac{b_{\text{max}}}{b_{\text{min}}} \right). \tag{A.20}
\]

To get the total emissivity, assume the electrons follow a Maxwell velocity distribution, and integrate over velocities. The Maxwell velocity distribution is given by

\[
f(v) = 4\pi \left( \frac{m_e}{2\pi k_B T} \right)^{3/2} v^2 e^{-m_e v^2/2k_B T} \propto v^2 e^{-m_e v^2/2k_B T}, \tag{A.21}
\]

where the \(4\pi\) comes from assuming isotropy, but the factors out front will cancel and only the dependence on velocity is necessary. The minimum velocity required for an electron to emit a photon is \(\frac{1}{2}m_ee^2 > h\nu\). Thus,

\[
v_{\text{min}}^2 = \frac{2h\nu}{m_e}. \tag{A.22}
\]

Therefore,

\[
dW \over dt \, d\omega \, dV = \int_{v_{\text{min}}}^{\infty} n_e n_i \frac{16Z^2 e^6}{3\sqrt{3}m_e^2 c^3 v^2} g_{\text{ff}}(v, \omega) v^2 e^{-m_e v^2/2k_B T} dv \over \int_{0}^{\infty} v^2 e^{-m_e v^2/2k_B T} dv. \tag{A.23}
\]

Evaluation of the integrals give:

\[
\int_{v_{\text{min}}}^{\infty} \frac{1}{v^2} e^{-m_e v^2/2k_B T} dv = \frac{1}{2} \left( \frac{m_e}{2k_B T} \right)^{-1} e^{-\nu/k_B T}. \tag{A.24}
\]
and

\[
\int_0^\infty v^2 e^{-m_e v^2/2k_B T} \, dv = \frac{\sqrt{\pi}}{4} \left( \frac{m_e}{2k_B T} \right)^{-3/2}.
\]  \hspace{1cm} (A.25)

Putting all the constants together gives

\[
\frac{dW}{dt \, d\omega \, dV} = n_e n_i \frac{16 Z^2 e^6}{3m_e^2 c^3} \left( \frac{2\pi m_e}{3k_B T} \right)^{1/2} e^{-h\nu/k_BT} g_{ff}(v, \omega).
\]  \hspace{1cm} (A.26)

Substituting \( \omega = 2\pi\nu \) and letting \( W \to E \) gives the desired result of the emissivity for thermal bremsstrahlung emission:

\[
\frac{dE}{dt \, d\nu \, dV} \equiv \varepsilon_{\nu} = \frac{32\pi Z^2 e^6}{3m_e^2 c^3} \left( \frac{2\pi m_e}{3k_B T} \right)^{1/2} n_e n_i e^{-h\nu/k_BT} g_{ff}(\nu).
\]  \hspace{1cm} (A.27)
APPENDIX B

COSMOLOGY

Cosmology is the study of the large scale Universe. The Friedmann equations describe the Universe, and are governed by Einstein’s Theory of General Relativity. The Friedmann equations are presented below and key ingredients are highlighted, while suppressing the tedious calculations. This will be a necessary equation for the next section where the Robertson-Walker metric is derived. Cosmologists often use \( c = 1 \) for convenience, but since this can lead to confusion, \( c \) is shown throughout these equations. In general, the metric can be written in a compact form as

\[
 ds^2 = g_{\mu\nu} dx^\mu dx^\nu. \tag{B.1}
\]

Here \( g_{\mu\nu} \) is the (covariant) metric tensor, and the Einstein summation convention is used where repeated indices are summed over (Greek letters are summed from 0 to 3 while Roman letters are summed from 1 to 3). In this notation, \((x^0, x^1, x^2, x^3) = (ct, r, \theta, \phi)\). For the Robertson-Walker metric, which assumes homogeneity and isotropy, \( g_{00} = 1, g_{11} = -R^2(t)/(1-kr^2), g_{22} = -R^2(t)r^2, g_{33} = -R^2(t)r^2 \sin^2 \theta, \) and all off diagonal elements are zero. This metric is derived in detail in Section C.1, but for
now, assume this is correct. The Christoffel symbol is defined as

\[ \Gamma^\lambda_{\mu\nu} = \frac{1}{2} g^{\lambda\alpha} \left( \partial_\mu g_{\nu\alpha} + \partial_\nu g_{\mu\alpha} - \partial_\alpha g_{\mu\nu} \right), \]  

(B.2)

where

\[ \partial_\mu \equiv \frac{\partial}{\partial x^\mu} \]  

(B.3)

and \( g^{\mu\nu} \) is the contravariant analog, i.e., the inverse matrix of \( g_{\mu\nu} \). Therefore, \( g^{\mu\alpha} g_{\alpha\nu} = \delta^\mu_\nu \), where \( \delta^\mu_\nu \) is the Kronecker delta. There are 19 non-zero Christoffel symbols, which are provided here:

\[ \Gamma^0_{01} = \frac{\dot{R}}{c R}, \quad \Gamma^0_{22} = \frac{\dot{R} r^2}{c}, \quad \Gamma^0_{33} = \frac{\dot{R} r^2 \sin^2 \theta}{c} \]  

(B.4)

\[ \Gamma^1_{01} = \frac{\dot{R}}{c R}, \quad \Gamma^1_{11} = \frac{k r}{(1 - k r^2)}, \quad \Gamma^1_{22} = -r (1 - k r^2), \quad \Gamma^1_{33} = -r (1 - k r^2) \sin^2 \theta \]  

(B.5)

\[ \Gamma^2_{02} = \frac{\dot{R}}{c R}, \quad \Gamma^2_{12} = \frac{1}{r}, \quad \Gamma^2_{32} = -\sin \theta \cos \theta \]  

(B.6)

\[ \Gamma^3_{03} = \frac{\dot{R}}{c R}, \quad \Gamma^3_{13} = \frac{1}{r}, \quad \Gamma^3_{23} = \frac{\cos \theta}{\sin \theta} \]  

(B.7)

The 19 elements come from the fact that \( \Gamma^\lambda_{\mu\nu} = \Gamma^\lambda_{\nu\mu} \). The Ricci tensor is given as

\[ R_{\mu\nu} = \partial_\lambda \Gamma^\lambda_{\mu\nu} - \partial_\nu \Gamma^\lambda_{\mu\lambda} + \Gamma^\lambda_{\mu\sigma} \Gamma^\sigma_{\lambda\nu} - \Gamma^\sigma_{\mu\lambda} \Gamma^\lambda_{\nu\sigma}, \]  

(B.8)
and is only non-zero when $\mu = \nu$.

\[
R_{00} = -\partial_0 \left( \Gamma^1_{01} + \Gamma^2_{02} + \Gamma^3_{03} \right) - \left( \Gamma^1_{01} \Gamma^1_{01} + \Gamma^2_{02} \Gamma^2_{02} + \Gamma^3_{03} \Gamma^3_{03} \right) \quad (B.9)
\]

\[
= \frac{-3\ddot{R}}{c^2R}
\]

\[
R_{11} = \partial_0 \Gamma^0_{11} - \Gamma^1_{10} \Gamma^0_{11} - \partial_1 \Gamma^2_{12} + \Gamma^0_{11} \Gamma^2_{02} + \Gamma^1_{11} \Gamma^2_{12} - \Gamma^2_{12} \Gamma^2_{12} \quad (B.10)
\]

\[
- \partial_1 \Gamma^3_{13} + \Gamma^0_{11} \Gamma^3_{03} + \Gamma^1_{11} \Gamma^3_{13} - \Gamma^3_{13} \Gamma^3_{13}
\]

\[
= \frac{\dot{R}\ddot{R} + 2\dot{R}^2 + 2kc^2}{c^2(1 - kr^2)}
\]

\[
R_{22} = \partial_0 \Gamma^0_{22} - \Gamma^2_{20} \Gamma^0_{22} + \partial_1 \Gamma^1_{22} + \Gamma^0_{22} \Gamma^1_{01} + \Gamma^1_{22} \Gamma^1_{11} - \Gamma^2_{21} \Gamma^1_{22} \quad (B.11)
\]

\[
- \partial_2 \Gamma^3_{23} + \Gamma^0_{22} \Gamma^3_{03} + \Gamma^1_{22} \Gamma^3_{13} - \Gamma^3_{23} \Gamma^3_{23}
\]

\[
= \frac{(R\ddot{R} + 2\dot{R}^2 + 2kc^2) r^2}{c^2}
\]

\[
R_{33} = \partial_0 \Gamma^0_{33} - \Gamma^3_{30} \Gamma^0_{33} + \partial_1 \Gamma^1_{33} + \Gamma^0_{33} \Gamma^1_{01} + \Gamma^1_{33} \Gamma^1_{11} - \Gamma^3_{31} \Gamma^1_{33} \quad (B.12)
\]

\[
+ \partial_2 \Gamma^2_{33} + \Gamma^0_{33} \Gamma^2_{02} + \Gamma^1_{33} \Gamma^2_{12} - \Gamma^3_{32} \Gamma^2_{33}
\]

\[
= \frac{(R\ddot{R} + 2\dot{R}^2 + 2kc^2) r^2 \sin^2 \theta}{c^2}.
\]

Thus, in general, the space components of the Ricci tensor can be written as

\[
R_{ij} = -g_{ij} \left( \frac{\dot{R}}{R} + \frac{2\dot{R}^2}{R^2} + \frac{2kc^2}{R^2} \right) \quad (B.13)
\]
The Ricci scalar is defined as
\[ \mathcal{R} = g^{\mu\nu} R_{\mu\nu}, \]  
(B.14)
and
\[ \mathcal{R} = \frac{-6}{c^2} \left( \frac{\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} + \frac{k c^2}{R^2} \right). \]  
(B.15)

The energy-momentum tensor is given by
\[ T_{\mu\nu} = \left( \rho + \frac{P}{c^2} \right) U_\mu U_\nu - P g_{\mu\nu}, \]  
(B.16)
where \( \rho \) is the density, \( U_\mu \) is the four-velocity, and \( P \) is the pressure. As seen from Equation B.16, \( T_{00} = \rho c^2 \) and \( T_{ij} = -P g_{ij} \). The Einstein tensor, \( G_{\mu\nu} \), is defined as
\[ G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \mathcal{R}, \]  
(B.17)
and can be solved with the above relations:
\[ G_{00} = \frac{3}{c^2} \left( \frac{\dot{R}^2}{R} + \frac{k c^2}{R^2} \right), \]  
(B.18)
\[ G_{ij} = \frac{g_{ij}}{c^2} \left( 2 \frac{\ddot{R}}{R} + \frac{2\dot{R}^2}{R^2} + \frac{k c^2}{R^2} \right). \]  
(B.19)

Now the Einstein Field equation is given as,
\[ G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}, \]  
(B.20)
where $G_{\mu\nu}$ is the Einstein tensor, $c$ is the speed of light, $G$ is the gravitational constant, and $T_{\mu\nu}$ is the energy-momentum tensor. Solving for $G_{00}$ and $G_{ij}$ gives the Friedmann equations:

\[
G_{00} \implies \left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G}{3} \rho - \frac{kc^2}{R^2}
\]  \hspace{1cm} (B.21)

\[
G_{ij} \implies \frac{\ddot{R}}{R} = -\frac{4\pi G}{3} \left(\rho + \frac{3P}{c^2}\right).
\]  \hspace{1cm} (B.22)

Add the cosmological constant, $\Lambda$, to Einstein’s equations to get

\[
G_{\mu\nu} - \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu},
\]  \hspace{1cm} (B.23)

which yields the familiar Friedmann equation

\[
\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G}{3} \rho - \frac{kc^2}{R^2} + \frac{\Lambda c^2}{3}.
\]  \hspace{1cm} (B.24)

Defining the Hubble parameter by $H = \dot{R}/R$ yields

\[
H^2 = \frac{8\pi G}{3} \rho - \frac{kc^2}{R^2} + \frac{\Lambda c^2}{3}.
\]  \hspace{1cm} (B.25)

Setting $k = 0$ in Equation B.21 gives the critical density

\[
\rho_{\text{crit}} = \frac{3H^2}{8\pi G}.
\]  \hspace{1cm} (B.26)
Now, define the Hubble parameter as $H(t) = H_0E(t)$, which allows the Friedmann equation to be written as

$$H^2 = H_0^2 \left( \frac{8\pi G}{3H_0^2}\rho - \frac{k c^2}{R^2 H_0^2} + \frac{\Lambda c^2}{3H_0^2} \right). \quad (B.27)$$

Defining the cosmological density parameter of matter as $\Omega_M = \rho/\rho_{\text{crit}}$ gives

$$\Omega_M = \frac{8\pi G\rho}{3H_0^2}. \quad (B.28)$$

Similarly, the cosmological density parameter of curvature is defined as

$$\Omega_k = -\frac{k c^2}{R^2 H_0^2}, \quad (B.29)$$

and the cosmological constant is given by

$$\Omega_\Lambda = \frac{\Lambda c^2}{3H_0^2}. \quad (B.30)$$

Therefore, the evolution function, $E(t)$, can be written in terms of the cosmological parameters as

$$E^2(t) = \Omega_M + \Omega_k + \Omega_\Lambda. \quad (B.31)$$

Defining the present day Hubble parameter as $H_0$, i.e., $H(0) = H_0 \implies E(0) = 1$, gives

$$1 = \Omega_M + \Omega_k + \Omega_\Lambda \implies \Omega_k = 1 - \Omega_M - \Omega_\Lambda. \quad (B.32)$$
It has been assumed that $R = R(t)$, but it is often more useful to write the Hubble parameter or the evolution function as a function of redshift, $z$, instead of time.

Consider a photon emitted by a nearby comoving source at a distance $R$. It will be Doppler shifted by

$$\frac{d\nu}{\nu} = \frac{-v}{c} = \frac{-HR}{c} = -H \, dt,$$

where $R = cd\tau$. Substituting $H \equiv \dot{R}/R$ into the above equation yields

$$\frac{d\nu}{\nu} = -\dot{R} \, dt \frac{R}{R} = -\frac{dR}{R} \frac{dt}{dt} \Rightarrow d\ln \nu = -d\ln R.$$  \hspace{1cm} (B.34)

Suppose the photon was emitted with frequency $\nu_e$ at time $t_e$ and observed at frequency $\nu_{\text{obs}}$ and time $t_{\text{obs}}$. Thus,

$$\int_{\nu_{\text{obs}}}^{\nu_e} d\ln \nu = -\int_{R_{\text{obs}}}^{R_e} d\ln R,$$

and

$$\frac{\nu_e}{\nu_{\text{obs}}} = \frac{R_{\text{obs}}}{R_e} = \frac{\lambda_{\text{obs}}}{\lambda_e} = (1 + z).$$  \hspace{1cm} (B.36)

It is standard to let $R_{\text{obs}} = 1$ which gives

$$R = \frac{1}{1 + z},$$  \hspace{1cm} (B.37)
where the subscript “e” has been dropped. Since $\rho \propto R^{-3} \implies \rho \propto (1 + z)^3$. Now, the evolution function can be written in terms of redshift:

$$E^2(z) = \Omega_M(1 + z)^3 + \Omega_k(1 + z)^2 + \Omega_\Lambda,$$

which is used in the equation for the angular diameter distance.
Here is a derivation of the angular diameter distance as given in Equation 3.48, but also for the general case allowing curvature ($\Omega_k \neq 0$).

### C.1 The Robertson-Walker Metric

Consider a three-sphere of radius $R$ embedded in four-dimensional Euclidean space. The coordinates are such that

\[ R^2 = x^2 + y^2 + z^2 + w^2, \]  

(C.1)

and in spherical polar coordinates this can be written as

\[
\begin{align*}
    w &= R \cos \chi \\
    z &= R \sin \chi \cos \theta \\
    x &= R \sin \chi \sin \theta \cos \phi \\
    y &= R \sin \chi \sin \theta \sin \phi.
\end{align*}
\]  

(C.2)
Here, \( R(t) \) is the scale factor, and \( \chi \) is the comoving coordinate. The line element, 
\[ d\ell^2 = dx^2 + dy^2 + dz^2 + dw^2 , \]
is given by 
\[ d\ell^2 = R^2 \left[ d\chi^2 + \sin^2\chi (d\theta^2 + \sin^2\theta \, d\phi^2) \right] . \quad (C.3) \]
The metric is given by 
\[ ds^2 = c^2 dt^2 - d\ell^2 , \]
so
\[ ds^2 = c^2 dt^2 - R^2(t) \left[ d\chi^2 + \sin^2\chi (d\theta^2 + \sin^2\theta \, d\phi^2) \right] . \quad (C.4) \]
This is the form of the Robertson-Walker (RW) metric for the case of positive curvature. This metric is also sometimes called the Friedmann-Lemaître-Robertson-Walker metric. Making the substitution for the new radial coordinate, \( r = \sin \chi \), yields 
\[ ds^2 = c^2 dt^2 - R^2(t) \left[ \frac{dr^2}{1 - r^2} + r^2(d\theta^2 + \sin^2\theta \, d\phi^2) \right] . \quad (C.5) \]
This metric can be converted to the form for constant negative curvature by letting \( R \to iR \) and \( \chi \to i\chi \). The line element in this case would be \[ d\ell^2 = R^2 \left[ d\chi^2 + \sinh^2\chi (d\theta^2 + \sin^2\theta \, d\phi^2) \right] . \quad (C.6) \]
Now, changing coordinates to \( r = \sinh \chi \) yields the metric for negative curvature:
\[ ds^2 = c^2 dt^2 - R^2(t) \left[ \frac{dr^2}{1 + r^2} + r^2(d\theta^2 + \sin^2\theta \, d\phi^2) \right] . \quad (C.7) \]
For the case of zero curvature, or flat space, the line element is given as

\[ d\ell^2 = R^2 \left[ d\chi^2 + \chi^2 \left( d\theta^2 + \sin^2 \theta \, d\phi^2 \right) \right], \quad (C.8) \]

and letting \( r = \chi \) yields

\[ ds^2 = c^2 dt^2 - R^2(t) \left[ dr^2 + r^2 \left( d\theta^2 + \sin^2 \theta \, d\phi^2 \right) \right]. \quad (C.9) \]

Thus, in general, the Robertson-Walker metric may be written in the following form:

\[ ds^2 = c^2 dt^2 - R^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2 \left( d\theta^2 + \sin^2 \theta \, d\phi^2 \right) \right], \quad (C.10) \]

where

\[ k = \begin{cases} 
-1, & \text{negative curvature} \\
0, & \text{zero curvature} \\
+1, & \text{positive curvature} 
\end{cases} \quad (C.11) \]

### C.2 Positive Curvature: Closed Universe

Consider the above metric for space with constant positive curvature, \( k = 1 \).

Light follows null geodesics, i.e., \( ds = 0 \), and along a radial line, \( d\theta = d\phi = 0 \).

Therefore,

\[ c^2 dt^2 = R^2(t) \left[ \frac{dr^2}{1 - kr^2} \right] \implies \frac{cdt}{R(t)} = \frac{dr}{\sqrt{1 - r^2}}. \quad (C.12) \]
Switch back to the original coordinate system by letting $r = \sin \chi$, so $dr = \cos \chi \, d\chi$.

Then,

$$\frac{dr}{\sqrt{1 - r^2}} = \frac{\cos \chi \, d\chi}{\sqrt{1 - \sin^2 \chi}} = d\chi,$$  \hfill (C.13)

and

$$c \int_{t_e}^{t_0} \frac{dt}{R(t)} = \int d\chi = \chi.$$  \hfill (C.14)

Note that

$$\frac{dt}{R(t)} = \frac{dR}{R(t) \dot{R}(t)},$$  \hfill (C.15)

and substitute this into the above equation to obtain

$$\chi = c \int_{t_e}^{t_0} \frac{dR}{R(t) \dot{R}(t)}.$$  \hfill (C.16)

The Hubble parameter is given as

$$H(t) \equiv \frac{\dot{R}(t)}{R(t)} = H_0 \, E(t) \implies \dot{R}(t) = R(t) \, H_0 \, E(t).$$  \hfill (C.17)

Recalling that

$$1 + z = \frac{R(t_0)}{R(t_e)},$$  \hfill (C.18)

and at $t = 0$

$$R(t) = \frac{R_0}{1 + z},$$  \hfill (C.19)

which results in

$$dR = \frac{-R_0}{(1 + z)^2} \, dz.$$  \hfill (C.20)
Therefore,
\[
\chi = c \int_{z_0}^{z_{\text{obs}}} \frac{(1 + z)^2}{R_0^2 H_0 E(z)} \frac{-R_0}{(1 + z)^2} \, dz = \frac{c}{R_0 H_0} \int_{z_{\text{obs}}}^{z_e} \frac{dz}{E(z)}. \tag{C.21}
\]

In the observer’s rest frame, let \(z_{\text{obs}} = 0\) and \(z_e = z\) giving the comoving distance
\[
r = \sin \chi = \sin \left( \frac{c}{R_0 H_0} \int_{0}^{z} \frac{dz}{E(z)} \right). \tag{C.22}
\]

The proper motion distance, or the physical distance, equals
\[
D_M(z) = R_0 (r_e - r_{\text{obs}}) = R_0 r \tag{C.23}
\]

where \(r_{\text{obs}}\) is taken to be 0. The angular diameter is related to the physical distance by
\[
D_A = \frac{1}{1 + z} D_M(z) = \frac{1}{1 + z} R_0 r \tag{C.24}
\]

(Carroll et al., 1992; Carroll, 2001). Conventionally, \(R_0 = 1\), and thus the angular diameter for a closed universe (positive curvature) is given as
\[
D_A = \frac{1}{(1 + z)} \sin \left( \frac{c}{H_0} \int_{0}^{z} \frac{dz}{E(z)} \right). \tag{C.25}
\]
C.3 Negative Curvature: Open Universe

For the negative curvature case \((k = -1)\), the metric is given as

\[
d s^2 = c^2 d t^2 - R^2(t) \left[ \frac{d r^2}{1 + k r^2} + r^2 (d \theta^2 + \sin^2 \theta \, d \phi^2) \right]. \tag{C.26}
\]

Making the substitution \(r = \sinh \chi\) again simplifies the integration since

\[
\frac{d r}{\sqrt{1 + k r^2}} = \frac{\cosh \chi \, d \chi}{\sqrt{1 + \sinh^2 \chi}} = d \chi. \tag{C.27}
\]

Now, the only difference from the above result is that \(\sin \chi \to \sinh \chi\). Therefore, the angular diameter distance for the open universe is given as

\[
D_A = \frac{1}{(1 + z)} \sinh \left( \frac{c}{H_0} \int_0^z \frac{dz}{E(z)} \right). \tag{C.28}
\]

C.4 Zero Curvature: Flat Universe

The latest results from WMAP indicate that the Universe is flat, or \(\Omega_k = 0\) (Komatsu et al., 2011). In this case, \(k = 0\), and the metric is simply

\[
d s^2 = c^2 d t^2 - R^2(t) \left[ d r^2 + r^2 (d \theta^2 + \sin^2 \theta \, d \phi^2) \right]. \tag{C.29}
\]

Following the same procedure as before, but instead for this case, \(r = \chi\). Therefore, compared to the above result for negative curvature, \(\sin \chi \to \chi\), and the angular
Figure C.1: A plot showing the angular diameter distance redshift relation for a flat Universe with $h = 0.702$, $\Omega_M = 0.27$, and $\Omega_\Lambda = 0.73$.

The angular diameter is found to be

$$D_A = \frac{c}{(1+z)H_0} \int_0^z \frac{dz}{E(z)}.$$  

A plot of the relationship between angular diameter distance, $D_A$, and redshift, $z$, is shown in Figure C.1 using cosmological parameters from WMAP7, i.e., $h = 0.702$, $\Omega_M = 0.27$, and $\Omega_\Lambda = 0.73$ (Komatsu et al., 2011). The angular diameter reaches a maximum around $z \approx 1.6$ and starts to decrease. $D_A$ decreases because the angular size will actually increase. Since the Universe is expanding, at large redshift objects were closer together, and thus the object will appear larger in the sky (relative to the size of the Universe).
C.5 General Case

The cosmological curvature parameter is given by

\[ |\Omega_k| = \frac{k c^2}{R_0^2 H_0^2} = \frac{k c^2}{H_0^2} \]  \hspace{1cm} (C.31)

for \( R_0 = 1 \). The angular diameter can be written in terms of \( \Omega_k \), and in general is given as

\[ D_A = \frac{c}{(1 + z) H_0} \frac{1}{\sqrt{|\Omega_k|}} \text{sinn} \left( \sqrt{|\Omega_k|} \int_0^z \frac{dz}{E(z)} \right), \]  \hspace{1cm} (C.32)

where

\[ \text{sinn} x = \begin{cases} 
\sinh x, & \text{negative curvature, } k = -1, \Omega_k < 0 \\
x, & \text{zero curvature, } k = 0, \Omega_k = 0 \\
\sin x, & \text{positive curvature, } k = +1, \Omega_k > 0.
\]  \hspace{1cm} (C.33)
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