2013

Monte Carlo assessment of solid propellant burning rate measurement

John A. Evans

MONTE CARLO ASSESSMENT OF SOLID PROPELLANT BURNING RATE MEASUREMENT

by

JOHN A. EVANS

A THESIS

Submitted in partial fulfillment of the requirements for the degree of Master of Science in The Department of Mechanical and Aerospace Engineering to The School of Graduate Studies of The University of Alabama in Huntsville

HUNTSVILLE, ALABAMA

2013

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We, the undersigned members of the Graduate Faculty of The University of Alabama in Huntsville, certify that we have advised and/or supervised the candidate of the work described in this thesis. We further certify that we have reviewed the thesis manuscript and approve it in partial fulfillment of the requirements for the degree of Master of Science in Engineering in Aerospace Engineering.

Robert A. Frederick, Jr.	5/6/13	Committee Chair
Dr. Robert A. Frederick, Jr.	(Date)	
HW Coleman	5/6/13	
Dr. Hugh Coleman	(Date)	
Kader Frendi	5/6/13	
Dr. Kader Frendi	(Date)	
D. Keith Hollingsworth	5/6/13	Department Chair
Dr. Keith Hollingsworth	(Date)	
Shankar Mahalingam	05/06/13	College Dean
Dr. Shankar Mahalingam	(Date)	
Rhonda Kay Gaede	5/8/13	Graduate Dean
Dr. Rhonda Gaede	(Date)	

ABSTRACT

School of Graduate Studies
The University of Alabama in Huntsville

Degree: Master of Science in Engineering College/Dept.: Engineering/Mechanical and Aerospace Engineering

Name of Candidate: John A. Evans

Title: Monte Carlo Assessment of Solid Propellant Burning Rate Measurement

Two new methods for determining the burn rate of solid propellant have been developed for the ultrasonic pulse-echo technique. These methods are digital as opposed to the analog method that had been used in the past. The work presented is an uncertainty analysis of the two new methods as well the old method. A previous uncertainty analysis assumed that the propagation time uncertainty was constant throughout the burn; however, this may not be the case. A parametric study was performed that varied the propagation time uncertainty over a range where it was thought to lie. Monte Carlo simulations were chosen because of their simplicity. The results showed that the uncertainty of the new digital methods, 4.2% to 5.7%, compared very well to the uncertainty of the analog method, 3.5% to 5%. An uncertainty analysis was also performed on the temperature sensitivity of the propellant. The results showed that uncertainty was high varying from 40% to 78% for the conditions investigated, but this could be attributed to the small number of samples that were tested.

Abstract Approval: Committee Chair Robert A. Frederick Jr
Dr. Robert A. Frederick, Jr.

Department Chair D. Keith Hollingsworth
Dr. Keith Hollingsworth

Graduate Dean Rhonda Kay Gaede 5/8/13
Dr. Rhonda Gaede

ACKNOWLEDGMENTS

This work was sponsored by NASA Constellation University Institutes Project under grant NCC3-989 with Claudia Meyer as the project manager. I would like to thank Dr. Robert Frederick for his guidance in this endeavor, and especially for his patience. I would also like to thank my mother and father for their support and my girlfriend for her enthusiastic encouragement.

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LIST OF SYMBOLS

a	Pressure coefficient
a_c	Regression slope of coupling material
a_o	Initial pressure coefficient
a_{p0}	Regression slope of propellant
A_b	Burn surface area
A_t	Throat area
b_c	Regression y-intercept of coupling material
b_{p0}	Regression y-intercept of propellant
B	Systematic error
P	Random error
r	Burn rate
T_i	Initial temperature
T_o	Reference temperature
U	Uncertainty
σ	Standard deviation
σ_p	Temperature sensitivity

LIST OF ACRONYMS

ARC	Atlantic Research Corporation
EDUM	Electronic Device for Ultrasonic Measurement
HTPB	Hydroxyl-terminated Polybutadiene
ONERA	Office National d’Etudes et de Recherches Aerospatiales
UAH	University of Alabama in Huntsville
UIUC	University of Illinois at Urbana-Champaign

CHAPTER 1

INTRODUCTION

Due to its simplicity, solid rocket propulsion was developed centuries ahead of liquid propulsion. The first development probably took place around 1000 A.D. after the discovery of black powder in China. The Chinese learned that through varying the proportion of the ingredients they were able to make either explosive or propulsive devices [1]. Today at the mention of solid rocket motors, the first images that usually come to mind are of the solid rocket booster motors used on the Space Shuttle. However, solid rockets motors are also commonly used for tactical missiles, in-space operations, and even for ejection seats.

1.1 Solid Rocket Motor Overview

A simple solid rocket motor can be seen in Figure 1.1. The motor is enclosed in a case made from metal or composite fiber materials. The inside surface of the case has an insulation layer to protect the case from the high propellant temperatures. The grain is the propellant and can be solid throughout the motor, or it can have a port of many different shapes running its length. At the forward end of the motor an igniter is in place to start the combustion process. Finally, at the aft end is the nozzle which accelerates the hot gas and provides the thrust for the rocket. Nozzles are typically made from carbon/phenolic, silica-phenolic, or carbon/carbon materials in order to minimize heat

and erosion problems. Most nozzles are fixed in place, but some have thrust-vector-control systems to help steer the rocket [2, 3].

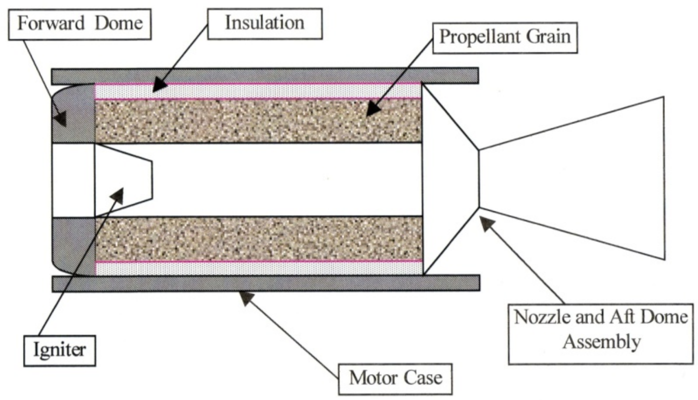

Figure 1.1: Cross section of a solid rocket motor

1.2 Burning Rate of Solid Propellants

The burning rate is one of the most important parameters to consider when designing or analyzing a solid rocket motor. When a motor is burning, the burning surface recedes in the direction normal to the burning surface. The rate of regression of the surface is called the burning rate, $\dot{r}$. From theory and experiment, the burning rate has been shown to be a function of the chamber pressure and is defined

$$\dot{r} = ap^n \,. \tag{1.1}$$

Equation (1.1) is known as St. Robert's Law, and typical values for $\dot{r}$ range from 0.5 in/s to 2 in/s. Values for the burning rate exponent generally range from $0.2 < n < 0.7$, although values of $n = 0$ and $n < 0$ are also possible [2]. The burning rate coefficient is

dependent on the propellant initial temperature, and will be discussed in Section 1.3. Figure 1.2 shows the burning rate behavior.

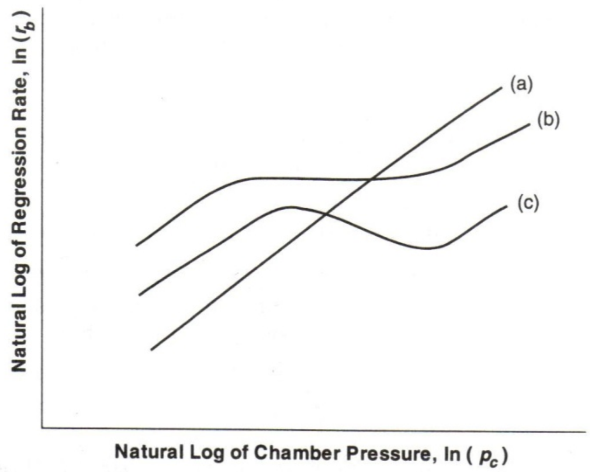

Figure 1.2: Behavior of (a) St. Robert's Law, (b) plateau burning, and (c) mesa burning [3]

1.3 Temperature Sensitivity of Solid Propellants

As previously mentioned, the burning rate coefficient, *a*, is dependent on the propellant initial temperature with [4]

$$a = a_0 \exp[\sigma_p (T_i - T_0)]\,. \tag{1.2}$$

In Equation (1.2) the term σ_p is the temperature sensitivity of burning rate at constant pressure, and is defined

$$\sigma_p = \left(\frac{\delta \ln r}{\delta T} \right)_p \tag{1.3}$$

with typical values ranging from 0.002 to 0.04 $°F^{-1}$ [3]. Another temperature coefficient is the temperature sensitivity of pressure at constant motor geometry

$$\pi_K = \left(\frac{\delta \ln p}{\delta T} \right)_K . \tag{1.4}$$

K is the ratio of the burning surface, A_b, to the nozzle throat area, A_t [3]. Since K is constant, π_K is used for neutral burning motors. Typical values for π_K range from 0.18 % to 0.36% $°C^{-1}$ [5]. The relationship between the two coefficients is [3]

$$\pi_K = \frac{\sigma_p}{1-n} . \tag{1.5}$$

From Equations (1.1) and (1.2), it can be observed that a lower initial propellant temperature means a propellant will have a lower burning rate, and thus a longer burning time, t_b [6]. Conversely, a propellant with a higher initial temperature will have a higher burning rate, and have a shorter burning time. These relationships are shown in Figure 1.3.

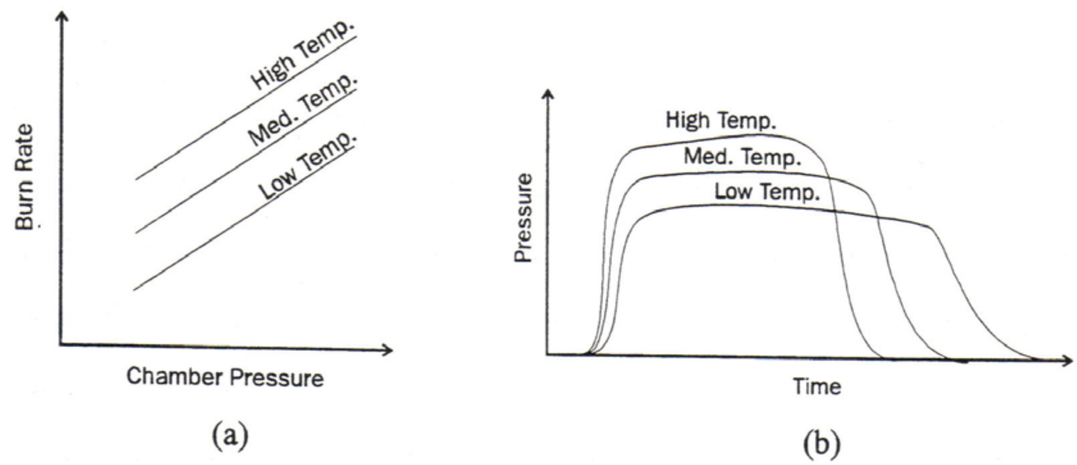

Figure 1.3 Initial propellant temperature dependence of (a) burning rate and (b) burning time. [3]

Motors using typical composite propellant show a burning time variation of 20% to 30% and a chamber pressure variation of 20% to 35% for initial propellant temperatures from -65°F to 160°F [3]. These are significant variations and would greatly affect the motor's performance and design.

1.4 Uncertainty Analysis

The burning rate and temperature sensitivity of a propellant are quantified by either measurement or calculation. With any measurement or calculation, there is always an implied uncertainty, U, that defines an interval in which the true value of the measured variable, X, will lie 95% of the time, $X \pm U$ [7]. The uncertainty usually is made up of two parts, systematic errors and random errors. The systematic error, B, is a fixed or constant component and is often referred to as bias. The random error, P, varies with each measurement and is often referred to as precision error. Both the systematic error

and random error can be made up of multiple errors called elemental errors. The total uncertainty is then defined as [7]

$$U = \sqrt{B^2 + P^2}\,. \tag{1.6}$$

In a calculation that requires measured variables, *X*, each of the variables will have an associated uncertainty, U_X, and must be accounted for. The result of the calculation, *r*, is called the data reduction equation [7]

$$r = r\left(X_1, X_2, \ldots, X_i\right). \tag{1.7}$$

The total uncertainty of the data reduction equation is [7]

$$U_r^2 = \left(\frac{\partial r}{\partial X_1}\right)^2 U_{X_1}^2 + \left(\frac{\partial r}{\partial X_2}\right)^2 U_{X_2}^2 + \ldots + \left(\frac{\partial r}{\partial X_i}\right)^2 U_{X_i}^2\,. \tag{1.8}$$

Another method that is often used to determine uncertainty is direct Monte Carlo simulations [7]. In this method, errors for measurement X_i are randomly drawn from a Gaussian distribution with a mean equal to zero and a standard deviation equal to one half its associated uncertainty. The errors are then added to the X_is, and the result, *r*, is calculated with the X_is. This process is repeated 10,000 times to get 10,000 values of *r*. The standard deviation, σ, of the *r* values is calculated and then multiplied by two in order to get the uncertainty, U_r [7].

$$U_r = 2\sigma \tag{1.9}$$

The 2σ interval will contain the true value of *r* 95% of the time. The obvious advantage of this method is not having to calculate the partial derivatives in Equation (1.8).

CHAPTER 2

BURN RATE MEASUREMENT TECHNIQUES

Numerous methods over the years have been used to determine the burn rate of solid propellant. An overview of three of the most common methods, strand burners, sub-scale motors, and the ultrasonic technique, is presented in the following sections. Where available, the uncertainty associated with those burn rate methods will also be discussed. The temperature uncertainty of the propellant, which is derived from the burn rate, will also be covered along its uncertainty.

2.1 Strand Burners

Strand burner tests involve burning a small strand of propellant in a pressure vessel [3, 8, 9]. During the test, the vessel is kept under constant pressure by using an exhaust nozzle or a remote, adjustable valve. Typically the strand is approximately 0.25 inches square and 6 inches long, and is coated with an epoxy on the sides and bottom to ensure end burning. Means of measuring the burning rate are through the use of timing wires. The wires are inserted into holes that are drilled at specified distances into the side of the strand for two to three wires. When the propellant is ignited, the timing wires are broken sequentially during the test. The wires are connected to a data acquisition system and with this, the time interval between when the wires are broken can be found. The burn rate can then be calculated using [9]

$$\dot{r} = \frac{x_2 - x_1}{t_2 - t_1} \quad . \tag{2.1}$$

An example of a strand burner is shown in Figure 2.1. A disadvantage of this technique is that since the test is performed at constant pressure, many different tests must be performed to fully characterize the burn rate versus pressure curve. Another drawback is that the small sample size leads to heat losses, which cause the burn rate to be less than in the full size motor [10,11].

The University of Minnesota conducted strand burner testing as early as 1947 [8]. They tested nineteen samples at 68°F and 1,000 psi. Taking the standard deviation of their test data, they estimated an uncertainty of 0.25% (1σ). Other research shows that with careful and consistent preparation of the strands, the relative uncertainty can be held to 0.5% [12].

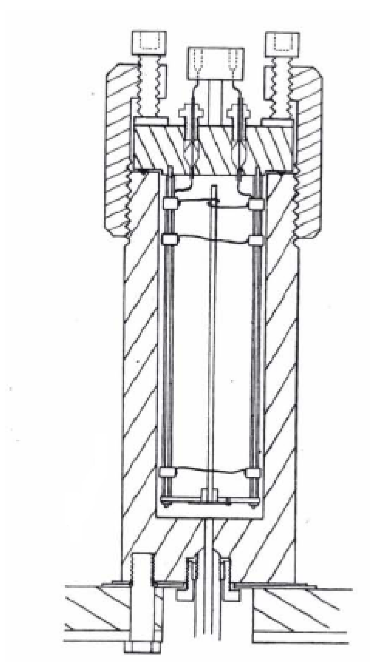

Figure 2.1: Schematic of a strand burner [8]

2.2 Sub-scale Motors

Another method to determine the propellant burn rate is to actually test fire a small sub-scale motor. The motor is weighed and the web thickness (distance from the initial burning surface to the insulated case wall) case is measured before firing. After firing, the chamber pressure and thrust are measured with respect to time. The burn rate can then be determined from the web thickness and the total burn time (assuming all the propellant was burned). Many tests must be performed to create an accurate burn rate vs. pressure curve just like the strand burner [10]. At typical sub-scale motor is shown in Figure 2.2. For the Redesigned Solid Rocket Motors used on the Space Shuttle, five inch center perforated motors are tested to determine the burn rate of the propellant batch.

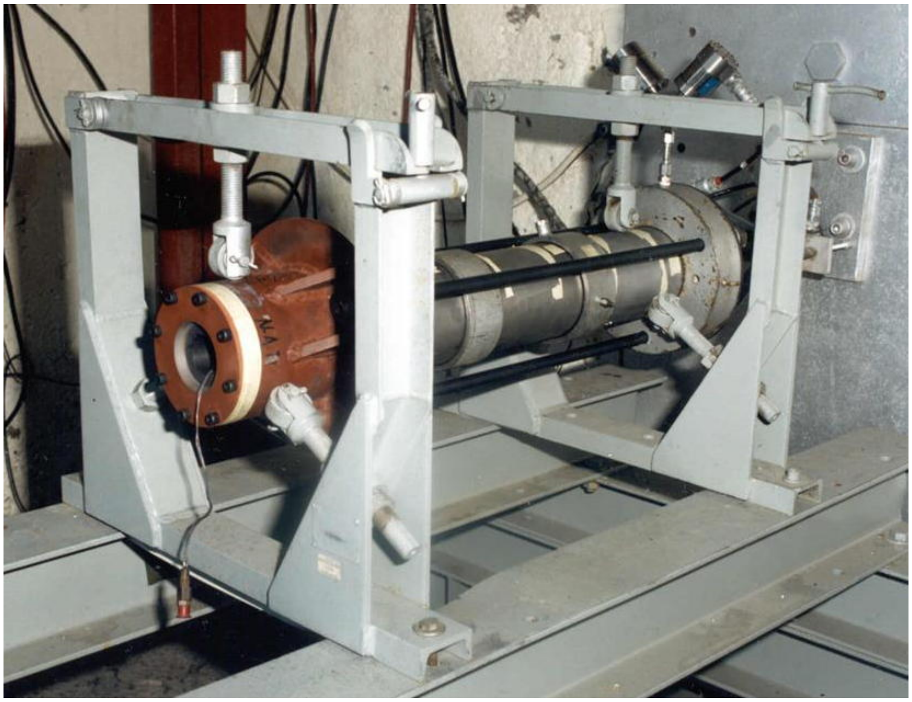

Figure 2.2: Sub-scale motor testing setup [13]

Three measurements are necessary for determining the burn rate of sub-scale motor: the propellant web thickness, the burn time, and the average pressure. Inaccuracies in these measurements can result in a burn rate error of up to 5% [13]. However, many manufacturers use this method and report uncertainties of about 1% (1σ) [12].

2.3 Ultrasonic Pulse-Echo Method

The ultrasonic pulse-echo method is based on measuring the propagation time of an ultrasonic wave through the propellant [9, 14-17]. An ultrasonic transducer sends out a pulse that travels through the propellant. When the pulse encounters a difference in impedance, like the propellant surface, part of the signal is reflected back and detected by the ultrasonic transducer [9]. Using the propagation time, τ, and the speed of sound in the propellant, c, the thickness of the propellant, t_P, can be calculated with [9]

$$t_P = \frac{\tau \cdot c}{2}. \tag{2.2}$$

The ultrasonic pulse-echo method was developed in France by the Office National d'Etudes et de Recherches Aerospatiales (ONERA) [14-18], and has been used many groups including the University of Illinois at Urbana-Champaign [19-22], the Atlantic Research Corporation [23], and the University of Alabama in Huntsville [9, 24-28].

2.3.1 ONERA

ONERA has used the ultrasonic pulse-echo method for testing small propellant samples and for sub-scale motors [14, 18]. For small samples two different setups are

used. The first setup uses a closed combustion bomb for testing at high pressures in the range of 5 MPa to 45 MPa [16]. The second setup also uses the closed combustion bomb but also has a nozzle mounted to an extension tube. This setup is designed for testing at pressures lower than 4 MPa [16]. Both setups use the Electronic Device for Ultrasonic Measurement (EDUM) to control the ultrasonic transducer [14, 17]. ONERA has not provided any information on the burn rate uncertainty of either setup.

ONERA has tested two different types of sub-scale motors. The first was a nozzleless motor that was used to determine the effects of grain geometries on the motor's performance [18]. The motor was instrumented with five ultrasonic transducers mounted axially along the rocket. The second type of sub-scale motor was a slab motor which was used to simulate a two-dimensional nozzleless motor. The purpose of this motor was also to determine the effects of grain geometries on the motor's performance [18]. Again ONERA has not provided any information on the burn rate uncertainty of either motor.

2.3.2 University of Illinois at Urbana-Champaign

The University of Illinois at Urbana-Champaign (UIUC) used the ultrasonic pulse-echo method to determine the burn rate perturbations due to oscillatory pressure changes, by testing small propellant samples in an oscillatory burner [21, 22]. UIUC used a digital method to determine the burn rate as compared to the analog method of ONERA. The ultrasonic transducer transmitted and received the ultrasonic wave. The waveforms were recorded by the data acquisition system and stored into a MATLAB program. A cross correlation algorithm was then performed on the waveforms to

determine the burn rate [21]. No information on the burn rate uncertainty of this method has been published by UIUC at this time.

2.3.3 Atlantic Research Corporation

The ultrasonic pulse-echo method developed by the Atlantic Research Corporation used a cross correlation technique to determine the burn rate. Small propellant samples were burned at low pressure, with tests lasting as long as 15 seconds. Although no information on the uncertainty of their method was released, they did state that the average burn rate obtained was approximately 7% lower than the reference strand burner data [23].

2.3.4. University of Alabama in Huntsville

The University of Alabama in Huntsville (UAH) developed and ultrasonic pulse-echo system in 1995, using an EDUM purchased from ONERA [14]. UAH uses both a closed and an open combustion bomb for their research. With a closed combustion bomb, the propellant burn rate can be determined over a wide pressure range in one test, as opposed to the multiple tests it would take for a strand burner. The research at UAH has included work on steady-state ballistics, propellant response function, transient ballistics, and temperature sensitivity [10, 14, 24-27]. The measurement uncertainty of the technique has also been investigated [9, 11, 28].

Traditionally the analog EDUM method was used at UAH to determine the propellant burn rate. Occasionally during a test the EDUM would lose the reflected

signal and would no longer track the propellant surface. In order to have a more reliable testing system, UAH recently developed two new digital methods for determining the burn rate [22]. The first is the Zero Crossing method which is similar to the EDUM method, and the other is the Cross Correlation method which is similar to the methods at UIUC and ARC.

An uncertainty analysis has previously been performed on the EDUM method at UAH. The analysis used a Monte Carlo simulation that showed the relative uncertainty in the burn rate to be 3.5% to 5% [9]. However, the uncertainty used for the propagation time measurement had been debated. The burn rate uncertainties for the new digital methods will be determined in this work.

2.4 Temperature Sensitivity Determination

Equation (1.3) shows that the temperature sensitivity can be calculated once burn rates at two different temperatures at the same pressure have been determined. Except for UAH, none of the previously mentioned institutions calculate the temperature sensitivity. Since UAH performs its testing in a temperature conditioning chamber, it has the ability to determine the temperature sensitivity.

Reported values for the uncertainty of the temperature sensitivity are commonly as high as 15% [12]. This high uncertainty could possibly be attributed to the fact that motor manufactures rarely perform more than three tests at each temperature [12]. The temperature sensitivity uncertainty determined at UAH will be examined in this work.

2.5 Objective

Even though the burn rate uncertainty for the EDUM method had been previously determined at UAH, there were still questions on the propagation time uncertainty used for the analysis. It was assumed the propagation time uncertainty remained constant throughout the burn; however it never clear this was true. Therefore, the objective of this work is to perform a parametric study in order to determine the effect of propagation time uncertainty on the burn rate uncertainty. A range of 0.1-0.5 μs will be used for the propagation time uncertainty. This range will be used to determine the burn rate uncertainty for the analogue EDUM method along with the new digital Zero Crossing and Cross Correlation methods. Also, the uncertainty of the temperature sensitivity will be determined.

CHAPTER 3

APPROACH

For solid propellant burn rate determination, the Propulsion Research Center utilizes a pulse-echo ultrasonic burn rate measurement technique. An ultrasonic transducer in conjunction with a closed combustion bomb allows samples of solid propellant to be burned while simultaneously raising the pressure. Thus, the determination of the burn rate of a sample over a wide range of pressures is possible with a single test. In addition, the facility has a temperature conditioning chamber which may be used to thermally soak propellant samples prior to testing in order to determine burn rate temperature sensitivity.

3.1 Propellant

The investigation used an ammonium perchlorate (AP)/Aluminized/hydroxyl-terminated polybutadiene (HTPB) composite propellant. The bimodal-oxidizer propellant was prepared in a one gallon mixer and cast into three cardboard tubes 12 in. long by 1.25 in. in diameter. After curing, the tubes were cut into approximately 0.5 in. thick samples with a lathe. The average mass of a sample was 20 grams. The mixing, casting, and preparation of the samples were performed by the U.S. Army's Aviation and Missile Research, Development and Engineering Center (AMRDEC) at the Redstone Arsenal. For this investigation, six samples were tested. Test 1, 2, and 3 were tested at 75°F, and Tests 4, 5, and 6 were tested at 145°F. Figure 3.1 shows a typical propellant sample.

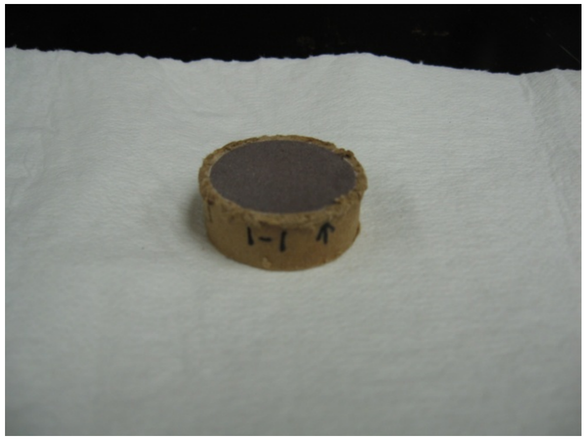

Figure 3.1: Propellant sample used for testing

3.2 Experimental Setup

The testing hardware and data acquisition system used for the investigation is discussed in the following sections. The main components of the testing hardware were a closed combustion bomb, a temperature test chamber, and a ventilation system. The data acquisition system included the EDUM, two National Instruments cards, an oscilloscope, and LabVIEW software.

3.2.1 Testing Hardware

The propellant samples were burned in a closed combustion bomb (Figures 3.2 and 3.2) that is rated up to 5,000 psi. The combustion bomb was placed inside a Russells temperature test chamber which allows the temperature of the propellant sample and the test hardware to be controlled at a constant level. The temperature test chamber sits in a 5’x 6.5’ test cell with one foot thick steel-reinforced walls with a 2.5” thick sliding steel

door. The combustion bomb is made of stainless steel since it has a high durability against the volatile gasses produced during the burning. In the event of over-pressurization, the combustion bomb has a check valve with a cracking pressure of 5,000 psi which will allow the excess gas to be vented into the test cell.

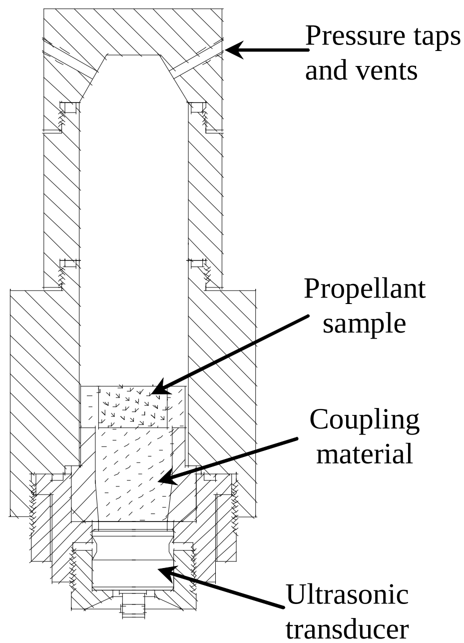

Figure 3.2: Schematic of combustion bomb showing the location of the propellant sample and ultrasonic transducer [9]

The combustion bomb has three ports for connecting two 5000 psi Setra pressure transducers and a ventilation system (Figure 3.2). One pressure transducer is connected directly to the combustion bomb inside the test chamber, while the other one is situated outside the test chamber and connected to the bomb through stainless steel tubing. The reason for the inside pressure transducer is minimize the pressure lag time of the outside pressure transducer due to the tubing connection. Before performing a test the outside

pressure transducer is calibrated. Using a transfer calibration from the outside to the inside transducer under lower pressurization rates, the outside transducer calibration is then transferred to the inside transducer.

A panel outside the test cell controls the ventilation system which is used to purge the combustion bomb of the volatile gasses produced during a test. During a purge, nitrogen is supplied to the combustion bomb at high pressure through the ventilation port, held for 2-3 seconds, and then exhausted back out the same port. A check valve on the supply line ensures the gasses stay in the exhaust line when exhausting. The gasses are then sent to a surge tank for depressurization. Finally, the gasses pass through a check valve into a tank filled with water where they are scrubbed and vented to the outside of the building. This process is repeated four times to get most of the volatile gasses out of the combustion bomb, so that it can be safely cleaned.

Figure 3.3: Combustion bomb inside the environmental test chamber

3.2.2 Data Acquisition System

The heart of the pulse-eco ultrasonic method is the EDUM developed by ONERA in France [17]. The EDUM sends a signal to the ultrasonic transducer (Panametrics V102 1.0/1.0) to send out an ultrasonic pulse at 1,000 Hz. The pulse travels through a coupling material in the propellant sample holder and then through the propellant (Figure 3.4). When the pulse encounters a difference in impedance, part of the signal is reflected back and detected by the ultrasonic transducer. The EDUM also receives the signal and measures the propagation time, τ, and outputs a voltage proportional to τ [17].

The reflections that the ultrasonic transducer detects occur at the coupling material/propellant interface and at the burning surface. The purpose of the coupling

material is to cause a delay in the burning surface echo return, so that the signal does not get lost in the internal echoes of the ultrasonic transducer. The coupling material also protects the ultrasonic transducer from the high temperature of the burning propellant. Figure 3.4 shows a schematic of the ultrasonic transducer setup and the waveform it produces as seen on an oscilloscope. Point 1 in Figure 3.4 is the signal produced by ultrasonic transducer and it travels in both directions. Point 2 is the reflection off of the back surface of the ultrasonic transducer. The back surface is generally attenuated to reduce the echo. Point 3 is the echo from the interface of the coupling material and the propellant sample. In order to reduce this echo the coupling material and propellant should have a similar density. Point 4 is the return echo from the propellant burning surface and is tracked during the test [17]. Using the propagation time of the signal and the speed of sound in the propellant and coupling material, the instantaneous thickness of the propellant can be found. The burning rate of the propellant can then be calculated by taking a time derivative of the instantaneous thickness.

Data are collected using two separate National Instruments PCI cards. The first card is a 14-bit National Instruments PCI-6224 data acquisition card. It is used to collect the combustion bomb internal pressure and the propagation time data output from the EDUM. The second card is a 12-bit National Instruments PCI-5122 High Speed Digitizer. It is used to record the actual waveforms of the ultrasonic pulse echo that were produced by the EDUM. These waveforms are similar to the waveform seen in the bottom of Figure 3.4. A Hewlett Packard 54602B oscilloscope is used to visualize the ultrasonic waveform during a test.

LabVIEW virtual instruments designed at the PRC were used to acquire and record all the data from a test. For a complete description of the virtual instruments and their computer code see Reference [22].

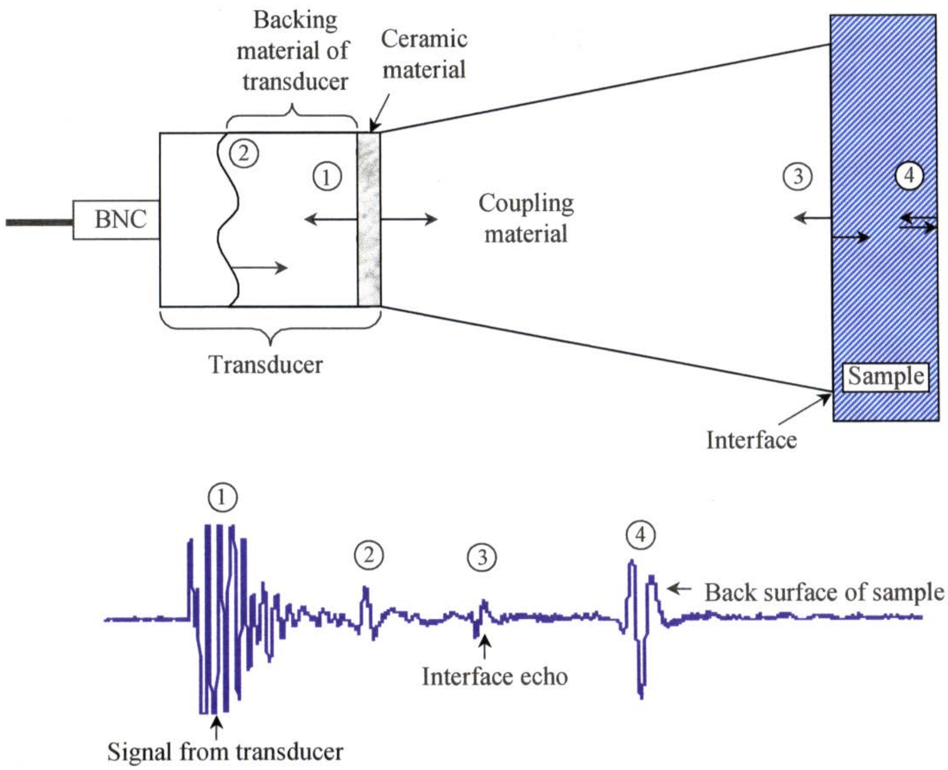

Figure 3.4: Schematic of the ultrasonic transducer setup and the waveform it produces [9]

3.3 Propagation Time Determination Methods

The propagation time, τ, of the ultrasonic signal was determined by three different methods: the EDUM method, the Zero Crossing method, and the Cross Correlation method. All three of these methods analyze each waveform in order to determine the location of the surface of the propellant and how thick a propellant sample and coupling

material are. The EDUM method is an analog method, whereas the Zero Crossing and Cross Correlation methods are digital methods.

3.3.1 EDUM Method

The EDUM method is an analog method of finding the signal propagation time through the propellant. The EDUM will output a waveform, a propagation time voltage (a voltage that is proportional to propagation time), two masks, and a propagation time gate. With the exception of the propagation time voltage, all of these outputs are displayed on an oscilloscope for use in the calibration of the EDUM. The EDUM works by finding the first zero crossing of the waveform after it detects the waveform peak associated with the propellant surface [17]. To help the EDUM identify the propellant surface peak, two masks and a threshold voltage are used. The masks "hide" peaks so that the EDUM does not lock onto them and produce false results (Figure 3.4). The first mask is calibrated so that the noise from the ultrasonic transducer and the peak from the reflection off the back of the transducer are covered. The second mask is calibrated to cover the same time as the first mask but to also cover the peak from the interface of the coupling material and the propellant sample. The time that these masks cover can be seen on the oscilloscope. Once these are set, the EDUM can then only lock onto a peak after these masks. After the second mask, there are many smaller peaks but only one considerable peak; this is the propellant surface peak (Figure 3.4). The threshold voltage is set at a voltage higher than the smaller peaks which appear before the propellant surface, so that the only peak that the EDUM can lock onto is the propellant surface peak. The propagation time gate indicates where the EDUM is taking its measurement for the propagation time, and is

shown on the oscilloscope which ensures the user that the correct peak is locked onto. Using this zero crossing, a voltage is returned by the EDUM that is proportional in time [17]. This time and voltage are displayed by the EDUM and are used in the calibration of the EDUM. During a firing, the EDUM continues to display the waveforms on the oscilloscope so that the operator can actually see the peak moving. As this peak moves, the zero crossing changes. As the zero crossing changes, the voltage outputted by the EDUM also changes and can be recorded. Using the calibration points that were recorded before the firing, propagation times for the burn can be found, [29].

3.3.2 Zero Crossing

The Zero Crossing method is a digital method for determining burn rate which is analogous to the analog EDUM method. A LabVIEW virtual instrument uses the PCI-5122 High Speed Digitizer to record the waveforms that are produced by the EDUM. These are the same waveforms that are seen on the oscilloscope. The waveforms are stored in an array whose dimensions are determined by the parameters set in the data acquisition system. A program written for MATLAB was created that would calculate the propagation time [22]. The algorithm for the Zero Crossing method, like the EDUM method, works off of two user inputs: a mask and a threshold. The mask is used to determine a start point in analyzing the data for the algorithm. It tells the code to ignore all the data (or every part of the waveform) in time before the mask [22]. The Zero Crossing method works on the principle that the surface of the propellant is the first zero crossing on the x-axis after the major peak of the surface echo. Since the waveform can cross the x-axis several times before it actually gets to this point, a threshold is set which

tells the program to ignore all zero crossings until the data are greater than this number. Once the algorithm has found where the data is past the mask, and where it is above the threshold, it will find the last cell in the array before the waveform crosses the x-axis and the cell in the array just after the waveform crosses the x-axis. Using the location of these cells, the algorithm interpolates a propagation time for the zero crossing and stores it in an array. The algorithm then repeats the process for the next waveform and continues until the propagation times for the total burn have been found [29].

3.3.3 Cross Correlation

The Cross Correlation method works by comparing the combined area under two waveforms [22]. The method assumes that the area under two combined waveforms will be the greatest when the two waveforms are aligned. A MATLAB program was written that compares the combined areas of the first waveform with subsequent waveforms [22]. It starts by comparing the two waveforms in their initial position. It then shifts the first waveform one time increment to the left and compares the combined area of the waveforms again. This shift is incremented and the combined areas are found until the first waveform is shifted to some maximum set point. Once the shifting is complete the algorithm evaluates where the combined area of the two waveforms is at a maximum and returns that shift value. The algorithm then increments to the next waveform and repeats the process until an entire set of waveforms have been compared with the initial waveform and the shift values have been recorded [29]. The shift values are converted to times and subtracted from the initial propagation time (which was determined by

averaging the first 20 propagation times found by the zero crossing algorithm) in order to determine the propagation time of the propellant surface throughout the burn [22].

3.4 Data Reduction

3.4.1 Burn Rate Determination

Once all the data has been collected for a test, it is reduced in order to find the propellant burn rate. A Mathcad worksheet is used for the data reduction; an example worksheet is provided in Appendix B. The pressure transducer voltages and propagation time voltages for each of the methods are imported into the worksheet. The initial thickness of the propellant sample, E_{p0}, which was measured with calipers before the test is entered into the worksheet. The pressure voltages from the test are then converted to pressures using data obtained from a pressure transducer calibration performed before the test using a dead weight tester. Next, the propagation time voltages from the EDUM are converted to propagation time using calibration data obtained from the EDUM before the test.

The propagation time that is measured is the propagation time through the coupling material and the propellant. Since the speed of sound in the coupling material and the propellant varies with pressure, a pre-test and post-test pressurization are conducted to determine their acoustic properties [9]. The two pressurization tests show that the propagation times through the coupling material and propellant are linear functions of pressure. Therefore, the propagation time through the coupling material, τ_c, can be expressed as a function of pressure, *P*, and two constants a_c and b_c, that represent the slope and y-intercept of the linear function [9]

$$\tau_c = a_c \cdot P + b_c . \tag{3.1}$$

Likewise, the propagation time through the coupling material and the propellant, τ_{p0}, can be expressed as a function of pressure, *P*, and two constants a_{p0} and b_{p0}, that represent the slope and y-intercept of the linear function [9]

$$\tau_{p0} = a_{p0} \cdot P + b_{p0} . \tag{3.2}$$

Using the initial propellant thickness, the pressure data, and the four regression constants, the instantaneous thickness of the propellant, E_p, can be found using

$$E_p = \frac{E_{p0} \cdot \left[\tau - \left(a_c \cdot P + b_c\right)\right]}{\left(a_{p0} \cdot P + b_{p0}\right) - \left(a_c \cdot P + b_c\right)} . \tag{3.3}$$

This equation represents the instantaneous thickness of the propellant as it burns during the test. See Reference [9] for a complete derivation of Equation (3.3).

For the burn rate determination, a running linear regression is applied to the instantaneous thickness data. A number of points, *N*, is taken from the start of the instantaneous thickness data, and the slope of these points is calculated as shown in Equation (3.4). This value is equated to the burn rate at the middle of the range of points. The range of points is then shifted one data point and the process is repeated. When the end of the range reaches the end of the instantaneous thickness data, a burn rate versus pressure curve can be created with the data. The entire process is done for each of the propagation time determination methods, and results in three burn rate curves.

$$r_k = \frac{N \sum_{i=k-\frac{N}{2}}^{i=k+\frac{N}{2}} E_i t_i - \sum_{i=k-\frac{N}{2}}^{i=k+\frac{N}{2}} E_i \sum_{i=k-\frac{N}{2}}^{i=k+\frac{N}{2}} t_i}{N \sum_{i=k-\frac{N}{2}}^{i=k+\frac{N}{2}} \left(t_i\right)^2 - \left(\sum_{i=k-\frac{N}{2}}^{i=k+\frac{N}{2}} t_i\right)} \tag{3.4}$$

3.4.2 Temperature Sensitivity

After determining the burn rates for all the tests, the temperature sensitivity was calculated for each propagation time method. First, the burn rate was found at 1,000 psi for each test. The temperature sensitivity was then calculated using

$$\sigma_P = \frac{\ln\left(\frac{r_4 + r_5 + r_6}{3}\right) - \ln\left(\frac{r_1 + r_2 + r_3}{3}\right)}{T_{145} - T_{75}} \qquad (3.5)$$

where r represents the burn rates at 1000 psi from each test, and T represents the test temperature. This was done for each propagation time method.

CHAPTER 4

UNCERTAINTY ANALYSIS

4.1 Uncertainties of Variables

Recall from Chapter 1, that with any measurement or calculation, there is always an implied uncertainty, *U*. The uncertainty usually is made up of two parts, systematic error, *B*, and random error, *P*, and both the systematic error and random error can be made up of multiple errors called elemental errors [7]. The following sections describe how the uncertainty of the initial propellant thickness, the pressure, the temperature, the propagation time, and the four regression constants were determined.

4.1.1 Uncertainty in Initial Propellant Thickness, U_{E0}

The propellant was cast and cured in cardboard tubes and cut into samples with a lathe. Each sample was approximately 0.5 in. thick and 1.25 in. in diameter. To the naked eye, the sample surface appeared to be flat. However, to allow for the possibility of a nonflat surface, a design criteria was created. It was assumed that the surface was off by 1° at most. With a diameter of 1.25 in. this would result in an increase in height across the surface of 0.022 in. Therefore the systematic error of the initial propellant thickness was

$$B_{Ep0} = 0.022in. \tag{4.1}$$

The random error of the initial propellant thickness was determined by measuring a sample four different times by three individuals. The random error was calculated to be

$$P_{Ep0} = 0.003in. \tag{4.2}$$

There using Equation (1.6) the total uncertainty of the initial propellant thickness was

$$U_{Ep0} = 0.022in. \tag{4.3}$$

4.1.2 Uncertainty in Pressure, U_p

The systematic error of the pressure measurement came from three different elemental error sources. The first comes from the pressure transducer. According to the transducer specification, the reading was accurate at 0.11% of full scale. Since the transducer could read up to 5000 psi, this provided an error 5.5 psi. The second source came from the digitization of the data acquisition system. The system used a 14 bit board, for a range of 5000 psi this provided an error of 0.305 psi. The third source came from the pressure transducer calibration on the dead weight tester. According to the manufacturer, the uncertainty of the tester is 0.1% of the pressure. Using the average range of 2,500 psi of the pressure transducer, this provided an error of 2.5 psi. Therefore the total systematic error is given by

$$B_P = \sqrt{5.5^2 + 0.305^2 + 2.5^2} = 6.04\,psi\ . \tag{4.4}$$

The random error for the pressure was found by applying known pressures to the pressure transducer with the dead weight tester. The pressure was then recorded by the data acquisition system for thousands of reading. The reading were plotted and resulted

in a Gaussian distribution. The random error was taken to be twice the standard deviation of the distribution and was

$$P_P = 2.7\,psi\,. \tag{4.5}$$

Therefore, using Equation (1.6) the total uncertainty in the pressure was

$$U_P = 7\,psi\,. \tag{4.6}$$

4.1.3 Uncertainty in Temperature, U_T

The temperature of the propellant sample and the test hardware was controlled at a constant level with a Russells GD-16-3-3-WC Temperature and Humidity Test Chamber. The uncertainty in the temperature was taken from the manufacturer specification and was

$$U_T = 2.2°F\,. \tag{4.7}$$

4.1.4 Uncertainty in Propagation Time, U_τ

As mentioned in Section 2.5, a range of propagation times was used in order to perform a parametric study. Since the propagation time uncertainty used in the previous research [9] was questioned, a parametric study was used to determine the effect of the propagation time uncertainty on the burn rate uncertainty. If the effect on the burn rate uncertainty was minimal, U_τ could be left at the previously used value of 0.3μs. If not, further research would be required to accurately determine U_τ. The range was used for all three propagation time determination methods and was 0.1-0.5 μs in increments of 0.025 μs.

$$U_\tau = 0.1 - 0.5\mu s\ . \qquad (4.8)$$

4.1.5 Uncertainty in the Regression Constants

The regression constants a_c, b_c, a_{p0}, and b_{p0}, are determined by a linear regression on the data from two pressurization tests. Since these constants have been determined by using numbers that have an uncertainty, P and τ, the constants will therefore have an uncertainty. The linear regression equations used to determine the constants are [9]

$$a_{p0} = \frac{N_p \sum_{k=1}^{N_p} P_{pk}\tau_{pk} - \sum_{k=1}^{N_p} P_{pk} \sum_{k=1}^{N_p} \tau_{pk}}{N_p \sum_{k=1}^{N_p}\left(P_{pk}{}^2\right) - \left(\sum_{k=1}^{N_p} P_{pk}\right)^2}$$

$$b_{p0} = \frac{\sum_{k=1}^{N_p} P_{pk}{}^2 \sum_{k=1}^{N_p} \tau_{pk} - \sum_{k=1}^{N_p} P_{pk} \sum_{k=1}^{Np} P_{pk}\tau_{pk}}{N_p \sum_{k=1}^{N_p}\left(P_{pk}{}^2\right) - \left(\sum_{k=1}^{N_p} P_{pk}\right)^2}$$

$$a_c = \frac{N_c \sum_{k=1}^{N_c} P_{ck}\tau_{ck} - \sum_{k=1}^{N_c} P_{ck} \sum_{k=1}^{N_c} \tau_{ck}}{N_c \sum_{k=1}^{N_c}\left(P_{ck}{}^2\right) - \left(\sum_{k=1}^{N_c} P_{ck}\right)^2}$$

$$b_c = \frac{\sum_{k=1}^{N_c} P_{ck}{}^2 \sum_{k=1}^{N_c} \tau_{ck} - \sum_{k=1}^{N_c} P_{ck} \sum_{k=1}^{N_c} P_{ck}\tau_{ck}}{N_c \sum_{k=1}^{N_c}\left(P_{ck}{}^2\right) - \left(\sum_{k=1}^{N_c} P_{ck}\right)^2}$$

where N_p and N_c are the number of data points used for the regression. Using Equation (1.8), the uncertainties of the regression constants are given by [9]

$$\begin{aligned} U_{a_{p0}}{}^2 = {} & \sum_{k=1}^{N_p}\left(\frac{\partial a_{p0}}{\partial \tau_{pk}}\right)^2 P_{\tau_{pk}}{}^2 + \sum_{k=1}^{N_p}\left(\frac{\partial a_{p0}}{\partial \tau_{pk}}\right)^2 B_{\tau_{pk}}{}^2 + \sum_{k=1}^{N_p}\left(\frac{\partial a_{p0}}{\partial P_{pk}}\right)^2 P_{P_{pk}}{}^2 + \sum_{k=1}^{N_p}\left(\frac{\partial a_{p0}}{\partial P_{pk}}\right)^2 B_{P_{pk}}{}^2 \\ & + 2\sum_{k=1}^{N_p-1}\sum_{i=k+1}^{N_p}\left(\frac{\partial a_{p0}}{\partial \tau_{pk}}\right)\left(\frac{\partial a_{p0}}{\partial \tau_{pi}}\right) B_{\tau_{pi}\tau_{pk}} + 2\sum_{k=1}^{N_p-1}\sum_{i=k+1}^{N_p}\left(\frac{\partial a_{p0}}{\partial P_{pk}}\right)\left(\frac{\partial a_{p0}}{\partial P_{pi}}\right) B_{P_{pi}P_{pk}} \end{aligned}$$

$$\begin{aligned} U_{b_{p0}}{}^2 = {} & \sum_{k=1}^{N_p}\left(\frac{\partial b_{p0}}{\partial \tau_{pk}}\right)^2 P_{\tau_{pk}}{}^2 + \sum_{k=1}^{N_p}\left(\frac{\partial b_{p0}}{\partial \tau_{pk}}\right)^2 B_{\tau_{pk}}{}^2 + \sum_{k=1}^{N_p}\left(\frac{\partial b_{p0}}{\partial P_{pk}}\right)^2 P_{P_{pk}}{}^2 + \sum_{k=1}^{N_p}\left(\frac{\partial b_{p0}}{\partial P_{pk}}\right)^2 B_{P_{pk}}{}^2 \\ & + 2\sum_{k=1}^{N_p-1}\sum_{i=k+1}^{N_p}\left(\frac{\partial b_{p0}}{\partial \tau_{pk}}\right)\left(\frac{\partial b_{p0}}{\partial \tau_{pi}}\right) B_{\tau_{pi}\tau_{pk}} + 2\sum_{k=1}^{N_p-1}\sum_{i=k+1}^{N_p}\left(\frac{\partial b_{p0}}{\partial P_{pk}}\right)\left(\frac{\partial b_{p0}}{\partial P_{pi}}\right) B_{P_{pi}P_{pk}} \end{aligned}$$

$$\begin{aligned} U_{a_c}{}^2 = {} & \sum_{k=1}^{N_c}\left(\frac{\partial a_c}{\partial \tau_{ck}}\right)^2 P_{\tau_{ck}}{}^2 + \sum_{k=1}^{N_c}\left(\frac{\partial a_c}{\partial \tau_{ck}}\right)^2 B_{\tau_{ck}}{}^2 + \sum_{k=1}^{N_c}\left(\frac{\partial a_c}{\partial P_{ck}}\right)^2 P_{P_{ck}}{}^2 + \sum_{k=1}^{N_c}\left(\frac{\partial a_c}{\partial P_{ck}}\right)^2 B_{P_{ck}}{}^2 \\ & + 2\sum_{k=1}^{N_c-1}\sum_{i=k+1}^{N_c}\left(\frac{\partial a_c}{\partial \tau_{ck}}\right)\left(\frac{\partial a_c}{\partial \tau_{ci}}\right) B_{\tau_{ci}\tau_{ck}} + 2\sum_{k=1}^{N_c-1}\sum_{i=k+1}^{N_c}\left(\frac{\partial a_c}{\partial P_{ck}}\right)\left(\frac{\partial a_c}{\partial P_{ci}}\right) B_{P_{ci}P_{ck}} \end{aligned}$$

$$\begin{aligned} U_{b_c}{}^2 = {} & \sum_{k=1}^{N_c}\left(\frac{\partial b_c}{\partial \tau_{ck}}\right)^2 P_{\tau_{ck}}{}^2 + \sum_{k=1}^{N_c}\left(\frac{\partial b_c}{\partial \tau_{ck}}\right)^2 B_{\tau_{ck}}{}^2 + \sum_{k=1}^{N_c}\left(\frac{\partial b_c}{\partial P_{ck}}\right)^2 P_{P_{ck}}{}^2 + \sum_{k=1}^{N_c}\left(\frac{\partial b_c}{\partial P_{ck}}\right)^2 B_{P_{ck}}{}^2 \\ & + 2\sum_{k=1}^{N_c-1}\sum_{i=k+1}^{N_c}\left(\frac{\partial b_c}{\partial \tau_{ck}}\right)\left(\frac{\partial b_c}{\partial \tau_{ci}}\right) B_{\tau_{ci}\tau_{ck}} + 2\sum_{k=1}^{N_c-1}\sum_{i=k+1}^{N_c}\left(\frac{\partial b_c}{\partial P_{ck}}\right)\left(\frac{\partial b_c}{\partial P_{ci}}\right) B_{P_{ci}P_{ck}} \end{aligned}$$

In the previous equations, the P^2 terms represent the random errors and the B terms represent the systematic errors. The terms $B_{\tau\tau}$ and B_{PP} represent the covariance of the correlated systematic errors in propagation time and pressure, respectfully. The covariance for all the correlated random errors and the covariance for the correlated systematic errors between propagation time and pressure are considered negligible.

Since a range of times was used for the propagation time uncertainty, the uncertainties for the regression constants also consisted of a range. The uncertainty of a_c ranged from 4.629e-8 to 8.387e-8 μs/psi, and the uncertainty of a_{p0} ranged from 8.118e-8 to 2.462e-7 μs/psi. The uncertainty for b_c and b_{p0} turned out to be equal to the propagation time uncertainty. Appendix C contains a copy of the Mathcad worksheet used to calculate the regression constants and their uncertainties.

Table 4.1 summarizes the uncertainties of the measured variables and of the regression constants.

Table 4.1: Summary Variable Uncertainties

Variable	Uncertainty
Initial Thickness, E_{P0}	0.022 in
Pressure, P	7 psi
Temperature, T	2.2°F
Propagation time, τ	0.1-0.5 μs
Regression constant, a_c	4.629e-8 to 8.387e-8 μs/psi
Regression constant, a_{p0}	8.118e-8 to 2.462e-7 μs/psi
Regression constant, b_c	0.1-0.5 μs
Regression constant, b_{p0}	0.1-0.5 μs

4.2 Uncertainty in the Propellant Burn Rate, U_r

With the uncertainties of the measured variables and regression constants known, the burn rate uncertainty was calculated. As outlined in Section 1.4, a direct Monte Carlo simulation was performed to determine the burn rate uncertainty. A MATLAB program was created to perform the Monte Carlo simulation. A copy of the program is shown in Appendix D. Depending on the amount of available data, the instantaneous thickness of the propellant, E_P, was calculated in increments of 101 or 151 points, this represented 0.1 and 0.15 seconds of data. The propellant thickness was calculated by substituting the test data into Equation (3.3). However, each variable was "nudged" by a value randomly drawn from a Gaussian distribution with a mean of zero and a standard deviation equal to one half the uncertainty value given in Table 4.1. The burn rate was then calculated for the increment by applying a running linear regression on the propellant thickness data. The calculated burn rate was assigned to the mean pressure of the increment. This process was repeated 10,000 times for this increment with new "nudges" each time. This resulted in 10,000 burn rates for the increment. The uncertainty of the burn rate, U_r, was then calculated by taking twice the standard deviation, σ, of the 10,000 burn rates as shown in Equation (1.9). The program then shifted the increment interval down the instantaneous thickness curve and kept repeating the process until then end of data was reached. A burn rate uncertainty was determined for each of the three propagation time determination methods.

4.3 Uncertainty in the Temperature Sensitivity, $U_{\sigma p}$

Once the propellant burn rates and their uncertainties were known, the temperature sensitivity and its uncertainty could be determined. The burn rate and its uncertainty were found at 1000 psi for each of the six tests. The temperature sensitivity was then found using Equation (3.5). A Monte Carlo simulation was then performed in Microsoft Excel to determine the temperature sensitivity uncertainty. The temperature uncertainty was calculated 10,000 times; however, each time the burn rates and temperatures in Equation (3.5) were “nudged” by a value randomly drawn from a Gaussian distribution with a mean of zero and a standard deviation equal to one half their uncertainty value. The temperature sensitivity uncertainty was then calculated to be twice the standard deviation of the calculated temperature sensitivities. The temperature sensitivity uncertainty was found for each of the three propagation time determination method at 1000 psi.

CHAPTER 5

RESULTS

5.1 Burn Rate Uncertainty

A set of six tests was conducted using solid propellant from the same batch. Since the EDUM method for determining the burn rate has been the standard burn rate characterization method at UAH, the results from the Zero Crossing method and the Cross Correlation method were compared against it. Since the results of all six tests were similar, only the results of Test 5, a 145°F test, are presented here. The rest of the results are shown in Appendix A.

As described earlier, each method uses the same data reduction equation, Equation (3.3), to determine the propellant thickness. Figure 5.1 shows a plot of the thickness as determined by each of the three methods. As can be seen, all three curves agree well with each other, with the Zero Crossing and Cross Correlation methods matching very well. Figure 5.2 shows the pressure range for the test. Figure 5.3 shows the burn rate for all three methods, again they show very good agreement.

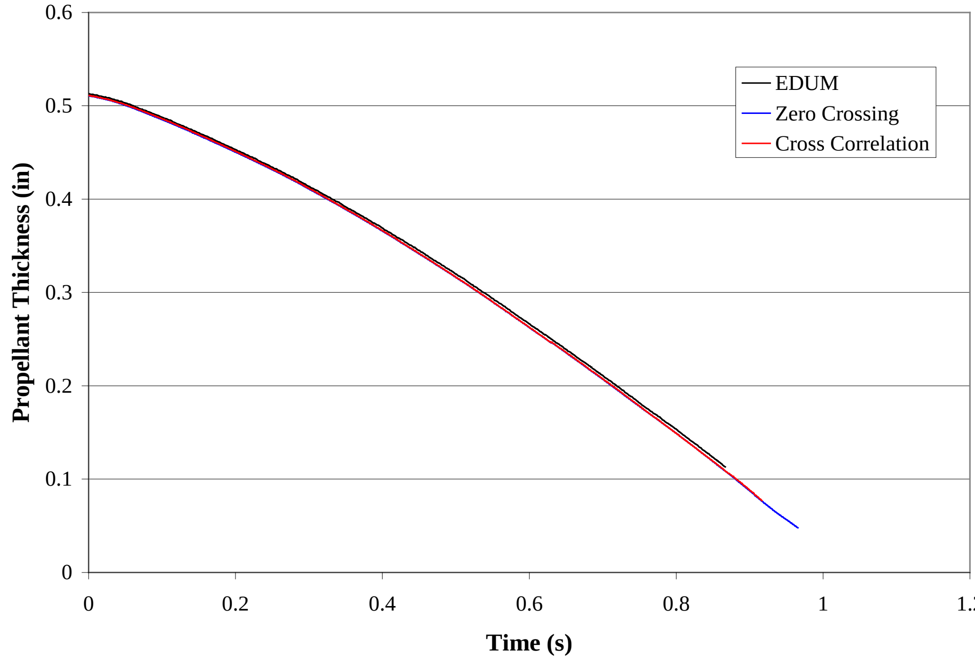

Figure 5.1: Propagation time versus test time for Test 5

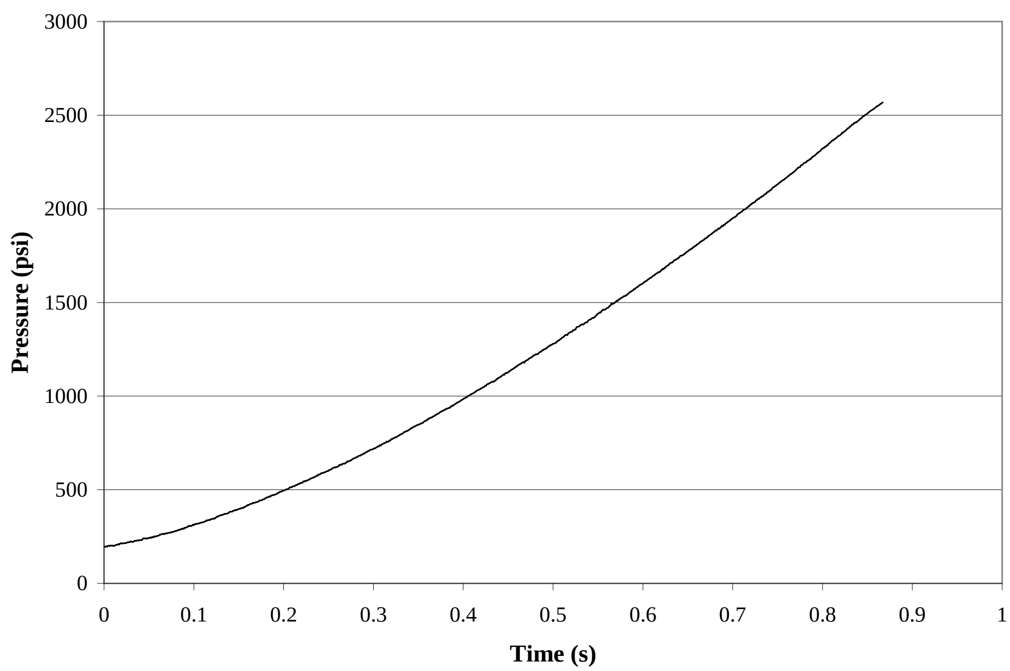

Figure 5.2: Pressure versus time for Test 5

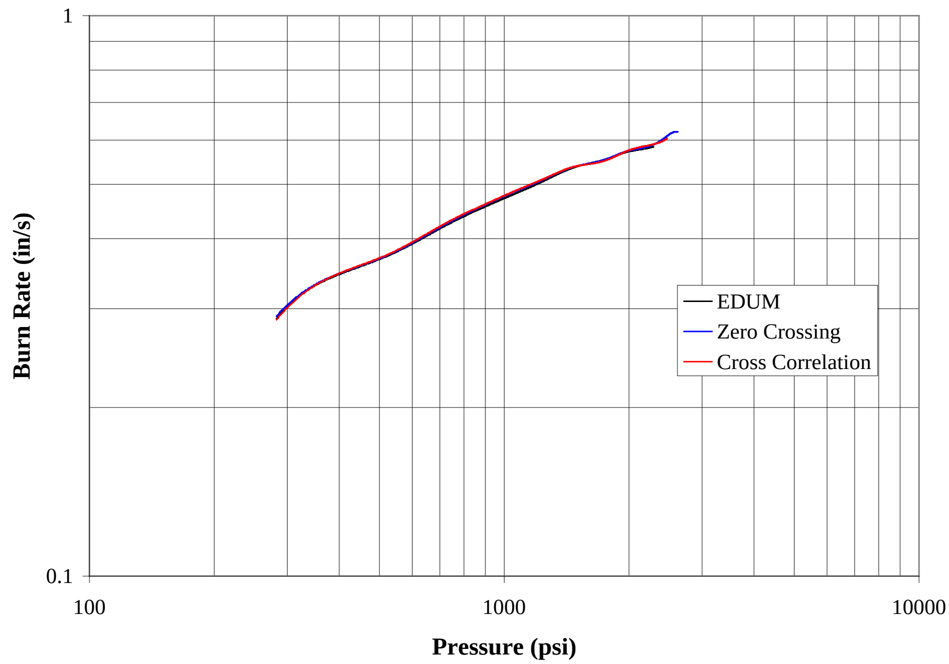

Figure 5.3: Burn rate curves for Test 5

The burn rate uncertainty of each method was determined by performing a parametric study on the propagation time uncertainty. For each method, the burn rate uncertainty was calculated over a range of propagation time uncertainties from 0.1 μs to 0.5 μs in 0.025 μs increments. Figure 5.4 shows the burn rate uncertainty produced by the EDUM method, and Figure 5.5 shows the relative burn rate uncertainty from the EDUM method. As can be seen, the uncertainty increases as the pressure increases, and as expected it increases at the propagation time uncertainty increases. The burn rate uncertainty ranged from 0.013 in/s to 0.032 in/s with a relative uncertainty of 4.28% to 5.58%. These results compare well with the previous research [9] performed at UAH that showed the burn rate

uncertainty of the EDUM method ranged from 0.01 in/s to 0.035 in/s with a relative uncertainty of 3.5% to 5% which was determined using a propagation time uncertainty of 0.3 μs. The burn rate and relative uncertainties for the Zero Crossing and Cross Correlation methods are shown in Figures 5.6-5.9.

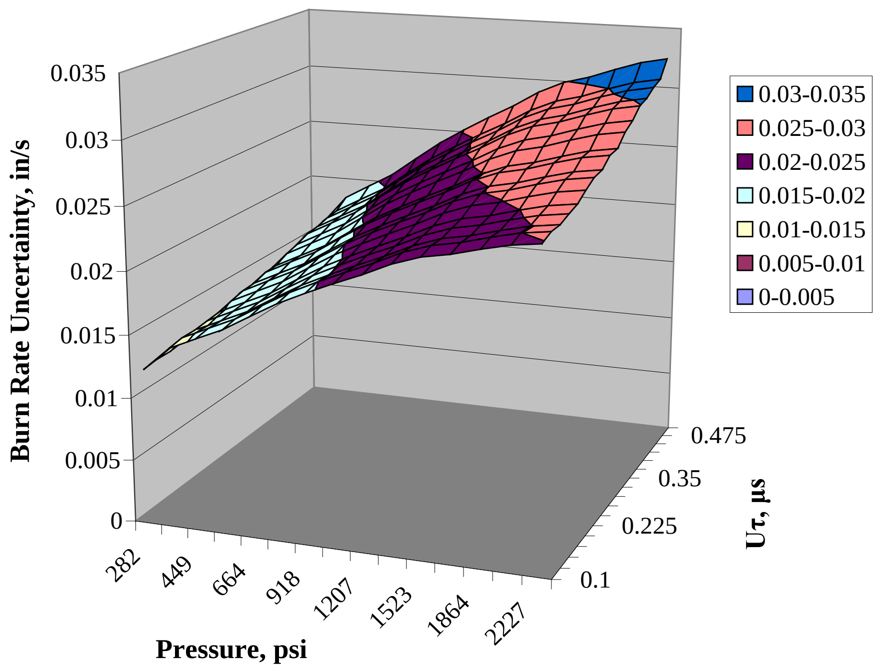

Figure 5.4: Burn rate uncertainty for the EDUM method.

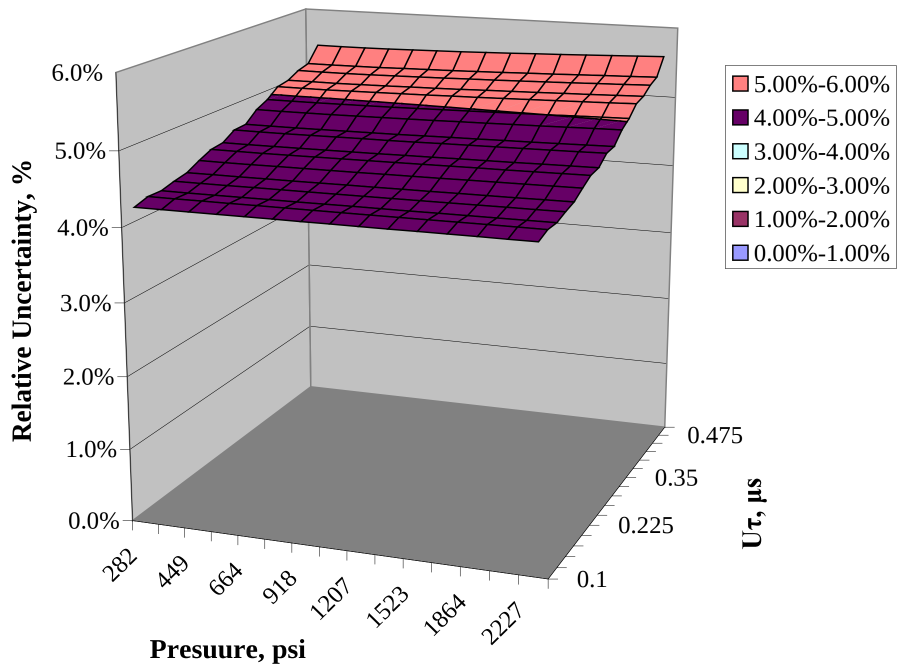

Figure 5.5: Relative uncertainty for the EDUM method

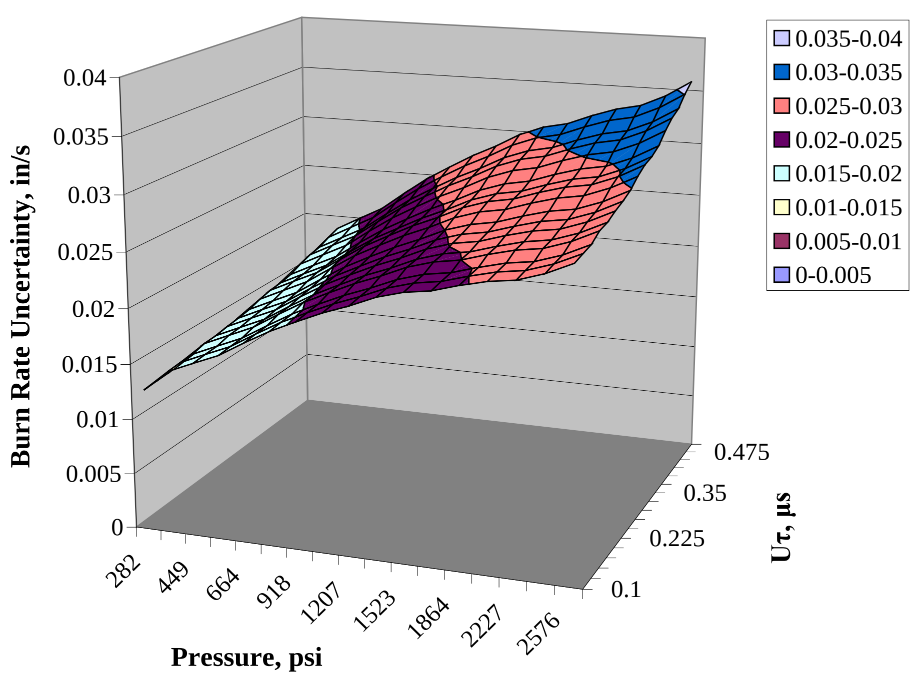

Figure 5.6: Burn rate uncertainty for the Zero Crossing method

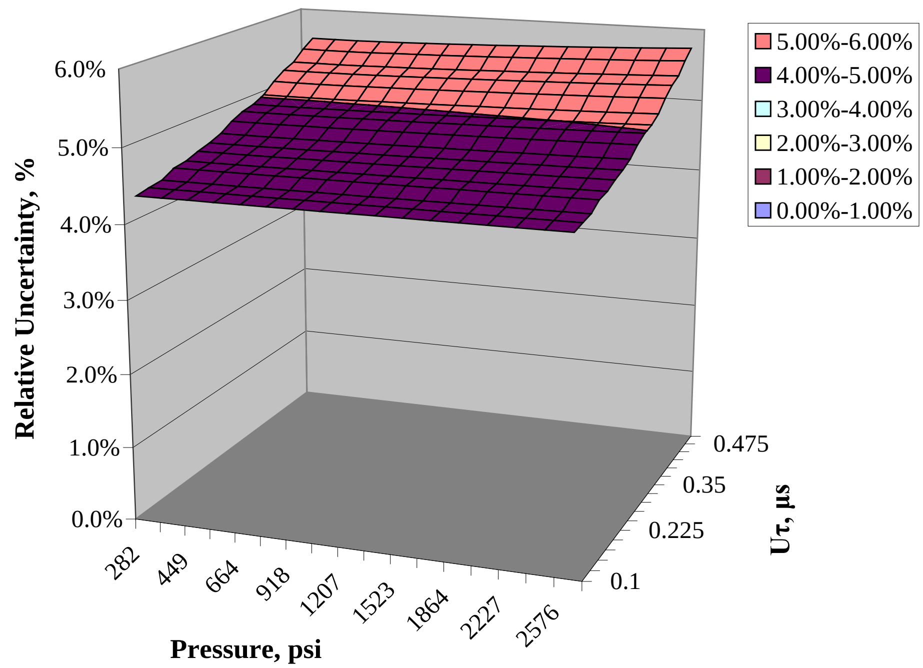

Figure 5.7: Relative uncertainty for the Zero Crossing method

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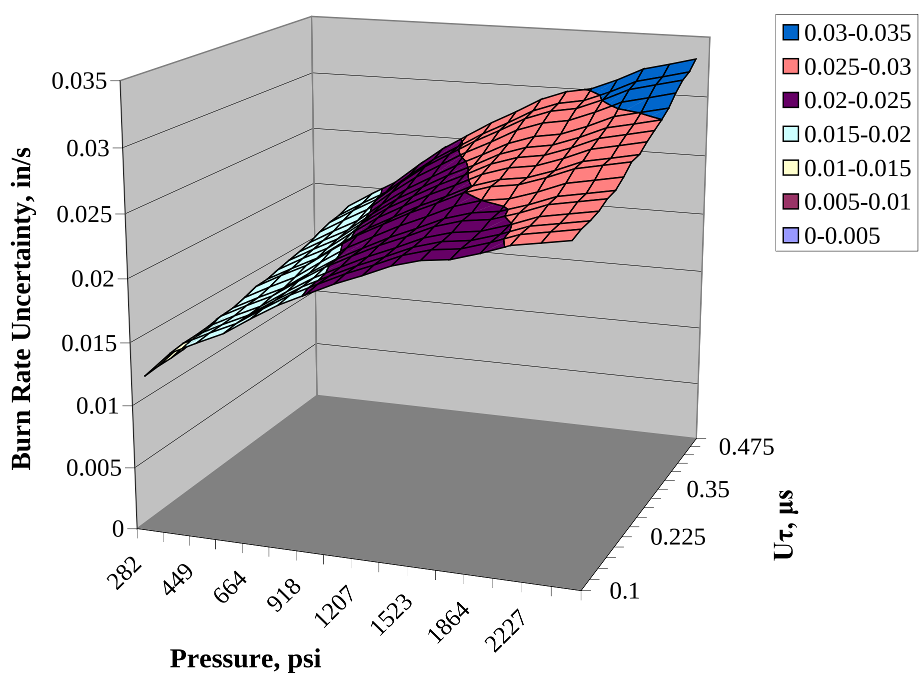


Figure 5.8 Burn rate uncertainty for the Cross Correlation method

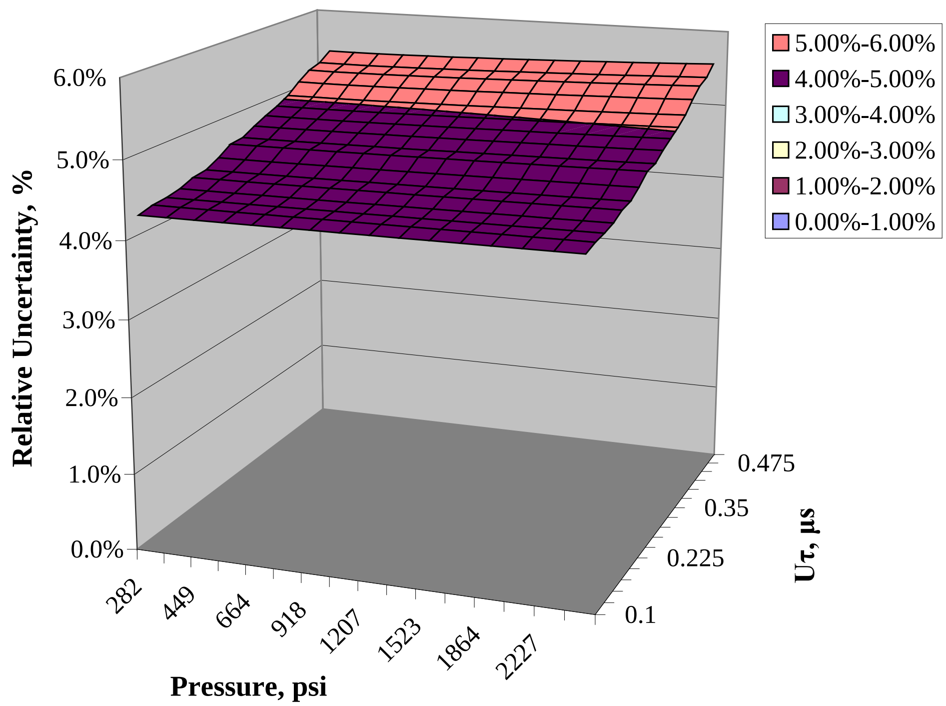


Figure 5.9: Relative uncertainty for the Cross Correlation method

From Figures 5.4, 5.6, and 5.8 it can be seen that the burn rate uncertainties of the three different methods are very similar; all three have the same overall shape. For the Zero Crossing, the range is from 0.012 in/s to 0.036 in/s and for the Cross Correlation the range is 0.012 in/s to 0.033 in/s. All three relative uncertainty plots are also similar. The Zero Crossing ranges from 4.39% to 5.73%, and the Cross Correlation ranges from 4.33% to 5.55%. These values compare well with the values determined by the EDUM method. The relative uncertainty plots are somewhat flat, showing that the propagation time uncertainty does not have a large effect on the burn rate uncertainty.

5.2 Temperature Sensitivity Uncertainty

The temperature uncertainty was calculated for the three propagation time methods at 1000 psi. The burn rates and burn rate uncertainties used for the analysis are given in Table 5.1 and the results are shown in Table 5.2. As can be seen, the relative uncertainty is fairly high, much higher than the 15% reported by motor manufacturers. This difference could be attributed to the fact that only three tests were performed at each temperature. Future work that would require the temperature sensitivity should use a greater population of motor tests.

Table 5.1: Burn Rate Data Used to Calculate Temperature Sensitivity Uncertainty

	EDUM		Zero Crossing		Cross Correlation	
Test	r (in/s)	U_r (in/s)	r (in/s)	U_r (in/s)	r (in/s)	U_r (in/s)
1	0.4718	0.0226	0.4524	0.0213	0.4523	0.0215
2	0.4385	0.0211	0.4391	0.0213	0.4474	0.0214
3	0.4410	0.0209	0.4484	0.0214	0.4000	0.0180
4	0.4770	0.0221	0.4782	0.0225	0.4794	0.0223
5	0.4705	0.0220	0.4757	0.0227	0.4770	0.0222
6	0.4726	0.0228	0.4734	0.0220	0.4739	0.0226

Table 5.2: Temperature Sensitivity Uncertainty Results

Method	σ_p (°F^{-1})	$U\sigma_p$ (°F^{-1})	Relative Uncertainty (%)
EDUM	0.000708	0.000553	78
Zero Crossing	0.000902	0.000554	61
Cross Correlation	0.001363	0.000549	40

CHAPTER 6

CONCLUSION

An uncertainty analysis was applied to three different methods for determining the burn rate of solid propellant. Two of the methods, the Zero Crossing method and the Cross Correlation method, are digital methods recently developed. The third method was the analog EDUM method that has been used extensively. Because it is not clear if the propagation time uncertainty is constant throughout the burn, a parametric study was performed to determine the effects of its uncertainty. The propagation time uncertainty was varied from 0.1-0.5 μs and was used for all three methods. Monte Carlo simulations were used to perform the uncertainty analysis. It was found that the burn rate uncertainty for the EDUM method ranged from 0.013 in/s to 0.032 in/s with a relative uncertainty of 4.28% to 5.58%. These numbers compare well to results from previous research. The Zero Crossing method ranged from 0.012 in/s to 0.036 in/s with relative uncertainties from 4.39% to 5.73%. The Cross Correlation method ranged from 0.012 in/s to 0.033 in/s with relative uncertainties from 4.33% to 5.55%. These results show the propagation time uncertainty does not have large effect on the burn rate from the range in which it is believed to lie.

The temperature sensitivity uncertainty of the propellant was also calculated. A Monte Carlo simulation was used for this as well. The uncertainties were high, ranging

from 40% to 78%. One cause of this could be the limited number of tests performed. Future work could entail performing more testing for the temperature sensitivity, although a greater number of tests should be used.

APPENDICES

APPENDIX A

RESULTS

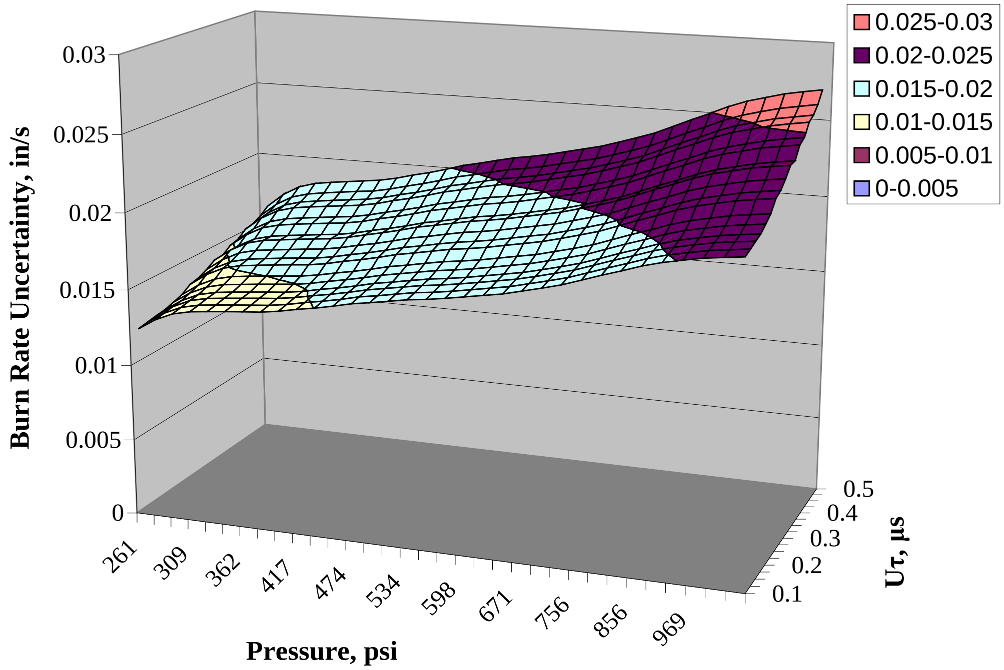


Figure A.1: Test 1 burn rate uncertainty for the EDUM method.

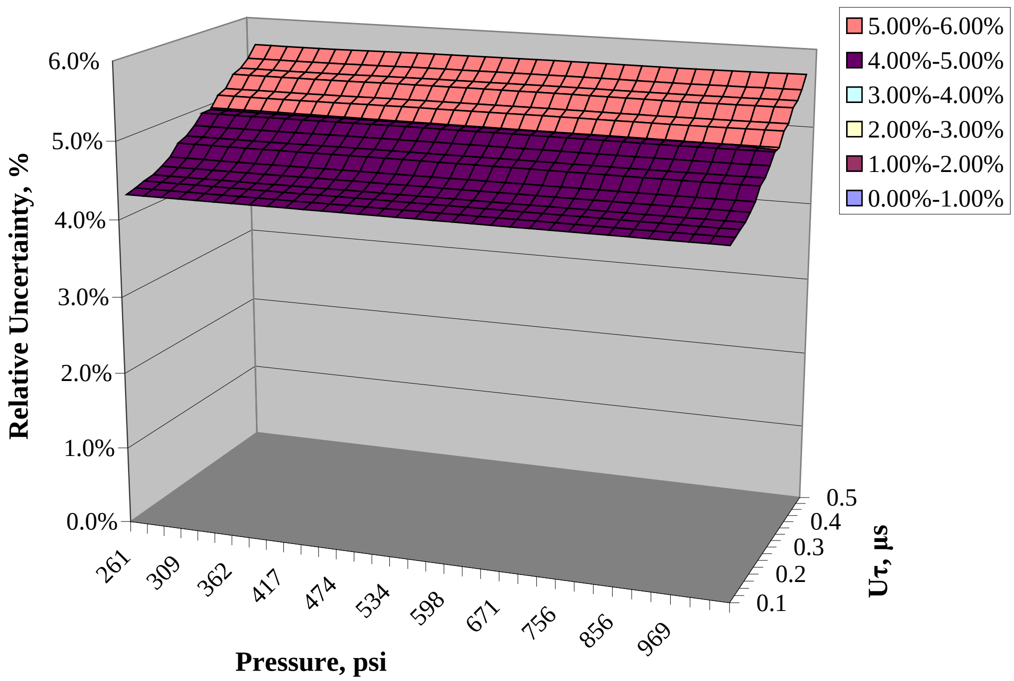


Figure A.2: Test 1 relative uncertainty for the EDUM method

Pressure, psi	Burn Rate, in/s	Propagation Time, µs																
		0.1	0.125	0.15	0.175	0.2	0.225	0.25	0.275	0.3	0.325	0.35	0.375	0.4	0.425	0.45	0.475	0.5
261	0.289	0.0125	0.0126	0.0127	0.0128	0.0130	0.0132	0.0136	0.0137	0.0140	0.0144	0.0144	0.0148	0.0150	0.0154	0.0155	0.0158	0.0162
276	0.305	0.0132	0.0133	0.0134	0.0135	0.0137	0.0139	0.0144	0.0145	0.0148	0.0152	0.0152	0.0157	0.0158	0.0163	0.0164	0.0167	0.0172
292	0.316	0.0137	0.0138	0.0139	0.0140	0.0142	0.0145	0.0149	0.0151	0.0154	0.0157	0.0158	0.0162	0.0164	0.0169	0.0170	0.0173	0.0178
309	0.322	0.0140	0.0141	0.0142	0.0143	0.0145	0.0147	0.0152	0.0153	0.0156	0.0160	0.0161	0.0165	0.0167	0.0172	0.0174	0.0177	0.0182
326	0.325	0.0141	0.0142	0.0143	0.0144	0.0146	0.0149	0.0153	0.0155	0.0158	0.0162	0.0162	0.0167	0.0169	0.0174	0.0175	0.0178	0.0183
344	0.328	0.0142	0.0143	0.0144	0.0145	0.0148	0.0150	0.0154	0.0156	0.0159	0.0163	0.0163	0.0168	0.0170	0.0175	0.0177	0.0180	0.0185
362	0.330	0.0143	0.0144	0.0146	0.0147	0.0149	0.0151	0.0156	0.0157	0.0160	0.0164	0.0165	0.0170	0.0171	0.0176	0.0178	0.0181	0.0186
381	0.333	0.0144	0.0145	0.0147	0.0148	0.0150	0.0152	0.0157	0.0158	0.0161	0.0166	0.0166	0.0171	0.0173	0.0178	0.0179	0.0182	0.0188
399	0.336	0.0146	0.0147	0.0148	0.0149	0.0151	0.0154	0.0159	0.0160	0.0163	0.0168	0.0168	0.0173	0.0175	0.0180	0.0181	0.0185	0.0190
417	0.342	0.0148	0.0149	0.0151	0.0152	0.0154	0.0156	0.0161	0.0163	0.0166	0.0170	0.0170	0.0176	0.0177	0.0183	0.0184	0.0188	0.0193
436	0.346	0.0150	0.0151	0.0153	0.0154	0.0156	0.0159	0.0163	0.0165	0.0168	0.0173	0.0173	0.0178	0.0180	0.0185	0.0187	0.0190	0.0196
455	0.352	0.0153	0.0154	0.0155	0.0156	0.0159	0.0161	0.0166	0.0168	0.0171	0.0175	0.0176	0.0181	0.0183	0.0188	0.0190	0.0193	0.0199
474	0.358	0.0155	0.0156	0.0158	0.0159	0.0161	0.0164	0.0169	0.0171	0.0174	0.0178	0.0179	0.0184	0.0186	0.0192	0.0193	0.0197	0.0202
493	0.362	0.0157	0.0158	0.0160	0.0161	0.0163	0.0166	0.0171	0.0173	0.0176	0.0181	0.0181	0.0187	0.0188	0.0194	0.0196	0.0199	0.0205
513	0.367	0.0159	0.0160	0.0162	0.0163	0.0165	0.0168	0.0173	0.0175	0.0179	0.0183	0.0183	0.0189	0.0191	0.0197	0.0198	0.0202	0.0208
534	0.372	0.0161	0.0162	0.0164	0.0165	0.0167	0.0170	0.0175	0.0177	0.0181	0.0185	0.0186	0.0191	0.0193	0.0199	0.0201	0.0205	0.0210
554	0.375	0.0163	0.0164	0.0165	0.0167	0.0169	0.0172	0.0177	0.0179	0.0182	0.0187	0.0187	0.0193	0.0195	0.0201	0.0203	0.0206	0.0212
576	0.379	0.0164	0.0166	0.0167	0.0168	0.0171	0.0174	0.0179	0.0181	0.0184	0.0189	0.0189	0.0195	0.0197	0.0203	0.0205	0.0209	0.0214
598	0.384	0.0166	0.0168	0.0169	0.0170	0.0173	0.0176	0.0181	0.0183	0.0187	0.0191	0.0192	0.0198	0.0200	0.0206	0.0207	0.0211	0.0217
621	0.389	0.0168	0.0170	0.0171	0.0173	0.0175	0.0178	0.0183	0.0185	0.0189	0.0194	0.0194	0.0200	0.0202	0.0208	0.0210	0.0214	0.0220
645	0.393	0.0170	0.0172	0.0173	0.0175	0.0177	0.0180	0.0186	0.0188	0.0191	0.0196	0.0196	0.0203	0.0205	0.0211	0.0213	0.0216	0.0223
671	0.400	0.0173	0.0175	0.0176	0.0178	0.0180	0.0183	0.0189	0.0191	0.0194	0.0199	0.0200	0.0206	0.0208	0.0214	0.0216	0.0220	0.0226
698	0.406	0.0176	0.0178	0.0179	0.0180	0.0183	0.0186	0.0192	0.0194	0.0198	0.0203	0.0203	0.0209	0.0211	0.0218	0.0220	0.0224	0.0230
726	0.414	0.0179	0.0181	0.0182	0.0184	0.0186	0.0189	0.0195	0.0197	0.0201	0.0206	0.0207	0.0213	0.0215	0.0222	0.0224	0.0228	0.0234
756	0.423	0.0183	0.0185	0.0186	0.0188	0.0191	0.0194	0.0200	0.0202	0.0206	0.0211	0.0211	0.0218	0.0220	0.0227	0.0229	0.0233	0.0239
788	0.432	0.0187	0.0189	0.0191	0.0192	0.0195	0.0198	0.0204	0.0206	0.0210	0.0216	0.0216	0.0223	0.0225	0.0232	0.0234	0.0238	0.0244
821	0.441	0.0191	0.0193	0.0194	0.0196	0.0199	0.0202	0.0208	0.0210	0.0214	0.0220	0.0220	0.0227	0.0229	0.0236	0.0238	0.0243	0.0249
856	0.450	0.0195	0.0197	0.0198	0.0200	0.0203	0.0206	0.0212	0.0215	0.0219	0.0225	0.0225	0.0232	0.0234	0.0241	0.0243	0.0248	0.0255
892	0.458	0.0198	0.0200	0.0202	0.0203	0.0206	0.0210	0.0216	0.0218	0.0223	0.0228	0.0229	0.0236	0.0238	0.0245	0.0247	0.0252	0.0259
930	0.463	0.0201	0.0202	0.0204	0.0206	0.0209	0.0212	0.0219	0.0221	0.0225	0.0231	0.0231	0.0239	0.0241	0.0248	0.0251	0.0255	0.0262
969	0.468	0.0203	0.0205	0.0206	0.0208	0.0211	0.0215	0.0221	0.0223	0.0228	0.0234	0.0234	0.0242	0.0244	0.0251	0.0253	0.0258	0.0265
1008	0.472	0.0205	0.0206	0.0208	0.0210	0.0213	0.0216	0.0223	0.0225	0.0230	0.0236	0.0236	0.0244	0.0246	0.0253	0.0256	0.0260	0.0267
1049	0.475	0.0206	0.0208	0.0209	0.0211	0.0214	0.0218	0.0224	0.0227	0.0231	0.0237	0.0238	0.0245	0.0248	0.0255	0.0257	0.0262	0.0269

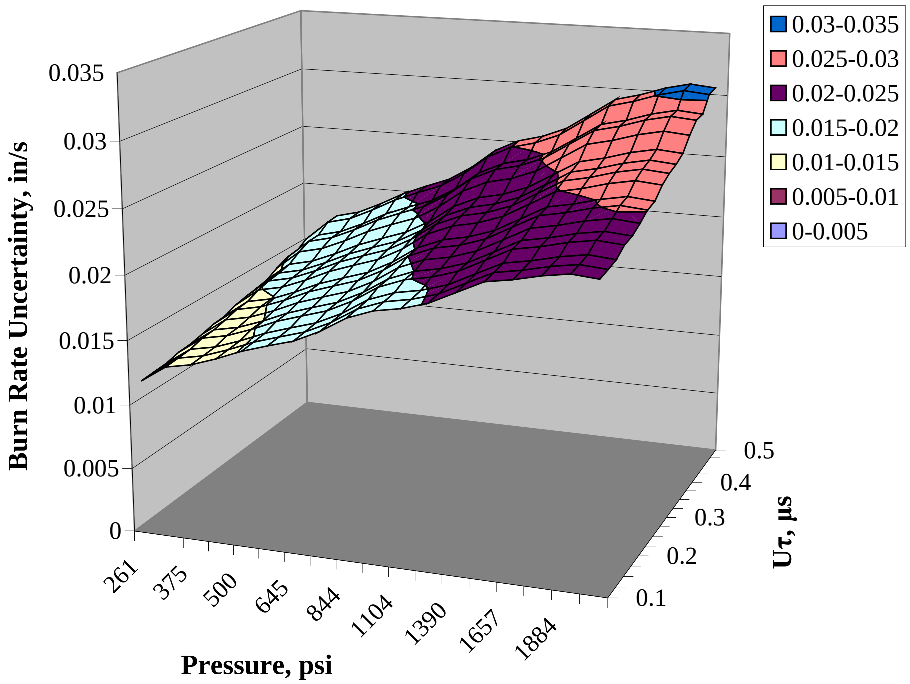


Figure A.3: Test 1 burn rate uncertainty for the Zero Crossing method.

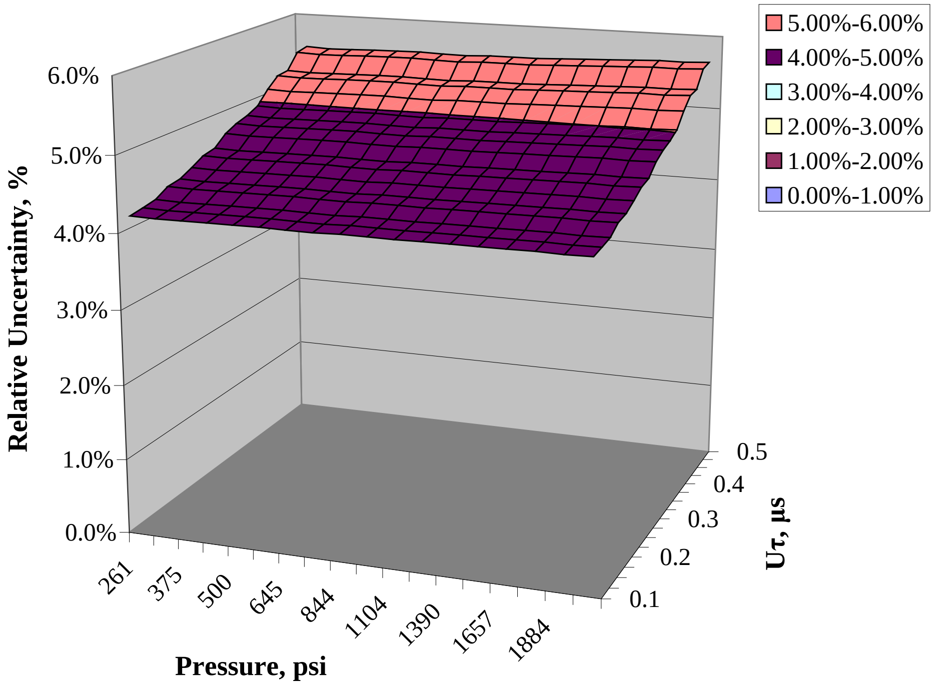


Figure A.4: Test 1 relative uncertainty for the Zero Crossing method.

Pressure, psi	Burn Rate, in/s	Propagation Time, µs																
		0.1	0.125	0.15	0.175	0.2	0.225	0.25	0.275	0.3	0.325	0.35	0.375	0.4	0.425	0.45	0.475	0.5
261	0.283	0.012	0.012	0.012	0.012	0.013	0.013	0.013	0.013	0.013	0.014	0.014	0.014	0.015	0.015	0.015	0.016	0.016
314	0.314	0.013	0.013	0.013	0.014	0.014	0.014	0.014	0.014	0.015	0.015	0.015	0.016	0.016	0.017	0.017	0.017	0.017
375	0.323	0.014	0.014	0.014	0.014	0.014	0.015	0.015	0.015	0.015	0.016	0.016	0.016	0.017	0.017	0.017	0.018	0.018
436	0.339	0.014	0.014	0.015	0.015	0.015	0.015	0.016	0.016	0.016	0.016	0.017	0.017	0.017	0.018	0.018	0.019	0.019
500	0.357	0.015	0.015	0.015	0.016	0.016	0.016	0.016	0.017	0.017	0.017	0.017	0.018	0.018	0.019	0.019	0.020	0.020
569	0.372	0.016	0.016	0.016	0.016	0.016	0.017	0.017	0.017	0.018	0.018	0.018	0.018	0.019	0.020	0.020	0.020	0.021
645	0.386	0.016	0.016	0.017	0.017	0.017	0.017	0.018	0.018	0.018	0.019	0.019	0.019	0.020	0.020	0.020	0.021	0.021
736	0.408	0.017	0.017	0.017	0.018	0.018	0.018	0.019	0.019	0.019	0.020	0.020	0.020	0.021	0.021	0.022	0.022	0.023
844	0.435	0.018	0.019	0.019	0.019	0.019	0.020	0.020	0.020	0.021	0.021	0.021	0.022	0.022	0.023	0.023	0.024	0.024
969	0.452	0.019	0.019	0.019	0.020	0.020	0.020	0.021	0.021	0.022	0.022	0.022	0.022	0.023	0.024	0.024	0.025	0.025
1104	0.461	0.020	0.020	0.020	0.020	0.020	0.021	0.021	0.021	0.022	0.022	0.023	0.023	0.024	0.024	0.024	0.025	0.026
1247	0.475	0.020	0.020	0.020	0.021	0.021	0.021	0.022	0.022	0.023	0.023	0.023	0.024	0.025	0.025	0.025	0.026	0.026
1390	0.498	0.021	0.021	0.021	0.022	0.022	0.022	0.023	0.023	0.024	0.024	0.024	0.025	0.026	0.026	0.027	0.028	0.028
1529	0.520	0.022	0.022	0.022	0.023	0.023	0.024	0.024	0.024	0.025	0.025	0.026	0.026	0.027	0.028	0.028	0.029	0.029
1657	0.528	0.022	0.023	0.023	0.023	0.023	0.024	0.024	0.025	0.025	0.026	0.026	0.026	0.027	0.028	0.028	0.029	0.030
1775	0.539	0.023	0.023	0.023	0.024	0.024	0.024	0.025	0.025	0.026	0.026	0.027	0.027	0.028	0.029	0.029	0.030	0.030
1884	0.547	0.023	0.023	0.024	0.024	0.024	0.025	0.025	0.025	0.026	0.027	0.027	0.027	0.028	0.029	0.029	0.031	0.031
1983	0.543	0.023	0.023	0.023	0.024	0.024	0.025	0.025	0.025	0.026	0.027	0.027	0.027	0.028	0.029	0.029	0.030	0.031

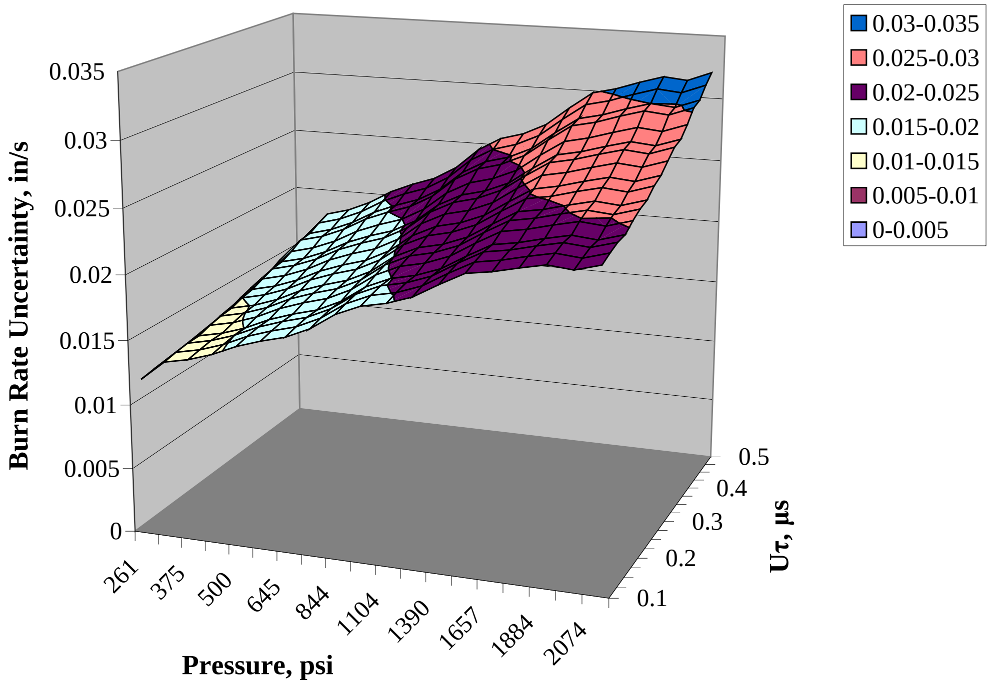


Figure A.5: Test 1 burn rate uncertainty for the Cross Correlation method.

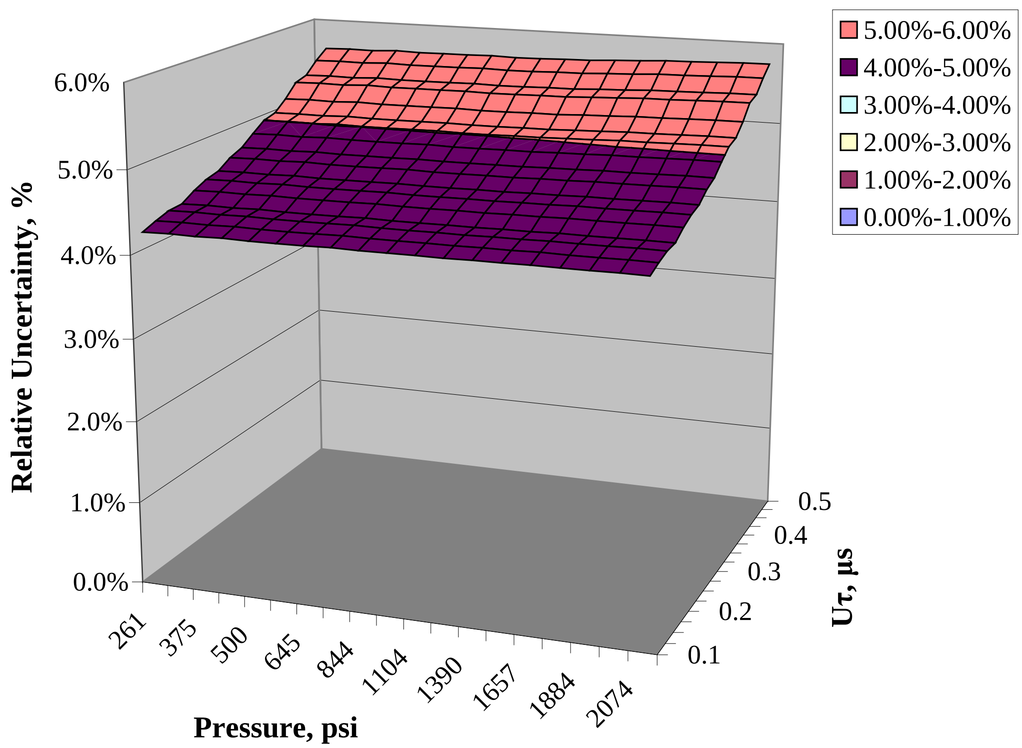


Figure A.6: Test 1 relative uncertainty for the Cross Correlation method.

Pressure, psi	Burn Rate, in/s	Propagation Time, μs																
		0.1	0.125	0.15	0.175	0.2	0.225	0.25	0.275	0.3	0.325	0.35	0.375	0.4	0.425	0.45	0.475	0.5
261	0.283	0.012	0.012	0.012	0.012	0.013	0.013	0.013	0.013	0.013	0.014	0.014	0.014	0.015	0.015	0.015	0.016	0.016
314	0.318	0.014	0.014	0.014	0.014	0.014	0.015	0.015	0.015	0.015	0.016	0.016	0.016	0.016	0.017	0.017	0.018	0.018
375	0.327	0.014	0.014	0.014	0.014	0.015	0.015	0.015	0.015	0.016	0.016	0.016	0.016	0.017	0.017	0.018	0.018	0.018
436	0.341	0.015	0.015	0.015	0.015	0.015	0.016	0.016	0.016	0.016	0.017	0.017	0.017	0.018	0.018	0.018	0.019	0.019
500	0.360	0.015	0.016	0.016	0.016	0.016	0.016	0.017	0.017	0.017	0.018	0.018	0.018	0.019	0.019	0.019	0.020	0.020
569	0.374	0.016	0.016	0.016	0.017	0.017	0.017	0.017	0.018	0.018	0.018	0.019	0.019	0.019	0.020	0.020	0.021	0.021
645	0.386	0.017	0.017	0.017	0.017	0.017	0.018	0.018	0.018	0.018	0.019	0.019	0.019	0.020	0.021	0.021	0.021	0.022
736	0.404	0.017	0.018	0.018	0.018	0.018	0.019	0.019	0.019	0.019	0.020	0.020	0.020	0.021	0.022	0.022	0.022	0.023
844	0.434	0.019	0.019	0.019	0.019	0.020	0.020	0.020	0.020	0.021	0.021	0.022	0.022	0.022	0.023	0.023	0.024	0.024
969	0.452	0.019	0.020	0.020	0.020	0.020	0.021	0.021	0.021	0.022	0.022	0.023	0.023	0.023	0.024	0.024	0.025	0.026
1104	0.461	0.020	0.020	0.020	0.020	0.021	0.021	0.021	0.022	0.022	0.023	0.023	0.023	0.024	0.025	0.025	0.026	0.026
1247	0.477	0.020	0.021	0.021	0.021	0.021	0.022	0.022	0.023	0.023	0.023	0.024	0.024	0.025	0.026	0.026	0.026	0.027
1390	0.501	0.022	0.022	0.022	0.022	0.023	0.023	0.023	0.024	0.024	0.025	0.025	0.025	0.026	0.027	0.027	0.028	0.028
1529	0.524	0.022	0.023	0.023	0.023	0.024	0.024	0.024	0.025	0.025	0.026	0.026	0.027	0.027	0.028	0.028	0.029	0.030
1657	0.530	0.023	0.023	0.023	0.023	0.024	0.024	0.025	0.025	0.026	0.026	0.027	0.027	0.028	0.029	0.029	0.030	0.030
1775	0.541	0.023	0.024	0.024	0.024	0.024	0.025	0.025	0.026	0.026	0.027	0.027	0.028	0.028	0.029	0.029	0.030	0.031
1884	0.550	0.024	0.024	0.024	0.024	0.025	0.025	0.026	0.026	0.027	0.027	0.028	0.028	0.029	0.030	0.030	0.031	0.031
1983	0.546	0.023	0.024	0.024	0.024	0.025	0.025	0.025	0.026	0.026	0.027	0.028	0.028	0.029	0.030	0.030	0.031	0.031
2074	0.558	0.024	0.024	0.025	0.025	0.025	0.026	0.026	0.027	0.027	0.028	0.028	0.029	0.029	0.030	0.031	0.031	0.032

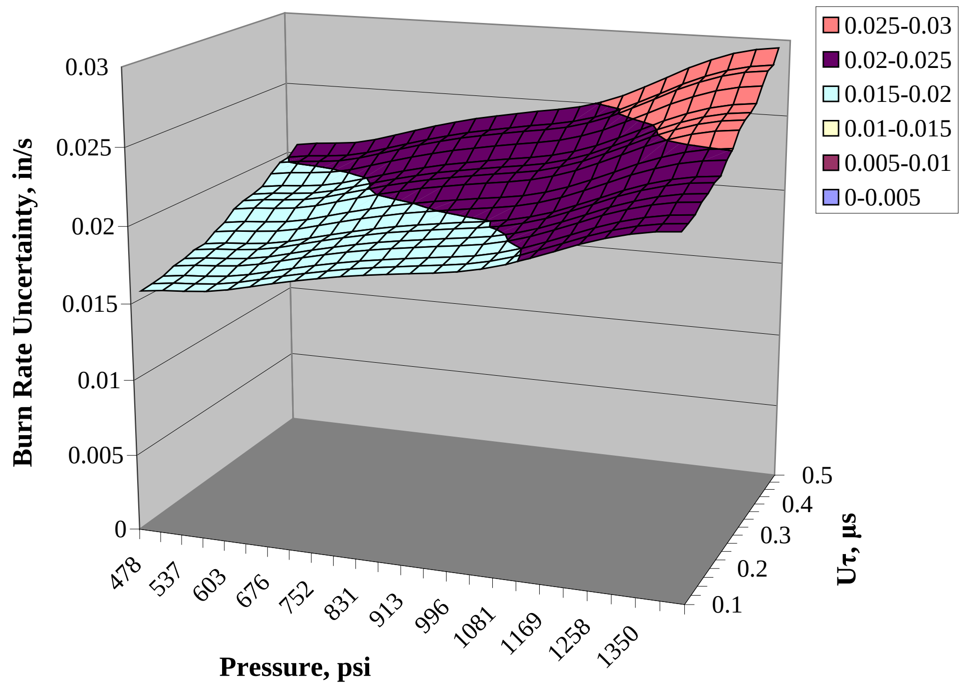


Figure A.7: Test 2 burn rate uncertainty for the EDUM method.

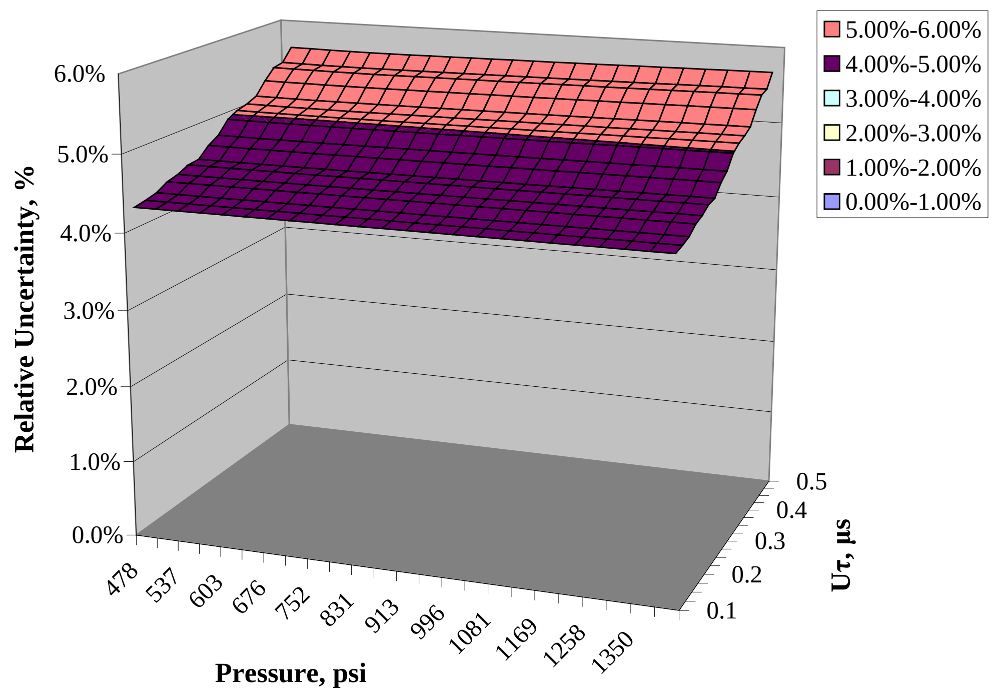


Figure A.8: Test 2 relative uncertainty for the EDUM method.

Pressure, psi	Burn Rate, in/s	Propagation Time, µs																
		0.1	0.125	0.15	0.175	0.2	0.225	0.25	0.275	0.3	0.325	0.35	0.375	0.4	0.425	0.45	0.475	0.5
478	0.367	0.016	0.016	0.016	0.016	0.017	0.017	0.017	0.017	0.018	0.018	0.018	0.019	0.019	0.019	0.020	0.020	0.021
507	0.371	0.016	0.016	0.016	0.017	0.017	0.017	0.017	0.018	0.018	0.018	0.019	0.019	0.019	0.020	0.020	0.020	0.021
537	0.373	0.016	0.016	0.016	0.017	0.017	0.017	0.017	0.018	0.018	0.018	0.019	0.019	0.019	0.020	0.020	0.020	0.021
569	0.376	0.016	0.016	0.017	0.017	0.017	0.017	0.017	0.018	0.018	0.019	0.019	0.019	0.019	0.020	0.020	0.020	0.021
603	0.381	0.017	0.017	0.017	0.017	0.017	0.017	0.018	0.018	0.018	0.019	0.019	0.019	0.020	0.020	0.021	0.021	0.021
639	0.389	0.017	0.017	0.017	0.017	0.018	0.018	0.018	0.018	0.019	0.019	0.020	0.020	0.020	0.021	0.021	0.021	0.022
676	0.397	0.017	0.017	0.017	0.018	0.018	0.018	0.018	0.019	0.019	0.020	0.020	0.020	0.020	0.021	0.021	0.022	0.022
713	0.404	0.018	0.018	0.018	0.018	0.018	0.018	0.019	0.019	0.019	0.020	0.020	0.020	0.021	0.021	0.022	0.022	0.023
752	0.410	0.018	0.018	0.018	0.018	0.019	0.019	0.019	0.019	0.020	0.020	0.021	0.021	0.021	0.022	0.022	0.022	0.023
791	0.416	0.018	0.018	0.018	0.019	0.019	0.019	0.019	0.020	0.020	0.021	0.021	0.021	0.021	0.022	0.023	0.023	0.023
831	0.421	0.018	0.018	0.019	0.019	0.019	0.019	0.019	0.020	0.020	0.021	0.021	0.021	0.022	0.022	0.023	0.023	0.024
872	0.426	0.018	0.019	0.019	0.019	0.019	0.019	0.020	0.020	0.020	0.021	0.021	0.022	0.022	0.023	0.023	0.023	0.024
913	0.430	0.019	0.019	0.019	0.019	0.019	0.020	0.020	0.020	0.021	0.021	0.022	0.022	0.022	0.023	0.023	0.023	0.024
954	0.434	0.019	0.019	0.019	0.019	0.020	0.020	0.020	0.021	0.021	0.022	0.022	0.022	0.022	0.023	0.024	0.024	0.024
996	0.439	0.019	0.019	0.019	0.020	0.020	0.020	0.020	0.021	0.021	0.022	0.022	0.022	0.023	0.023	0.024	0.024	0.025
1038	0.446	0.019	0.019	0.020	0.020	0.020	0.020	0.020	0.021	0.021	0.022	0.022	0.023	0.023	0.024	0.024	0.024	0.025
1081	0.455	0.020	0.020	0.020	0.020	0.021	0.021	0.021	0.022	0.022	0.023	0.023	0.023	0.023	0.024	0.025	0.025	0.026
1125	0.467	0.020	0.020	0.021	0.021	0.021	0.021	0.021	0.022	0.022	0.023	0.024	0.024	0.024	0.025	0.025	0.026	0.026
1169	0.479	0.021	0.021	0.021	0.021	0.022	0.022	0.022	0.023	0.023	0.024	0.024	0.024	0.025	0.025	0.026	0.026	0.027
1213	0.492	0.021	0.021	0.022	0.022	0.022	0.023	0.023	0.023	0.024	0.024	0.025	0.025	0.025	0.026	0.027	0.027	0.028
1258	0.503	0.022	0.022	0.022	0.023	0.023	0.023	0.023	0.024	0.024	0.025	0.025	0.026	0.026	0.027	0.027	0.028	0.028
1304	0.511	0.022	0.022	0.023	0.023	0.023	0.023	0.024	0.024	0.025	0.025	0.026	0.026	0.026	0.027	0.028	0.028	0.029
1350	0.517	0.022	0.023	0.023	0.023	0.023	0.024	0.024	0.025	0.025	0.026	0.026	0.026	0.027	0.028	0.028	0.028	0.029
1396	0.520	0.023	0.023	0.023	0.023	0.024	0.024	0.024	0.025	0.025	0.026	0.026	0.027	0.027	0.028	0.028	0.029	0.029

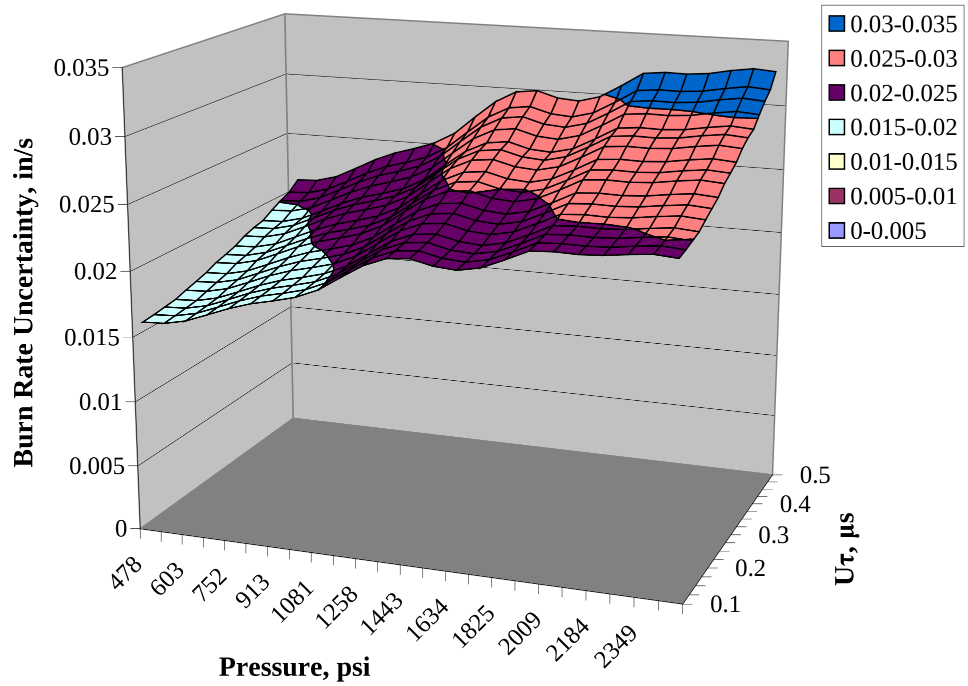


Figure A.9: Test 2 burn rate uncertainty for the Zero Crossing method.

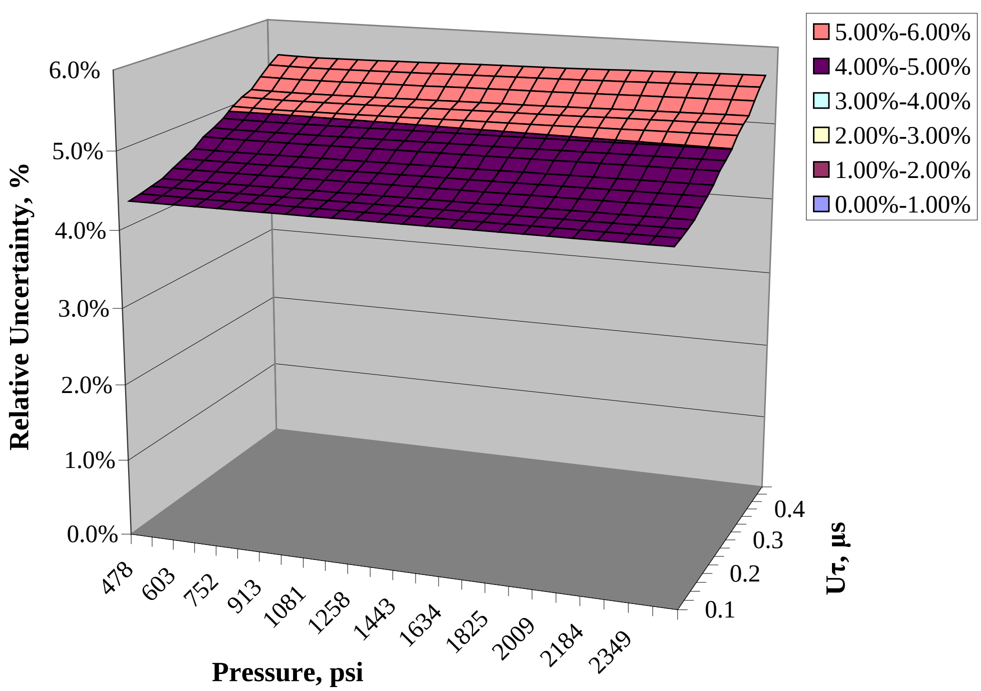


Figure A.10: Test 2 relative uncertainty for the Zero Crossing method.

Pressure, psi	Burn Rate, in/s	Propagation Time, μs																
		0.1	0.125	0.15	0.175	0.2	0.225	0.25	0.275	0.3	0.325	0.35	0.375	0.4	0.425	0.45	0.475	0.5
478	0.370	0.016	0.016	0.016	0.017	0.017	0.017	0.017	0.018	0.018	0.018	0.019	0.019	0.019	0.020	0.020	0.020	0.021
537	0.372	0.016	0.016	0.017	0.017	0.017	0.017	0.017	0.018	0.018	0.018	0.019	0.019	0.019	0.020	0.020	0.020	0.021
603	0.380	0.017	0.017	0.017	0.017	0.017	0.018	0.018	0.018	0.018	0.019	0.019	0.019	0.020	0.020	0.021	0.021	0.022
676	0.394	0.017	0.017	0.018	0.018	0.018	0.018	0.019	0.019	0.019	0.019	0.020	0.020	0.020	0.021	0.021	0.022	0.022
752	0.409	0.018	0.018	0.018	0.018	0.019	0.019	0.019	0.020	0.020	0.020	0.021	0.021	0.021	0.022	0.022	0.023	0.023
831	0.421	0.018	0.019	0.019	0.019	0.019	0.019	0.020	0.020	0.020	0.021	0.021	0.022	0.022	0.022	0.023	0.023	0.024
913	0.429	0.019	0.019	0.019	0.019	0.020	0.020	0.020	0.021	0.021	0.021	0.022	0.022	0.022	0.023	0.023	0.024	0.025
996	0.439	0.019	0.019	0.020	0.020	0.020	0.020	0.021	0.021	0.021	0.022	0.022	0.023	0.023	0.023	0.024	0.024	0.025
1081	0.455	0.020	0.020	0.020	0.020	0.021	0.021	0.021	0.022	0.022	0.022	0.023	0.023	0.024	0.024	0.025	0.025	0.026
1169	0.479	0.021	0.021	0.021	0.022	0.022	0.022	0.023	0.023	0.023	0.024	0.024	0.025	0.025	0.026	0.026	0.027	0.027
1258	0.503	0.022	0.022	0.022	0.023	0.023	0.023	0.024	0.024	0.025	0.025	0.025	0.026	0.026	0.027	0.027	0.028	0.029
1350	0.518	0.023	0.023	0.023	0.023	0.024	0.024	0.024	0.025	0.025	0.026	0.026	0.027	0.027	0.028	0.028	0.029	0.030
1443	0.521	0.023	0.023	0.023	0.023	0.024	0.024	0.025	0.025	0.025	0.026	0.026	0.027	0.027	0.028	0.028	0.029	0.030
1538	0.512	0.022	0.023	0.023	0.023	0.023	0.024	0.024	0.025	0.025	0.025	0.026	0.026	0.027	0.027	0.028	0.028	0.029
1634	0.509	0.022	0.022	0.023	0.023	0.023	0.024	0.024	0.025	0.025	0.025	0.026	0.026	0.027	0.027	0.028	0.028	0.029
1730	0.517	0.023	0.023	0.023	0.023	0.024	0.024	0.024	0.025	0.025	0.026	0.026	0.027	0.027	0.028	0.028	0.029	0.030
1825	0.533	0.023	0.024	0.024	0.024	0.024	0.025	0.025	0.026	0.026	0.026	0.027	0.028	0.028	0.029	0.029	0.030	0.031
1918	0.551	0.024	0.024	0.025	0.025	0.025	0.026	0.026	0.027	0.027	0.027	0.028	0.028	0.029	0.030	0.030	0.031	0.032
2009	0.553	0.024	0.024	0.025	0.025	0.025	0.026	0.026	0.027	0.027	0.028	0.028	0.029	0.029	0.030	0.030	0.031	0.032
2098	0.552	0.024	0.024	0.025	0.025	0.025	0.026	0.026	0.027	0.027	0.028	0.028	0.029	0.029	0.030	0.030	0.031	0.032
2184	0.554	0.024	0.024	0.025	0.025	0.025	0.026	0.026	0.027	0.027	0.028	0.028	0.029	0.029	0.030	0.030	0.031	0.032
2268	0.559	0.025	0.025	0.025	0.025	0.026	0.026	0.026	0.027	0.027	0.028	0.029	0.029	0.029	0.030	0.031	0.031	0.032
2349	0.563	0.025	0.025	0.025	0.025	0.026	0.026	0.027	0.027	0.028	0.028	0.029	0.029	0.030	0.030	0.031	0.032	0.033
2427	0.560	0.025	0.025	0.025	0.025	0.026	0.026	0.026	0.027	0.027	0.028	0.029	0.029	0.029	0.030	0.031	0.031	0.033

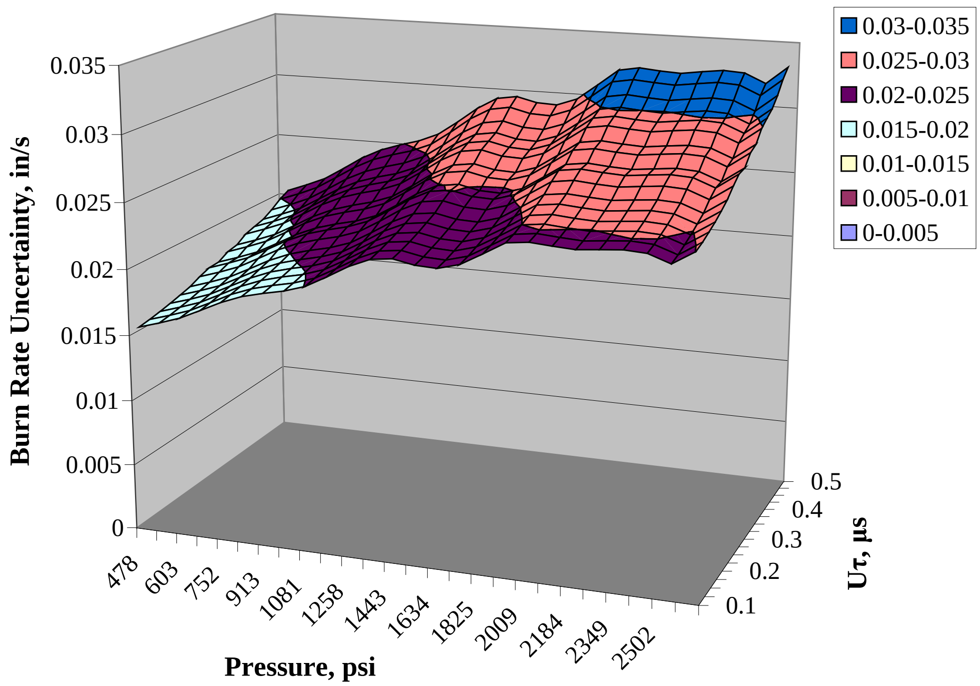


Figure A.11: Test 2 burn rate uncertainty for the Cross Correlation method.

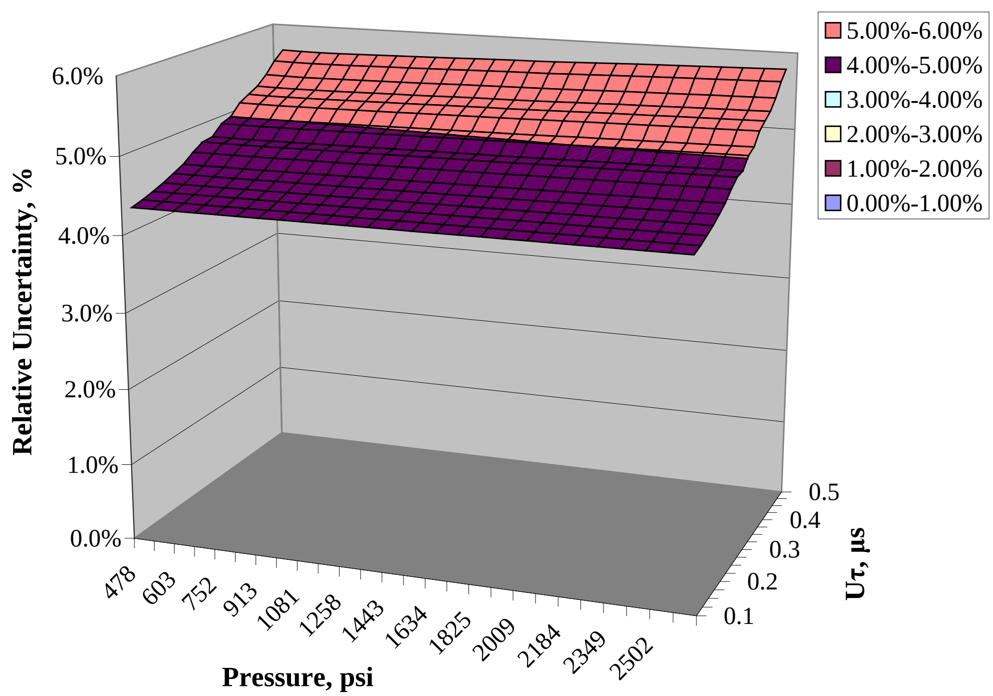


Figure A.12: Test 2 relative uncertainty for the Cross Correlation method.

Pressure, psi	Burn Rate, in/s	Propagation Time, μs																
		0.1	0.125	0.15	0.175	0.2	0.225	0.25	0.275	0.3	0.325	0.35	0.375	0.4	0.425	0.45	0.475	0.5
478	0.360	0.016	0.016	0.016	0.016	0.016	0.017	0.017	0.017	0.017	0.018	0.018	0.019	0.019	0.019	0.019	0.020	0.020
537	0.371	0.016	0.016	0.017	0.017	0.017	0.017	0.018	0.018	0.018	0.018	0.019	0.019	0.019	0.020	0.020	0.021	0.021
603	0.383	0.017	0.017	0.017	0.017	0.017	0.018	0.018	0.018	0.019	0.019	0.019	0.020	0.020	0.020	0.021	0.021	0.022
676	0.400	0.017	0.018	0.018	0.018	0.018	0.019	0.019	0.019	0.019	0.020	0.020	0.021	0.021	0.021	0.022	0.022	0.023
752	0.417	0.018	0.018	0.019	0.019	0.019	0.019	0.020	0.020	0.020	0.021	0.021	0.021	0.022	0.022	0.022	0.023	0.024
831	0.431	0.019	0.019	0.019	0.019	0.020	0.020	0.020	0.021	0.021	0.021	0.022	0.022	0.022	0.023	0.023	0.024	0.024
913	0.440	0.019	0.019	0.020	0.020	0.020	0.020	0.021	0.021	0.021	0.022	0.022	0.023	0.023	0.023	0.024	0.024	0.025
996	0.447	0.020	0.020	0.020	0.020	0.020	0.021	0.021	0.022	0.022	0.022	0.022	0.023	0.023	0.024	0.024	0.025	0.025
1081	0.459	0.020	0.020	0.020	0.021	0.021	0.021	0.022	0.022	0.022	0.023	0.023	0.024	0.024	0.024	0.025	0.025	0.026
1169	0.478	0.021	0.021	0.021	0.022	0.022	0.022	0.023	0.023	0.023	0.024	0.024	0.025	0.025	0.025	0.026	0.027	0.027
1258	0.498	0.022	0.022	0.022	0.022	0.023	0.023	0.024	0.024	0.024	0.025	0.025	0.026	0.026	0.026	0.027	0.028	0.028
1350	0.514	0.022	0.023	0.023	0.023	0.024	0.024	0.024	0.025	0.025	0.026	0.026	0.027	0.027	0.027	0.028	0.029	0.029
1443	0.518	0.023	0.023	0.023	0.023	0.024	0.024	0.025	0.025	0.025	0.026	0.026	0.027	0.027	0.028	0.028	0.029	0.029
1538	0.511	0.022	0.023	0.023	0.023	0.023	0.024	0.024	0.025	0.025	0.025	0.026	0.026	0.027	0.027	0.028	0.029	0.029
1634	0.509	0.022	0.022	0.023	0.023	0.023	0.024	0.024	0.025	0.025	0.025	0.026	0.026	0.027	0.027	0.028	0.028	0.029
1730	0.518	0.023	0.023	0.023	0.023	0.024	0.024	0.025	0.025	0.025	0.026	0.026	0.027	0.027	0.028	0.028	0.029	0.030
1825	0.539	0.024	0.024	0.024	0.024	0.025	0.025	0.026	0.026	0.026	0.027	0.027	0.028	0.028	0.029	0.029	0.030	0.031
1918	0.560	0.024	0.025	0.025	0.025	0.026	0.026	0.027	0.027	0.027	0.028	0.028	0.029	0.029	0.030	0.031	0.031	0.032
2009	0.564	0.025	0.025	0.025	0.025	0.026	0.026	0.027	0.027	0.027	0.028	0.029	0.029	0.030	0.030	0.031	0.032	0.032
2098	0.561	0.025	0.025	0.025	0.025	0.026	0.026	0.027	0.027	0.027	0.028	0.028	0.029	0.030	0.030	0.031	0.032	0.032
2184	0.559	0.024	0.025	0.025	0.025	0.026	0.026	0.027	0.027	0.027	0.028	0.028	0.029	0.030	0.030	0.031	0.031	0.032
2268	0.562	0.025	0.025	0.025	0.025	0.026	0.026	0.027	0.027	0.027	0.028	0.028	0.029	0.030	0.030	0.031	0.032	0.032
2349	0.565	0.025	0.025	0.025	0.026	0.026	0.026	0.027	0.027	0.028	0.028	0.029	0.029	0.030	0.030	0.031	0.032	0.033
2427	0.563	0.025	0.025	0.025	0.025	0.026	0.026	0.027	0.027	0.027	0.028	0.029	0.029	0.030	0.030	0.031	0.032	0.032
2502	0.550	0.024	0.024	0.025	0.025	0.025	0.026	0.026	0.027	0.027	0.028	0.028	0.029	0.029	0.030	0.030	0.031	0.032
2573	0.572	0.025	0.025	0.026	0.026	0.026	0.027	0.027	0.028	0.028	0.029	0.029	0.030	0.030	0.031	0.031	0.032	0.033

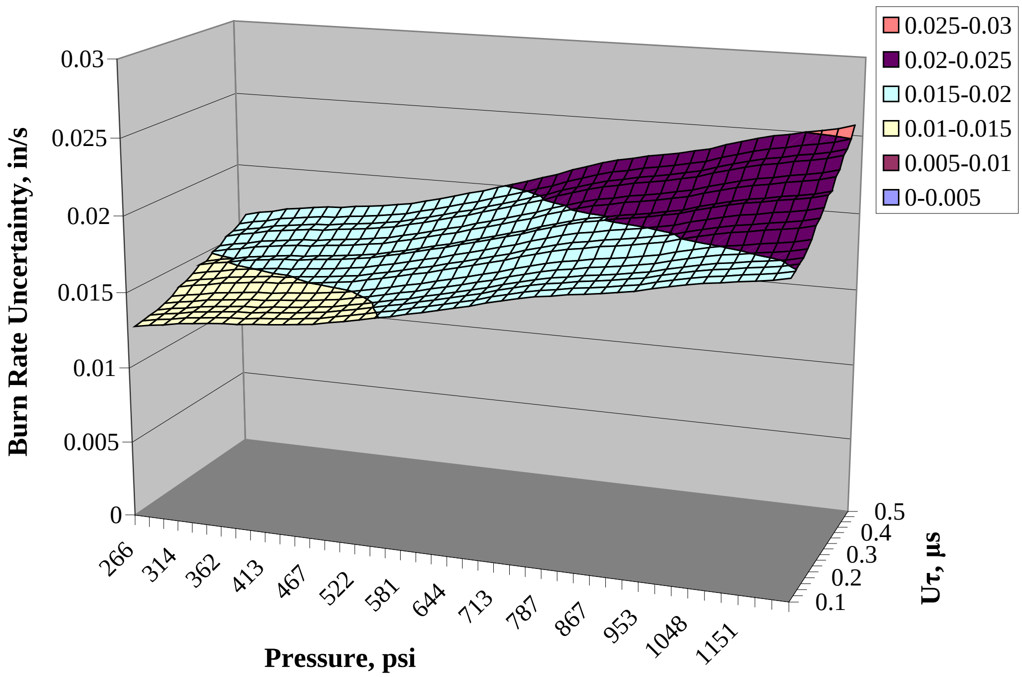


Figure A.13: Test 3 burn rate uncertainty for the EDUM method.

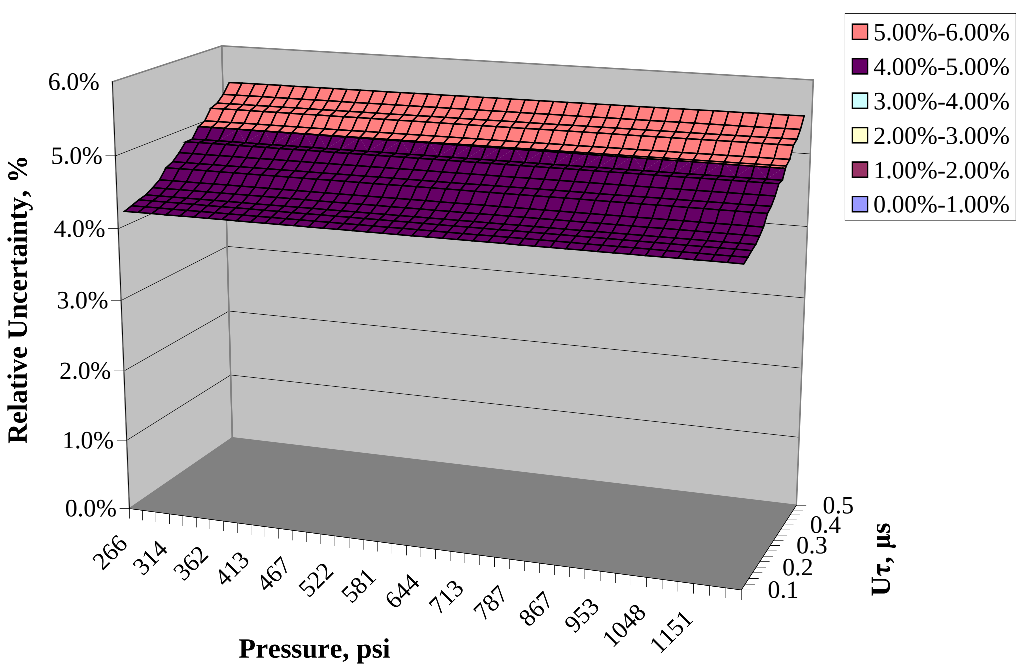


Figure A.14: Test 3 relative uncertainty for the EDUM method.

Pressure, psi	Burn Rate, in/s	Propagation Time, µs																
		0.1	0.125	0.15	0.175	0.2	0.225	0.25	0.275	0.3	0.325	0.35	0.375	0.4	0.425	0.45	0.475	0.5
266	0.302	0.013	0.013	0.013	0.013	0.013	0.013	0.014	0.014	0.014	0.015	0.015	0.015	0.015	0.016	0.016	0.016	0.016
282	0.306	0.013	0.013	0.013	0.013	0.013	0.014	0.014	0.014	0.014	0.015	0.015	0.015	0.015	0.016	0.016	0.016	0.017
298	0.309	0.013	0.013	0.013	0.013	0.014	0.014	0.014	0.014	0.015	0.015	0.015	0.015	0.016	0.016	0.016	0.016	0.017
314	0.313	0.013	0.013	0.014	0.014	0.014	0.014	0.014	0.015	0.015	0.015	0.015	0.016	0.016	0.016	0.016	0.017	0.017
330	0.316	0.013	0.014	0.014	0.014	0.014	0.014	0.015	0.015	0.015	0.015	0.015	0.016	0.016	0.016	0.017	0.017	0.017
346	0.318	0.014	0.014	0.014	0.014	0.014	0.014	0.015	0.015	0.015	0.015	0.015	0.016	0.016	0.017	0.017	0.017	0.017
362	0.320	0.014	0.014	0.014	0.014	0.014	0.014	0.015	0.015	0.015	0.016	0.016	0.016	0.016	0.017	0.017	0.017	0.018
379	0.321	0.014	0.014	0.014	0.014	0.014	0.014	0.015	0.015	0.015	0.016	0.016	0.016	0.016	0.017	0.017	0.017	0.018
396	0.324	0.014	0.014	0.014	0.014	0.014	0.014	0.015	0.015	0.015	0.016	0.016	0.016	0.016	0.017	0.017	0.017	0.018
413	0.326	0.014	0.014	0.014	0.014	0.014	0.015	0.015	0.015	0.015	0.016	0.016	0.016	0.016	0.017	0.017	0.017	0.018
431	0.328	0.014	0.014	0.014	0.014	0.014	0.015	0.015	0.015	0.016	0.016	0.016	0.016	0.017	0.017	0.017	0.017	0.018
449	0.331	0.014	0.014	0.014	0.014	0.015	0.015	0.015	0.015	0.016	0.016	0.016	0.017	0.017	0.017	0.017	0.018	0.018
467	0.333	0.014	0.014	0.014	0.014	0.015	0.015	0.015	0.015	0.016	0.016	0.016	0.017	0.017	0.017	0.017	0.018	0.018
485	0.338	0.014	0.014	0.015	0.015	0.015	0.015	0.016	0.016	0.016	0.016	0.016	0.017	0.017	0.018	0.018	0.018	0.019
503	0.342	0.015	0.015	0.015	0.015	0.015	0.015	0.016	0.016	0.016	0.017	0.017	0.017	0.017	0.018	0.018	0.018	0.019
522	0.347	0.015	0.015	0.015	0.015	0.015	0.016	0.016	0.016	0.016	0.017	0.017	0.017	0.018	0.018	0.018	0.019	0.019
541	0.352	0.015	0.015	0.015	0.015	0.016	0.016	0.016	0.016	0.017	0.017	0.017	0.018	0.018	0.018	0.018	0.019	0.019
561	0.356	0.015	0.015	0.015	0.015	0.016	0.016	0.016	0.017	0.017	0.017	0.017	0.018	0.018	0.019	0.019	0.019	0.020
581	0.362	0.015	0.016	0.016	0.016	0.016	0.016	0.017	0.017	0.017	0.018	0.018	0.018	0.018	0.019	0.019	0.019	0.020
602	0.367	0.016	0.016	0.016	0.016	0.016	0.016	0.017	0.017	0.017	0.018	0.018	0.018	0.019	0.019	0.019	0.020	0.020
623	0.372	0.016	0.016	0.016	0.016	0.016	0.017	0.017	0.017	0.018	0.018	0.018	0.019	0.019	0.019	0.020	0.020	0.020
644	0.378	0.016	0.016	0.016	0.016	0.017	0.017	0.017	0.018	0.018	0.018	0.018	0.019	0.019	0.020	0.020	0.020	0.021
667	0.383	0.016	0.016	0.017	0.017	0.017	0.017	0.018	0.018	0.018	0.019	0.019	0.019	0.019	0.020	0.020	0.020	0.021
689	0.390	0.017	0.017	0.017	0.017	0.017	0.017	0.018	0.018	0.019	0.019	0.019	0.020	0.020	0.020	0.020	0.021	0.021
713	0.396	0.017	0.017	0.017	0.017	0.017	0.018	0.018	0.018	0.019	0.019	0.019	0.020	0.020	0.021	0.021	0.021	0.022
737	0.401	0.017	0.017	0.017	0.017	0.018	0.018	0.019	0.019	0.019	0.020	0.020	0.020	0.020	0.021	0.021	0.021	0.022
762	0.406	0.017	0.017	0.018	0.018	0.018	0.018	0.019	0.019	0.019	0.020	0.020	0.020	0.021	0.021	0.021	0.022	0.022
787	0.409	0.017	0.018	0.018	0.018	0.018	0.018	0.019	0.019	0.019	0.020	0.020	0.021	0.021	0.021	0.022	0.022	0.022
813	0.414	0.018	0.018	0.018	0.018	0.018	0.019	0.019	0.019	0.020	0.020	0.020	0.021	0.021	0.022	0.022	0.022	0.023
839	0.417	0.018	0.018	0.018	0.018	0.018	0.019	0.019	0.019	0.020	0.020	0.020	0.021	0.021	0.022	0.022	0.022	0.023
867	0.420	0.018	0.018	0.018	0.018	0.019	0.019	0.019	0.020	0.020	0.020	0.020	0.021	0.021	0.022	0.022	0.022	0.023
895	0.424	0.018	0.018	0.018	0.018	0.019	0.019	0.020	0.020	0.020	0.021	0.021	0.021	0.021	0.022	0.022	0.023	0.023
924	0.427	0.018	0.018	0.018	0.019	0.019	0.019	0.020	0.020	0.020	0.021	0.021	0.021	0.022	0.022	0.022	0.023	0.023
953	0.434	0.018	0.019	0.019	0.019	0.019	0.019	0.020	0.020	0.021	0.021	0.021	0.022	0.022	0.023	0.023	0.023	0.024
984	0.439	0.019	0.019	0.019	0.019	0.019	0.020	0.020	0.020	0.021	0.021	0.021	0.022	0.022	0.023	0.023	0.023	0.024
1015	0.443	0.019	0.019	0.019	0.019	0.020	0.020	0.020	0.021	0.021	0.022	0.022	0.022	0.022	0.023	0.023	0.024	0.024
1048	0.448	0.019	0.019	0.019	0.019	0.020	0.020	0.021	0.021	0.021	0.022	0.022	0.022	0.023	0.023	0.024	0.024	0.025
1081	0.451	0.019	0.019	0.019	0.020	0.020	0.020	0.021	0.021	0.021	0.022	0.022	0.023	0.023	0.024	0.024	0.024	0.025
1116	0.455	0.019	0.019	0.020	0.020	0.020	0.020	0.021	0.021	0.022	0.022	0.022	0.023	0.023	0.024	0.024	0.024	0.025
1151	0.457	0.019	0.020	0.020	0.020	0.020	0.020	0.021	0.021	0.022	0.022	0.022	0.023	0.023	0.024	0.024	0.025	0.025
1188	0.461	0.020	0.020	0.020	0.020	0.020	0.021	0.021	0.021	0.022	0.022	0.022	0.023	0.023	0.024	0.024	0.025	0.025
1225	0.466	0.020	0.020	0.020	0.020	0.021	0.021	0.022	0.022	0.022	0.023	0.023	0.023	0.024	0.024	0.025	0.025	0.026

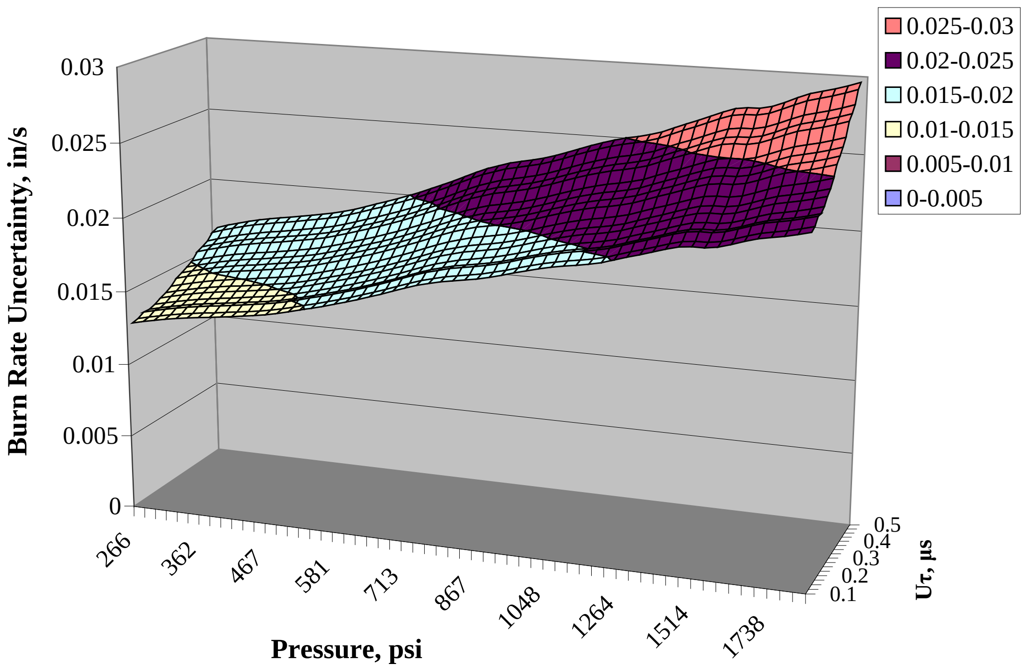


Figure A.15: Test 3 burn rate uncertainty for the Zero Crossing method.

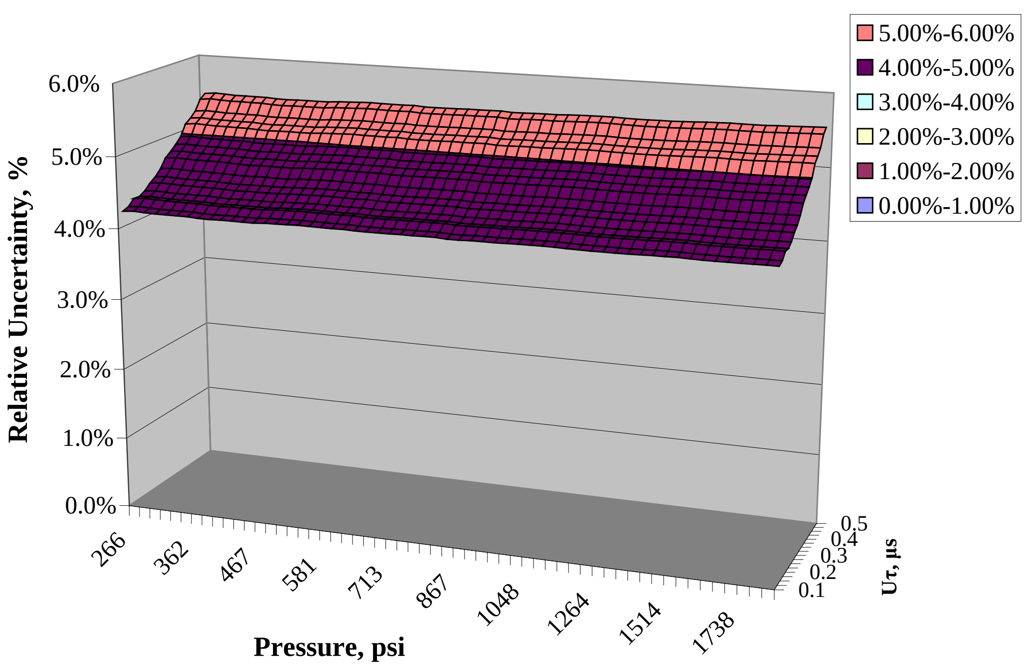


Figure A.16: Test 3 relative uncertainty for the Zero Crossing method.

Pressure, psi	Burn Rate, in/s	Propagation Time, μs																
		0.1	0.125	0.15	0.175	0.2	0.225	0.25	0.275	0.3	0.325	0.35	0.375	0.4	0.425	0.45	0.475	0.5
266	0.303	0.013	0.013	0.013	0.013	0.013	0.014	0.014	0.014	0.014	0.015	0.015	0.015	0.016	0.016	0.016	0.016	0.017
282	0.307	0.013	0.013	0.013	0.013	0.014	0.014	0.014	0.014	0.015	0.015	0.015	0.015	0.016	0.016	0.016	0.017	0.017
298	0.311	0.013	0.013	0.014	0.014	0.014	0.014	0.014	0.014	0.015	0.015	0.015	0.015	0.016	0.016	0.016	0.017	0.017
314	0.315	0.013	0.013	0.014	0.014	0.014	0.014	0.014	0.015	0.015	0.015	0.015	0.016	0.016	0.016	0.017	0.017	0.017
330	0.318	0.014	0.014	0.014	0.014	0.014	0.014	0.014	0.015	0.015	0.015	0.016	0.016	0.016	0.016	0.017	0.017	0.017
346	0.321	0.014	0.014	0.014	0.014	0.014	0.014	0.015	0.015	0.015	0.015	0.016	0.016	0.016	0.017	0.017	0.017	0.017
362	0.323	0.014	0.014	0.014	0.014	0.014	0.014	0.015	0.015	0.015	0.016	0.016	0.016	0.017	0.017	0.017	0.017	0.018
379	0.326	0.014	0.014	0.014	0.014	0.014	0.015	0.015	0.015	0.015	0.016	0.016	0.016	0.017	0.017	0.017	0.018	0.018
396	0.329	0.014	0.014	0.014	0.014	0.014	0.015	0.015	0.015	0.016	0.016	0.016	0.016	0.017	0.017	0.017	0.018	0.018
413	0.331	0.014	0.014	0.014	0.014	0.015	0.015	0.015	0.015	0.016	0.016	0.016	0.016	0.017	0.017	0.017	0.018	0.018
431	0.333	0.014	0.014	0.015	0.014	0.015	0.015	0.015	0.015	0.016	0.016	0.016	0.017	0.017	0.017	0.017	0.018	0.018
449	0.336	0.014	0.014	0.015	0.015	0.015	0.015	0.015	0.015	0.016	0.016	0.016	0.017	0.017	0.017	0.018	0.018	0.018
467	0.339	0.014	0.014	0.015	0.015	0.015	0.015	0.015	0.016	0.016	0.016	0.017	0.017	0.017	0.018	0.018	0.018	0.018
485	0.342	0.015	0.015	0.015	0.015	0.015	0.015	0.016	0.016	0.016	0.016	0.017	0.017	0.018	0.018	0.018	0.018	0.019
503	0.346	0.015	0.015	0.015	0.015	0.015	0.016	0.016	0.016	0.016	0.017	0.017	0.017	0.018	0.018	0.018	0.019	0.019
522	0.351	0.015	0.015	0.015	0.015	0.015	0.016	0.016	0.016	0.017	0.017	0.017	0.017	0.018	0.018	0.018	0.019	0.019
541	0.355	0.015	0.015	0.016	0.015	0.016	0.016	0.016	0.016	0.017	0.017	0.017	0.018	0.018	0.018	0.019	0.019	0.019
561	0.360	0.015	0.015	0.016	0.016	0.016	0.016	0.016	0.017	0.017	0.017	0.018	0.018	0.018	0.019	0.019	0.019	0.020
581	0.365	0.016	0.016	0.016	0.016	0.016	0.016	0.017	0.017	0.017	0.018	0.018	0.018	0.019	0.019	0.019	0.020	0.020
602	0.370	0.016	0.016	0.016	0.016	0.016	0.017	0.017	0.017	0.018	0.018	0.018	0.018	0.019	0.019	0.019	0.020	0.020
623	0.376	0.016	0.016	0.016	0.016	0.017	0.017	0.017	0.017	0.018	0.018	0.018	0.019	0.019	0.019	0.020	0.020	0.021
644	0.382	0.016	0.016	0.017	0.017	0.017	0.017	0.017	0.018	0.018	0.018	0.019	0.019	0.020	0.020	0.020	0.021	0.021
667	0.388	0.016	0.017	0.017	0.017	0.017	0.017	0.018	0.018	0.018	0.019	0.019	0.019	0.020	0.020	0.020	0.021	0.021
689	0.395	0.017	0.017	0.017	0.017	0.017	0.018	0.018	0.018	0.019	0.019	0.019	0.020	0.020	0.020	0.021	0.021	0.022
713	0.401	0.017	0.017	0.017	0.017	0.018	0.018	0.018	0.019	0.019	0.019	0.020	0.020	0.021	0.021	0.021	0.022	0.022
737	0.407	0.017	0.017	0.018	0.018	0.018	0.018	0.018	0.019	0.019	0.020	0.020	0.020	0.021	0.021	0.021	0.022	0.022
762	0.411	0.017	0.018	0.018	0.018	0.018	0.018	0.019	0.019	0.020	0.020	0.020	0.020	0.021	0.021	0.022	0.022	0.022
787	0.416	0.018	0.018	0.018	0.018	0.018	0.019	0.019	0.019	0.020	0.020	0.020	0.021	0.021	0.022	0.022	0.022	0.023
813	0.419	0.018	0.018	0.018	0.018	0.018	0.019	0.019	0.019	0.020	0.020	0.020	0.021	0.021	0.022	0.022	0.023	0.023
839	0.423	0.018	0.018	0.018	0.018	0.019	0.019	0.019	0.020	0.020	0.020	0.021	0.021	0.022	0.022	0.022	0.023	0.023
867	0.426	0.018	0.018	0.019	0.019	0.019	0.019	0.019	0.020	0.020	0.020	0.021	0.021	0.022	0.022	0.022	0.023	0.023
895	0.430	0.018	0.018	0.019	0.019	0.019	0.019	0.019	0.020	0.020	0.021	0.021	0.021	0.022	0.022	0.023	0.023	0.023
924	0.434	0.018	0.019	0.019	0.019	0.019	0.019	0.020	0.020	0.021	0.021	0.021	0.022	0.022	0.023	0.023	0.023	0.024
953	0.439	0.019	0.019	0.019	0.019	0.019	0.020	0.020	0.020	0.021	0.021	0.021	0.022	0.023	0.023	0.023	0.024	0.024
984	0.444	0.019	0.019	0.019	0.019	0.020	0.020	0.020	0.021	0.021	0.021	0.022	0.022	0.023	0.023	0.023	0.024	0.024
1015	0.448	0.019	0.019	0.020	0.020	0.020	0.020	0.020	0.021	0.021	0.022	0.022	0.022	0.023	0.023	0.024	0.024	0.025
1048	0.452	0.019	0.019	0.020	0.020	0.020	0.020	0.021	0.021	0.022	0.022	0.022	0.023	0.023	0.024	0.024	0.025	0.025
1081	0.456	0.019	0.020	0.020	0.020	0.020	0.020	0.021	0.021	0.022	0.022	0.022	0.023	0.023	0.024	0.024	0.025	0.025
1116	0.458	0.020	0.020	0.020	0.020	0.020	0.021	0.021	0.021	0.022	0.022	0.022	0.023	0.024	0.024	0.024	0.025	0.025
1151	0.462	0.020	0.020	0.020	0.020	0.020	0.021	0.021	0.021	0.022	0.022	0.023	0.023	0.024	0.024	0.024	0.025	0.025
1188	0.466	0.020	0.020	0.020	0.020	0.021	0.021	0.021	0.022	0.022	0.022	0.023	0.023	0.024	0.024	0.025	0.025	0.026
1225	0.472	0.020	0.020	0.021	0.021	0.021	0.021	0.021	0.022	0.022	0.023	0.023	0.024	0.024	0.025	0.025	0.026	0.026
1264	0.478	0.020	0.020	0.021	0.021	0.021	0.021	0.022	0.022	0.023	0.023	0.023	0.024	0.025	0.025	0.025	0.026	0.026
1304	0.483	0.021	0.021	0.021	0.021	0.021	0.022	0.022	0.022	0.023	0.023	0.024	0.024	0.025	0.025	0.026	0.026	0.027
1345	0.489	0.021	0.021	0.021	0.021	0.022	0.022	0.022	0.023	0.023	0.024	0.024	0.024	0.025	0.025	0.026	0.027	0.027
1387	0.494	0.021	0.021	0.022	0.022	0.022	0.022	0.022	0.023	0.024	0.024	0.024	0.025	0.025	0.026	0.026	0.027	0.027
1429	0.499	0.021	0.021	0.022	0.022	0.022	0.022	0.023	0.023	0.024	0.024	0.024	0.025	0.026	0.026	0.026	0.027	0.027
1471	0.500	0.021	0.021	0.022	0.022	0.022	0.022	0.023	0.023	0.024	0.024	0.025	0.025	0.026	0.026	0.026	0.027	0.028
1514	0.500	0.021	0.021	0.022	0.022	0.022	0.022	0.023	0.023	0.024	0.024	0.025	0.025	0.026	0.026	0.027	0.027	0.028
1555	0.503	0.021	0.022	0.022	0.022	0.022	0.023	0.023	0.023	0.024	0.024	0.025	0.025	0.026	0.026	0.027	0.027	0.028
1596	0.509	0.022	0.022	0.022	0.022	0.022	0.023	0.023	0.024	0.024	0.025	0.025	0.025	0.026	0.027	0.027	0.028	0.028
1634	0.515	0.022	0.022	0.022	0.022	0.023	0.023	0.023	0.024	0.025	0.025	0.025	0.026	0.027	0.027	0.027	0.028	0.028
1671	0.520	0.022	0.022	0.023	0.023	0.023	0.023	0.024	0.024	0.025	0.025	0.026	0.026	0.027	0.027	0.028	0.028	0.029
1706	0.524	0.022	0.022	0.023	0.023	0.023	0.024	0.024	0.024	0.025	0.025	0.026	0.026	0.027	0.027	0.028	0.029	0.029
1738	0.527	0.022	0.023	0.023	0.023	0.023	0.024	0.024	0.024	0.025	0.026	0.026	0.026	0.027	0.028	0.028	0.029	0.029
1768	0.531	0.023	0.023	0.023	0.023	0.023	0.024	0.024	0.025	0.025	0.026	0.026	0.027	0.027	0.028	0.028	0.029	0.029
1795	0.535	0.023	0.023	0.023	0.023	0.024	0.024	0.024	0.025	0.026	0.026	0.026	0.027	0.028	0.028	0.028	0.029	0.030

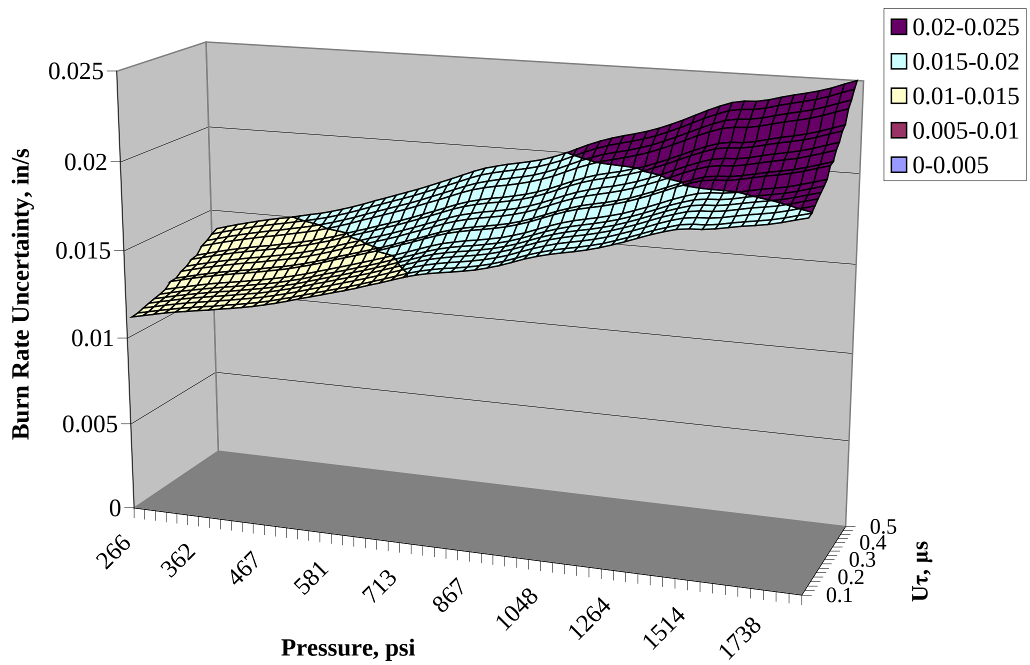


Figure A.17: Test 3 burn rate uncertainty for the Cross Correlation method.

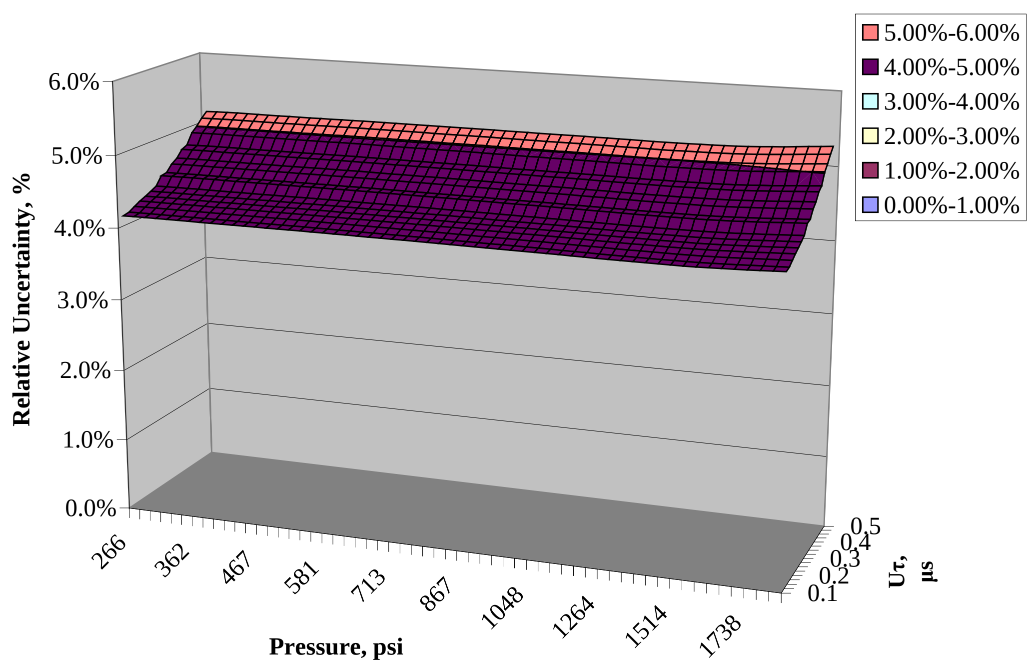


Figure A.18: Test 3 relative uncertainty for the Cross Correlation method.

Pressure, psi	Burn Rate, in/s	Propagation Time, μs																
		0.1	0.125	0.15	0.175	0.2	0.225	0.25	0.275	0.3	0.325	0.35	0.375	0.4	0.425	0.45	0.475	0.5
266	0.270	0.011	0.011	0.011	0.012	0.012	0.012	0.012	0.012	0.012	0.012	0.013	0.013	0.013	0.013	0.013	0.014	0.014
282	0.273	0.011	0.011	0.012	0.012	0.012	0.012	0.012	0.012	0.012	0.013	0.013	0.013	0.013	0.013	0.014	0.014	0.014
298	0.276	0.012	0.012	0.012	0.012	0.012	0.012	0.012	0.012	0.012	0.013	0.013	0.013	0.013	0.014	0.014	0.014	0.014
314	0.279	0.012	0.012	0.012	0.012	0.012	0.012	0.012	0.013	0.013	0.013	0.013	0.013	0.013	0.014	0.014	0.014	0.014
330	0.282	0.012	0.012	0.012	0.012	0.012	0.012	0.012	0.013	0.013	0.013	0.013	0.013	0.014	0.014	0.014	0.014	0.015
346	0.285	0.012	0.012	0.012	0.012	0.012	0.012	0.013	0.013	0.013	0.013	0.013	0.014	0.014	0.014	0.014	0.015	0.015
362	0.287	0.012	0.012	0.012	0.012	0.012	0.012	0.013	0.013	0.013	0.013	0.013	0.014	0.014	0.014	0.014	0.015	0.015
379	0.290	0.012	0.012	0.012	0.012	0.012	0.013	0.013	0.013	0.013	0.013	0.014	0.014	0.014	0.014	0.014	0.015	0.015
396	0.292	0.012	0.012	0.012	0.013	0.013	0.013	0.013	0.013	0.013	0.013	0.014	0.014	0.014	0.014	0.015	0.015	0.015
413	0.295	0.012	0.012	0.012	0.013	0.013	0.013	0.013	0.013	0.013	0.014	0.014	0.014	0.014	0.015	0.015	0.015	0.015
431	0.297	0.012	0.012	0.013	0.013	0.013	0.013	0.013	0.013	0.013	0.014	0.014	0.014	0.014	0.015	0.015	0.015	0.015
449	0.300	0.013	0.013	0.013	0.013	0.013	0.013	0.013	0.014	0.014	0.014	0.014	0.014	0.014	0.015	0.015	0.015	0.016
467	0.304	0.013	0.013	0.013	0.013	0.013	0.013	0.013	0.014	0.014	0.014	0.014	0.014	0.015	0.015	0.015	0.015	0.016
485	0.308	0.013	0.013	0.013	0.013	0.013	0.013	0.014	0.014	0.014	0.014	0.014	0.015	0.015	0.015	0.015	0.016	0.016
503	0.312	0.013	0.013	0.013	0.013	0.013	0.014	0.014	0.014	0.014	0.014	0.015	0.015	0.015	0.015	0.016	0.016	0.016
522	0.316	0.013	0.013	0.013	0.014	0.014	0.014	0.014	0.014	0.014	0.015	0.015	0.015	0.015	0.016	0.016	0.016	0.016
541	0.320	0.013	0.013	0.014	0.014	0.014	0.014	0.014	0.014	0.014	0.015	0.015	0.015	0.015	0.016	0.016	0.016	0.017
561	0.325	0.014	0.014	0.014	0.014	0.014	0.014	0.014	0.015	0.015	0.015	0.015	0.015	0.016	0.016	0.016	0.017	0.017
581	0.329	0.014	0.014	0.014	0.014	0.014	0.014	0.014	0.015	0.015	0.015	0.015	0.016	0.016	0.016	0.017	0.017	0.017
602	0.334	0.014	0.014	0.014	0.014	0.014	0.015	0.015	0.015	0.015	0.015	0.016	0.016	0.016	0.016	0.017	0.017	0.017
623	0.338	0.014	0.014	0.014	0.014	0.015	0.015	0.015	0.015	0.015	0.016	0.016	0.016	0.016	0.017	0.017	0.017	0.017
644	0.343	0.014	0.014	0.015	0.015	0.015	0.015	0.015	0.015	0.016	0.016	0.016	0.016	0.016	0.017	0.017	0.018	0.018
667	0.348	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.016	0.016	0.016	0.016	0.017	0.017	0.017	0.017	0.018	0.018
689	0.353	0.015	0.015	0.015	0.015	0.015	0.015	0.016	0.016	0.016	0.016	0.016	0.017	0.017	0.017	0.018	0.018	0.018
713	0.358	0.015	0.015	0.015	0.015	0.015	0.016	0.016	0.016	0.016	0.017	0.017	0.017	0.017	0.018	0.018	0.018	0.019
737	0.363	0.015	0.015	0.015	0.016	0.016	0.016	0.016	0.016	0.016	0.017	0.017	0.017	0.017	0.018	0.018	0.019	0.019
762	0.366	0.015	0.015	0.015	0.016	0.016	0.016	0.016	0.016	0.017	0.017	0.017	0.017	0.018	0.018	0.018	0.019	0.019
787	0.369	0.015	0.015	0.016	0.016	0.016	0.016	0.016	0.017	0.017	0.017	0.017	0.018	0.018	0.018	0.018	0.019	0.019
813	0.371	0.015	0.016	0.016	0.016	0.016	0.016	0.016	0.017	0.017	0.017	0.017	0.018	0.018	0.018	0.019	0.019	0.019
839	0.374	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.017	0.017	0.017	0.017	0.018	0.018	0.018	0.019	0.019	0.019
867	0.377	0.016	0.016	0.016	0.016	0.016	0.016	0.017	0.017	0.017	0.017	0.018	0.018	0.018	0.019	0.019	0.019	0.019
895	0.381	0.016	0.016	0.016	0.016	0.016	0.017	0.017	0.017	0.017	0.018	0.018	0.018	0.018	0.019	0.019	0.019	0.020
924	0.386	0.016	0.016	0.016	0.016	0.017	0.017	0.017	0.017	0.017	0.018	0.018	0.018	0.018	0.019	0.019	0.020	0.020
953	0.391	0.016	0.016	0.017	0.017	0.017	0.017	0.017	0.018	0.018	0.018	0.018	0.019	0.019	0.019	0.020	0.020	0.020
984	0.397	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.018	0.018	0.018	0.018	0.019	0.019	0.020	0.020	0.020	0.021
1015	0.402	0.017	0.017	0.017	0.017	0.017	0.017	0.018	0.018	0.018	0.019	0.019	0.019	0.019	0.020	0.020	0.020	0.021
1048	0.406	0.017	0.017	0.017	0.017	0.017	0.018	0.018	0.018	0.018	0.019	0.019	0.019	0.019	0.020	0.020	0.021	0.021
1081	0.409	0.017	0.017	0.017	0.017	0.018	0.018	0.018	0.018	0.018	0.019	0.019	0.019	0.020	0.020	0.021	0.021	0.021
1116	0.413	0.017	0.017	0.017	0.018	0.018	0.018	0.018	0.019	0.019	0.019	0.019	0.020	0.020	0.020	0.021	0.021	0.021
1151	0.416	0.017	0.017	0.018	0.018	0.018	0.018	0.018	0.019	0.019	0.019	0.019	0.020	0.020	0.021	0.021	0.021	0.022
1188	0.421	0.017	0.018	0.018	0.018	0.018	0.018	0.018	0.019	0.019	0.019	0.020	0.020	0.020	0.021	0.021	0.021	0.022
1225	0.426	0.018	0.018	0.018	0.018	0.018	0.018	0.019	0.019	0.019	0.020	0.020	0.020	0.020	0.021	0.021	0.022	0.022
1264	0.431	0.018	0.018	0.018	0.018	0.018	0.019	0.019	0.019	0.019	0.020	0.020	0.020	0.021	0.021	0.022	0.022	0.022
1304	0.437	0.018	0.018	0.018	0.019	0.019	0.019	0.019	0.020	0.020	0.020	0.020	0.021	0.021	0.022	0.022	0.022	0.023
1345	0.443	0.018	0.018	0.019	0.019	0.019	0.019	0.019	0.020	0.020	0.020	0.021	0.021	0.021	0.022	0.022	0.023	0.023
1386	0.448	0.019	0.019	0.019	0.019	0.019	0.019	0.020	0.020	0.020	0.021	0.021	0.021	0.021	0.022	0.022	0.023	0.023
1429	0.452	0.019	0.019	0.019	0.019	0.019	0.020	0.020	0.020	0.020	0.021	0.021	0.021	0.022	0.022	0.023	0.023	0.023
1471	0.454	0.019	0.019	0.019	0.019	0.019	0.020	0.020	0.020	0.020	0.021	0.021	0.022	0.022	0.022	0.023	0.023	0.024
1514	0.455	0.019	0.019	0.019	0.019	0.020	0.020	0.020	0.020	0.020	0.021	0.021	0.022	0.022	0.022	0.023	0.023	0.024
1555	0.457	0.019	0.019	0.019	0.019	0.020	0.020	0.020	0.021	0.021	0.021	0.021	0.022	0.022	0.023	0.023	0.023	0.024
1596	0.459	0.019	0.019	0.019	0.020	0.020	0.020	0.020	0.021	0.021	0.021	0.021	0.022	0.022	0.023	0.023	0.024	0.024
1634	0.462	0.019	0.019	0.019	0.020	0.020	0.020	0.020	0.021	0.021	0.021	0.022	0.022	0.022	0.023	0.023	0.024	0.024
1671	0.464	0.019	0.019	0.020	0.020	0.020	0.020	0.020	0.021	0.021	0.021	0.022	0.022	0.022	0.023	0.023	0.024	0.024
1706	0.465	0.019	0.019	0.020	0.020	0.020	0.020	0.020	0.021	0.021	0.022	0.022	0.022	0.022	0.023	0.024	0.024	0.024
1738	0.468	0.020	0.020	0.020	0.020	0.020	0.020	0.021	0.021	0.021	0.022	0.022	0.022	0.023	0.023	0.024	0.024	0.025
1768	0.472	0.020	0.020	0.020	0.020	0.020	0.021	0.021	0.021	0.021	0.022	0.022	0.023	0.023	0.024	0.024	0.024	0.025
1795	0.475	0.020	0.020	0.020	0.020	0.021	0.021	0.021	0.022	0.022	0.022	0.022	0.023	0.023	0.024	0.024	0.025	0.025

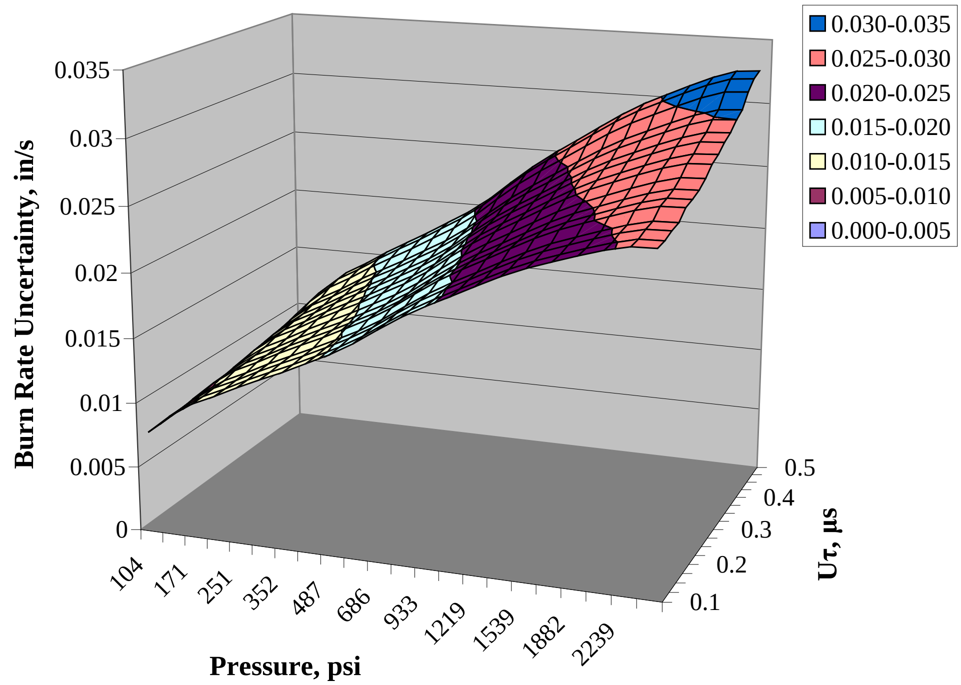


Figure A.19: Test 4 burn rate uncertainty for the EDUM method.

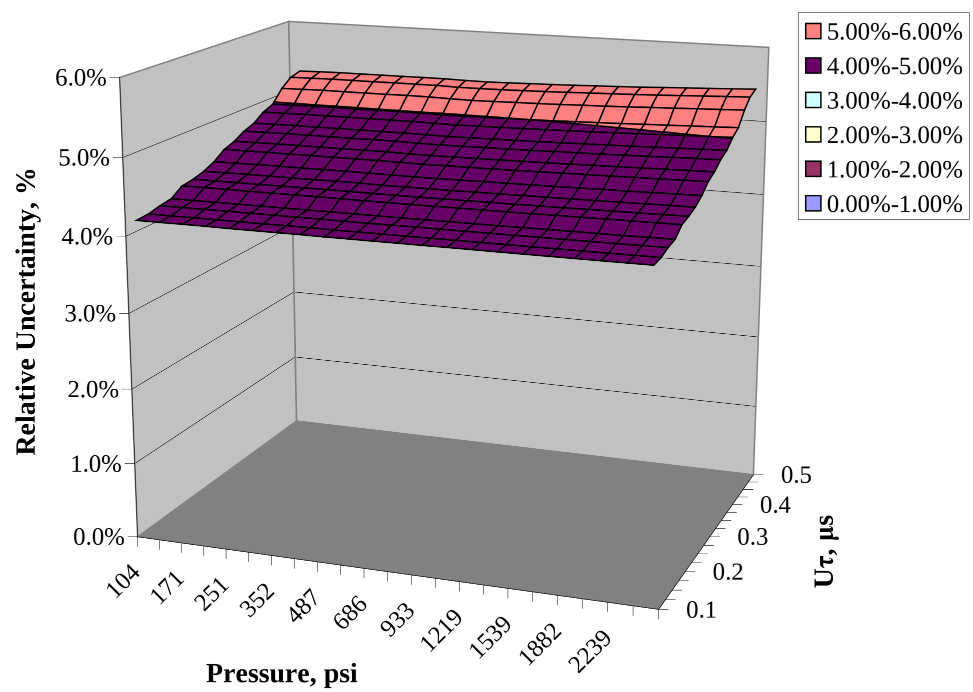


Figure A.20: Test 4 relative uncertainty for the EDUM method.

Pressure, psi	Burn Rate, in/s	Propagation Time, µs																
		0.1	0.125	0.15	0.175	0.2	0.225	0.25	0.275	0.3	0.325	0.35	0.375	0.4	0.425	0.45	0.475	0.5
104	0.186	0.008	0.008	0.008	0.008	0.008	0.008	0.008	0.008	0.009	0.009	0.009	0.009	0.009	0.009	0.010	0.010	0.010
135	0.221	0.009	0.009	0.009	0.009	0.010	0.010	0.010	0.010	0.010	0.010	0.011	0.011	0.011	0.011	0.011	0.012	0.012
171	0.248	0.010	0.010	0.011	0.011	0.011	0.011	0.011	0.011	0.011	0.012	0.012	0.012	0.012	0.012	0.013	0.013	0.013
209	0.267	0.011	0.011	0.011	0.011	0.012	0.012	0.012	0.012	0.012	0.012	0.013	0.013	0.013	0.013	0.014	0.014	0.014
251	0.287	0.012	0.012	0.012	0.012	0.013	0.013	0.013	0.013	0.013	0.013	0.014	0.014	0.014	0.014	0.015	0.015	0.015
299	0.306	0.013	0.013	0.013	0.013	0.013	0.014	0.014	0.014	0.014	0.014	0.015	0.015	0.015	0.015	0.016	0.016	0.016
352	0.325	0.014	0.014	0.014	0.014	0.014	0.014	0.014	0.015	0.015	0.015	0.015	0.016	0.016	0.016	0.017	0.017	0.017
412	0.344	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.016	0.016	0.016	0.016	0.017	0.017	0.017	0.018	0.018	0.018
487	0.365	0.015	0.015	0.016	0.016	0.016	0.016	0.016	0.017	0.017	0.017	0.017	0.018	0.018	0.018	0.019	0.019	0.019
579	0.389	0.016	0.016	0.017	0.017	0.017	0.017	0.017	0.018	0.018	0.018	0.019	0.019	0.019	0.019	0.020	0.020	0.021
686	0.417	0.018	0.018	0.018	0.018	0.018	0.018	0.019	0.019	0.019	0.020	0.020	0.020	0.021	0.021	0.021	0.022	0.022
805	0.444	0.019	0.019	0.019	0.019	0.019	0.020	0.020	0.020	0.021	0.021	0.021	0.021	0.022	0.022	0.023	0.023	0.024
933	0.467	0.020	0.020	0.020	0.020	0.021	0.021	0.021	0.021	0.022	0.022	0.022	0.023	0.023	0.023	0.024	0.025	0.025
1071	0.487	0.021	0.021	0.021	0.021	0.021	0.022	0.022	0.022	0.023	0.023	0.023	0.024	0.024	0.024	0.025	0.026	0.026
1219	0.509	0.021	0.022	0.022	0.022	0.022	0.023	0.023	0.023	0.024	0.024	0.024	0.025	0.025	0.026	0.026	0.027	0.027
1376	0.529	0.022	0.022	0.023	0.023	0.023	0.023	0.024	0.024	0.025	0.025	0.025	0.026	0.026	0.027	0.027	0.028	0.028
1539	0.547	0.023	0.023	0.023	0.024	0.024	0.024	0.024	0.025	0.025	0.026	0.026	0.027	0.027	0.028	0.028	0.029	0.029
1709	0.562	0.024	0.024	0.024	0.024	0.025	0.025	0.025	0.026	0.026	0.027	0.027	0.027	0.028	0.028	0.029	0.030	0.030
1882	0.576	0.024	0.024	0.025	0.025	0.025	0.026	0.026	0.026	0.027	0.027	0.028	0.028	0.029	0.029	0.030	0.031	0.031
2059	0.588	0.025	0.025	0.025	0.025	0.026	0.026	0.026	0.027	0.027	0.028	0.028	0.029	0.029	0.030	0.031	0.031	0.032
2239	0.598	0.025	0.025	0.026	0.026	0.026	0.027	0.027	0.027	0.028	0.028	0.029	0.029	0.030	0.030	0.031	0.032	0.032
2423	0.599	0.025	0.025	0.026	0.026	0.026	0.027	0.027	0.027	0.028	0.028	0.029	0.029	0.030	0.030	0.031	0.032	0.033

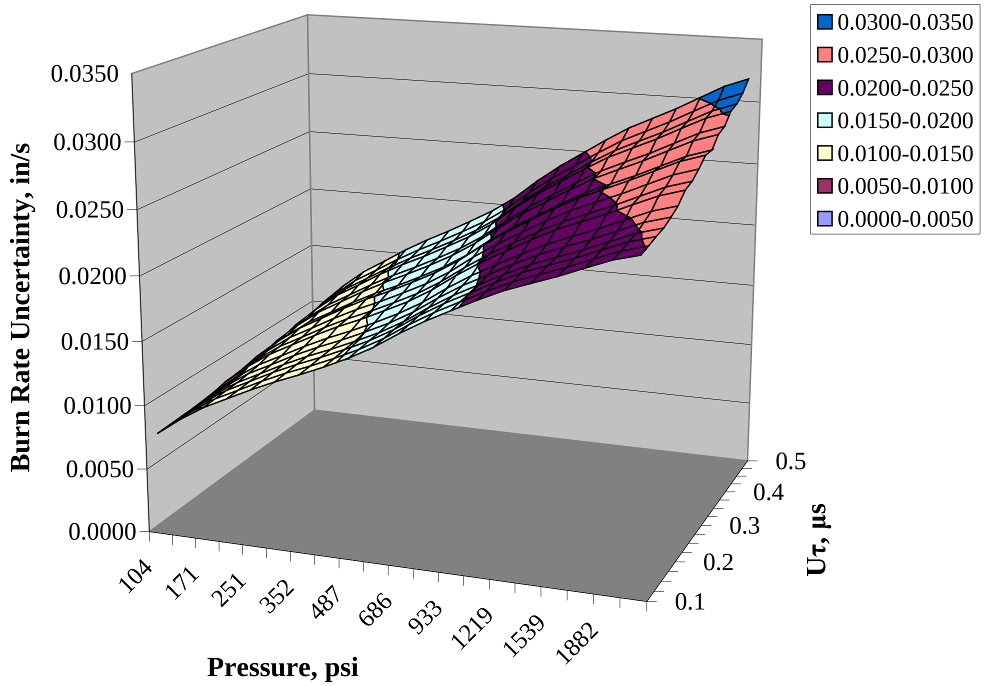


Figure A.21: Test 4 burn rate uncertainty for the Zero Crossing method.

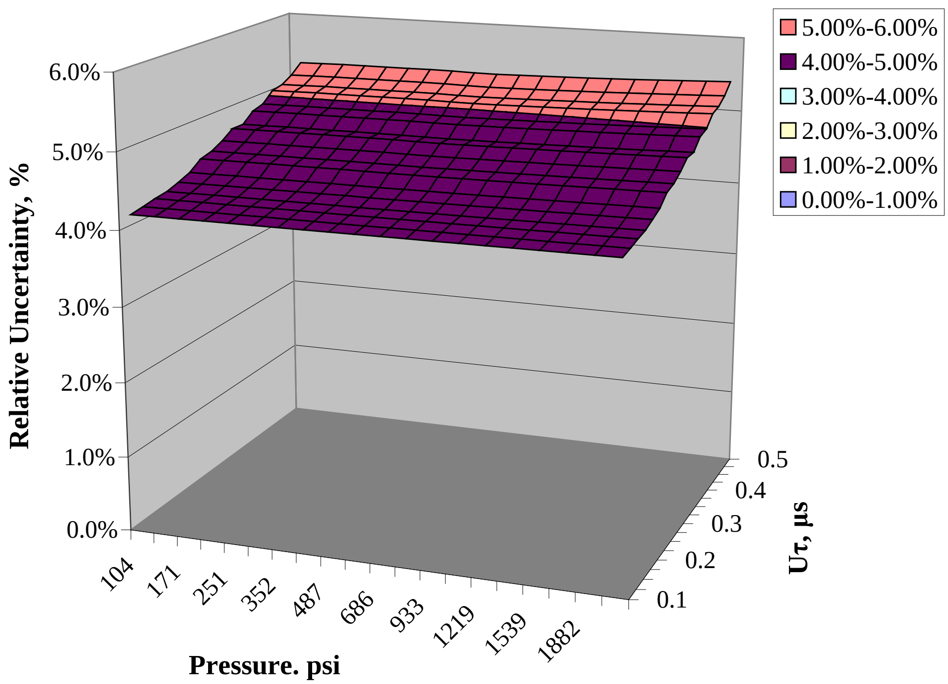


Figure A.22: Test 4 relative uncertainty for the Zero Crossing method.

Pressure, psi	Burn Rate, in/s	Propagation Time, µs																
		0.1	0.125	0.15	0.175	0.2	0.225	0.25	0.275	0.3	0.325	0.35	0.375	0.4	0.425	0.45	0.475	0.5
104	0.188	0.008	0.008	0.008	0.008	0.008	0.008	0.009	0.009	0.009	0.009	0.009	0.009	0.009	0.009	0.010	0.010	0.010
135	0.219	0.009	0.009	0.009	0.009	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.011	0.011	0.011	0.011	0.011	0.012
171	0.247	0.010	0.010	0.011	0.011	0.011	0.011	0.011	0.011	0.011	0.012	0.012	0.012	0.012	0.012	0.013	0.013	0.013
209	0.268	0.011	0.011	0.011	0.012	0.012	0.012	0.012	0.012	0.012	0.013	0.013	0.013	0.013	0.014	0.014	0.014	0.014
251	0.289	0.012	0.012	0.012	0.012	0.013	0.013	0.013	0.013	0.013	0.014	0.014	0.014	0.014	0.015	0.015	0.015	0.015
299	0.309	0.013	0.013	0.013	0.013	0.013	0.014	0.014	0.014	0.014	0.015	0.015	0.015	0.015	0.016	0.016	0.016	0.016
352	0.326	0.014	0.014	0.014	0.014	0.014	0.014	0.015	0.015	0.015	0.016	0.016	0.016	0.016	0.017	0.017	0.017	0.017
412	0.345	0.015	0.015	0.015	0.015	0.015	0.015	0.016	0.016	0.016	0.016	0.016	0.017	0.017	0.017	0.018	0.018	0.018
487	0.366	0.015	0.016	0.016	0.016	0.016	0.016	0.017	0.017	0.017	0.017	0.017	0.018	0.018	0.018	0.019	0.019	0.019
579	0.390	0.016	0.017	0.017	0.017	0.017	0.017	0.018	0.018	0.018	0.019	0.019	0.019	0.019	0.020	0.020	0.020	0.021
686	0.419	0.018	0.018	0.018	0.018	0.018	0.018	0.019	0.019	0.019	0.020	0.020	0.020	0.021	0.021	0.021	0.022	0.022
805	0.444	0.019	0.019	0.019	0.019	0.019	0.020	0.020	0.020	0.021	0.021	0.021	0.022	0.022	0.023	0.023	0.023	0.024
933	0.467	0.020	0.020	0.020	0.020	0.020	0.021	0.021	0.021	0.022	0.022	0.022	0.023	0.023	0.024	0.024	0.024	0.025
1071	0.489	0.021	0.021	0.021	0.021	0.021	0.022	0.022	0.022	0.023	0.023	0.023	0.024	0.024	0.025	0.025	0.025	0.026
1219	0.510	0.022	0.022	0.022	0.022	0.022	0.023	0.023	0.023	0.024	0.024	0.024	0.025	0.025	0.026	0.026	0.027	0.027
1376	0.526	0.022	0.022	0.023	0.023	0.023	0.023	0.024	0.024	0.025	0.025	0.025	0.026	0.026	0.027	0.027	0.027	0.028
1539	0.542	0.023	0.023	0.023	0.023	0.024	0.024	0.025	0.025	0.025	0.026	0.026	0.027	0.027	0.028	0.028	0.028	0.029
1709	0.560	0.024	0.024	0.024	0.024	0.024	0.025	0.026	0.026	0.026	0.027	0.027	0.028	0.028	0.029	0.029	0.029	0.030
1882	0.578	0.024	0.025	0.025	0.025	0.025	0.026	0.026	0.027	0.027	0.028	0.028	0.029	0.029	0.030	0.030	0.030	0.031
2059	0.589	0.025	0.025	0.025	0.025	0.026	0.026	0.027	0.027	0.028	0.028	0.028	0.029	0.029	0.030	0.030	0.031	0.032

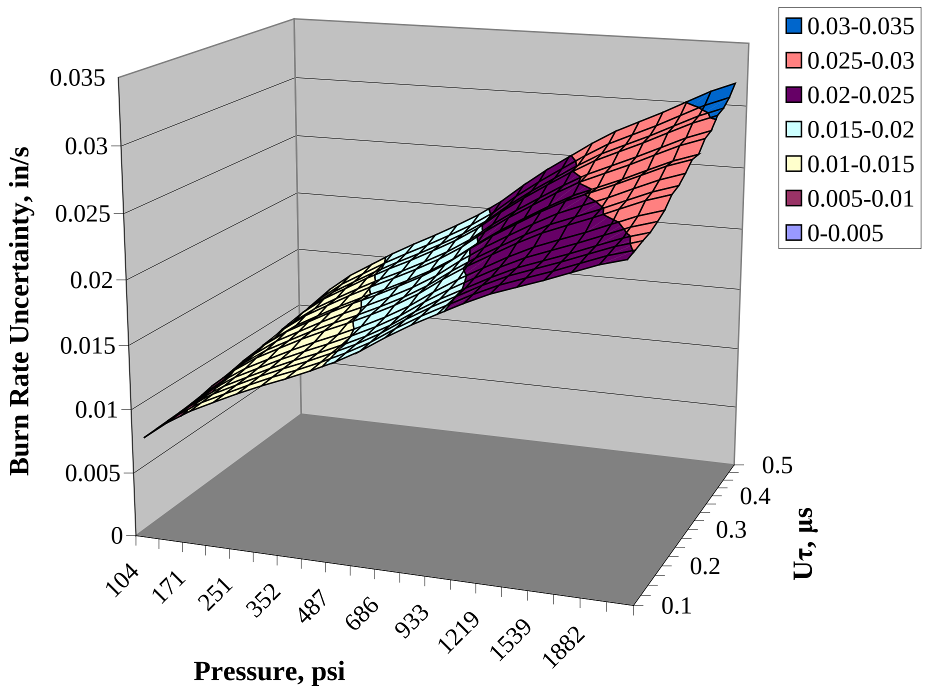


Figure A.23: Test 4 burn rate uncertainty for the Cross Correlation method.

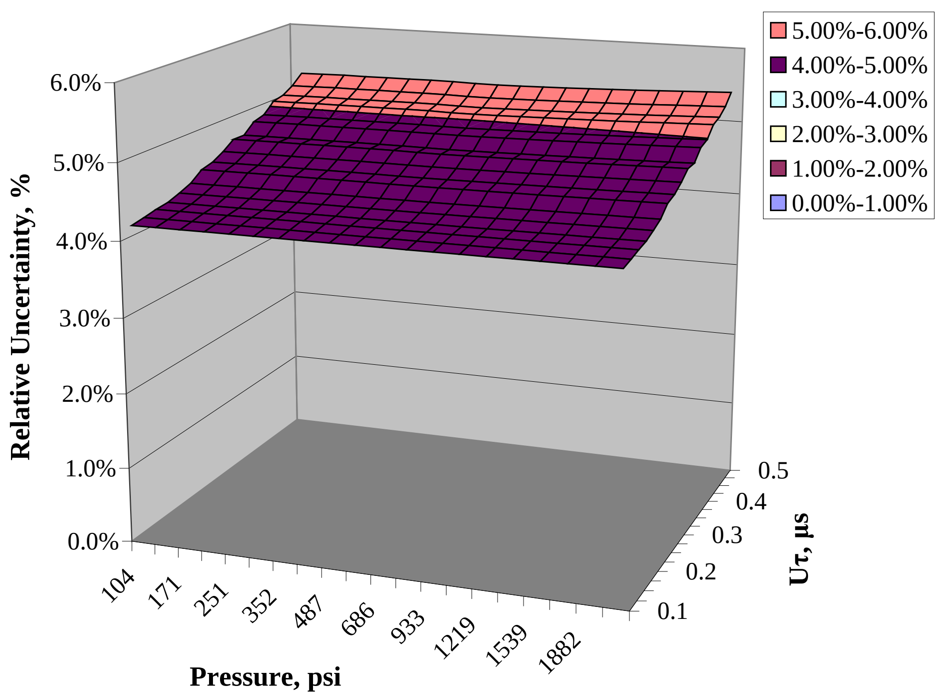


Figure A.24: Test 4 relative uncertainty for the Cross Correlation method.

Pressure,	Burn	Propagation Time, µs																
psi	Rate, in/s	0.1	0.125	0.15	0.175	0.2	0.225	0.25	0.275	0.3	0.325	0.35	0.375	0.4	0.425	0.45	0.475	0.5
104	0.187	0.008	0.008	0.008	0.008	0.008	0.008	0.008	0.009	0.009	0.009	0.009	0.009	0.009	0.009	0.010	0.010	0.010
135	0.220	0.009	0.009	0.009	0.009	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.011	0.011	0.011	0.011	0.011	0.012
171	0.247	0.010	0.010	0.011	0.011	0.011	0.011	0.011	0.011	0.011	0.012	0.012	0.012	0.012	0.013	0.013	0.013	0.013
209	0.269	0.011	0.011	0.011	0.012	0.012	0.012	0.012	0.012	0.012	0.013	0.013	0.013	0.013	0.014	0.014	0.014	0.014
251	0.289	0.012	0.012	0.012	0.012	0.013	0.013	0.013	0.013	0.013	0.014	0.014	0.014	0.014	0.015	0.015	0.015	0.015
299	0.309	0.013	0.013	0.013	0.013	0.013	0.014	0.014	0.014	0.014	0.015	0.015	0.015	0.015	0.016	0.016	0.016	0.016
352	0.326	0.014	0.014	0.014	0.014	0.014	0.014	0.015	0.015	0.015	0.016	0.016	0.016	0.016	0.017	0.017	0.017	0.017
412	0.346	0.015	0.015	0.015	0.015	0.015	0.015	0.016	0.016	0.016	0.016	0.016	0.017	0.017	0.017	0.018	0.018	0.018
487	0.366	0.015	0.016	0.016	0.016	0.016	0.016	0.017	0.017	0.017	0.017	0.017	0.018	0.018	0.019	0.019	0.019	0.019
579	0.390	0.016	0.017	0.017	0.017	0.017	0.017	0.018	0.018	0.018	0.019	0.019	0.019	0.019	0.020	0.020	0.020	0.021
686	0.419	0.018	0.018	0.018	0.018	0.018	0.018	0.019	0.019	0.019	0.020	0.020	0.020	0.021	0.021	0.021	0.022	0.022
805	0.445	0.019	0.019	0.019	0.019	0.019	0.020	0.020	0.020	0.021	0.021	0.021	0.022	0.022	0.023	0.023	0.023	0.024
933	0.468	0.020	0.020	0.020	0.020	0.020	0.021	0.021	0.021	0.022	0.022	0.022	0.023	0.023	0.024	0.024	0.024	0.025
1071	0.491	0.021	0.021	0.021	0.021	0.021	0.022	0.022	0.022	0.023	0.023	0.023	0.024	0.024	0.025	0.025	0.025	0.026
1219	0.511	0.022	0.022	0.022	0.022	0.022	0.023	0.023	0.023	0.024	0.024	0.024	0.025	0.025	0.026	0.026	0.027	0.027
1376	0.527	0.022	0.022	0.023	0.023	0.023	0.023	0.024	0.024	0.025	0.025	0.025	0.026	0.026	0.027	0.027	0.027	0.028
1539	0.543	0.023	0.023	0.023	0.023	0.024	0.024	0.025	0.025	0.025	0.026	0.026	0.027	0.027	0.028	0.028	0.028	0.029
1709	0.560	0.024	0.024	0.024	0.024	0.024	0.025	0.025	0.026	0.026	0.027	0.027	0.028	0.028	0.029	0.029	0.029	0.030
1882	0.576	0.024	0.025	0.025	0.025	0.025	0.026	0.026	0.027	0.027	0.028	0.028	0.028	0.029	0.029	0.030	0.030	0.031
2059	0.589	0.025	0.025	0.025	0.025	0.026	0.026	0.027	0.027	0.028	0.028	0.028	0.029	0.029	0.030	0.030	0.031	0.032

Pressure,	Burn	Propagation Time, µs																
psi	Rate, in/s	0.1	0.125	0.15	0.175	0.2	0.225	0.25	0.275	0.3	0.325	0.35	0.375	0.4	0.425	0.45	0.475	0.5
282	0.289	0.012	0.013	0.013	0.013	0.013	0.013	0.013	0.013	0.014	0.014	0.014	0.014	0.015	0.015	0.015	0.015	0.016
358	0.335	0.014	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.016	0.016	0.016	0.017	0.017	0.017	0.017	0.018	0.018
449	0.357	0.015	0.015	0.015	0.016	0.016	0.016	0.016	0.016	0.017	0.017	0.017	0.018	0.018	0.018	0.019	0.019	0.019
552	0.379	0.016	0.016	0.016	0.017	0.017	0.017	0.017	0.017	0.018	0.018	0.018	0.019	0.019	0.019	0.020	0.020	0.021
664	0.408	0.017	0.018	0.018	0.018	0.018	0.018	0.019	0.019	0.019	0.019	0.020	0.020	0.021	0.021	0.021	0.021	0.022
786	0.436	0.019	0.019	0.019	0.019	0.019	0.020	0.020	0.020	0.021	0.021	0.021	0.022	0.022	0.022	0.023	0.023	0.024
918	0.459	0.020	0.020	0.020	0.020	0.020	0.021	0.021	0.021	0.022	0.022	0.022	0.023	0.023	0.024	0.024	0.024	0.025
1059	0.481	0.021	0.021	0.021	0.021	0.021	0.022	0.022	0.022	0.023	0.023	0.024	0.024	0.025	0.025	0.025	0.025	0.026
1207	0.501	0.021	0.022	0.022	0.022	0.022	0.023	0.023	0.023	0.024	0.024	0.025	0.025	0.026	0.026	0.026	0.026	0.028
1362	0.524	0.022	0.023	0.023	0.023	0.023	0.024	0.024	0.024	0.025	0.025	0.026	0.026	0.027	0.027	0.027	0.028	0.029
1523	0.541	0.023	0.024	0.023	0.024	0.024	0.024	0.025	0.025	0.026	0.026	0.027	0.027	0.028	0.028	0.028	0.029	0.030
1690	0.550	0.024	0.024	0.024	0.024	0.024	0.025	0.025	0.025	0.026	0.026	0.027	0.027	0.028	0.028	0.029	0.029	0.030
1864	0.563	0.024	0.024	0.024	0.025	0.025	0.026	0.026	0.026	0.027	0.027	0.028	0.028	0.029	0.029	0.030	0.030	0.031
2042	0.574	0.025	0.025	0.025	0.025	0.025	0.026	0.027	0.027	0.027	0.027	0.028	0.029	0.030	0.030	0.030	0.031	0.032
2227	0.581	0.025	0.025	0.025	0.026	0.026	0.026	0.027	0.027	0.028	0.028	0.029	0.029	0.030	0.030	0.031	0.031	0.032

Pressure, psi	Burn Rate, in/s	Propagation Time, µs																
		0.1	0.125	0.15	0.175	0.2	0.225	0.25	0.275	0.3	0.325	0.35	0.375	0.4	0.425	0.45	0.475	0.5
282	0.292	0.013	0.013	0.013	0.013	0.013	0.013	0.014	0.014	0.014	0.014	0.014	0.015	0.015	0.015	0.016	0.016	0.016
358	0.336	0.015	0.015	0.015	0.015	0.015	0.016	0.016	0.016	0.016	0.016	0.017	0.017	0.017	0.018	0.018	0.018	0.019
449	0.358	0.016	0.016	0.016	0.016	0.016	0.017	0.017	0.017	0.017	0.018	0.018	0.018	0.018	0.019	0.019	0.020	0.020
552	0.380	0.017	0.017	0.017	0.017	0.017	0.018	0.018	0.018	0.018	0.019	0.019	0.019	0.020	0.020	0.020	0.021	0.021
664	0.410	0.018	0.018	0.018	0.019	0.019	0.019	0.019	0.019	0.020	0.020	0.020	0.021	0.021	0.022	0.022	0.022	0.023
786	0.439	0.019	0.019	0.020	0.020	0.020	0.020	0.021	0.021	0.021	0.022	0.022	0.022	0.023	0.023	0.023	0.024	0.025
918	0.463	0.020	0.020	0.021	0.021	0.021	0.021	0.022	0.022	0.022	0.023	0.023	0.023	0.024	0.024	0.025	0.025	0.026
1059	0.486	0.021	0.022	0.022	0.022	0.022	0.022	0.023	0.023	0.024	0.024	0.024	0.024	0.025	0.026	0.026	0.027	0.027
1207	0.505	0.022	0.022	0.022	0.023	0.023	0.023	0.024	0.024	0.025	0.025	0.025	0.025	0.026	0.027	0.027	0.028	0.028
1362	0.527	0.023	0.023	0.023	0.024	0.024	0.024	0.025	0.025	0.026	0.026	0.026	0.027	0.027	0.028	0.028	0.029	0.030
1523	0.542	0.024	0.024	0.024	0.025	0.025	0.025	0.025	0.026	0.026	0.027	0.027	0.027	0.028	0.029	0.029	0.030	0.031
1690	0.549	0.024	0.024	0.024	0.025	0.025	0.025	0.026	0.026	0.027	0.027	0.027	0.028	0.029	0.029	0.030	0.030	0.031
1864	0.564	0.025	0.025	0.025	0.026	0.026	0.026	0.026	0.027	0.028	0.028	0.028	0.029	0.029	0.030	0.030	0.031	0.032
2042	0.577	0.025	0.026	0.026	0.026	0.026	0.027	0.027	0.027	0.028	0.029	0.029	0.029	0.030	0.031	0.031	0.032	0.033
2227	0.584	0.026	0.026	0.026	0.027	0.027	0.027	0.027	0.028	0.029	0.029	0.029	0.030	0.031	0.031	0.032	0.033	0.033
2411	0.601	0.026	0.027	0.027	0.027	0.028	0.028	0.028	0.029	0.029	0.030	0.030	0.031	0.032	0.032	0.033	0.034	0.034
2576	0.625	0.027	0.028	0.028	0.029	0.029	0.029	0.029	0.030	0.031	0.031	0.031	0.032	0.033	0.034	0.034	0.035	0.036

Pressure, psi	Burn Rate, in/s	Propagation Time, µs																
		0.1	0.125	0.15	0.175	0.2	0.225	0.25	0.275	0.3	0.325	0.35	0.375	0.4	0.425	0.45	0.475	0.5
282	0.288	0.012	0.013	0.013	0.013	0.013	0.013	0.013	0.014	0.014	0.014	0.014	0.014	0.014	0.015	0.015	0.015	0.016
358	0.335	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.016	0.016	0.016	0.016	0.017	0.017	0.017	0.018	0.018	0.018
449	0.359	0.016	0.016	0.016	0.016	0.016	0.016	0.017	0.017	0.017	0.017	0.018	0.018	0.018	0.019	0.019	0.019	0.019
552	0.381	0.017	0.017	0.017	0.017	0.017	0.017	0.018	0.018	0.018	0.018	0.019	0.019	0.019	0.020	0.020	0.020	0.021
664	0.412	0.018	0.018	0.018	0.018	0.019	0.019	0.019	0.019	0.019	0.020	0.020	0.020	0.021	0.021	0.022	0.022	0.022
786	0.441	0.019	0.019	0.019	0.020	0.020	0.020	0.020	0.021	0.021	0.021	0.022	0.022	0.022	0.023	0.023	0.023	0.024
918	0.465	0.020	0.020	0.020	0.021	0.021	0.021	0.021	0.022	0.022	0.022	0.023	0.023	0.023	0.024	0.025	0.025	0.025
1059	0.487	0.021	0.021	0.021	0.022	0.022	0.022	0.022	0.023	0.023	0.024	0.024	0.024	0.025	0.025	0.026	0.026	0.027
1207	0.507	0.022	0.022	0.022	0.022	0.023	0.023	0.023	0.024	0.024	0.024	0.025	0.025	0.026	0.026	0.027	0.027	0.028
1362	0.528	0.023	0.023	0.023	0.023	0.024	0.024	0.024	0.025	0.025	0.026	0.026	0.026	0.027	0.027	0.028	0.028	0.029
1523	0.542	0.023	0.024	0.024	0.024	0.024	0.025	0.025	0.026	0.026	0.026	0.027	0.027	0.027	0.028	0.029	0.029	0.030
1690	0.548	0.024	0.024	0.024	0.024	0.025	0.025	0.025	0.026	0.026	0.027	0.027	0.027	0.028	0.029	0.029	0.029	0.030
1864	0.562	0.024	0.025	0.025	0.025	0.025	0.025	0.026	0.027	0.027	0.027	0.028	0.028	0.029	0.029	0.030	0.030	0.031
2042	0.580	0.025	0.025	0.026	0.026	0.026	0.026	0.027	0.028	0.028	0.028	0.029	0.029	0.030	0.030	0.031	0.031	0.032
2227	0.588	0.026	0.026	0.026	0.026	0.027	0.027	0.027	0.028	0.028	0.029	0.029	0.030	0.030	0.031	0.032	0.032	0.033
2411	0.596	0.026	0.026	0.026	0.026	0.027	0.027	0.028	0.028	0.028	0.029	0.030	0.030	0.031	0.031	0.032	0.032	0.033

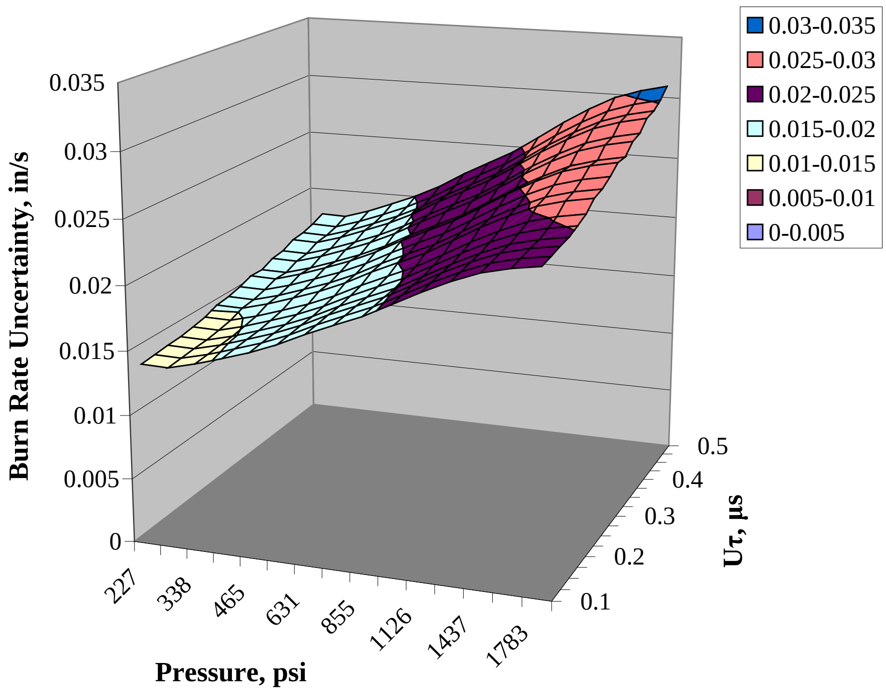


Figure A.25: Test 6 burn rate uncertainty for the EDUM method.

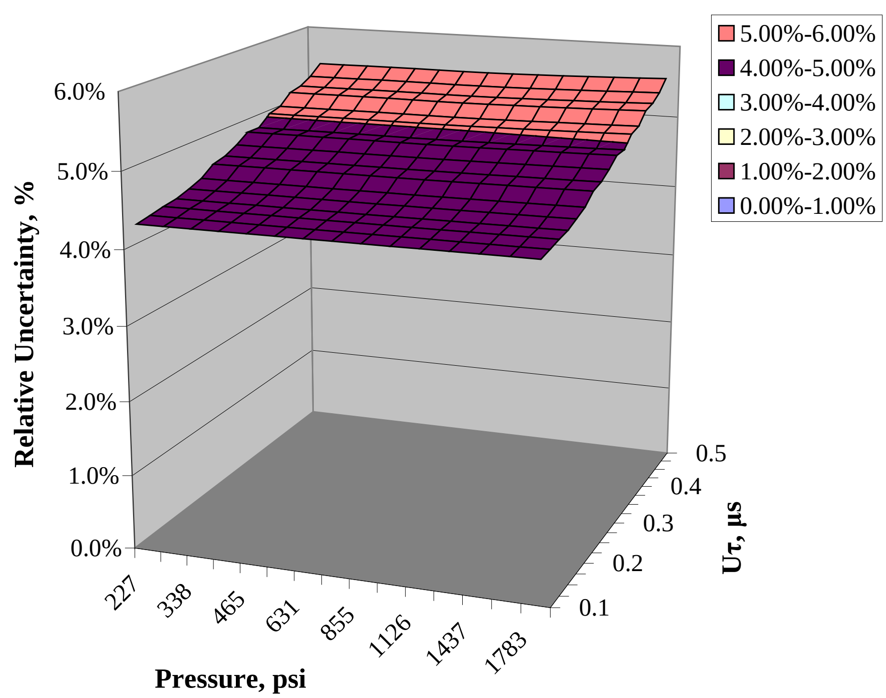


Figure A.26: Test 6 relative uncertainty for the EDUM method.

Pressure,	Burn	Propagation Time, μs																
psi	Rate, in/s	0.1	0.125	0.15	0.175	0.2	0.225	0.25	0.275	0.3	0.325	0.35	0.375	0.4	0.425	0.45	0.475	0.5
227	0.325	0.014	0.014	0.014	0.014	0.015	0.015	0.015	0.015	0.016	0.016	0.016	0.016	0.016	0.017	0.017	0.017	0.018
281	0.324	0.014	0.014	0.014	0.014	0.015	0.015	0.015	0.015	0.016	0.016	0.016	0.016	0.016	0.017	0.017	0.017	0.018
338	0.336	0.015	0.015	0.015	0.015	0.015	0.015	0.016	0.016	0.016	0.016	0.016	0.017	0.017	0.018	0.018	0.018	0.018
399	0.351	0.015	0.015	0.015	0.016	0.016	0.016	0.016	0.017	0.017	0.017	0.017	0.018	0.018	0.018	0.018	0.019	0.019
465	0.366	0.016	0.016	0.016	0.016	0.016	0.017	0.017	0.017	0.018	0.018	0.018	0.018	0.019	0.019	0.019	0.020	0.020
541	0.385	0.017	0.017	0.017	0.017	0.017	0.018	0.018	0.018	0.018	0.019	0.019	0.019	0.020	0.020	0.020	0.021	0.021
631	0.408	0.018	0.018	0.018	0.018	0.018	0.019	0.019	0.019	0.020	0.020	0.020	0.021	0.021	0.021	0.021	0.022	0.022
737	0.428	0.019	0.019	0.019	0.019	0.019	0.019	0.020	0.020	0.021	0.021	0.021	0.022	0.022	0.022	0.022	0.023	0.023
855	0.448	0.019	0.020	0.020	0.020	0.020	0.020	0.021	0.021	0.021	0.022	0.022	0.023	0.023	0.023	0.024	0.024	0.025
985	0.473	0.021	0.021	0.021	0.021	0.021	0.022	0.022	0.022	0.023	0.023	0.023	0.024	0.024	0.025	0.025	0.025	0.026
1126	0.498	0.022	0.022	0.022	0.022	0.022	0.023	0.023	0.024	0.024	0.024	0.024	0.025	0.025	0.026	0.026	0.027	0.027
1276	0.520	0.023	0.023	0.023	0.023	0.023	0.024	0.024	0.025	0.025	0.026	0.026	0.026	0.027	0.027	0.027	0.028	0.029
1437	0.539	0.023	0.024	0.024	0.024	0.024	0.025	0.025	0.025	0.026	0.027	0.027	0.027	0.028	0.028	0.029	0.029	0.030
1607	0.551	0.024	0.024	0.024	0.024	0.025	0.025	0.026	0.026	0.027	0.027	0.027	0.028	0.028	0.029	0.029	0.030	0.030
1783	0.559	0.024	0.024	0.025	0.025	0.025	0.026	0.026	0.026	0.027	0.028	0.028	0.028	0.029	0.029	0.030	0.030	0.031

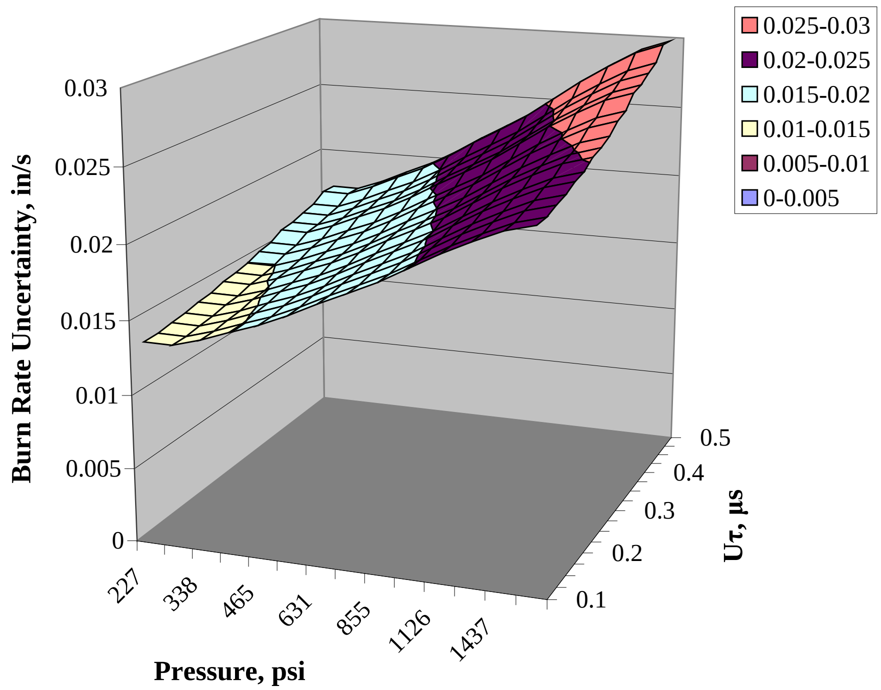


Figure A.27: Test 6 burn rate uncertainty for the Zero Crossing method.

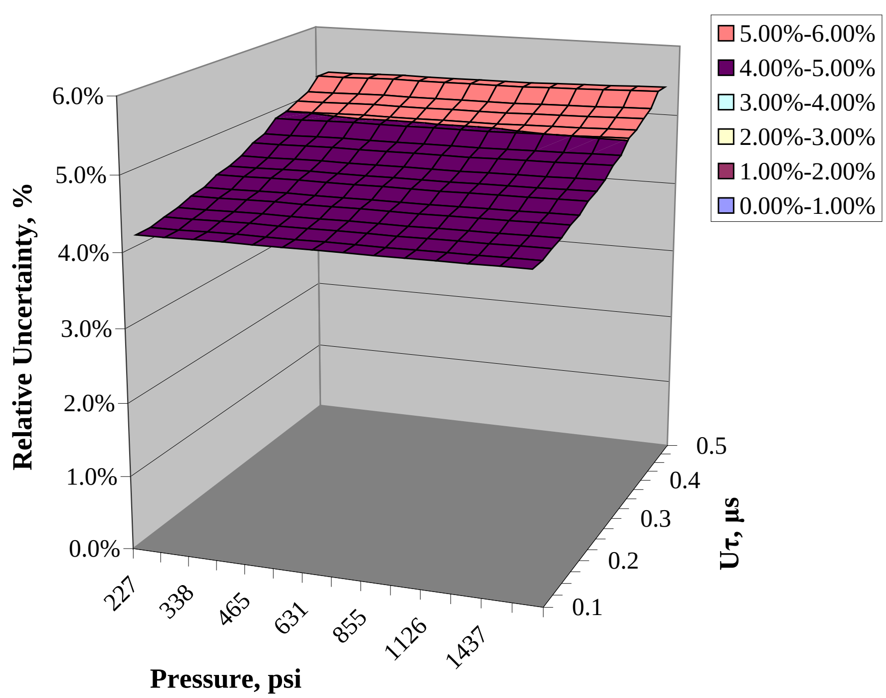


Figure A.28: Test 6 relative uncertainty for the Zero Crossing method.

Pressure, psi	Burn Rate, in/s	Propagation Time, µs																
		0.1	0.125	0.15	0.175	0.2	0.225	0.25	0.275	0.3	0.325	0.35	0.375	0.4	0.425	0.45	0.475	0.5
227	0.323	0.014	0.014	0.014	0.014	0.014	0.014	0.015	0.015	0.015	0.015	0.016	0.016	0.016	0.016	0.017	0.017	0.017
281	0.322	0.014	0.014	0.014	0.014	0.014	0.014	0.015	0.015	0.015	0.015	0.015	0.016	0.016	0.016	0.017	0.017	0.017
338	0.335	0.014	0.014	0.014	0.015	0.015	0.015	0.015	0.015	0.016	0.016	0.016	0.017	0.017	0.017	0.017	0.018	0.018
399	0.350	0.015	0.015	0.015	0.015	0.016	0.016	0.016	0.016	0.016	0.017	0.017	0.017	0.018	0.018	0.018	0.019	0.019
465	0.366	0.016	0.016	0.016	0.016	0.016	0.016	0.017	0.017	0.017	0.017	0.018	0.018	0.018	0.019	0.019	0.020	0.020
541	0.386	0.016	0.016	0.017	0.017	0.017	0.017	0.018	0.018	0.018	0.018	0.019	0.019	0.019	0.020	0.020	0.021	0.021
631	0.407	0.017	0.017	0.018	0.018	0.018	0.018	0.019	0.019	0.019	0.019	0.020	0.020	0.020	0.021	0.021	0.022	0.022
737	0.428	0.018	0.018	0.018	0.019	0.019	0.019	0.019	0.020	0.020	0.020	0.021	0.021	0.021	0.022	0.022	0.023	0.023
855	0.449	0.019	0.019	0.019	0.020	0.020	0.020	0.020	0.021	0.021	0.021	0.022	0.022	0.023	0.023	0.023	0.024	0.024
985	0.473	0.020	0.020	0.020	0.021	0.021	0.021	0.022	0.022	0.022	0.023	0.023	0.024	0.024	0.024	0.025	0.025	0.025
1126	0.498	0.021	0.021	0.022	0.022	0.022	0.022	0.023	0.023	0.023	0.024	0.024	0.025	0.025	0.026	0.026	0.027	0.027
1276	0.520	0.022	0.022	0.022	0.023	0.023	0.023	0.024	0.024	0.024	0.025	0.025	0.026	0.026	0.027	0.027	0.028	0.028
1437	0.539	0.023	0.023	0.023	0.024	0.024	0.024	0.025	0.025	0.025	0.026	0.026	0.027	0.027	0.028	0.028	0.029	0.029
1607	0.551	0.023	0.024	0.024	0.024	0.024	0.025	0.025	0.025	0.026	0.026	0.027	0.028	0.028	0.028	0.029	0.030	0.030

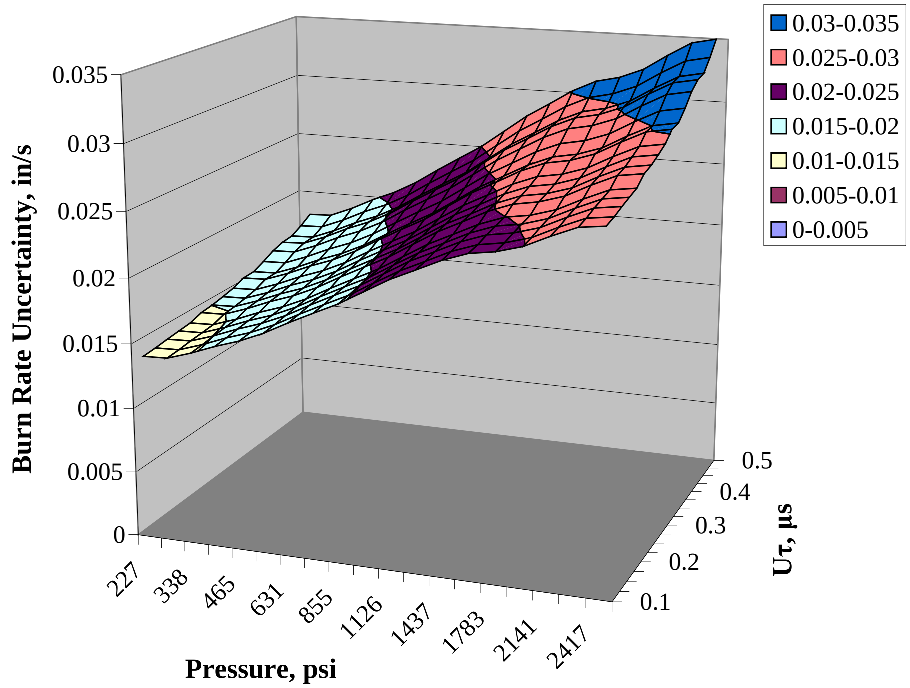


Figure A.29: Test 6 burn rate uncertainty for the Cross Correlation method.

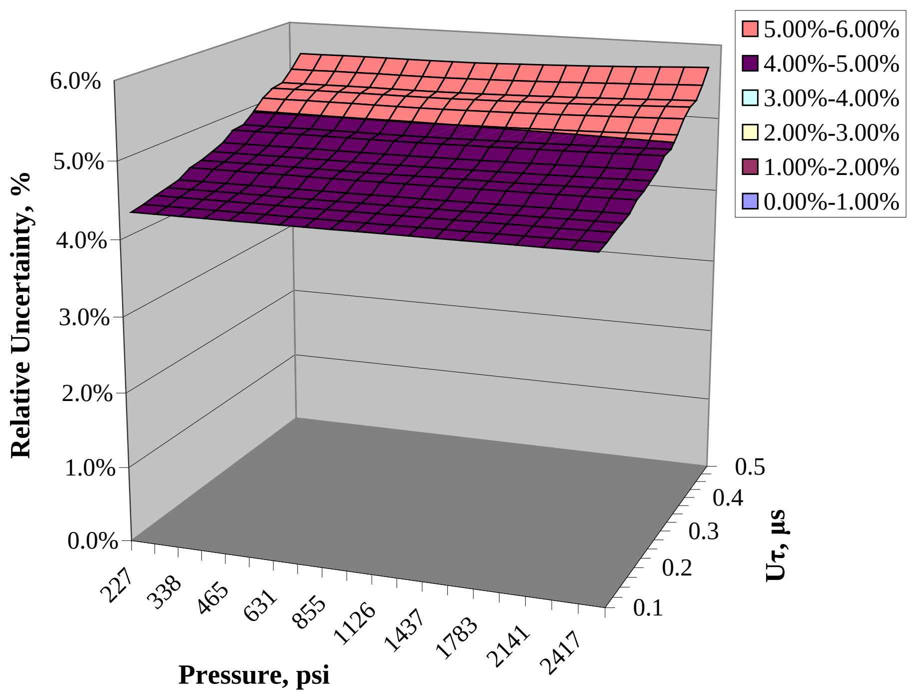


Figure A.30: Test 6 relative uncertainty for the Cross Correlation method.

Pressure, psi	Burn Rate, in/s	Propagation Time, μs																
		0.1	0.125	0.15	0.175	0.2	0.225	0.25	0.275	0.3	0.325	0.35	0.375	0.4	0.425	0.45	0.475	0.5
227	0.324	0.014	0.014	0.014	0.014	0.015	0.015	0.015	0.015	0.015	0.016	0.016	0.016	0.017	0.017	0.017	0.017	0.018
281	0.325	0.014	0.014	0.014	0.015	0.015	0.015	0.015	0.015	0.015	0.016	0.016	0.016	0.017	0.017	0.017	0.018	0.018
338	0.339	0.015	0.015	0.015	0.015	0.015	0.016	0.016	0.016	0.016	0.016	0.017	0.017	0.017	0.018	0.018	0.018	0.019
399	0.355	0.015	0.016	0.016	0.016	0.016	0.016	0.016	0.017	0.017	0.017	0.017	0.018	0.018	0.019	0.019	0.019	0.020
465	0.369	0.016	0.016	0.016	0.016	0.017	0.017	0.017	0.017	0.018	0.018	0.018	0.018	0.019	0.019	0.019	0.020	0.021
541	0.387	0.017	0.017	0.017	0.017	0.017	0.018	0.018	0.018	0.018	0.019	0.019	0.019	0.020	0.020	0.020	0.021	0.022
631	0.409	0.018	0.018	0.018	0.018	0.018	0.019	0.019	0.019	0.019	0.020	0.020	0.020	0.021	0.021	0.021	0.022	0.023
737	0.429	0.019	0.019	0.019	0.019	0.019	0.020	0.020	0.020	0.020	0.021	0.021	0.021	0.022	0.022	0.023	0.023	0.024
855	0.449	0.020	0.020	0.020	0.020	0.020	0.021	0.021	0.021	0.021	0.022	0.022	0.022	0.023	0.023	0.024	0.024	0.025
985	0.474	0.021	0.021	0.021	0.021	0.021	0.022	0.022	0.022	0.023	0.023	0.023	0.024	0.024	0.025	0.025	0.026	0.026
1126	0.498	0.022	0.022	0.022	0.022	0.022	0.023	0.023	0.023	0.024	0.024	0.024	0.025	0.026	0.026	0.026	0.027	0.028
1276	0.518	0.023	0.023	0.023	0.023	0.023	0.024	0.024	0.024	0.025	0.025	0.025	0.026	0.027	0.027	0.027	0.028	0.029
1437	0.538	0.023	0.024	0.024	0.024	0.024	0.025	0.025	0.025	0.026	0.026	0.026	0.027	0.028	0.028	0.028	0.029	0.030
1607	0.553	0.024	0.024	0.025	0.025	0.025	0.026	0.026	0.026	0.026	0.027	0.027	0.028	0.029	0.029	0.029	0.030	0.031
1783	0.559	0.024	0.025	0.025	0.025	0.025	0.026	0.026	0.026	0.027	0.027	0.027	0.028	0.029	0.029	0.030	0.030	0.031
1962	0.571	0.025	0.025	0.025	0.026	0.026	0.026	0.027	0.027	0.027	0.028	0.028	0.029	0.030	0.030	0.030	0.031	0.032
2141	0.591	0.026	0.026	0.026	0.027	0.027	0.027	0.028	0.028	0.028	0.029	0.029	0.030	0.031	0.031	0.031	0.032	0.033
2299	0.609	0.027	0.027	0.027	0.027	0.028	0.028	0.028	0.029	0.029	0.030	0.030	0.031	0.032	0.032	0.032	0.033	0.035
2417	0.615	0.027	0.027	0.027	0.028	0.028	0.028	0.029	0.029	0.029	0.030	0.030	0.031	0.032	0.033	0.033	0.034	0.035

APPENDIX B

MATHCAD CODE FOR TEST DATA REDUCTION

The following pages show the Mathcad data reduction worksheet for Test 5.

DIGITAL ULTRASONIC DATA REDUCTION PROGRAM

$PreTest_{EDUM} :=$

	0	1	2	3
0	0	0.04	0.044	0.276
1	0	0.039	0.042	0.278
2	0	0.039	0.042	0.277
3	0	0.04	0.042	0.273
4	0	0.04	0.042	0.273
5	0	0.041	0.043	0.273
6	0	0.039	0.042	0.278
7	0	0.038	0.042	0.277
8	0	0.04	0.042	0.277
9	0	0.037	0.041	0.276
10	0	0.04	0.041	0.278

Outside Inside PropTime

$PreTest_{CC} :=$

	0
0	$5.525 \cdot 10^{-5}$
1	$5.525 \cdot 10^{-5}$
2	$5.524 \cdot 10^{-5}$
3	$5.525 \cdot 10^{-5}$
4	$5.525 \cdot 10^{-5}$
5	$5.525 \cdot 10^{-5}$
6	$5.525 \cdot 10^{-5}$
7	$5.524 \cdot 10^{-5}$
8	$5.525 \cdot 10^{-5}$
9	$5.525 \cdot 10^{-5}$

$PreTest_{ZC} :=$

	0
0	$5.525 \cdot 10^{-5}$
1	$5.524 \cdot 10^{-5}$
2	$5.524 \cdot 10^{-5}$
3	$5.525 \cdot 10^{-5}$
4	$5.525 \cdot 10^{-5}$
5	$5.525 \cdot 10^{-5}$
6	$5.525 \cdot 10^{-5}$
7	$5.524 \cdot 10^{-5}$
8	$5.525 \cdot 10^{-5}$
9	$5.525 \cdot 10^{-5}$

$\text{rows}(PreTest_{CC}) = 1 \times 10^{4}$ $\quad$ $\text{rows}(PreTest_{ZC}) = 1 \times 10^{4}$

$\text{rows}(PreTest_{EDUM}) = 1 \times 10^{4}$

$PreTest_{EDUM} := \text{augment}\left(PreTest_{EDUM}^{\langle 1 \rangle}, PreTest_{EDUM}^{\langle 2 \rangle}, PreTest_{EDUM}^{\langle 3 \rangle}\right)$

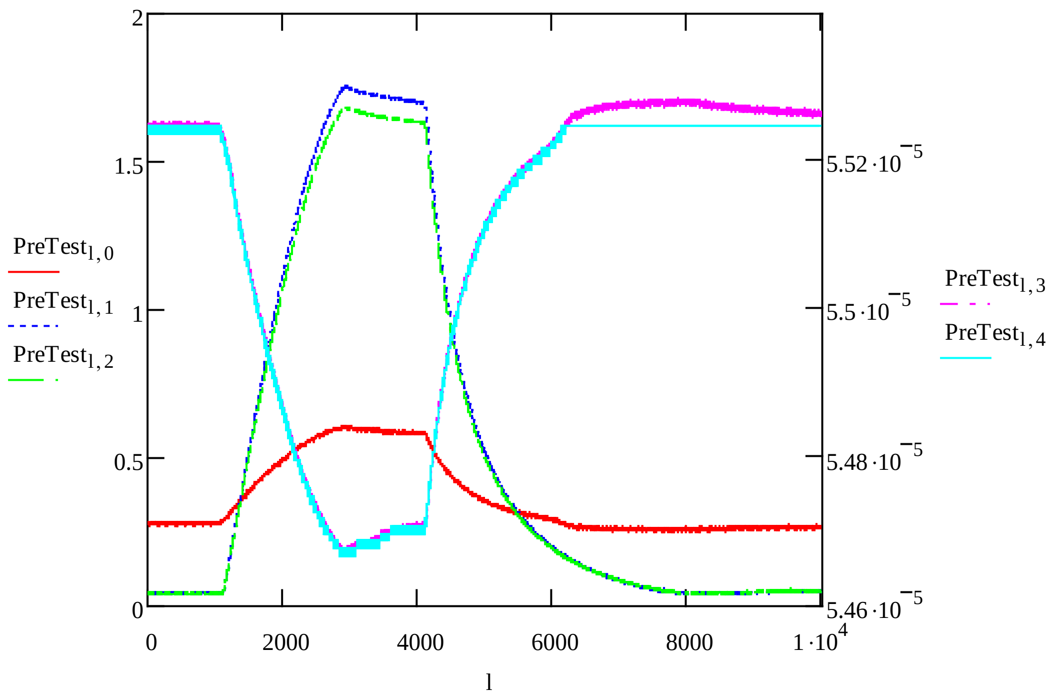
Eliminates empty first column

$\text{rows}(PreTest_{EDUM}) = 1 \times 10^{4}$ $\quad$ $i := 0 .. \text{rows}(PreTest_{EDUM}) - 1$

$PreTest := augment\left(PreTest_{EDUM}^{\langle 2\rangle}, PreTest_{EDUM}^{\langle 1\rangle}, PreTest_{EDUM}^{\langle 0\rangle}, PreTest_{ZC}, PreTest_{CC}\right)$

EDUM Prop Time, Inside Pressure, Outside Pressure, ZC Prop Time, CC Prop Time

$l := 0..\ rows(PreTest) - 1$



$Test_{EDUM} :=$

	0	1	2	3
0	0	0.055	0.041	0.243
1	0	0.054	0.041	0.239
2	0	0.053	0.048	0.237
3	0	0.052	0.041	0.24
4	0	0.051	0.041	0.241
5	0	0.054	0.042	0.245
6	0	0.05	0.043	0.241
7	0	0.05	0.042	0.238
8	0	0.048	0.043	0.235
9	0	0.049	0.044	0.236
10	0	0.048	0.043	0.237

Outside Inside PropTime

$Test_{ZC} :=$

	0
0	$5.527 \cdot 10^{-5}$
1	$5.527 \cdot 10^{-5}$
2	$5.527 \cdot 10^{-5}$
3	$5.527 \cdot 10^{-5}$
4	$5.527 \cdot 10^{-5}$
5	$5.527 \cdot 10^{-5}$
6	$5.527 \cdot 10^{-5}$
7	$5.527 \cdot 10^{-5}$
8	$5.527 \cdot 10^{-5}$
9	$5.527 \cdot 10^{-5}$

$Test_{CC} :=$

	0
0	$5.527 \cdot 10^{-5}$
1	$5.527 \cdot 10^{-5}$
2	$5.527 \cdot 10^{-5}$
3	$5.527 \cdot 10^{-5}$
4	$5.527 \cdot 10^{-5}$
5	$5.527 \cdot 10^{-5}$
6	$5.527 \cdot 10^{-5}$
7	$5.527 \cdot 10^{-5}$
8	$5.527 \cdot 10^{-5}$
9	$5.527 \cdot 10^{-5}$

$$Test_{ZC} := \mathrm{submatrix}\left(Test_{ZC}, 0, \mathrm{rows}\left(Test_{EDUM}\right) - 1, 0, 0\right)$$

$$Test_{CC} := \mathrm{submatrix}\left(Test_{CC}, 0, \mathrm{rows}\left(Test_{EDUM}\right) - 1, 0, 0\right)$$

$\mathrm{rows}\left(Test_{ZC}\right) = 1 \times 10^4$ $\qquad$ $\mathrm{rows}\left(Test_{CC}\right) = 1 \times 10^4$

$$Test_{EDUM} := \mathrm{augment}\left(Test_{EDUM}^{\langle 1 \rangle}, Test_{EDUM}^{\langle 2 \rangle}, Test_{EDUM}^{\langle 3 \rangle}\right)$$

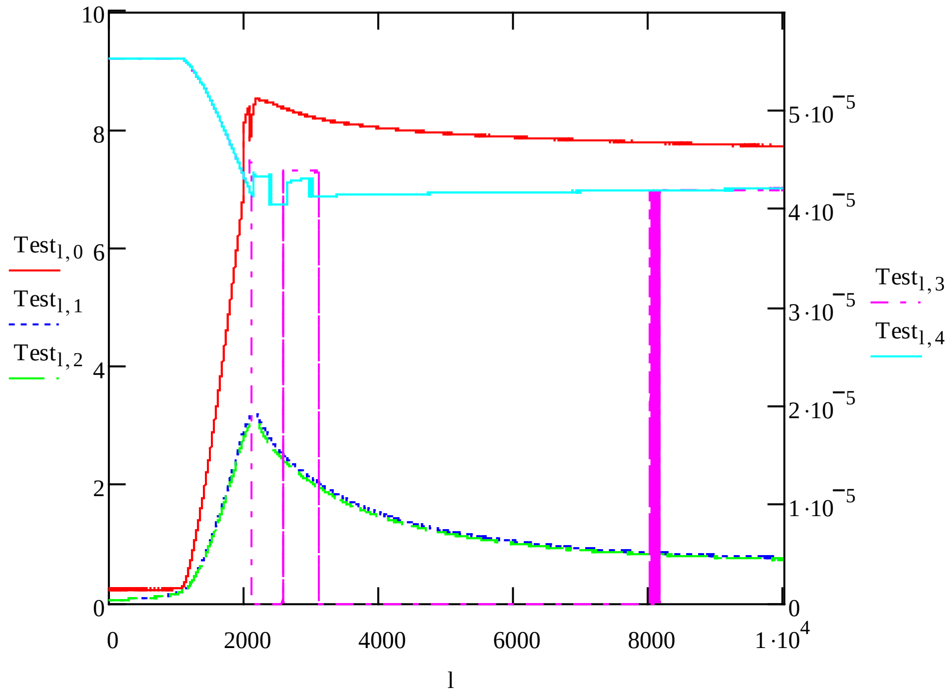
Eliminates empty first column

$\mathrm{rows}\left(Test_{EDUM}\right) = 1 \times 10^4$ $\qquad$ $i := 0 .. \mathrm{rows}\left(Test_{EDUM}\right) - 1$

$$Test := \mathrm{augment}\left(Test_{EDUM}^{\langle 2 \rangle}, Test_{EDUM}^{\langle 1 \rangle}, Test_{EDUM}^{\langle 0 \rangle}, Test_{ZC}, Test_{CC}\right)$$

EDUM Prop Time, Inside Pressure, Outside Pressure, ZC Prop Time, CC Prop Time

$l := 0 .. \mathrm{rows}(Test) - 1$



$PostTest_{EDUM} :=$

	0	1	2	3
0	0	0.046	0.043	8.382
1	0	0.046	0.041	8.38
2	0	0.043	0.043	8.385
3	0	0.045	0.042	8.386
4	0	0.046	0.043	8.388
5	0	0.045	0.04	8.386
6	0	0.044	0.042	8.386
7	0	0.043	0.043	8.386
8	0	0.045	0.043	8.386
9	0	0.045	0.041	8.388
10	0	0.044	0.043	8.386

Outside Inside PropTime

$PostTest_{ZC} :=$

	0
0	$4.093 \cdot 10^{-5}$
1	$4.093 \cdot 10^{-5}$
2	$4.092 \cdot 10^{-5}$
3	$4.093 \cdot 10^{-5}$
4	$4.092 \cdot 10^{-5}$
5	$4.093 \cdot 10^{-5}$
6	$4.093 \cdot 10^{-5}$
7	$4.092 \cdot 10^{-5}$
8	$4.093 \cdot 10^{-5}$
9	$4.093 \cdot 10^{-5}$

$PostTest_{CC} :=$

	0
0	$4.093 \cdot 10^{-5}$
1	$4.093 \cdot 10^{-5}$
2	$4.092 \cdot 10^{-5}$
3	$4.093 \cdot 10^{-5}$
4	$4.092 \cdot 10^{-5}$
5	$4.093 \cdot 10^{-5}$
6	$4.093 \cdot 10^{-5}$
7	$4.092 \cdot 10^{-5}$
8	$4.093 \cdot 10^{-5}$
9	$4.093 \cdot 10^{-5}$

$rows(PostTest_{ZC}) = 1 \times 10^{4}$

$rows(PostTest_{CC}) = 1 \times 10^{4}$

$$PostTest_{EDUM} := augment\left(PostTest_{EDUM}^{\langle 1 \rangle}, PostTest_{EDUM}^{\langle 2 \rangle}, PostTest_{EDUM}^{\langle 3 \rangle}\right)$$

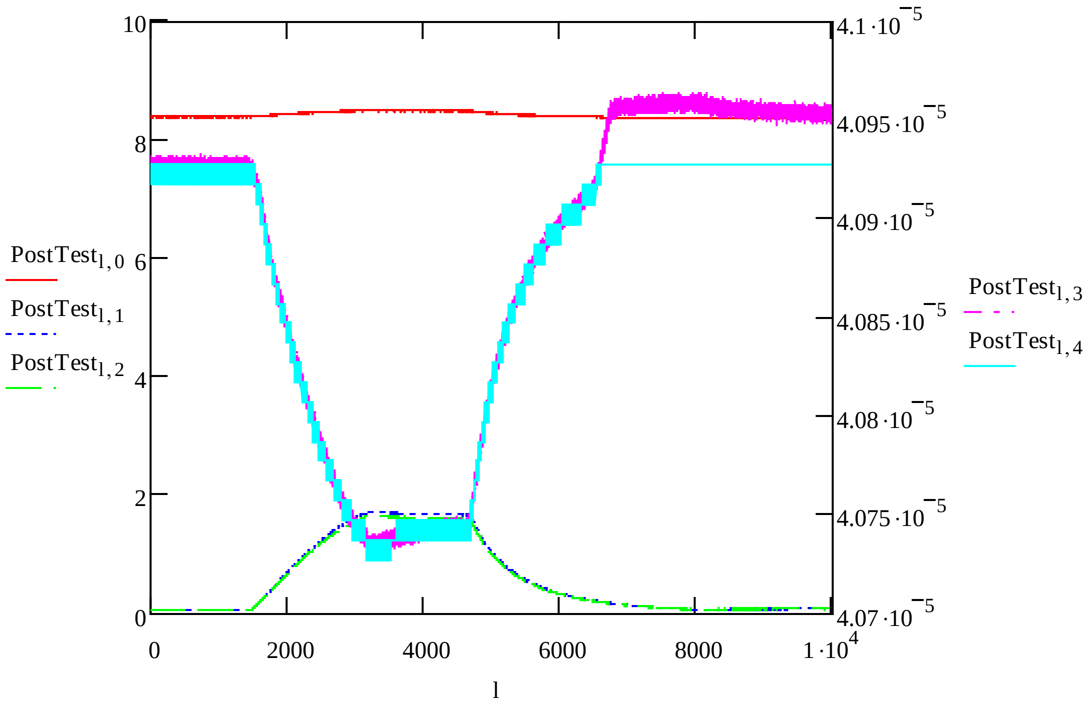
Eliminates empty first column

$rows(PostTest_{EDUM}) = 1 \times 10^{4}$

$$PostTest := augment\left(PostTest_{EDUM}^{\langle 2 \rangle}, PostTest_{EDUM}^{\langle 1 \rangle}, PostTest_{EDUM}^{\langle 0 \rangle}, PostTest_{ZC}, PostTest_{CC}\right)$$

EDUM Prop Time, Inside Pressure, Outside Pressure, ZC Prop Time, CC Prop Time

$l := 0 .. rows(PostTest) - 1$



ENTER the initial thickness: Propellant Thickness: $E_{p0} := 0.5125$

ENTER the ultrasonic frequency: $freq := 1000$

ENTER the pressure calibration factors for the inside transducer:

Inside Pressure	Outside Pressure
$m_{P1} := 895.258$	$m2_P := 933.76$
$b_{P1} := -9.353$	$b2_P := -26.332$

ENTER the Ultrasonic calibration points:

$\tau_1 := 39.1 \cdot 10^{-6}$ $V_1 := 8.800$

$\tau_2 := 54.3 \cdot 10^{-6}$ $V_2 := 0.300$

DIRECTIONS FOR TRANSFERRING PRESSURE CALIBRATION

Enter 1 into mp1, Enter 0 into bp1
Enter slope and intercept from pressure transducer calibration into mp2 and bp2
Enter value from v4 into mp1 (below)
Enter value from v3 into bp1 (below)
Values for v4 and v3 should now be 1 and 0

$P(V) := \left(m_{P1} \cdot V + b_{P1} + 0\right)$ $\qquad P2(V) := \left(m2_P \cdot V + b2_P + 14.7\right)$

$$m_\tau := \frac{\tau_2 - \tau_1}{V_2 - V_1} \qquad \tau(V) := \left[\left(V - V_1\right) \cdot m_\tau + \tau_1\right]$$

$PreTest^{\langle 0 \rangle} := \tau\left(PreTest^{\langle 0 \rangle}\right)$

$PreTest^{\langle 1 \rangle} := P\left(PreTest^{\langle 1 \rangle}\right)$

$PreTest^{\langle 2 \rangle} := P2\left(PreTest^{\langle 2 \rangle}\right)$

$Test^{\langle 0 \rangle} := \tau\left(Test^{\langle 0 \rangle}\right)$

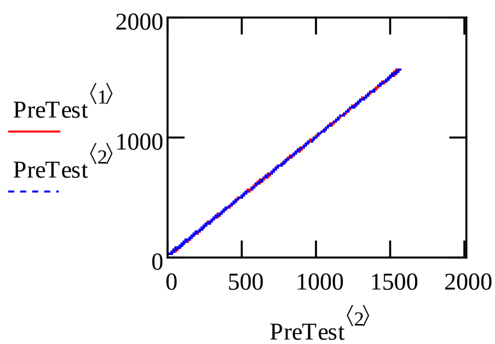
$Test^{\langle 1 \rangle} := P\left(Test^{\langle 1 \rangle}\right)$

$Test^{\langle 2 \rangle} := P2\left(Test^{\langle 2 \rangle}\right)$

$PostTest^{\langle 0 \rangle} := \tau\left(PostTest^{\langle 0 \rangle}\right)$

$PostTest^{\langle 1 \rangle} := P\left(PostTest^{\langle 1 \rangle}\right)$

$PostTest^{\langle 2 \rangle} := P2\left(PostTest^{\langle 2 \rangle}\right)$



compare inside (1) and outside (2) transducers

If calibrations are the same V4 should be 1 and V3 should be 0

$v := \text{regress}\left(PreTest^{\langle 1 \rangle}, PreTest^{\langle 2 \rangle}, 1\right)$

$v_4 = 1 \qquad v_3 = 3.938 \times 10^{-4}$

POST TEST (COUPLING MATERIAL) DATA

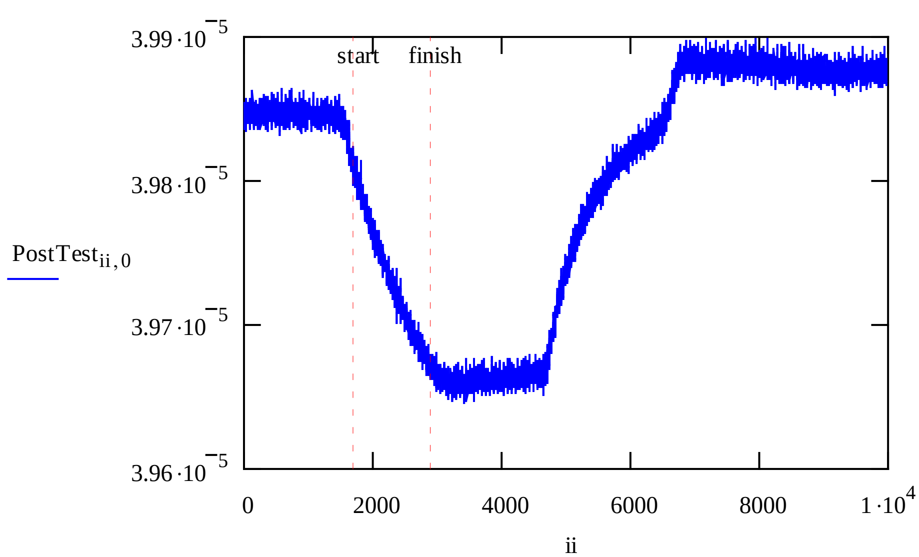
Edum Method

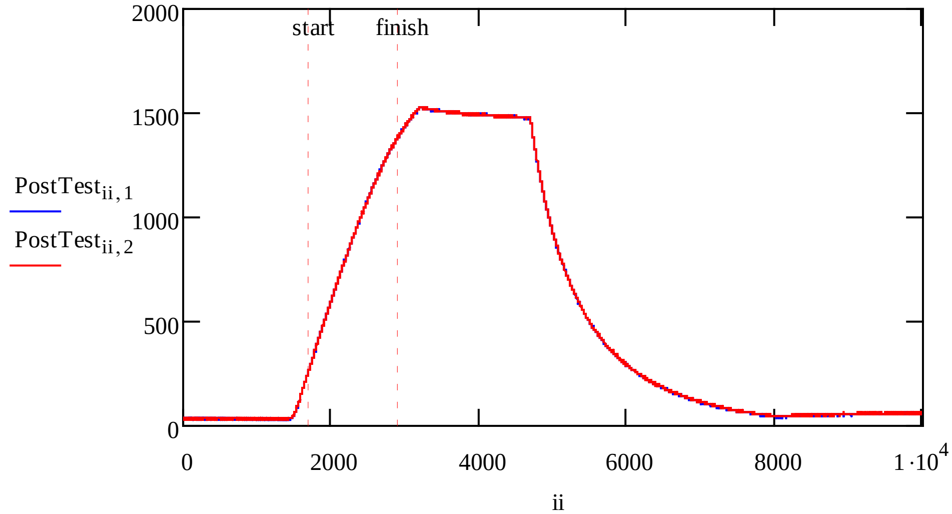
PostTest =

	0	1	2	3	4
0	$3.985 \cdot 10^{-5}$	28.696	31.682	$4.093 \cdot 10^{-5}$	$4.093 \cdot 10^{-5}$
1	$3.985 \cdot 10^{-5}$	27.681	30.926	$4.093 \cdot 10^{-5}$	$4.093 \cdot 10^{-5}$
2	$3.984 \cdot 10^{-5}$	28.696	28.809	$4.092 \cdot 10^{-5}$	$4.092 \cdot 10^{-5}$
3	$3.984 \cdot 10^{-5}$	28.261	30.775	$4.093 \cdot 10^{-5}$	$4.093 \cdot 10^{-5}$
4	$3.984 \cdot 10^{-5}$	28.841	31.229	$4.092 \cdot 10^{-5}$	$4.092 \cdot 10^{-5}$
5	$3.984 \cdot 10^{-5}$	26.232	30.17	$4.093 \cdot 10^{-5}$	$4.093 \cdot 10^{-5}$
6	$3.984 \cdot 10^{-5}$	28.406	29.868	$4.093 \cdot 10^{-5}$	$4.093 \cdot 10^{-5}$
7	$3.984 \cdot 10^{-5}$	28.696	28.507	$4.092 \cdot 10^{-5}$	$4.092 \cdot 10^{-5}$
8	$3.984 \cdot 10^{-5}$	28.986	30.623	$4.093 \cdot 10^{-5}$	$4.093 \cdot 10^{-5}$
9	$3.984 \cdot 10^{-5}$	27.102	30.623	$4.093 \cdot 10^{-5}$	$4.093 \cdot 10^{-5}$
10	$3.984 \cdot 10^{-5}$	28.841	29.565	$4.092 \cdot 10^{-5}$	$4.092 \cdot 10^{-5}$
11	$3.984 \cdot 10^{-5}$	27.392	27.751	$4.093 \cdot 10^{-5}$	$4.093 \cdot 10^{-5}$
12	$3.984 \cdot 10^{-5}$	28.841	28.356	$4.092 \cdot 10^{-5}$	$4.093 \cdot 10^{-5}$
13	$3.985 \cdot 10^{-5}$	27.826	27.902	$4.093 \cdot 10^{-5}$	$4.093 \cdot 10^{-5}$
14	$3.985 \cdot 10^{-5}$	27.681	30.926	$4.093 \cdot 10^{-5}$	$4.093 \cdot 10^{-5}$
15	$3.985 \cdot 10^{-5}$	28.116	28.809	$4.092 \cdot 10^{-5}$	$4.092 \cdot 10^{-5}$

$start := 1700$ $\qquad$ $finish := 2900$

$rows(PostTest) = 1 \times 10^4$ $\qquad$ $ii := 0 .. rows(PostTest) - 1$





$p := 1$ order of the regression

$PostTest1 := submatrix(PostTest, start, finish - 1, 0, 1)$

$v := regress\left(PostTest1^{\langle 1 \rangle}, PostTest1^{\langle 0 \rangle}, p\right)$ Regression

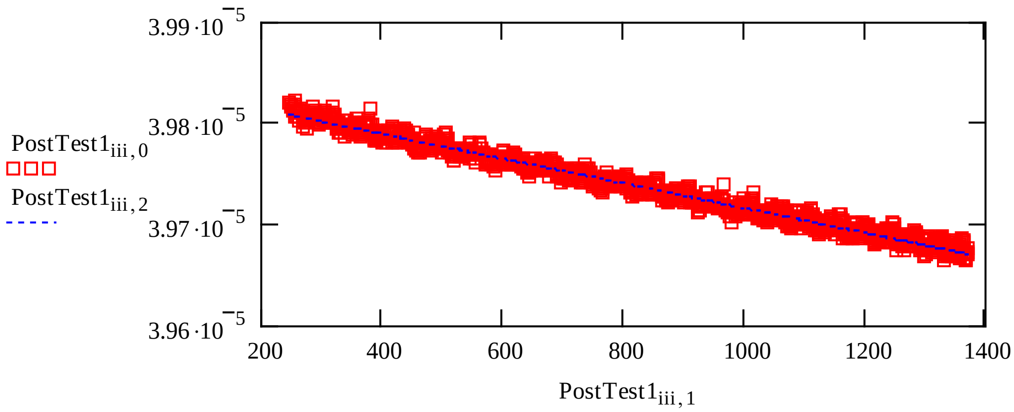
$a_c := v_4$ $b_c := v_3$ $a_c = -1.223 \times 10^{-10}$ $b_c = 3.984 \times 10^{-5}$

$iii := 0 .. rows(PostTest1) - 1$

$PostTest1_{iii,2} := v_4 \cdot PostTest1_{iii,1} + v_3$

PostTest1 =

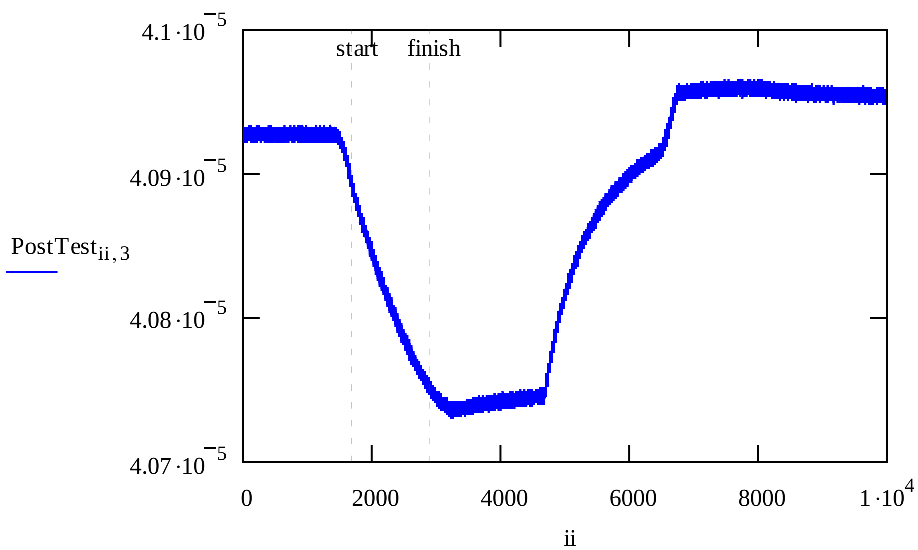
	0	1
0	$3.982 \cdot 10^{-5}$	249.311
1	$3.982 \cdot 10^{-5}$	249.891
2	$3.982 \cdot 10^{-5}$	250.906
3	$3.981 \cdot 10^{-5}$	252.935
4	$3.981 \cdot 10^{-5}$	255.399
5	$3.982 \cdot 10^{-5}$	256.124
6	$3.982 \cdot 10^{-5}$	256.413
7	$3.982 \cdot 10^{-5}$	257.283
8	$3.981 \cdot 10^{-5}$	256.703
9	$3.981 \cdot 10^{-5}$	259.458
10	$3.981 \cdot 10^{-5}$	258.878
11	$3.981 \cdot 10^{-5}$	260.762
12	$3.981 \cdot 10^{-5}$	261.922

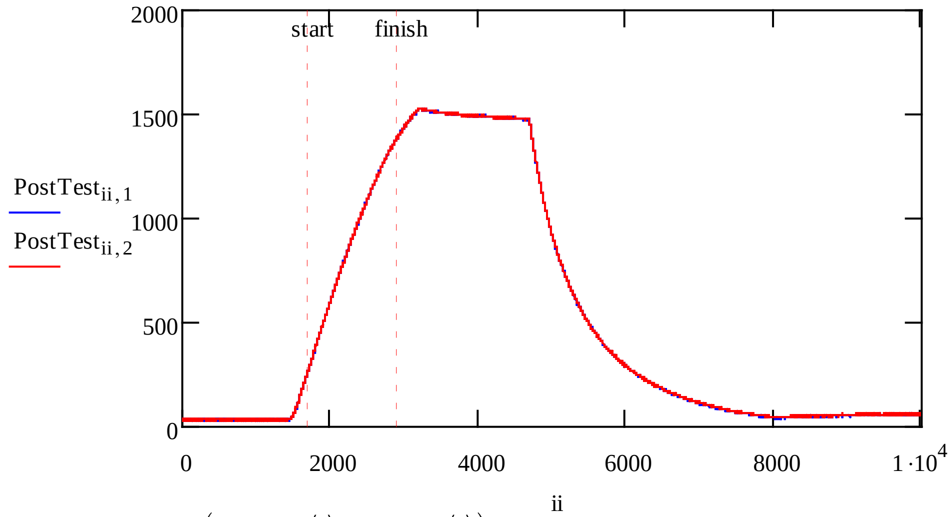


The column 2 of Coupling1 contains the interpolation of the speed of sound in the Epoxy as a function of Pressure.

ZeroCrossing Method

start := 1700   finish := 2900





$PostTest_{zc} := augment\left(PostTest^{\langle 3 \rangle}, PostTest^{\langle 1 \rangle}\right)$

$PostTest1_{zc} := submatrix\left(PostTest_{zc}, start, finish - 1, 0, 1\right)$

$v_{zc} := regress\left(PostTest1_{zc}^{\langle 1 \rangle}, PostTest1_{zc}^{\langle 0 \rangle}, p\right)$

$a_{czc} := v_{zc_4}$ $\qquad$ $b_{czc} := v_{zc_3}$

$a_{czc} = -1.229 \times 10^{-10}$

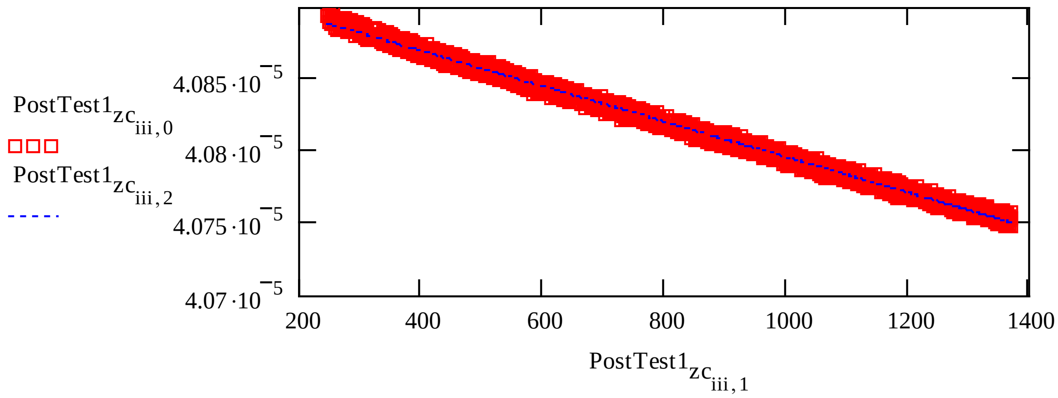
$b_{czc} = 4.092 \times 10^{-5}$

$PostTest1_{zc} =$

	0
0	$4.089 \cdot 10^{-5}$
1	$4.089 \cdot 10^{-5}$
2	$4.089 \cdot 10^{-5}$
3	$4.089 \cdot 10^{-5}$
4	$4.089 \cdot 10^{-5}$
5	$4.089 \cdot 10^{-5}$
6	$4.089 \cdot 10^{-5}$
7	$4.09 \cdot 10^{-5}$
8	$4.089 \cdot 10^{-5}$
9	$4.089 \cdot 10^{-5}$
10	$4.089 \cdot 10^{-5}$
11	$4.089 \cdot 10^{-5}$

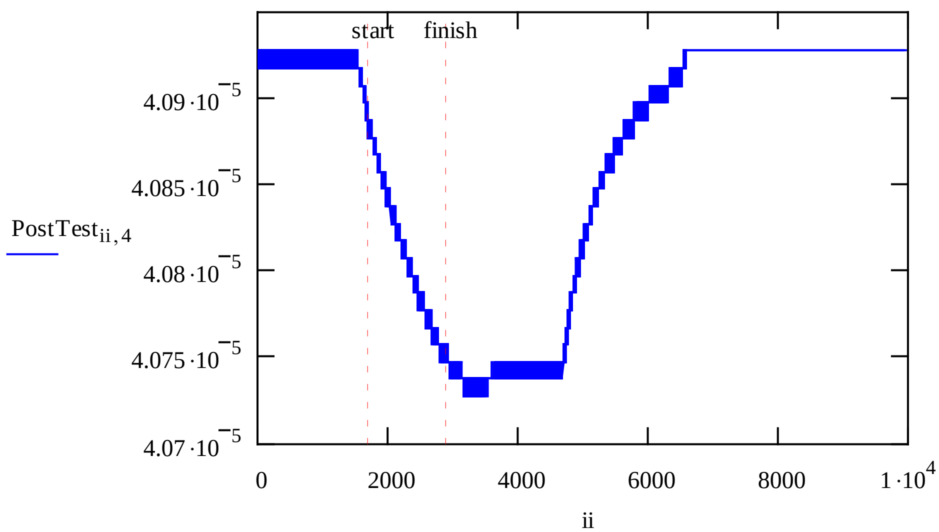
$iii := 0 .. rows\left(PostTest1_{zc}\right) - 1$

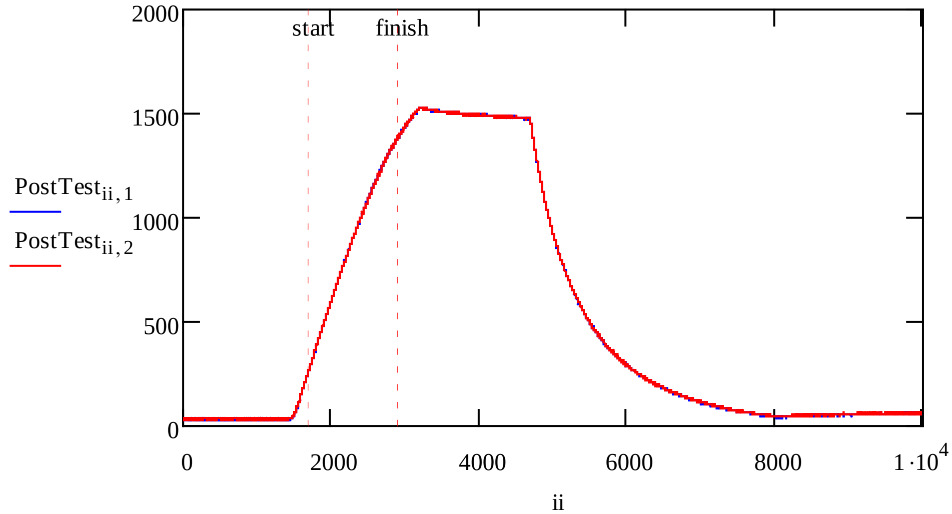
$PostTest1_{zc_{iii,2}} := v_{zc_4} \cdot PostTest1_{zc_{iii,1}} + v_{zc_3}$



CrossCorrelation Method

start := 1700 finish := 2900





$PostTest_{cc} := augment\left(PostTest^{\langle 4 \rangle}, PostTest^{\langle 1 \rangle}\right)$

$PostTest1_{cc} := submatrix\left(PostTest_{cc}, start, finish - 1, 0, 1\right)$

$v_{cc} := regress\left(PostTest1_{cc}^{\langle 1 \rangle}, PostTest1_{cc}^{\langle 0 \rangle}, p\right)$

$a_{ccc} := v_{cc_4}$ $\qquad$ $b_{ccc} := v_{cc_3}$

$a_{ccc} = -1.221 \times 10^{-10}$

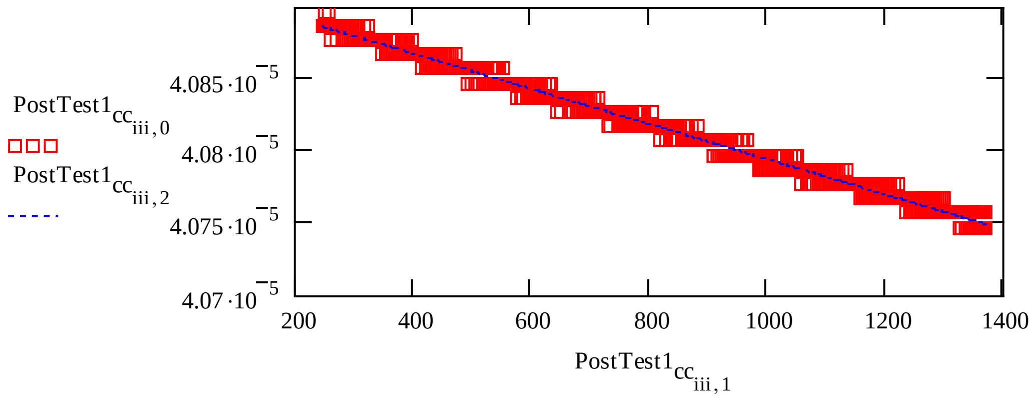
$b_{ccc} = 4.092 \times 10^{-5}$

$iii := 0..\, rows\left(PostTest1_{cc}\right) - 1$

$PostTest1_{cc_{iii,2}} := v_{cc_4} \cdot PostTest1_{cc_{iii,1}} + v_{cc_3}$

$PostTest1_{cc} =$

	0
0	$4.08871 \cdot 10^{-5}$
1	$4.08871 \cdot 10^{-5}$
2	$4.08971 \cdot 10^{-5}$
3	$4.08871 \cdot 10^{-5}$
4	$4.08871 \cdot 10^{-5}$
5	$4.08871 \cdot 10^{-5}$
6	$4.08871 \cdot 10^{-5}$
7	$4.08971 \cdot 10^{-5}$
8	$4.08871 \cdot 10^{-5}$
9	$4.08871 \cdot 10^{-5}$



PRETEST (PROPELLANT) DATA

Edum Method

$h := 0..\, \mathrm{rows}(PreTest) - 1$

Subtract the propagation time in the epoxy. The result gives the real propagation time in the propellant as a function of Pressure

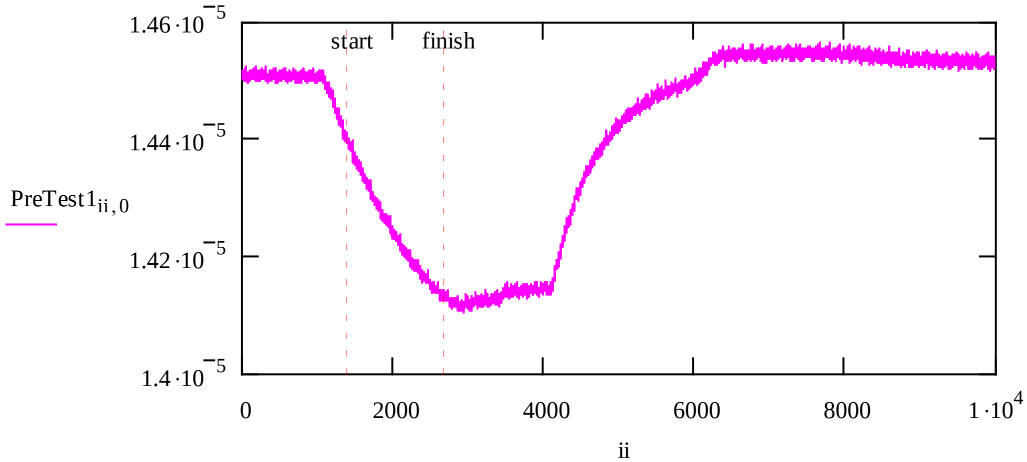
$PreTest1_{h,0} := PreTest_{h,0} - \left(a_c \cdot PreTest_{h,1} + b_c\right)$

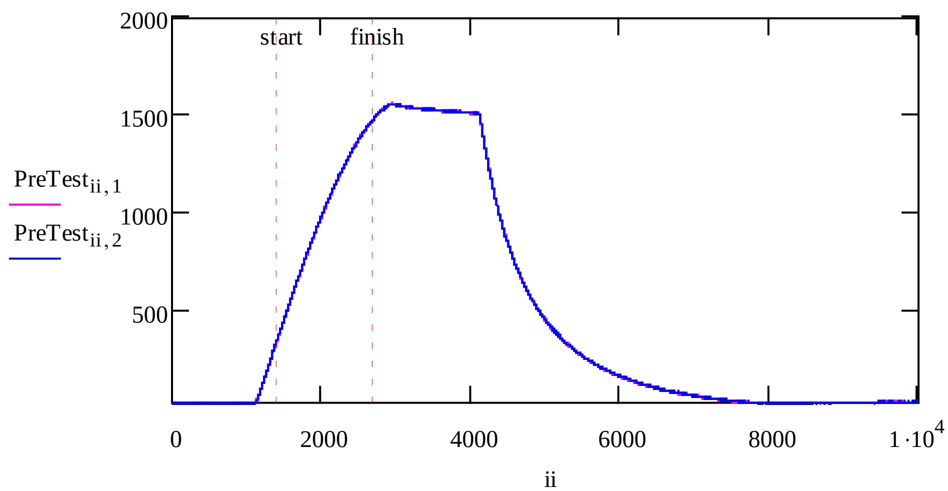
$PreTest1_{h,1} := PreTest_{h,1}$

$\mathrm{rows}(PreTest1) = 1 \times 10^4$

$ii := 0..\, \mathrm{rows}(PreTest) - 1$

$start := 1400$ $\qquad$ $finish := 2700$ $\qquad$ **Reduce data set**





$PreTest2 := \mathrm{submatrix}(PreTest1, start, finish - 1, 0, 1)$

$w := \mathrm{regress}\left(PreTest2^{\langle 1\rangle}, PreTest2^{\langle 0\rangle}, p\right)$ regression to get the coefficients ap and bp

$a_p := w_4$ $\qquad$ $b_p := w_3$

$a_p = -2.336 \times 10^{-10}$

$b_p = 1.447 \times 10^{-5}$

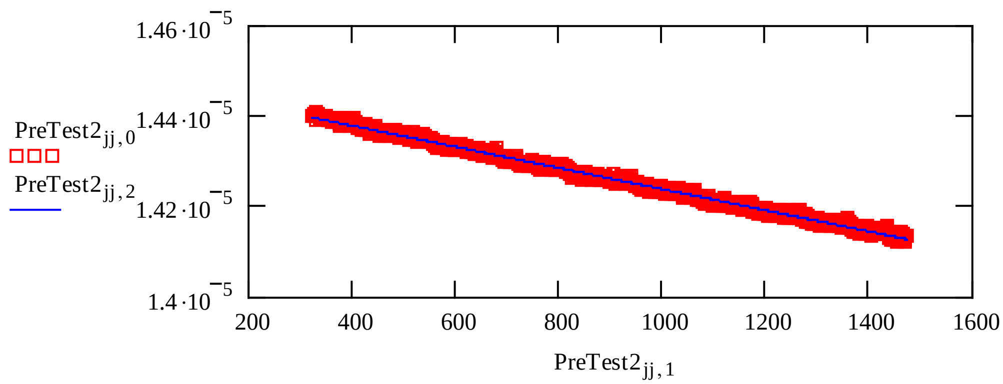
Coefficients representing the behavior of the speed of sound in the propellant as a function of Pessure

$jj := 0 .. rows(PreTest2) - 1$

$PreTest2_{jj,2} := w_4 \cdot PreTest2_{jj,1} + w_3$

PreTest2 =

	0	1
0	$1.43987 \cdot 10^{-5}$	326.13509
1	$1.43994 \cdot 10^{-5}$	326.85935
2	$1.44054 \cdot 10^{-5}$	329.17896
3	$1.44097 \cdot 10^{-5}$	330.48336
4	$1.44012 \cdot 10^{-5}$	332.22284
5	$1.44038 \cdot 10^{-5}$	334.97666
6	$1.43987 \cdot 10^{-5}$	335.41175
7	$1.43921 \cdot 10^{-5}$	333.67226
8	$1.43977 \cdot 10^{-5}$	337.00621
9	$1.44012 \cdot 10^{-5}$	337.00621
10	$1.44013 \cdot 10^{-5}$	337.73136
11	$1.43939 \cdot 10^{-5}$	338.89072



Zero Crossing Method

$h := 0 .. rows(PreTest) - 1$

Subtract the propagation time in the epoxy. The result gives the real propagation time in the propellant as a function of Pressure

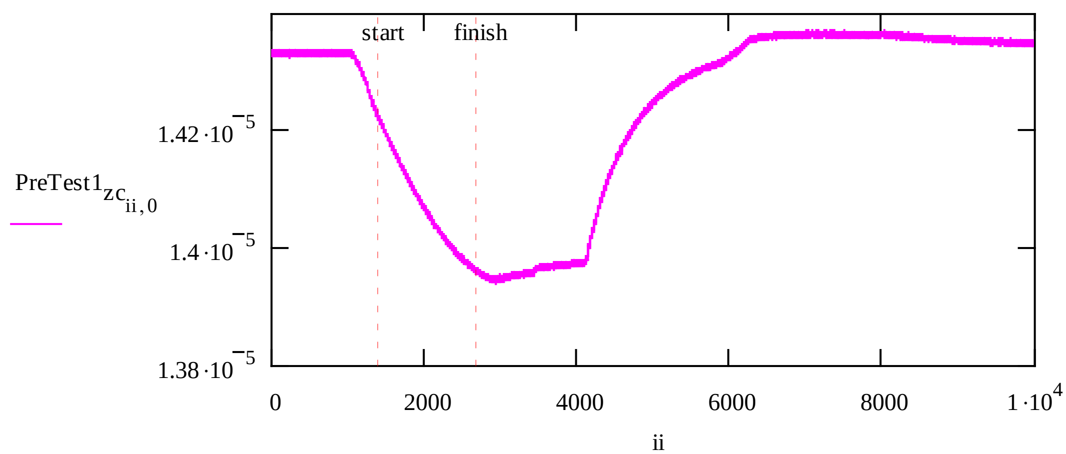
$PreTest1zc_{h,0} := PreTest_{h,3} - \left(a_{czc} \cdot PreTest_{h,1} + b_{czc}\right)$

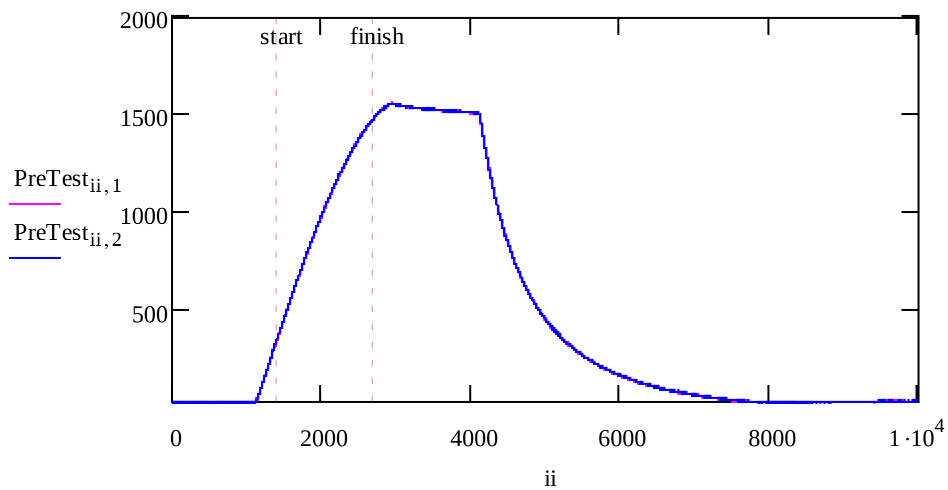
$PreTest1zc_{h,1} := PreTest_{h,1}$

$rows(PreTest1_{zc}) = 1 \times 10^4$

$ii := 0 .. rows(PreTest1_{zc}) - 1$

$start := 1400$ $finish := 2700$ **Reduce data set**





$PreTest2_{zc} := submatrix(PreTest1_{zc}, start, finish - 1, 0, 1)$

$w_{zc} := regress\left(PreTest2_{zc}^{\langle 1 \rangle}, PreTest2_{zc}^{\langle 0 \rangle}, p\right)$ regression to get the coefficients ap and bp

$a_{pzc} := w_{zc_4}$ $\qquad$ $b_{pzc} := w_{zc_3}$

$a_{pzc} = -2.309 \times 10^{-10}$

$b_{pzc} = 1.43 \times 10^{-5}$

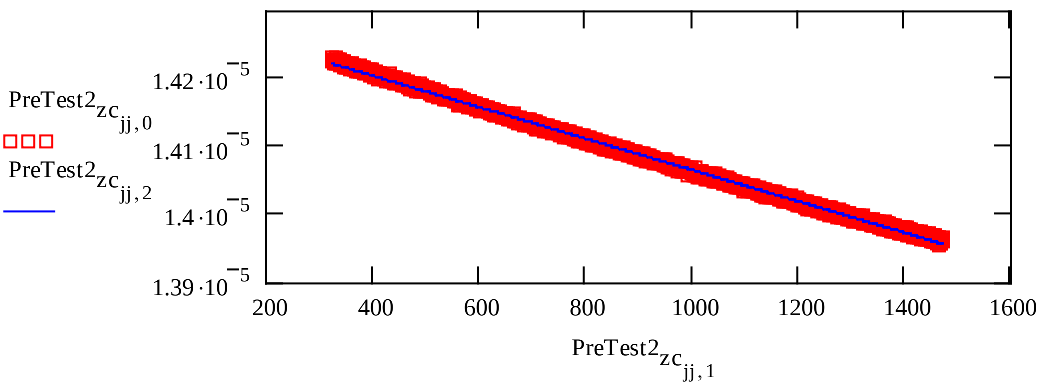
Coefficients representing the behavior of the speed of sound in the propellant as a function of Pessure

$jj := 0 .. \mathrm{rows}(PreTest2_{zc}) - 1$

$PreTest2_{zc_{jj,2}} := w_{zc_4} \cdot PreTest2_{zc_{jj,1}} + w_{zc_3}$

$PreTest2_{zc} =$

	0	1	2
0	$1.423 \cdot 10^{-5}$	326.135	$1.422 \cdot 10^{-5}$
1	$1.423 \cdot 10^{-5}$	326.859	$1.422 \cdot 10^{-5}$
2	$1.422 \cdot 10^{-5}$	329.179	$1.422 \cdot 10^{-5}$
3	$1.423 \cdot 10^{-5}$	330.483	$1.422 \cdot 10^{-5}$
4	$1.422 \cdot 10^{-5}$	332.223	$1.422 \cdot 10^{-5}$
5	$1.423 \cdot 10^{-5}$	334.977	$1.422 \cdot 10^{-5}$
6	$1.423 \cdot 10^{-5}$	335.412	$1.422 \cdot 10^{-5}$
7	$1.422 \cdot 10^{-5}$	333.672	$1.422 \cdot 10^{-5}$
8	$1.423 \cdot 10^{-5}$	337.006	$1.422 \cdot 10^{-5}$
9	$1.422 \cdot 10^{-5}$	337.006	$1.422 \cdot 10^{-5}$
10	$1.422 \cdot 10^{-5}$	337.731	$1.422 \cdot 10^{-5}$
11	$1.423 \cdot 10^{-5}$	338.891	$1.422 \cdot 10^{-5}$
12	$1.422 \cdot 10^{-5}$	340.92	$1.422 \cdot 10^{-5}$
13	$1.422 \cdot 10^{-5}$	340.775	$1.422 \cdot 10^{-5}$
14	$1.422 \cdot 10^{-5}$	344.978	$1.422 \cdot 10^{-5}$
15	$1.422 \cdot 10^{-5}$	345.414	$1.422 \cdot 10^{-5}$



$h := 0..\,\mathrm{rows}(PreTest) - 1$

Subtract the propagation time in the epoxy. The result gives the real propagation time in the propellant as a function of Pressure

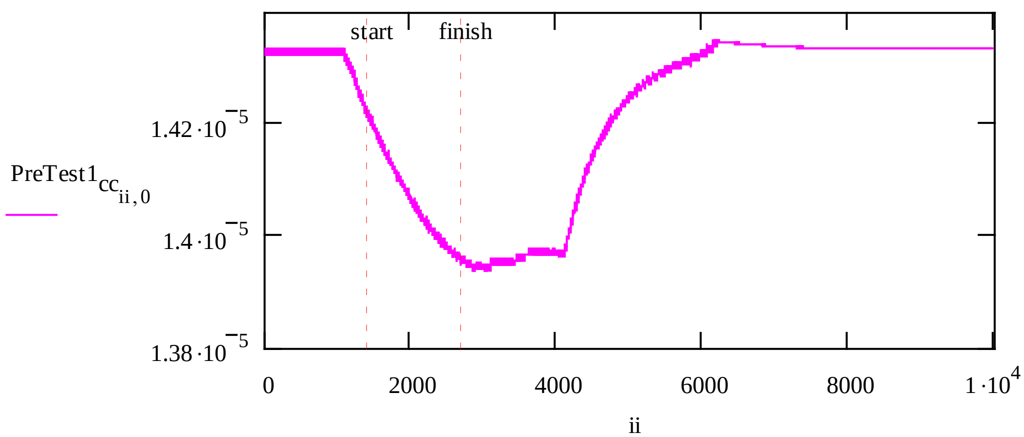
$PreTest1cc_{h,0} := PreTest_{h,4} - \left(a_{ccc} \cdot PreTest_{h,1} + b_{ccc}\right)$

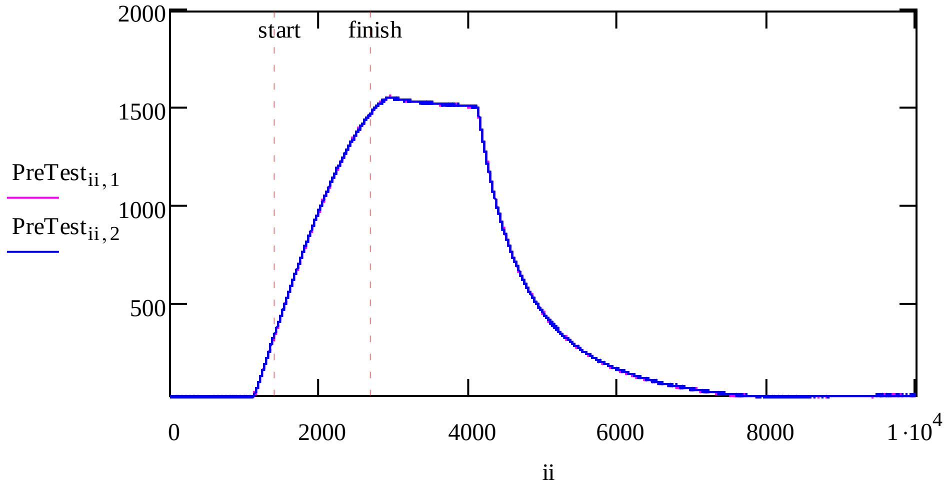
$PreTest1cc_{h,1} := PreTest_{h,1}$

$\mathrm{rows}\left(PreTest1_{cc}\right) = 1 \times 10^{4}$

$ii := 0..\,\mathrm{rows}\left(PreTest1_{cc}\right) - 1$

$start := 1400$ $finish := 2700$ **Reduce data set**





$PreTest2_{cc} := submatrix(PreTest1_{cc}, start, finish - 1, 0, 1)$

$w_{cc} := regress\left(PreTest2_{cc}^{\langle 1 \rangle}, PreTest2_{cc}^{\langle 0 \rangle}, p\right)$ regression to get the coefficients ap and bp

$a_{pcc} := w_{cc_4}$ $b_{pcc} := w_{cc_3}$

$a_{pcc} = -2.312 \times 10^{-10}$

$b_{pcc} = 1.43 \times 10^{-5}$

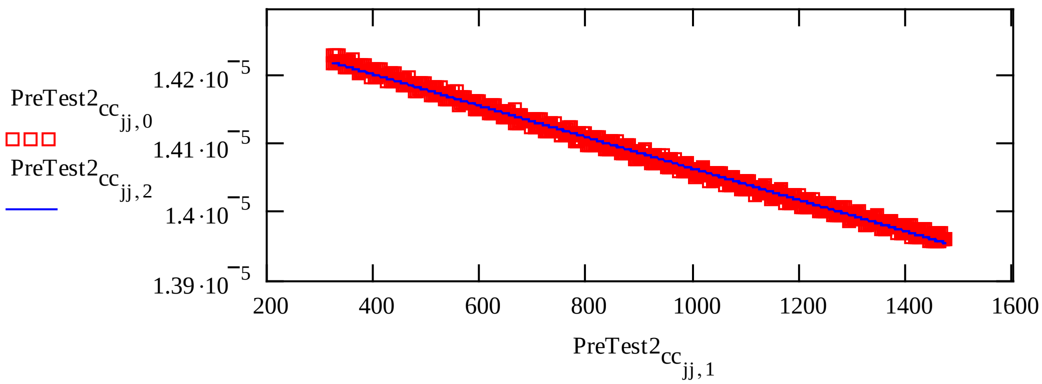
Coefficients representing the behavior of the speed of sound in the propellant as a function of Pessure

$jj := 0..\, rows(PreTest2_{cc}) - 1$

$PreTest2_{cc_{jj,2}} := w_{cc_4} \cdot PreTest2_{cc_{jj,1}} + w_{cc_3}$

$PreTest2_{cc} =$

	0
0	$1.42298 \cdot 10^{-5}$
1	$1.42199 \cdot 10^{-5}$
2	$1.42202 \cdot 10^{-5}$
3	$1.42304 \cdot 10^{-5}$
4	$1.42206 \cdot 10^{-5}$
5	$1.42309 \cdot 10^{-5}$
6	$1.4221 \cdot 10^{-5}$
7	$1.42208 \cdot 10^{-5}$
8	$1.42312 \cdot 10^{-5}$
9	$1.42212 \cdot 10^{-5}$
10	$1.42213 \cdot 10^{-5}$



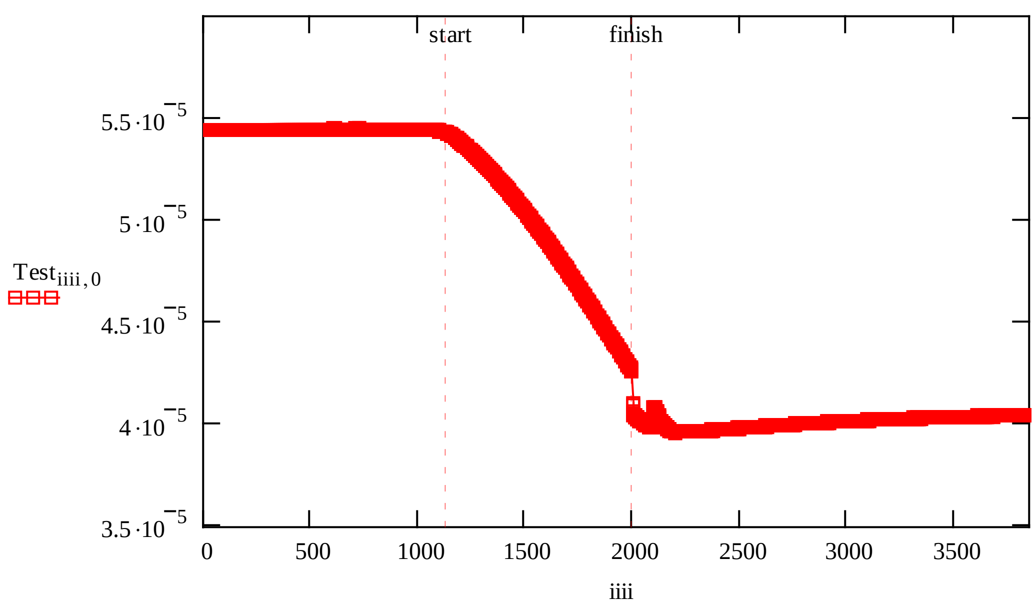
TEST (FIRING) DATA

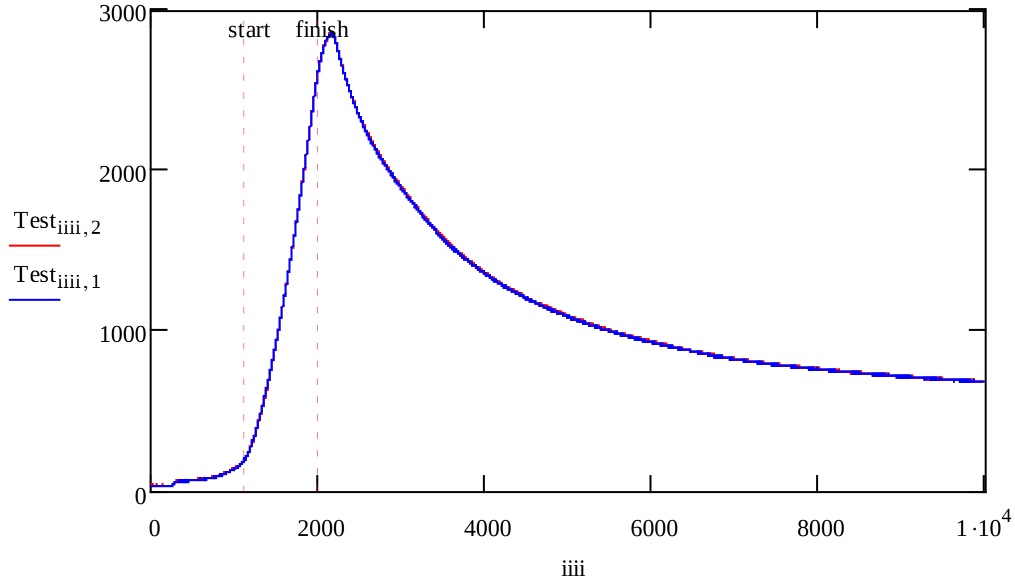
Edum Method

$start := 1135$ $finish := 2003$

$rows(Test) = 1 \times 10^{4}$

$iiii := 1 .. rows(Test) - 1$





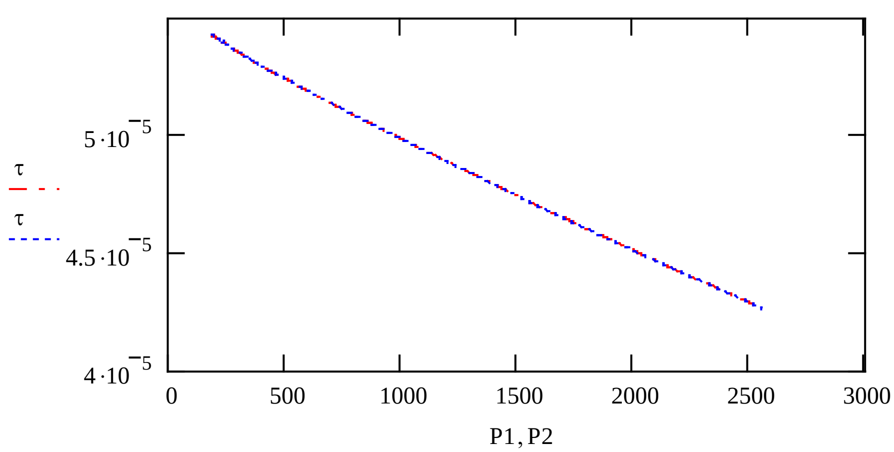
$\tau := \text{submatrix}(Test, start, finish - 1, 0, 0)$

$P2 := \text{submatrix}(Test, start, finish - 1, 2, 2)$

$P1 := \text{submatrix}(Test, start, finish - 1, 1, 1)$

$\Delta t := \dfrac{1}{freq}$

$k := 0 .. \text{rows}(\tau) - 1$



Calculate propellant thickness

$$E1p_k := \frac{E_{p0}}{a_p \cdot P1_k + b_p} \cdot \left[\tau_k - \left(a_c \cdot P1_k + b_c\right)\right]$$

$$E2p_k := \frac{E_{p0}}{a_p \cdot P2_k + b_p} \cdot \left[\tau_k - \left(a_c \cdot P2_k + b_c\right)\right]$$

$$t_k := \frac{1}{freq} \cdot k$$

Smoothing function

$$E1_p := \mathrm{supsmooth}\left(t, E1_p\right)$$

$$E2_p := \mathrm{supsmooth}\left(t, E2_p\right)$$

$E1_p =$

	0
0	0.5134
1	0.5132
2	0.513
3	0.5128
4	0.5126
5	0.51241
6	0.51221
7	0.51201
8	0.51181
9	0.51161
10	0.51141
11	0.51121
12	0.51101
13	0.51081
14	0.51062
15	0.51042

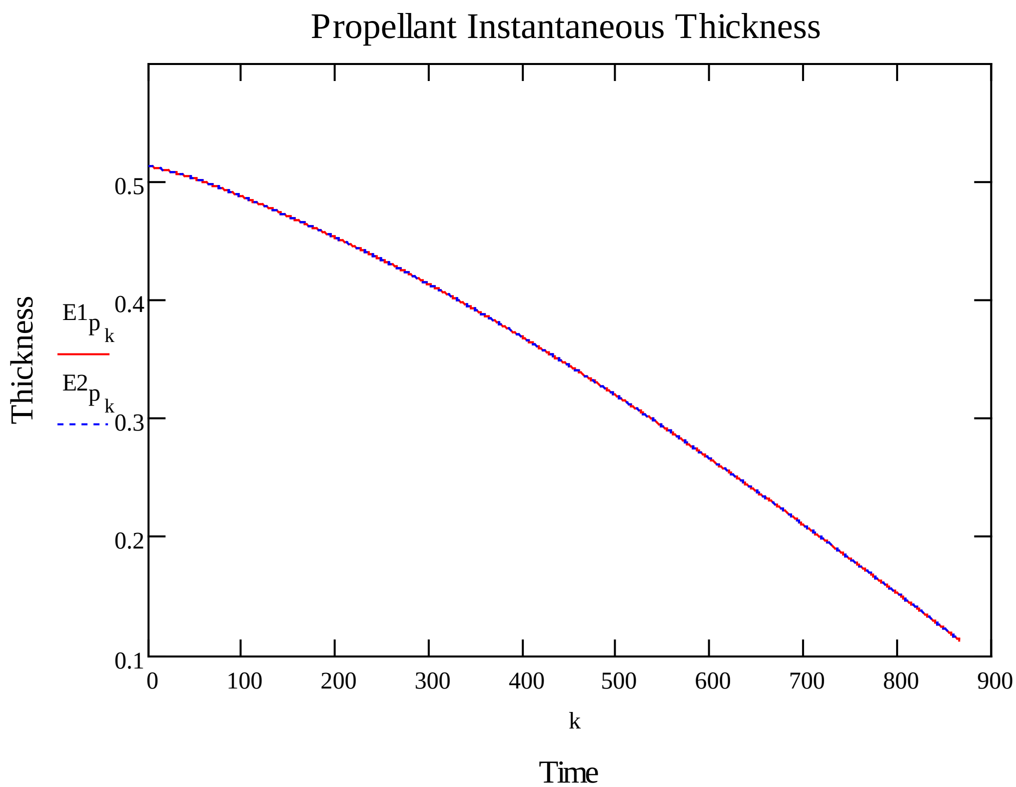
$P1 =$

	0
0	197.8536
1	195.679
2	197.1293
3	197.5635
4	198.4337
5	200.6074
6	199.8831
7	201.6226
8	200.6074
9	202.4919
10	201.3325
11	200.3182
12	202.0568
13	204.8115
14	205.5358
15	206.406

$\tau =$

	0
0	$5.42592 \cdot 10^{-5}$
1	$5.42453 \cdot 10^{-5}$
2	$5.42424 \cdot 10^{-5}$
3	$5.42296 \cdot 10^{-5}$
4	$5.42284 \cdot 10^{-5}$
5	$5.42307 \cdot 10^{-5}$
6	$5.42278 \cdot 10^{-5}$
7	$5.42203 \cdot 10^{-5}$
8	$5.42151 \cdot 10^{-5}$
9	$5.42116 \cdot 10^{-5}$
10	$5.42093 \cdot 10^{-5}$
11	$5.4211 \cdot 10^{-5}$
12	$5.42023 \cdot 10^{-5}$
13	$5.41948 \cdot 10^{-5}$
14	$5.41849 \cdot 10^{-5}$
15	$5.41797 \cdot 10^{-5}$

$$\mathrm{rows}\left(E1_p\right) = 868$$



Calculate burning rate

number of points to use in linear regression. (Must be an odd number.)

$N := 151$

$M := \frac{N-1}{2}$ $\quad q := 0..N-1$ $\quad time_q := \Delta t \cdot q$

$i := M..rows(E1p) - 1 - M$

linear regression to get the burning rate.

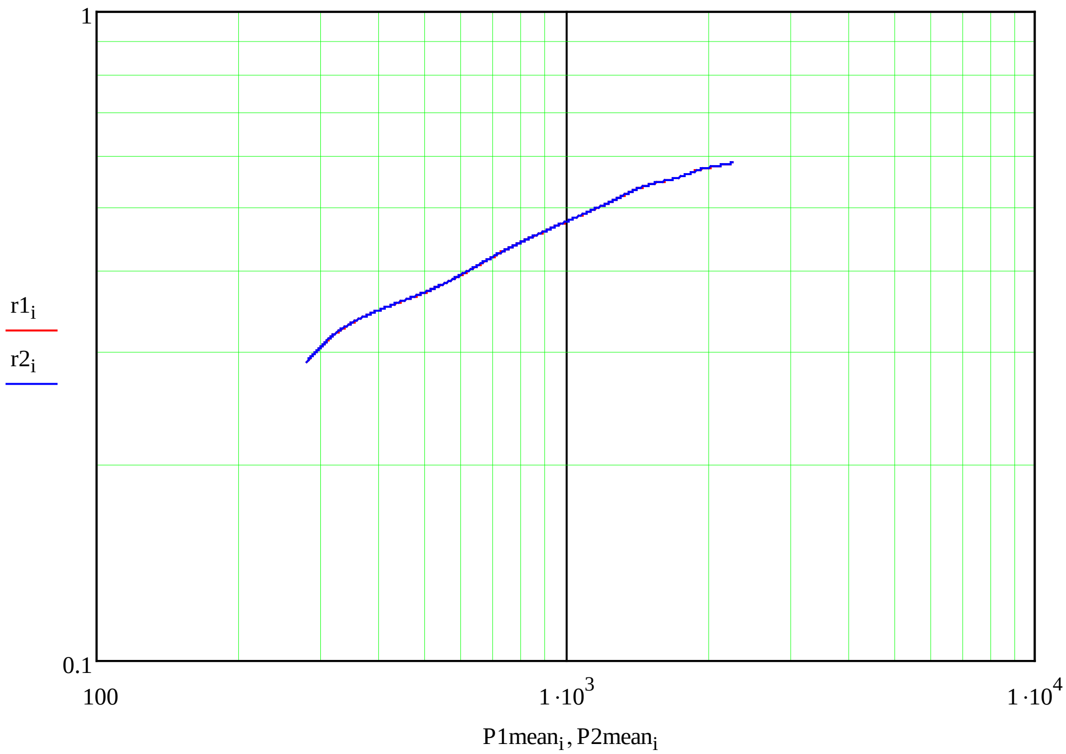
$r1_i := -slope(time, submatrix(E1p, i-M, i+M, 0, 0))$

$r2_i := -slope(time, submatrix(E2p, i-M, i+M, 0, 0))$

$P1mean_i := mean(submatrix(P1, i-M, i+M, 0, 0))$

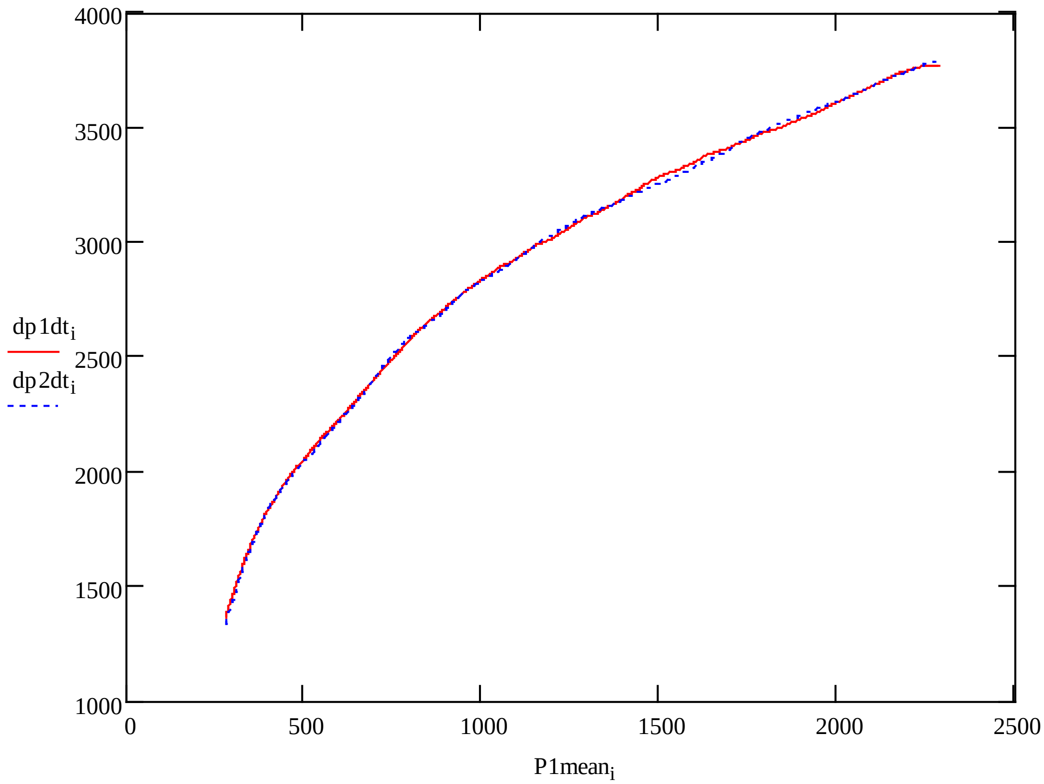
$P2mean_i := mean(submatrix(P2, i-M, i+M, 0, 0))$

EDUM Burn Rate



$dp1dt_i := slope(time, submatrix(P1, i - M, i + M, 0, 0))$

$dp2dt_i := slope(time, submatrix(P2, i - M, i + M, 0, 0))$



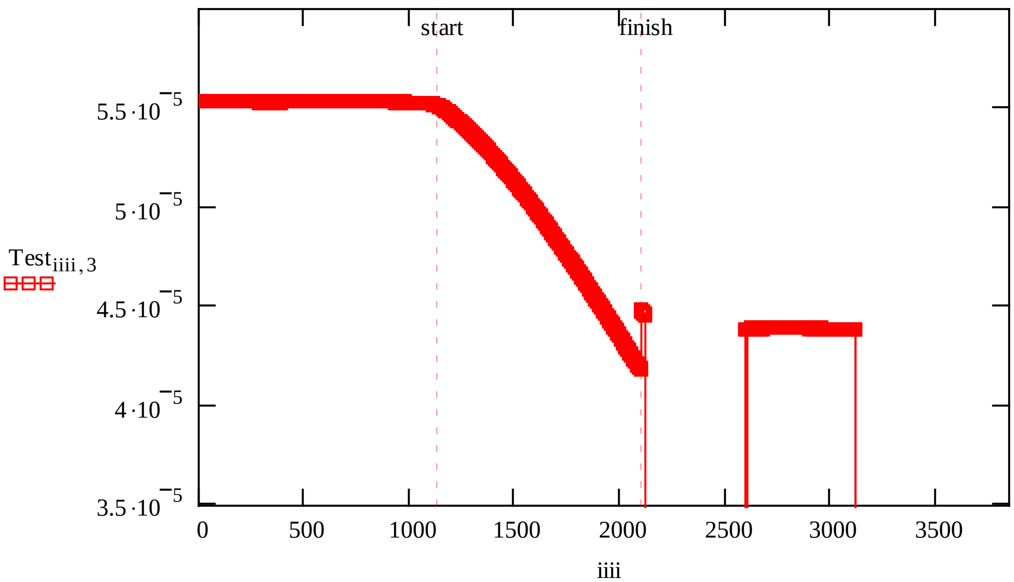
TEST (FIRING) DATA ZERO CROSSING

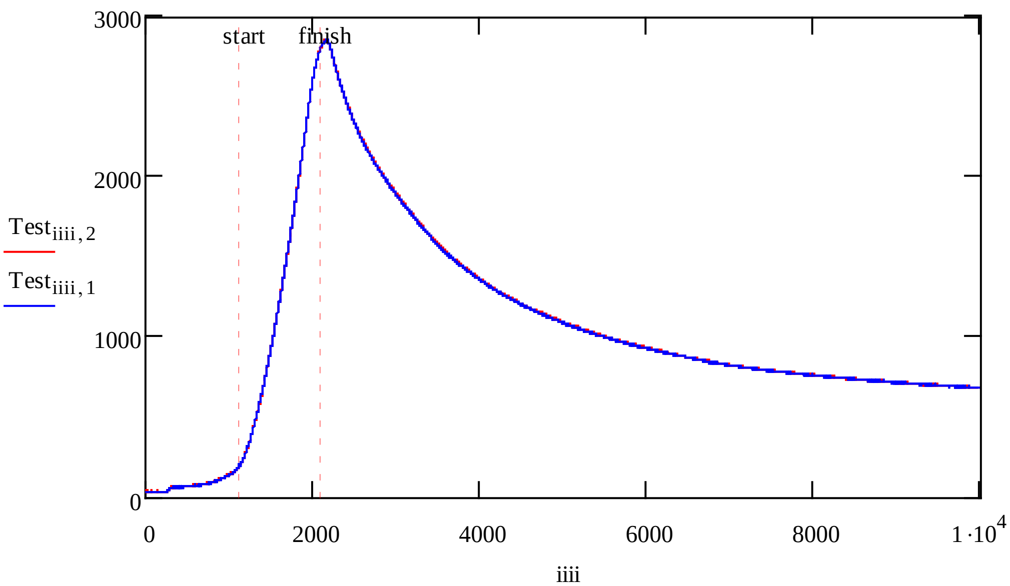
Zero Crossing Method

start := 1135 finish := 2102

$\text{rows}(\text{Test}) = 1 \times 10^{4}$

$\text{iiii} := 1 .. \text{rows}(\text{Test}) - 1$



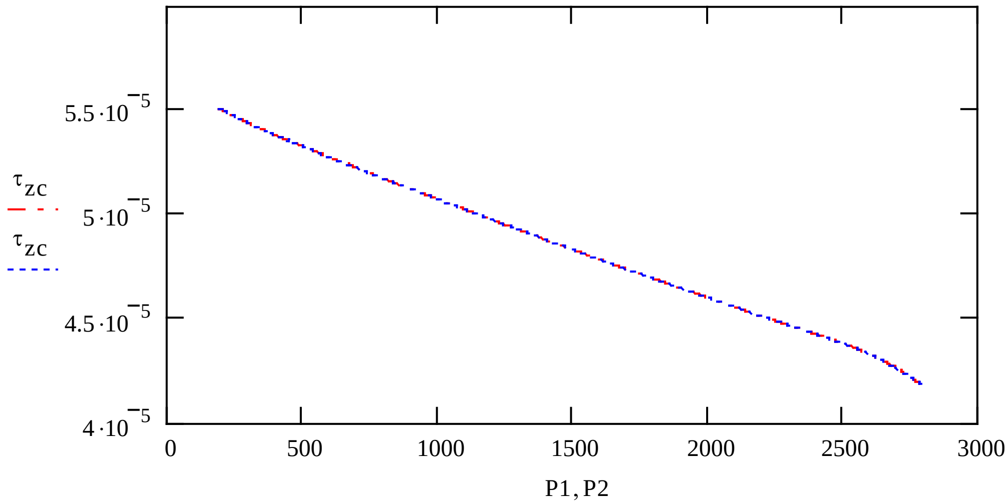


$\tau_{zc} := \text{submatrix}(\text{Test}, \text{start}, \text{finish} - 1, 3, 3)$

$P2 := \text{submatrix}(\text{Test}, \text{start}, \text{finish} - 1, 2, 2)$

$P1 := \text{submatrix}(\text{Test}, \text{start}, \text{finish} - 1, 1, 1)$

$\Delta t := \dfrac{1}{\text{freq}}$ $\quad k := 0 .. \text{rows}(\tau_{zc}) - 1$ $\quad \text{rows}(t) = 868$



Calculate propellant thickness

$$E1_{pzc_k} := \frac{E_{p0}}{a_{pzc}\cdot P1_k + b_{pzc}} \cdot \left[\tau_{zc_k} - \left(a_{czc}\cdot P1_k + b_{czc}\right)\right]$$

$$E2_{pzc_k} := \frac{E_{p0}}{a_{pzc}\cdot P2_k + b_{pzc}} \cdot \left[\tau_{zc_k} - \left(a_{czc}\cdot P2_k + b_{czc}\right)\right]$$

Smoothing function

$$j_k := \frac{k}{freq}$$

$$E1_{pzc} := \mathrm{supsmooth}\left(j, E1_{pzc}\right) \qquad E2_{pzc} := \mathrm{supsmooth}\left(j, E2_{pzc}\right)$$

$E1_{pzc} =$

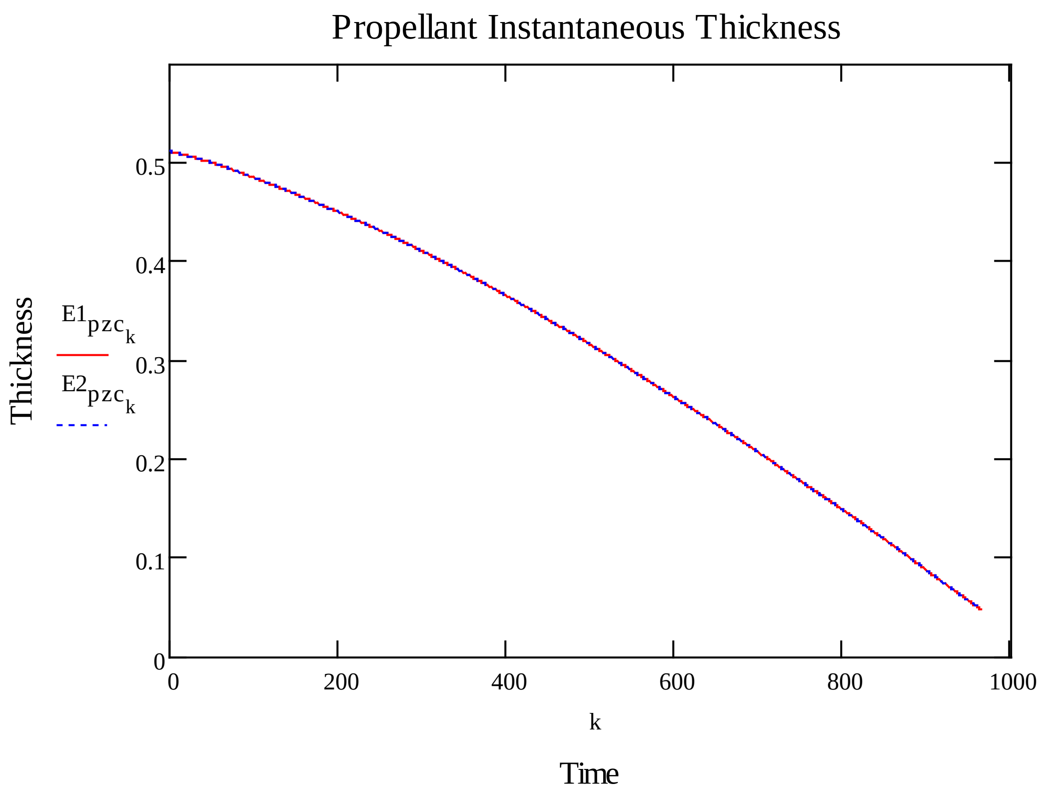
	0
0	0.51145
1	0.51124
2	0.51102
3	0.51081
4	0.5106
5	0.51039
6	0.51018
7	0.50996
8	0.50975
9	0.50954
10	0.50933
11	0.50911
12	0.5089
13	0.50869
14	0.50848

$P1 =$

	0
0	197.854
1	195.679
2	197.129
3	197.564
4	198.434
5	200.607
6	199.883
7	201.623
8	200.607
9	202.492
10	201.333
11	200.318
12	202.057
13	204.812
14	205.536

$\tau_{zc} =$

	0
0	$5.5095\cdot10^{-5}$
1	$5.50943\cdot10^{-5}$
2	$5.50907\cdot10^{-5}$
3	$5.5083\cdot10^{-5}$
4	$5.50844\cdot10^{-5}$
5	$5.50775\cdot10^{-5}$
6	$5.50693\cdot10^{-5}$
7	$5.50711\cdot10^{-5}$
8	$5.50635\cdot10^{-5}$
9	$5.50566\cdot10^{-5}$
10	$5.50589\cdot10^{-5}$
11	$5.5051\cdot10^{-5}$
12	$5.504\cdot10^{-5}$
13	$5.50445\cdot10^{-5}$
14	$5.50354\cdot10^{-5}$



Calculate burning rate

number of points to use in linear regression. (Must be an odd number.)

$N := 151$ $\quad M := \dfrac{N-1}{2}$ $\quad q := 0..\,N-1$ $\quad time_q := \Delta t \cdot q$

$i := M..\,rows\left(E1_{pzc}\right) - 1 - M$

linear regression to get the burning rate.

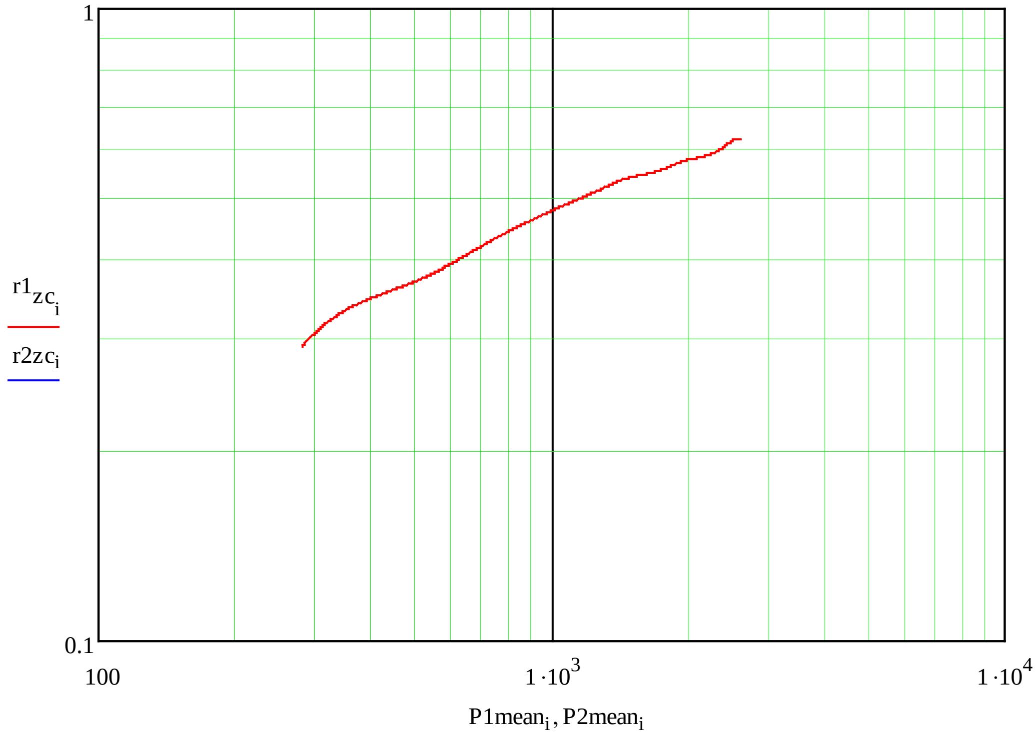
$r1_{zc_i} := -slope\left(time, submatrix\left(E1_{pzc}, i-M, i+M, 0, 0\right)\right)$

$r2_{zc_i} := -slope\left(time, submatrix\left(E2_{pzc}, i-M, i+M, 0, 0\right)\right)$

$P1mean_i := mean(submatrix(P1, i-M, i+M, 0, 0))$

$P2mean_i := mean(submatrix(P2, i-M, i+M, 0, 0))$

Zero Crossing Burn Rate

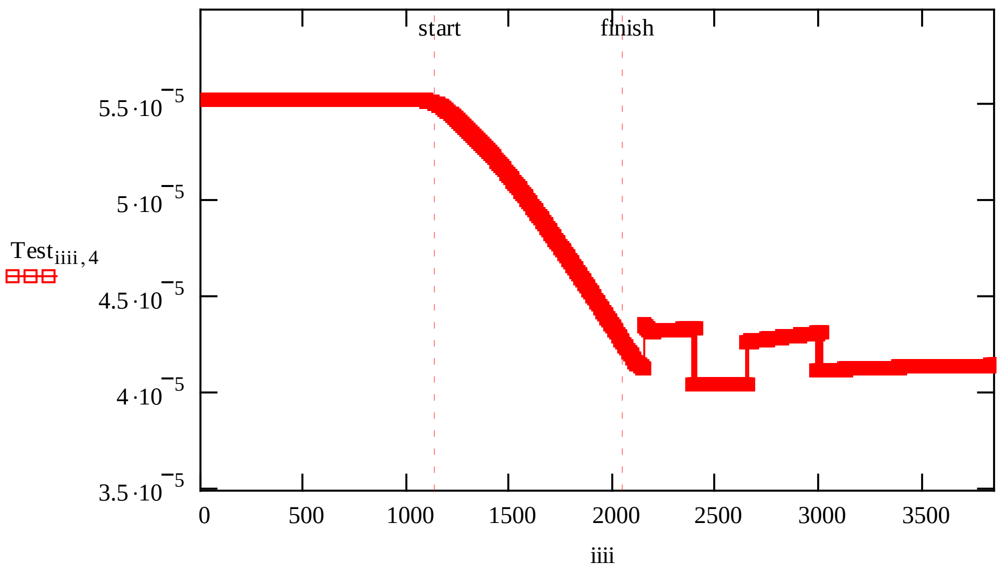


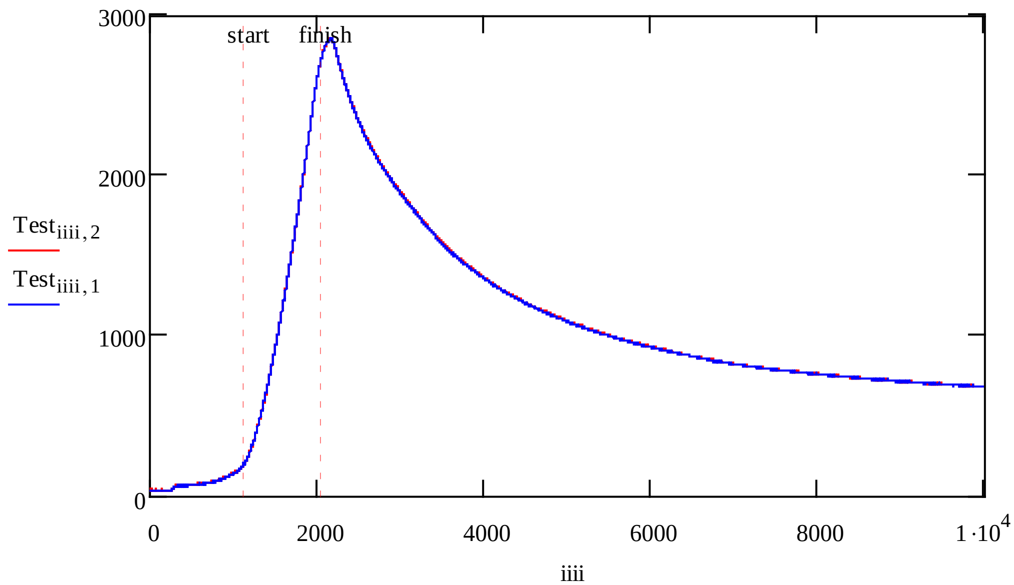
TEST (FIRING) DATA CROSS CORRELATION

Cross Correlation Method

$\text{rows}(\text{Test}) = 1 \times 10^4$ $\quad$ $\text{start} := 1135$ $\quad$ $\text{finish} := 2053$

$\text{iiii} := 1 .. \text{rows}(\text{Test}) - 1$



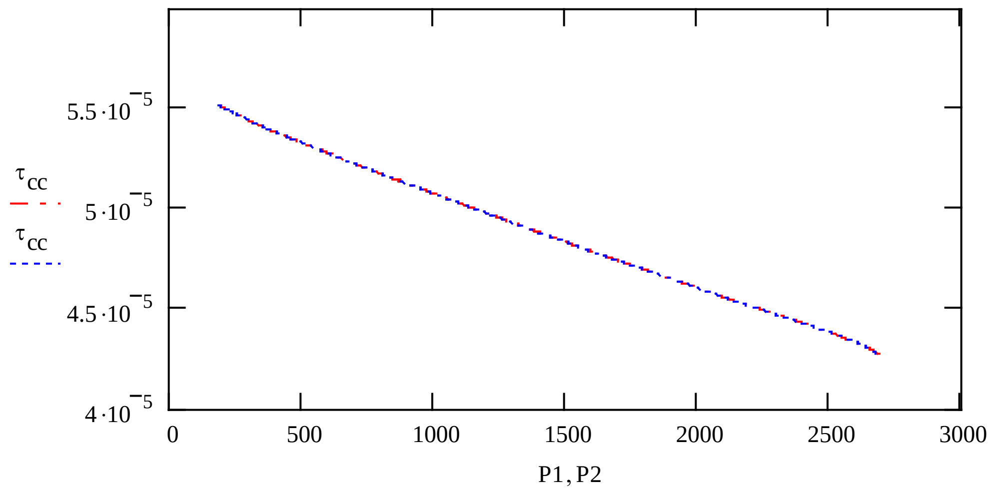


$\tau_{cc} := \mathrm{submatrix}(Test, start, finish - 1, 4, 4)$

$P2 := \mathrm{submatrix}(Test, start, finish - 1, 2, 2)$

$P1 := \mathrm{submatrix}(Test, start, finish - 1, 1, 1)$

$$\Delta t := \frac{1}{freq} \qquad k := 0..\,\mathrm{rows}(\tau_{cc}) - 1$$



Calculate propellant thickness

$$E1_{pcc_k} := \frac{E_{p0}}{a_{pcc} \cdot P1_k + b_{pcc}} \cdot \left[\tau_{cc_k} - \left(a_{ccc} \cdot P1_k + b_{ccc}\right)\right]$$

$$E2_{pcc_k} := \frac{E_{p0}}{a_{pcc} \cdot P2_k + b_{pcc}} \cdot \left[\tau_{cc_k} - \left(a_{ccc} \cdot P2_k + b_{ccc}\right)\right]$$

$E1_{pcc} =$

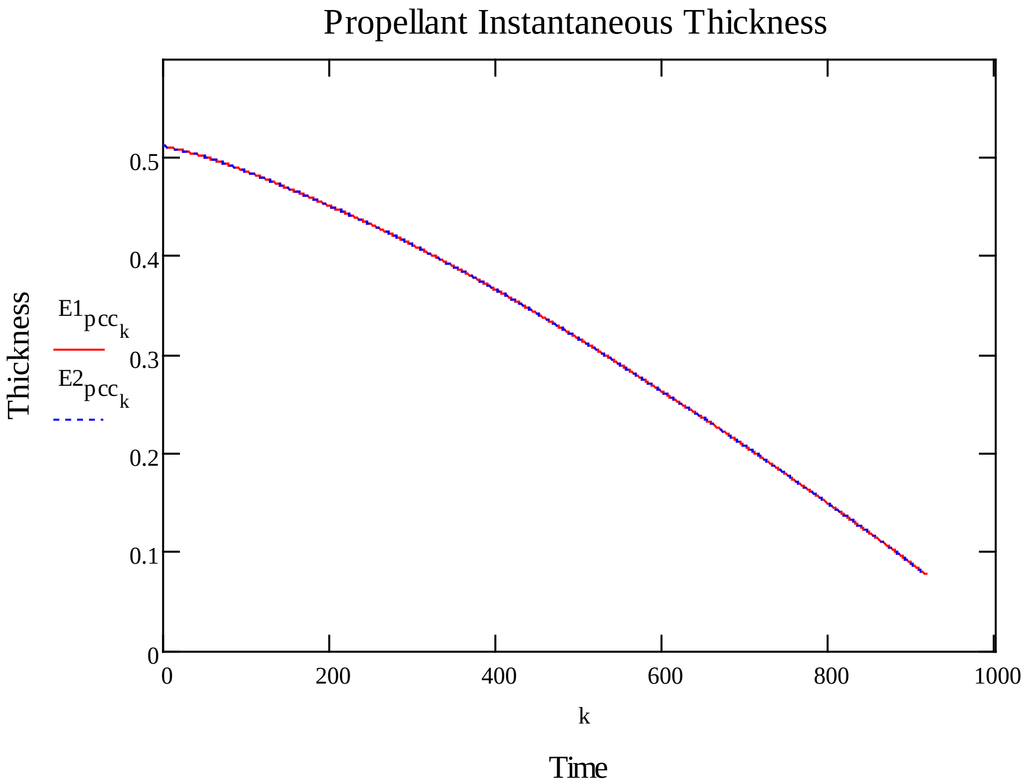
	0
0	0.511
1	0.51097
2	0.51099
3	0.51064
4	0.51065
5	0.51031
6	0.51031
7	0.51033
8	0.50995
9	0.50962
10	0.50996
11	0.50959
12	0.50925
13	0.50929
14	0.50894
15	0.50859

$$l_k := \frac{1}{freq} \cdot k$$

Smoothing function

$$E1_{pcc} := \mathrm{supsmooth}\left(l, E1_{pcc}\right)$$

$$E2_{pcc} := \mathrm{supsmooth}\left(l, E2_{pcc}\right)$$



Calculate burning rate

number of points to use in linear regression. (Must be an odd number.)

$N := 151$ $\quad M := \frac{N-1}{2}$ $\quad q := 0..\,N-1$ $\quad time_q := \Delta t \cdot q$

$i := M..\,rows(E1_{pcc}) - 1 - M$

linear regression to get the burning rate.

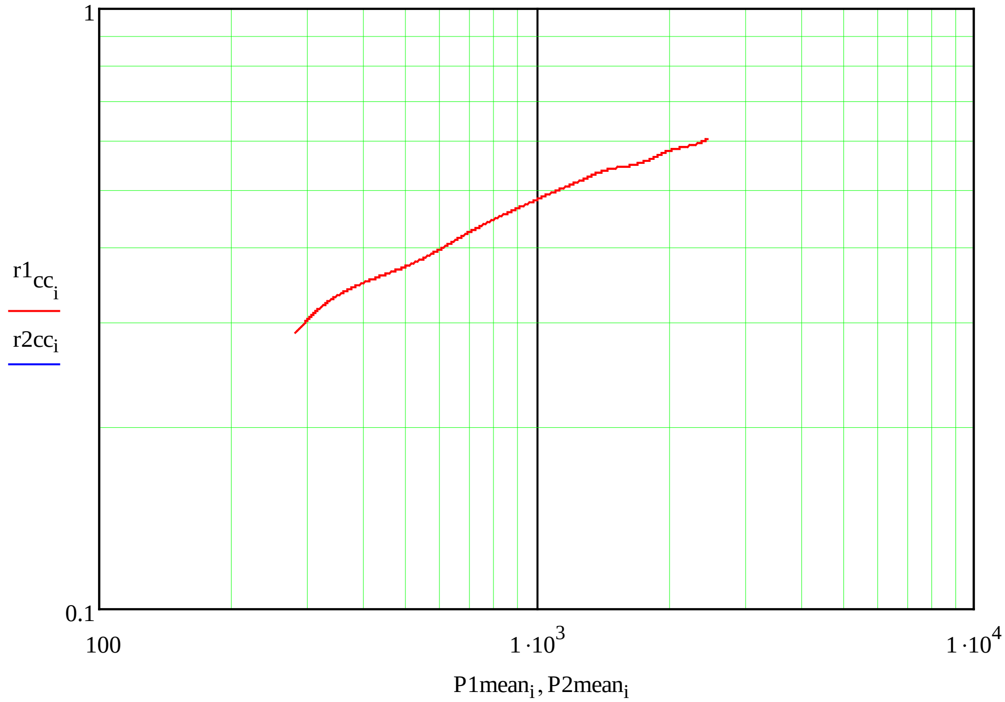
$$r1_{cc_i} := -\mathrm{slope}\left(\mathrm{time}, \mathrm{submatrix}\left(E1_{pcc}, i-M, i+M, 0, 0\right)\right)$$

$$r2_{cc_i} := -\mathrm{slope}\left(\mathrm{time}, \mathrm{submatrix}\left(E2_{pcc}, i-M, i+M, 0, 0\right)\right)$$

$$P1mean_i := \mathrm{mean}(\mathrm{submatrix}(P1, i-M, i+M, 0, 0))$$

$$P2mean_i := \mathrm{mean}(\mathrm{submatrix}(P2, i-M, i+M, 0, 0))$$

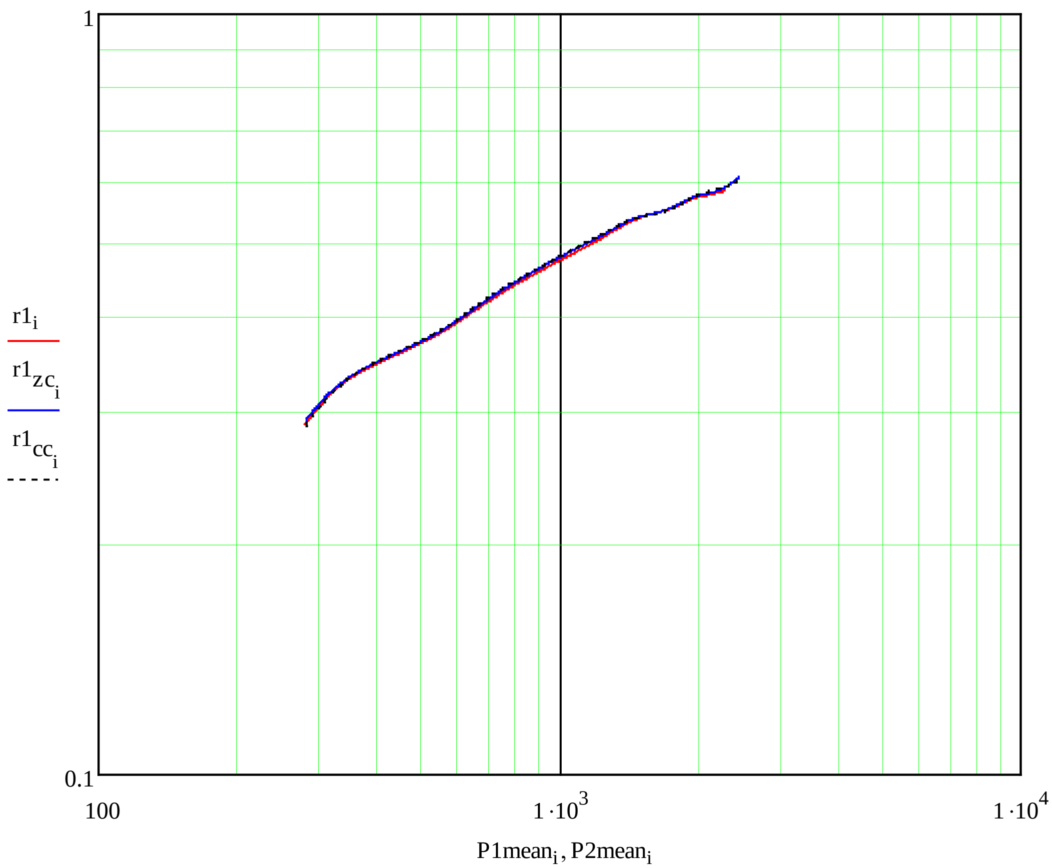
Cross Correlation Burn Rate



$$dp1dt_i := \mathrm{slope}(\mathrm{time}, \mathrm{submatrix}(P1, i-M, i+M, 0, 0))$$

$$dp2dt_i := \mathrm{slope}(\mathrm{time}, \mathrm{submatrix}(P2, i-M, i+M, 0, 0))$$

Burn Rate For All Three Methods



APPENDIX C

REGRESSION CONTSTANT UNCERTAINTIES

The following section shows the Mathcad worksheet that was used to calculate the regression constant uncertainties for Test 5.

$ORIGIN := 1$

$CD :=$

	1	2
1	$4.089 \cdot 10^{-5}$	249.311
2	$4.089 \cdot 10^{-5}$	249.891
3	$4.09 \cdot 10^{-5}$	250.906
4	$4.089 \cdot 10^{-5}$	252.935
5	$4.089 \cdot 10^{-5}$	255.399
6	$4.089 \cdot 10^{-5}$	256.124
7	$4.089 \cdot 10^{-5}$	256.413
8	$4.09 \cdot 10^{-5}$	257.283
9	$4.089 \cdot 10^{-5}$	256.703
10	$4.089 \cdot 10^{-5}$	259.458
11	$4.089 \cdot 10^{-5}$	258.878
12	$4.089 \cdot 10^{-5}$	260.762
13	$4.089 \cdot 10^{-5}$	261.922
14	$4.089 \cdot 10^{-5}$	261.197
15	$4.089 \cdot 10^{-5}$	262.647
16	$4.088 \cdot 10^{-5}$	263.371

$PD :=$

	1	2
1	$1.423 \cdot 10^{-5}$	326.135
2	$1.422 \cdot 10^{-5}$	326.859
3	$1.422 \cdot 10^{-5}$	329.179
4	$1.423 \cdot 10^{-5}$	330.483
5	$1.422 \cdot 10^{-5}$	332.223
6	$1.423 \cdot 10^{-5}$	334.977
7	$1.422 \cdot 10^{-5}$	335.412
8	$1.422 \cdot 10^{-5}$	333.672
9	$1.423 \cdot 10^{-5}$	337.006
10	$1.422 \cdot 10^{-5}$	337.006
11	$1.422 \cdot 10^{-5}$	337.731
12	$1.422 \cdot 10^{-5}$	338.891
13	$1.422 \cdot 10^{-5}$	340.92
14	$1.422 \cdot 10^{-5}$	340.775
15	$1.422 \cdot 10^{-5}$	344.978
16	$1.422 \cdot 10^{-5}$	345.414

$Nc := rows(CD)$

$Nc = 1.2 \times 10^{3}$

$U_{\tau} := 0.3 \cdot 10^{-6}$

$U_{PB} := 6.04$

$U_{PP} := 2.7$

$Np := rows(PD)$

$Np = 1.3 \times 10^{3}$

$Pc := CD^{\langle 2 \rangle}$ $\quad\quad \tau c := CD^{\langle 1 \rangle}$

$$ac := \frac{Nc \cdot \left[\sum_{i=1}^{Nc} \left(Pc_i \cdot \tau c_i\right)\right] - \left(\sum_{i=1}^{Nc} Pc_i\right) \cdot \left(\sum_{i=1}^{Nc} \tau c_i\right)}{\left[Nc \cdot \sum_{i=1}^{Nc} \left(Pc_i\right)^2\right] - \left(\sum_{i=1}^{Nc} Pc_i\right)^2}$$

$$ac = -1.22098 \times 10^{-10}$$

$Pp := PD^{\langle 2 \rangle}$ $\quad\quad \tau p := PD^{\langle 1 \rangle}$

$$bc := \frac{\left[\sum_{i=1}^{Nc} \left(Pc_i\right)^2\right] \cdot \left(\sum_{i=1}^{Nc} \tau c_i\right) - \left(\sum_{i=1}^{Nc} Pc_i\right) \cdot \left[\sum_{i=1}^{Nc} \left(Pc_i \cdot \tau c_i\right)\right]}{\left[Nc \cdot \sum_{i=1}^{Nc} \left(Pc_i\right)^2\right] - \left(\sum_{i=1}^{Nc} Pc_i\right)^2}$$

$$bc = 4.09168 \times 10^{-5}$$

$i := 1, 2 .. Nc$

$$DacD\tau_i := \frac{\left(Nc \cdot Pc_i\right) - \sum_{i=1}^{Nc} Pc_i}{\left[Nc \cdot \sum_{i=1}^{Nc} \left(Pc_i\right)^2\right] - \left(\sum_{i=1}^{Nc} Pc_i\right)^2}$$

$$DbcD\tau_i := \frac{\left[\sum_{i=1}^{Nc} \left(Pc_i\right)^2\right] - Pc_i \cdot \left(\sum_{i=1}^{Nc} Pc_i\right)}{\left[Nc \cdot \sum_{i=1}^{Nc} \left(Pc_i\right)^2\right] - \left(\sum_{i=1}^{Nc} Pc_i\right)^2}$$

$$DacDP_i := \frac{\left(Nc\cdot\tau c_i - \sum_{i=1}^{Nc}\tau c_i\right)\cdot\left[\left[Nc\cdot\sum_{i=1}^{Nc}\left(Pc_i\right)^2\right] - \left(\sum_{i=1}^{Nc}Pc_i\right)^2\right]\ \ldots \\ + -\left[Nc\cdot\left[\sum_{i=1}^{Nc}\left(Pc_i\cdot\tau c_i\right)\right] - \left(\sum_{i=1}^{Nc}Pc_i\right)\cdot\left(\sum_{i=1}^{Nc}\tau c_i\right)\right]\cdot\left[\left(2\cdot Nc\cdot Pc_i\right) - 2\cdot\sum_{i=1}^{Nc}Pc_i\right]}{\left[\left[Nc\cdot\sum_{i=1}^{Nc}\left(Pc_i\right)^2\right] - \left(\sum_{i=1}^{Nc}Pc_i\right)^2\right]^2}$$

$$DbcDP_i := \frac{\left[\left(2\cdot Pc_i\cdot\sum_{i=1}^{Nc}\tau c_i\right) - \left[\left[\sum_{i=1}^{Nc}\left(Pc_i\cdot\tau c_i\right)\right] + \left(\tau c_i\cdot\sum_{i=1}^{Nc}Pc_i\right)\right]\right]\cdot\left[\left[Nc\cdot\sum_{i=1}^{Nc}\left(Pc_i\right)^2\right] - \left(\sum_{i=1}^{Nc}Pc_i\right)^2\right]\ \ldots \\ + -\left[\left[\sum_{i=1}^{Nc}\left(Pc_i\right)^2\right]\cdot\left(\sum_{i=1}^{Nc}\tau c_i\right) - \left(\sum_{i=1}^{Nc}Pc_i\right)\cdot\left[\sum_{i=1}^{Nc}\left(Pc_i\cdot\tau c_i\right)\right]\right]\cdot\left[\left(2\cdot Nc\cdot Pc_i\right) - 2\cdot\sum_{i=1}^{Nc}Pc_i\right]}{\left[\left[Nc\cdot\sum_{i=1}^{Nc}\left(Pc_i\right)^2\right] - \left(\sum_{i=1}^{Nc}Pc_i\right)^2\right]^2}$$

$$Uac := \left[\sum_{i=1}^{Nc}\left[\left(DacD\tau_i\right)^2\cdot U_\tau{}^2 + \left(DacDP_i\right)^2\cdot U_{PB}{}^2 + \left(DacDP_i\right)^2\cdot U_{PP}{}^2\right] \ldots \\ + 2\cdot\left[\sum_{i=1}^{Nc-1}\left[\sum_{j=i+1}^{Nc}\left[\left(DacD\tau_i\right)\cdot\left(DacD\tau_j\right)\cdot U_\tau{}^2 + \left(DacDP_i\right)\cdot\left(DacDP_j\right)\cdot U_{PB}{}^2\right]\right]\right]\right]^{.5}$$

$$Uac = 2.89933\times 10^{-14} \qquad \left|\frac{Uac}{ac}\right| = 0.02375\,\%$$

$$Ubc := \left[\sum_{i=1}^{Nc}\left[\left(DbcD\tau_i\right)^2\cdot U_\tau{}^2 + \left(DbcDP_i\right)^2\cdot U_{PB}{}^2 + \left(DbcDP_i\right)^2\cdot U_{PP}{}^2\right] \ldots \\ + 2\cdot\sum_{i=1}^{Nc-1}\left[\sum_{j=i+1}^{Nc}\left[\left(DbcD\tau_i\right)\cdot\left(DbcD\tau_j\right)\cdot U_\tau{}^2 + \left(DbcDP_i\right)\cdot\left(DbcDP_j\right)\cdot U_{PB}{}^2\right]\right]\right]^{.5}$$

$$Ubc = 3.00001\times 10^{-7} \qquad \left|\frac{Ubc}{bc}\right| = 0.7332\,\%$$

$$ap := \frac{Np \cdot \left[\sum_{i=1}^{Np} \left(Pp_i \cdot \tau p_i\right)\right] - \left(\sum_{i=1}^{Np} Pp_i\right) \cdot \left(\sum_{i=1}^{Np} \tau p_i\right)}{\left[Np \cdot \sum_{i=1}^{Np} \left(Pp_i\right)^2\right] - \left(\sum_{i=1}^{Np} Pp_i\right)^2}$$

$$ap = -2.3119 \times 10^{-10}$$

$$bp := \frac{\left[\sum_{i=1}^{Np} \left(Pp_i\right)^2\right] \cdot \left(\sum_{i=1}^{Np} \tau p_i\right) - \left(\sum_{i=1}^{Np} Pp_i\right) \cdot \left[\sum_{i=1}^{Np} \left(Pp_i \cdot \tau p_i\right)\right]}{\left[Np \cdot \sum_{i=1}^{Np} \left(Pp_i\right)^2\right] - \left(\sum_{i=1}^{Np} Pp_i\right)^2}$$

$$bp = 1.42957 \times 10^{-5}$$

$i := 1, 2 .. Np$

$$DapD\tau_i := \frac{\left(Np \cdot Pp_i\right) - \sum_{i=1}^{Np} Pp_i}{\left[Np \cdot \sum_{i=1}^{Np} \left(Pp_i\right)^2\right] - \left(\sum_{i=1}^{Np} Pp_i\right)^2}$$

$$DbpD\tau_i := \frac{\left[\sum_{i=1}^{Np} \left(Pp_i\right)^2\right] - Pp_i \cdot \left(\sum_{i=1}^{Np} Pp_i\right)}{\left[Np \cdot \sum_{i=1}^{Np} \left(Pp_i\right)^2\right] - \left(\sum_{i=1}^{Np} Pp_i\right)^2}$$

$$DapDP_i := \frac{\left(Np\cdot\tau p_i - \sum_{i=1}^{Np}\tau p_i\right)\cdot\left[\left[Np\cdot\sum_{i=1}^{Np}\left(Pp_i\right)^2\right] - \left(\sum_{i=1}^{Np}Pp_i\right)^2\right] \cdots \\ + -\left[Np\cdot\left[\sum_{i=1}^{Np}\left(Pp_i\cdot\tau p_i\right)\right] - \left(\sum_{i=1}^{Np}Pp_i\right)\cdot\left(\sum_{i=1}^{Np}\tau p_i\right)\right]\cdot\left[\left(2\cdot Np\cdot Pp_i\right) - 2\cdot\sum_{i=1}^{Np}Pp_i\right]}{\left[\left[Np\cdot\sum_{i=1}^{Np}\left(Pp_i\right)^2\right] - \left(\sum_{i=1}^{Np}Pp_i\right)^2\right]^2}$$

$$DbpDP_i := \frac{\left[\left(2\cdot Pp_i\cdot\sum_{i=1}^{Np}\tau p_i\right) - \left[\left[\sum_{i=1}^{Np}\left(Pp_i\cdot\tau p_i\right)\right] + \left(\tau p_i\cdot\sum_{i=1}^{Np}Pp_i\right)\right]\right]\cdot\left[\left[Np\cdot\sum_{i=1}^{Np}\left(Pp_i\right)^2\right] - \left(\sum_{i=1}^{Np}Pp_i\right)^2\right] \cdots \\ + -\left[\left[\sum_{i=1}^{Np}\left(Pp_i\right)^2\right]\cdot\left(\sum_{i=1}^{Np}\tau p_i\right) - \left(\sum_{i=1}^{Np}Pp_i\right)\cdot\left[\sum_{i=1}^{Np}\left(Pp_i\cdot\tau p_i\right)\right]\right]\cdot\left[\left(2\cdot Np\cdot Pp_i\right) - 2\cdot\sum_{i=1}^{Np}Pp_i\right]}{\left[\left[Np\cdot\sum_{i=1}^{Np}\left(Pp_i\right)^2\right] - \left(\sum_{i=1}^{Np}Pp_i\right)^2\right]^2}$$

$$Uap := \left[\left[\sum_{i=1}^{Np}\left[\left(DapD\tau_i\right)^2\cdot U_\tau{}^2 + \left(DapDP_i\right)^2\cdot U_{PB}{}^2 + \left(DapDP_i\right)^2\cdot U_{PP}{}^2\right]\right] \ldots \\ + 2\cdot\left[\sum_{i=1}^{Np-1}\left[\sum_{j=i+1}^{Np}\left[\left(DapD\tau_i\right)\cdot\left(DapD\tau_j\right)\cdot U_\tau{}^2 + \left(DapDP_i\right)\cdot\left(DapDP_j\right)\cdot U_{PB}{}^2\right]\right]\right]\right]^{.5}$$

$$Uap = 5.11023\times 10^{-14} \qquad \left|\frac{Uap}{ap}\right| = 0.0221\,\%$$

$$Ubp := \left[\left[\sum_{i=1}^{Np}\left[\left(DbpD\tau_i\right)^2\cdot U_\tau{}^2 + \left(DbpDP_i\right)^2\cdot U_{PB}{}^2 + \left(DbpDP_i\right)^2\cdot U_{PP}{}^2\right]\right] \ldots \\ + 2\cdot\sum_{i=1}^{Np-1}\left[\sum_{j=i+1}^{Np}\left[\left(DbpD\tau_i\right)\cdot\left(DbpD\tau_j\right)\cdot U_\tau{}^2 + \left(DbpDP_i\right)\cdot\left(DbpDP_j\right)\cdot U_{PB}{}^2\right]\right]\right]^{.5}$$

$$Ubp = 3.00003\times 10^{-7} \qquad \left|\frac{Ubp}{bp}\right| = 2.09856\,\%$$

$ac = -1.22098 \times 10^{-10}$ $\quad Uac = 2.89933 \times 10^{-14}$ $\quad \left|\frac{Uac}{ac}\right| = 0.02375\,\%$

$bc = 4.09168 \times 10^{-5}$ $\quad Ubc = 3.00001 \times 10^{-7}$ $\quad \left|\frac{Ubc}{bc}\right| = 0.7332\,\%$

$ap = -2.3119 \times 10^{-10}$ $\quad Uap = 5.11023 \times 10^{-14}$ $\quad \left|\frac{Uap}{ap}\right| = 0.0221\,\%$

$bp = 1.42957 \times 10^{-5}$ $\quad Ubp = 3.00003 \times 10^{-7}$ $\quad \left|\frac{Ubp}{bp}\right| = 2.09856\,\%$

$Constants_1 := ac$ $\quad Constants_5 := Uac$ $\quad Constants_9 := \left|\frac{Uac}{ac}\right|$

$Constants_2 := bc$ $\quad Constants_6 := Ubc$ $\quad Constants_{10} := \left|\frac{Ubc}{bc}\right|$

$Constants_3 := ap$ $\quad Constants_7 := Uap$ $\quad Constants_{11} := \left|\frac{Uap}{ap}\right|$

$Constants_4 := bp$ $\quad Constants_8 := Ubp$ $\quad Constants_{12} := \left|\frac{Ubp}{bp}\right|$

Constants =

	1
1	$-1.22098 \cdot 10^{-10}$
2	$4.09168 \cdot 10^{-5}$
3	$-2.3119 \cdot 10^{-10}$
4	$1.42957 \cdot 10^{-5}$
5	$2.89933 \cdot 10^{-14}$
6	$3.00001 \cdot 10^{-7}$
7	$5.11023 \cdot 10^{-14}$
8	$3.00003 \cdot 10^{-7}$
9	$2.37459 \cdot 10^{-4}$
10	$7.33198 \cdot 10^{-3}$
11	$2.2104 \cdot 10^{-4}$
12	0.02099

out := WRITEPRN("EDUM_Constants.xls" , Constants)

APPENDIX D

MATLAB UNCERTAINTY CODE

The following section contains the MATLAB code used to determine the burn rate uncertainty, U_r.

```
clear
clc

data1 = xlsread('Test64EDUM.xls');
random = xlsread('EDUMRandomNumbers_E64.xls');

P1=data1(:,1);
tau1=data1(:,2);
E1=data1(:,3);
bc1=data1(3,4);
bp1=data1(5,4);


Epo1=random(:,1);
ac1=random(:,2);
ap1=random(:,3);
XP1=random(:,4);

Nu1=151;
M1=(Nu1-1)/2;
Nb1=length(data1(:,1));
iterations=5000;
istep=50;

dt1=1/1000;
q1=1:1:Nu1;
time1=dt1*q1;


h1=1:istep:Nb1-Nu1+1;
r1=zeros(length(h1),2);
k1=length(h1);

for h1=1:istep:Nb1-Nu1+1;
    rate1=-polyfit(time1',E1(h1:h1+Nu1-1),1);
    r1(h1,:)=rate1;
    Pressure_mean1=mean(P1(h1:h1+Nu1-1));
    Pmean1(h1)=Pressure_mean1;
end

figure
r1=r1(:,1);
loglog(Pmean1,r1,'-');
xlabel('Pressure, psi');
ylabel('Burn Rate, in/s');
grid on
hold

%%%%%%%%%% BEGIN UNCERTAINTY ANALYSIS %%%%%%%%%%

%%%%% Utau=.1 %%%%%
P1=data1(:,1);
tau1=data1(:,2);
Utau=.1;
```

```
Ubc=Utau;
Ubpo=Utau;
bc1=data1(3,4);
bp1=data1(5,4);

bc1=bc1+(.5*Ubc*randn(iterations,1));
bp1=bp1+(.5*Ubpo*randn(iterations,1));
Xtau1=.5*Utau*randn(iterations,1);


for i=1:istep:Nb1-Nu1+1;
    for j=1:1:iterations;
        for k=1:1:Nu1;
            P(k,1)=P1(k+i-1,1)+XP1(j,1);
            tau(k,1)=tau1(k+i-1,1)+Xtau1(j,1);
            Ep(k,1)=(Epo1(j,1)*(tau(k,1)-
(ac1(j,1)*P(k,1)+bc1(j,1))))/(ap1(j,1)*P(k,1)+bp1(j,1));
        end
        r(j,:)=-polyfit(time1',Ep,1);
    end
    Ur(i,:)=2*std(r(:,1));
end

Pmean1=Pmean1';
Data1(:,1)=Pmean1(:,1);
Data1(:,2)=r1(:,1);
Data1(:,3)=Ur(:,1);

%%%% Utau=.125 %%%%
P1=data1(:,1);
tau1=data1(:,2);
Utau=.125;
Ubc=Utau;
Ubpo=Utau;
bc1=data1(3,4);
bp1=data1(5,4);

bc1=bc1+(.5*Ubc*randn(iterations,1));
bp1=bp1+(.5*Ubpo*randn(iterations,1));
Xtau1=.5*Utau*randn(iterations,1);


for i=1:istep:Nb1-Nu1+1;
    for j=1:1:iterations;
        for k=1:1:Nu1;
            P(k,1)=P1(k+i-1,1)+XP1(j,1);
            tau(k,1)=tau1(k+i-1,1)+Xtau1(j,1);
            Ep(k,1)=(Epo1(j,1)*(tau(k,1)-
(ac1(j,1)*P(k,1)+bc1(j,1))))...
                /(ap1(j,1)*P(k,1)+bp1(j,1));
        end
        r(j,:)=-polyfit(time1',Ep,1);
    end
    Ur(i,:)=2*std(r(:,1));
end
```

```
Data1(:,4)=Ur(:,1);

%%%% Utau=.15 %%%%
P1=data1(:,1);
tau1=data1(:,2);
Utau=.15;
Ubc=Utau;
Ubpo=Utau;
bc1=data1(3,4);
bp1=data1(5,4);

bc1=bc1+(.5*Ubc*randn(iterations,1));
bp1=bp1+(.5*Ubpo*randn(iterations,1));
Xtau1=.5*Utau*randn(iterations,1);


for i=1:istep:Nb1-Nu1+1;
    for j=1:1:iterations;
        for k=1:1:Nu1;
            P(k,1)=P1(k+i-1,1)+XP1(j,1);
            tau(k,1)=tau1(k+i-1,1)+Xtau1(j,1);
            Ep(k,1)=(Epo1(j,1)*(tau(k,1)-
(ac1(j,1)*P(k,1)+bc1(j,1))))...
                /(ap1(j,1)*P(k,1)+bp1(j,1));
        end
        r(j,:)=-polyfit(time1',Ep,1);
    end
    Ur(i,:)=2*std(r(:,1));
end

Data1(:,5)=Ur(:,1);

%%%% Utau=.175 %%%%
P1=data1(:,1);
tau1=data1(:,2);
Utau=.175;
Ubc=Utau;
Ubpo=Utau;
bc1=data1(3,4);
bp1=data1(5,4);

bc1=bc1+(.5*Ubc*randn(iterations,1));
bp1=bp1+(.5*Ubpo*randn(iterations,1));
Xtau1=.5*Utau*randn(iterations,1);


for i=1:istep:Nb1-Nu1+1;
    for j=1:1:iterations;
        for k=1:1:Nu1;
            P(k,1)=P1(k+i-1,1)+XP1(j,1);
            tau(k,1)=tau1(k+i-1,1)+Xtau1(j,1);
            Ep(k,1)=(Epo1(j,1)*(tau(k,1)-
(ac1(j,1)*P(k,1)+bc1(j,1))))...
                /(ap1(j,1)*P(k,1)+bp1(j,1));
```

```
        end
        r(j,:)=-polyfit(time1',Ep,1);
    end
    Ur(i,:)=2*std(r(:,1));
end

Data1(:,6)=Ur(:,1);

%%%% Utau=.2 %%%%
P1=data1(:,1);
tau1=data1(:,2);
Utau=.2;
Ubc=Utau;
Ubpo=Utau;
bc1=data1(3,4);
bp1=data1(5,4);

bc1=bc1+(.5*Ubc*randn(iterations,1));
bp1=bp1+(.5*Ubpo*randn(iterations,1));
Xtau1=.5*Utau*randn(iterations,1);


for i=1:istep:Nb1-Nu1+1;
    for j=1:1:iterations;
        for k=1:1:Nu1;
            P(k,1)=P1(k+i-1,1)+XP1(j,1);
            tau(k,1)=tau1(k+i-1,1)+Xtau1(j,1);
            Ep(k,1)=(Epo1(j,1)*(tau(k,1)-
(ac1(j,1)*P(k,1)+bc1(j,1))))...
                /(ap1(j,1)*P(k,1)+bp1(j,1));
        end
        r(j,:)=-polyfit(time1',Ep,1);
    end
    Ur(i,:)=2*std(r(:,1));
end

Data1(:,7)=Ur(:,1);

%%%% Utau=.225 %%%%
P1=data1(:,1);
tau1=data1(:,2);
Utau=.225;
Ubc=Utau;
Ubpo=Utau;
bc1=data1(3,4);
bp1=data1(5,4);

bc1=bc1+(.5*Ubc*randn(iterations,1));
bp1=bp1+(.5*Ubpo*randn(iterations,1));
Xtau1=.5*Utau*randn(iterations,1);


for i=1:istep:Nb1-Nu1+1;
    for j=1:1:iterations;
        for k=1:1:Nu1;
```

```
            P(k,1)=P1(k+i-1,1)+XP1(j,1);
            tau(k,1)=tau1(k+i-1,1)+Xtau1(j,1);
            Ep(k,1)=(Epo1(j,1)*(tau(k,1)-
(ac1(j,1)*P(k,1)+bc1(j,1))))...
                /(ap1(j,1)*P(k,1)+bp1(j,1));
        end
        r(j,:)=-polyfit(time1',Ep,1);
    end
    Ur(i,:)=2*std(r(:,1));
end

Data1(:,8)=Ur(:,1);

%%%% Utau=.25 %%%%
P1=data1(:,1);
tau1=data1(:,2);
Utau=.25;
Ubc=Utau;
Ubpo=Utau;
bc1=data1(3,4);
bp1=data1(5,4);

bc1=bc1+(.5*Ubc*randn(iterations,1));
bp1=bp1+(.5*Ubpo*randn(iterations,1));
Xtau1=.5*Utau*randn(iterations,1);


for i=1:istep:Nb1-Nu1+1;
    for j=1:1:iterations;
        for k=1:1:Nu1;
            P(k,1)=P1(k+i-1,1)+XP1(j,1);
            tau(k,1)=tau1(k+i-1,1)+Xtau1(j,1);
            Ep(k,1)=(Epo1(j,1)*(tau(k,1)-
(ac1(j,1)*P(k,1)+bc1(j,1))))...
                /(ap1(j,1)*P(k,1)+bp1(j,1));
        end
        r(j,:)=-polyfit(time1',Ep,1);
    end
    Ur(i,:)=2*std(r(:,1));
end

Data1(:,9)=Ur(:,1);

%%%% Utau=..275 %%%%
P1=data1(:,1);
tau1=data1(:,2);
Utau=.275;
Ubc=Utau;
Ubpo=Utau;
bc1=data1(3,4);
bp1=data1(5,4);

bc1=bc1+(.5*Ubc*randn(iterations,1));
bp1=bp1+(.5*Ubpo*randn(iterations,1));
Xtau1=.5*Utau*randn(iterations,1);
```

```
for i=1:istep:Nb1-Nu1+1;
    for j=1:1:iterations;
        for k=1:1:Nu1;
            P(k,1)=P1(k+i-1,1)+XP1(j,1);
            tau(k,1)=tau1(k+i-1,1)+Xtau1(j,1);
            Ep(k,1)=(Epo1(j,1)*(tau(k,1)-
(ac1(j,1)*P(k,1)+bc1(j,1))))...
                /(ap1(j,1)*P(k,1)+bp1(j,1));
        end
        r(j,:)=-polyfit(time1',Ep,1);
    end
    Ur(i,:)=2*std(r(:,1));
end

Data1(:,10)=Ur(:,1);

%%%% Utau=.3 %%%%
P1=data1(:,1);
tau1=data1(:,2);
Utau=.3;
Ubc=Utau;
Ubpo=Utau;
bc1=data1(3,4);
bp1=data1(5,4);

bc1=bc1+(.5*Ubc*randn(iterations,1));
bp1=bp1+(.5*Ubpo*randn(iterations,1));
Xtau1=.5*Utau*randn(iterations,1);


for i=1:istep:Nb1-Nu1+1;
    for j=1:1:iterations;
        for k=1:1:Nu1;
            P(k,1)=P1(k+i-1,1)+XP1(j,1);
            tau(k,1)=tau1(k+i-1,1)+Xtau1(j,1);
            Ep(k,1)=(Epo1(j,1)*(tau(k,1)-
(ac1(j,1)*P(k,1)+bc1(j,1))))...
                /(ap1(j,1)*P(k,1)+bp1(j,1));
        end
        r(j,:)=-polyfit(time1',Ep,1);
    end
    Ur(i,:)=2*std(r(:,1));
end

Data1(:,11)=Ur(:,1);

%%%% Utau=.325 %%%%
P1=data1(:,1);
tau1=data1(:,2);
Utau=.325;
Ubc=Utau;
Ubpo=Utau;
bc1=data1(3,4);
```

```
bp1=data1(5,4);

bc1=bc1+(.5*Ubc*randn(iterations,1));
bp1=bp1+(.5*Ubpo*randn(iterations,1));
Xtau1=.5*Utau*randn(iterations,1);


for i=1:istep:Nb1-Nu1+1;
    for j=1:1:iterations;
        for k=1:1:Nu1;
            P(k,1)=P1(k+i-1,1)+XP1(j,1);
            tau(k,1)=tau1(k+i-1,1)+Xtau1(j,1);
            Ep(k,1)=(Epo1(j,1)*(tau(k,1)-
(ac1(j,1)*P(k,1)+bc1(j,1))))...
                /(ap1(j,1)*P(k,1)+bp1(j,1));
        end
        r(j,:)=-polyfit(time1',Ep,1);
    end
    Ur(i,:)=2*std(r(:,1));
end

Data1(:,12)=Ur(:,1);

%%%% Utau=.35 %%%%
P1=data1(:,1);
tau1=data1(:,2);
Utau=.35;
Ubc=Utau;
Ubpo=Utau;
bc1=data1(3,4);
bp1=data1(5,4);

bc1=bc1+(.5*Ubc*randn(iterations,1));
bp1=bp1+(.5*Ubpo*randn(iterations,1));
Xtau1=.5*Utau*randn(iterations,1);


for i=1:istep:Nb1-Nu1+1;
    for j=1:1:iterations;
        for k=1:1:Nu1;
            P(k,1)=P1(k+i-1,1)+XP1(j,1);
            tau(k,1)=tau1(k+i-1,1)+Xtau1(j,1);
            Ep(k,1)=(Epo1(j,1)*(tau(k,1)-
(ac1(j,1)*P(k,1)+bc1(j,1))))...
                /(ap1(j,1)*P(k,1)+bp1(j,1));
        end
        r(j,:)=-polyfit(time1',Ep,1);
    end
    Ur(i,:)=2*std(r(:,1));
end

Data1(:,13)=Ur(:,1);

%%%% Utau=.375 %%%%
P1=data1(:,1);
```

```
tau1=data1(:,2);
Utau=.375;
Ubc=Utau;
Ubpo=Utau;
bc1=data1(3,4);
bp1=data1(5,4);

bc1=bc1+(.5*Ubc*randn(iterations,1));
bp1=bp1+(.5*Ubpo*randn(iterations,1));
Xtau1=.5*Utau*randn(iterations,1);


for i=1:istep:Nb1-Nu1+1;
    for j=1:1:iterations;
        for k=1:1:Nu1;
            P(k,1)=P1(k+i-1,1)+XP1(j,1);
            tau(k,1)=tau1(k+i-1,1)+Xtau1(j,1);
            Ep(k,1)=(Epo1(j,1)*(tau(k,1)-
(ac1(j,1)*P(k,1)+bc1(j,1))))...
                /(ap1(j,1)*P(k,1)+bp1(j,1));
        end
        r(j,:)=-polyfit(time1',Ep,1);
    end
    Ur(i,:)=2*std(r(:,1));
end

Data1(:,14)=Ur(:,1);

%%%% Utau=.4 %%%%
P1=data1(:,1);
tau1=data1(:,2);
Utau=.4;
Ubc=Utau;
Ubpo=Utau;
bc1=data1(3,4);
bp1=data1(5,4);

bc1=bc1+(.5*Ubc*randn(iterations,1));
bp1=bp1+(.5*Ubpo*randn(iterations,1));
Xtau1=.5*Utau*randn(iterations,1);


for i=1:istep:Nb1-Nu1+1;
    for j=1:1:iterations;
        for k=1:1:Nu1;
            P(k,1)=P1(k+i-1,1)+XP1(j,1);
            tau(k,1)=tau1(k+i-1,1)+Xtau1(j,1);
            Ep(k,1)=(Epo1(j,1)*(tau(k,1)-
(ac1(j,1)*P(k,1)+bc1(j,1))))...
                /(ap1(j,1)*P(k,1)+bp1(j,1));
        end
        r(j,:)=-polyfit(time1',Ep,1);
    end
    Ur(i,:)=2*std(r(:,1));
end
```

```
Data1(:,15)=Ur(:,1);

%%%% Utau=.425 %%%%
P1=data1(:,1);
tau1=data1(:,2);
Utau=.425;
Ubc=Utau;
Ubpo=Utau;
bc1=data1(3,4);
bp1=data1(5,4);

bc1=bc1+(.5*Ubc*randn(iterations,1));
bp1=bp1+(.5*Ubpo*randn(iterations,1));
Xtau1=.5*Utau*randn(iterations,1);


for i=1:istep:Nb1-Nu1+1;
    for j=1:1:iterations;
        for k=1:1:Nu1;
            P(k,1)=P1(k+i-1,1)+XP1(j,1);
            tau(k,1)=tau1(k+i-1,1)+Xtau1(j,1);
            Ep(k,1)=(Epo1(j,1)*(tau(k,1)-
(ac1(j,1)*P(k,1)+bc1(j,1))))...
                /(ap1(j,1)*P(k,1)+bp1(j,1));
        end
        r(j,:)=-polyfit(time1',Ep,1);
    end
    Ur(i,:)=2*std(r(:,1));
end

Data1(:,16)=Ur(:,1);

%%%% Utau=.45 %%%%
P1=data1(:,1);
tau1=data1(:,2);
Utau=.45;
Ubc=Utau;
Ubpo=Utau;
bc1=data1(3,4);
bp1=data1(5,4);

bc1=bc1+(.5*Ubc*randn(iterations,1));
bp1=bp1+(.5*Ubpo*randn(iterations,1));
Xtau1=.5*Utau*randn(iterations,1);


for i=1:istep:Nb1-Nu1+1;
    for j=1:1:iterations;
        for k=1:1:Nu1;
            P(k,1)=P1(k+i-1,1)+XP1(j,1);
            tau(k,1)=tau1(k+i-1,1)+Xtau1(j,1);
            Ep(k,1)=(Epo1(j,1)*(tau(k,1)-
(ac1(j,1)*P(k,1)+bc1(j,1))))...
                /(ap1(j,1)*P(k,1)+bp1(j,1));
```

```
        end
        r(j,:)=-polyfit(time1',Ep,1);
    end
    Ur(i,:)=2*std(r(:,1));
end

Data1(:,17)=Ur(:,1);

%%%% Utau=.475 %%%%
P1=data1(:,1);
tau1=data1(:,2);
Utau=.475;
Ubc=Utau;
Ubpo=Utau;
bc1=data1(3,4);
bp1=data1(5,4);

bc1=bc1+(.5*Ubc*randn(iterations,1));
bp1=bp1+(.5*Ubpo*randn(iterations,1));
Xtau1=.5*Utau*randn(iterations,1);


for i=1:istep:Nb1-Nu1+1;
    for j=1:1:iterations;
        for k=1:1:Nu1;
            P(k,1)=P1(k+i-1,1)+XP1(j,1);
            tau(k,1)=tau1(k+i-1,1)+Xtau1(j,1);
            Ep(k,1)=(Epo1(j,1)*(tau(k,1)-
(ac1(j,1)*P(k,1)+bc1(j,1))))...
                /(ap1(j,1)*P(k,1)+bp1(j,1));
        end
        r(j,:)=-polyfit(time1',Ep,1);
    end
    Ur(i,:)=2*std(r(:,1));
end

Data1(:,18)=Ur(:,1);

%%%% Utau=.5 %%%%
P1=data1(:,1);
tau1=data1(:,2);
Utau=.5;
Ubc=Utau;
Ubpo=Utau;
bc1=data1(3,4);
bp1=data1(5,4);

bc1=bc1+(.5*Ubc*randn(iterations,1));
bp1=bp1+(.5*Ubpo*randn(iterations,1));
Xtau1=.5*Utau*randn(iterations,1);


for i=1:istep:Nb1-Nu1+1;
    for j=1:1:iterations;
        for k=1:1:Nu1;
```

```
            P(k,1)=P1(k+i-1,1)+XP1(j,1);
            tau(k,1)=tau1(k+i-1,1)+Xtau1(j,1);
            Ep(k,1)=(Epo1(j,1)*(tau(k,1)-
(ac1(j,1)*P(k,1)+bc1(j,1))))...
                /(ap1(j,1)*P(k,1)+bp1(j,1));
        end
        r(j,:)=-polyfit(time1',Ep,1);
    end
    Ur(i,:)=2*std(r(:,1));
end

Data1(:,19)=Ur(:,1);


kk=1;
for i=1:1:length(Data1);
    if Data1(i,1)~=0;
        DATA1(kk,:)=Data1(i,:);
        kk=kk+1;
    end
end

kk=1;
for i=1:1:length(Ur);
    if Ur(i,1)~=0;
        UR(kk,1)=Ur(i,1);
        kk=kk+1;
    end
end

figure
plot(DATA1(:,1),DATA1(:,3))
xlabel('Pressure, psi');
ylabel('Burn Rate Uncertainty, in/s');
hold


xlswrite('EDUM_Uncertainty_Test64_Rev03', DATA1)
```

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