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Joshua Bonner

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DETECTION OF PERSONNEL AND SMALL ARMS FIRE USING PULSE RADAR

by

JOSHUA BONNER

A THESIS

Submitted in partial fulfillment of the requirements for the degree of Master of Science in The Department of Physics to The School of Graduate Studies of The University of Alabama in Huntsville

HUNTSVILLE, ALABAMA

2013
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THESIS APPROVAL FORM

Submitted by Joshua Bonner in partial fulfillment of the requirements for the degree of Master of Science in Physics and accepted on behalf of the Faculty of the School of Graduate Studies by the thesis committee.

We, the undersigned members of the Graduate Faculty of The University of Alabama in Huntsville, certify that we have advised and/or supervised the candidate on the work described in this thesis. We further certify that we have reviewed the thesis manuscript and approve it in partial fulfillment of the requirements for the degree of Master of Science in Physics.

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ABSTRACT

The School of Graduate Studies

The University of Alabama in Huntsville

Degree: Master of Science College/Dept. Department of Physics

Name of Candidate: Joshua Bonner

Title: DETECTION OF PERSONNEL AND SMALL ARMS FIRE USING PULSE RADAR

The scope is to fill in a gap of detecting targets using low power, small size radars. The main objective of this thesis is to show, through simulation, that a small, low power radar can be theoretically designed using standard radar principles to accomplish said task.

The primary method used in this thesis is to ascertain the conclusion with two separate Matlab Simulations. First, the design of a radar to detect human sized targets moving along the ground and secondly, detection of small arms fire aimed towards helicopters were used as the primary examples.

The results show that a human sized target can be detected up to a maximum of 4 km away using a radar with a power of 150 watts and a bullet up to several hundred meters using 1000 watts of power.
The simulations show that such a radar is theoretical and mathematically plausible within the constraints.
ACKNOWLEDGMENTS

The work described in this thesis would not have been possible without the assistance of a number of people who deserve special mention. First, I would like to thank Dr. Jacob Heerikhuisen for his steadfast guidance, assistance and patience. Second, the other members of my committee, Dr. Lingze Duan and Dr. Larry Carey, have both been incredibly helpful with comments and suggestions.

Those who paved the way for my research have my utmost respect and without their diligence and expertise, my thesis would not have been possible. There are far too many to name but all of them are remarkable and highly intelligent individuals.

I would also like to thank my family who encouraged me from the beginning to never settle for the status quo in life and to always work towards a goal. I would like to make a special thank you to my father who was always behind me in whatever endeavor I pursued and who never wavered in his support nor in his guidance.
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I. QUALITATIVE AND QUANTITATIVE DISCUSSION OF KEY RADAR CONCEPTS

1. History

As early as 1886, Heinrich Hertz showed that radio waves could be reflected from solid objects and then in 1895 Alexander Popov, a physics instructor at the Imperial Russian Navy school in Kronstadt, developed an apparatus using a coherer tube for detecting distant lightning strikes. The next year, he added a spark-gap transmitter. In 1897, while testing this in communications between two ships in the Baltic Sea, he took note of an interference beat caused by the passage of a third vessel. In his report, Popov wrote that this phenomenon might be used for detecting objects, but he did nothing more with this observation.

Next came Christian Huelsmeyer, who was the first to use radio waves to detect the presence of distant metallic objects. In 1904 he demonstrated the feasibility of detecting a ship in dense fog but not its distance. He would eventually get a patent on September 23, 1904 for the first full radar application, which he called telemobiloscope.

In 1922, Hoyt Taylor and Leo Young, while working as researchers with the U.S. Navy, discovered that when radio waves were broadcast at 60 MHz it was possible to determine the range and bearing of nearby ships in the Potomac River. Despite Taylor's suggestion that this method could be used in low visibility, the Navy did not immediately
continue the work. Serious investigation began eight years later after the discovery that radar could be used to track airplanes.

In 1934, the Frenchman Émile Girardeau stated he was building an obstacle-locating radio apparatus conceived according to the principles stated by Tesla and a part of which was installed on the Normandie liner in 1936. During the same year, the Soviet military engineer P.K. Oschepkov, in collaboration with Leningrad Electro-physical Institute, produced an experimental apparatus capable of detecting an aircraft within 3 km of a receiver. The French and Soviet systems, however, used continuous-wave operation and could not give the full performance that was ultimately at the center of modern radar.

Before the Second World War, researchers in France, Germany, Italy, Japan, the Netherlands, the Soviet Union, the United Kingdom, and the United States, developed technologies that led to the modern version of radar. Australia, Canada, New Zealand, and South Africa followed prewar Great Britain, and Hungary had similar developments during the war.

Full radar evolved as a pulsed system, and the first such elementary apparatus was demonstrated in December 1934 by American Robert M. Page while working at the Naval Research Laboratory. The following year, the United States Army successfully tested a primitive surface to surface radar to aim coastal battery search lights at night. This was followed by a pulsed system demonstrated in May 1935 by Rudolf Kühnhold and the firm GEMA in Germany and then one in June 1935 by an Air Ministry team led by Robert A. Watson Watt in Great Britain. Later, in 1943, Page greatly improved radar with the monopulse technique that was used for many years in most radar applications.
The British were the first to fully exploit radar as a defense against aircraft attack. The Air Ministry asked British scientists in 1934 to investigate the possibility of propagating electromagnetic energy and the likely effect. Following a study, they concluded that detection of aircraft appeared feasible. Robert Watt's team demonstrated to his superiors the capabilities of a working prototype and then patented the device. It served as the basis for the Chain Home network of radars to defend Great Britain. The war precipitated research to find better resolution, more portability, and more features for radar.

![Three Chain Home Transmit Towers, near Dover](Cite Ref 15)

**Figure 1.1 Sept 2006 Photograph of Three Chain Home Transmit Towers, near Dover**

World War II showed that radar was viable and would be a staple of technology used in the future. In the decades that followed, radars were refined to the point where they could detect, track and in some cases, image, weather systems, vehicles on
roadways, small projectiles such as mortar shells and even smaller asteroids as they pass the Earth
2. Background

2.1 Radar Basics

The word RADAR is an abbreviation for RAdio Detection And Ranging. In general, radar systems use modulated waveforms and directive antennas to transmit electromagnetic energy into a specific volume in space to search for targets. Targets within a search volume of space will reflect portions of this Electro-Magnetic energy in the form of radar returns or echoes, back to the radar. These returns are then processed by the radar receiver to extract target information such as range, velocity, angular position, and other target identifying characteristics.

Radars can be classified as ground based, airborne, space based, or ship based radar systems. They can also be classified into numerous sub-categories based on the specific radar characteristics, such as the frequency band, antenna type, and waveforms utilized. Another classification is concerned with the mission and/or the functionality of the radar. This includes: weather, acquisition and search, tracking, track-while-scan, fire control, early warning, over the horizon, terrain following, and terrain avoidance radars.
In Figure 2.1, I we see 4 of the radars used by the Ballistic Missile Defense System. Upper left-Aegis SPY-1, upper right-AN-TPY 2, lower left-SBIRS, lower right-Sea Based X-Band

Radar waves scatter in a variety of ways depending on the size (wavelength) of the radio wave and the shape of the target. If the wavelength is much shorter than the target's size, the wave will bounce off in a way similar to the way light is reflected by a mirror. If the wavelength is much longer than the size of the target, the target may not be visible because of poor reflection. Low Frequency radar technology is dependent on resonances for detection, but not identification, of targets. This is described by Rayleigh scattering, an effect that creates the Earth's blue sky and red sunsets. When the two length
scales are comparable, there may be resonances. Early radars used very long wavelengths that were larger than the targets and received a vague signal, whereas some modern systems use shorter wavelengths (a few centimeters or shorter) that can image objects as small as a loaf of bread.

Short radio waves reflect from curves and corners, in a way similar to glint from a rounded piece of glass. The most reflective targets for short wavelengths have 90° angles between the reflective surfaces. A structure consisting of three flat surfaces meeting at a single corner, like the corner on a box, will reflect waves entering its opening directly back at the source. These so-called corner reflectors are commonly used as radar reflectors to make otherwise difficult-to-detect objects easier to detect and are often found on boats in order to improve their detection in a rescue situation and to reduce collisions. For similar reasons, objects attempting to avoid detection will angle their surfaces in a way to eliminate inside corners and avoid surfaces and edges perpendicular to likely detection directions, which leads to stealth aircraft. These precautions do not completely eliminate reflection because of diffraction, especially at longer wavelengths. Half wavelength long wires or strips of conducting material, such as chaff, are very reflective but do not direct the scattered energy back toward the source. The extent to which an object reflects or scatters radio waves is called its radar cross section.
Most objects that man has built to fly have a rounded shape. This shape creates a very efficient radar reflector which means that no matter where the radar signal hits the plane, some of the signal gets reflected back to the radar. These objects include among other things, airplanes and missiles. However, as is often the case, there are exceptions to this rule. (Cite Ref. 13)
A stealth aircraft are made of completely flat surfaces and very sharp edges. When a radar signal hits a stealth plane, the signal reflects away from the radar at an angle, thus avoiding detection. Surfaces on a stealth aircraft also can absorb radar energy as well. (Cite Ref. 13)
In Figure 2.4, we easily see the incident wave being reflected back towards the radar by the smooth surfaces of the object. However, Figure 2.4 explains the origin of these reflected waves as being “reflected” by currents in the object that were setup by the incident wave.

Electronic steering is achieved by controlling the phase of the electric current feeding the array elements, and thus the name phased array is adopted. Phased array radars utilize phased array antennas, and are often called multifunction radars. A phased array is a composite antenna formed from two or more basic radiators. Array antennas synthesize narrow directive beams that may be steered mechanically or electronically.
Radars are most often classified by the types of waveforms they use, or by their operating frequency. Considering the waveforms first, radars can be Continuous Wave or Pulsed Radars.

Continuous Wave radars are those that continuously emit electromagnetic energy, and use separate transmit and receive antennas. Un-modulated Continuous Wave radars can accurately measure target radial velocity (i.e. Doppler shift) and angular position. However, target range information cannot be extracted without utilizing some form of modulation. The primary use of un-modulated Continuous Wave radars is in target velocity search and track, and in missile guidance. Because of this, I will focus on Continuous Wave radars and only briefly mention Pulse radar.

Pulsed radars use a train of modulated, pulsed waveforms. In this category, radar systems can be classified on the basis of the Pulse Repetition Frequency (PRF) as low PRF, medium PRF, and high PRF radars. Low PRF radars are primarily used for ranging where Doppler shift is not of interest. High PRF radars are mainly used to measure target velocity. Continuous wave as well as pulsed radars can measure both target range and radial velocity by utilizing different modulation schemes. Figure 2.5 below has the radar classifications based on the operating frequency. High Frequency (HF) radars utilize the electromagnetic waves' reflection off the ionosphere to detect targets beyond the horizon. Very High Frequency (VHF) and Ultra High Frequency (UHF) bands are used for very long range Early Warning Radars (EWR) such as for Ballistic Missile Defense. Because of the very large wavelength and the sensitivity requirements for very long range measurements, large apertures are needed in such radar systems such as the Sea Based X-Band pictured in the lower right hand corner of Figure 2.1 or the Cobra Dane radar in
Alaska. Radars in the L-band are primarily ground based and ship based systems that are used in long range military and air traffic control search operations. Most ground and ship based medium range radars operate in the S-band while most weather detection radar systems are C-band radars. Medium range search and fire control military radars and metric instrumentation radars are also C-band. The X-band is used for radar systems where the size of the antenna constitutes a physical limitation; this includes most military radars such as the Aegis SPY-1 and the Army AN-TPY3. The higher frequency bands (Ku, K, and Ka) suffer severe weather and atmospheric attenuation. Therefore, radars utilizing these frequency bands are limited to short range applications, such as police traffic radar, short range terrain avoidance, and terrain following radar. Milli-Meter Wave (MMW) radars are mainly limited to very short range Radio Frequency (RF) seekers and experimental radar systems.

<table>
<thead>
<tr>
<th>Band</th>
<th>Frequency (GHz)</th>
<th>Wavelength (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>0.255 - 0.390</td>
<td>133 - 76.9</td>
</tr>
<tr>
<td>L</td>
<td>0.390 - 1.550</td>
<td>76.9 - 19.3</td>
</tr>
<tr>
<td>S</td>
<td>1.550 - 4.20</td>
<td>19.3 - 7.1</td>
</tr>
<tr>
<td>C</td>
<td>4.20 - 5.75</td>
<td>7.1 - 5.2</td>
</tr>
<tr>
<td>X</td>
<td>5.75 - 10.90</td>
<td>5.2 - 2.7</td>
</tr>
<tr>
<td>K</td>
<td>10.90 - 36.0</td>
<td>2.7 - 0.83</td>
</tr>
<tr>
<td>Ku</td>
<td>10.90 - 22.0</td>
<td>2.7 - 1.36</td>
</tr>
<tr>
<td>Ka</td>
<td>22.0 - 36.0</td>
<td>1.36 - 0.83</td>
</tr>
<tr>
<td>Q</td>
<td>36.0 - 46.0</td>
<td>0.83 - 0.65</td>
</tr>
<tr>
<td>V</td>
<td>46.0 - 56.0</td>
<td>0.65 - 0.53</td>
</tr>
<tr>
<td>W</td>
<td>56.0 - 100.0</td>
<td>0.53 - 0.30</td>
</tr>
</tbody>
</table>

Figure 2.5 Radar Classifications
Figure 2.5 is a tabular representation showing the different bands of radio that are used by radar devices. As with all other EM radiation, frequency is the inverse of wavelength. As one increases, the other decreases. The ranges of each band of radar can be calculated using equations presented later in this paper.

Radars use Doppler frequency to extract target radial velocity as well as to distinguish between moving and stationary targets or objects such as debris cluster. The Doppler phenomenon describes the shift in the center frequency of an incident waveform due to the target motion with respect to the source of radiation. Depending on the direction of the target's motion, this frequency shift may be positive or negative. A waveform incident on a target has equiphasic wave fronts separated by a wavelength $\lambda$.

![Diagram](image)

**Figure 2.6 Effect of target motion on the reflected equiphasic waveforms.**

(Cite Ref 1)

In Figure 2.6, notice that the target aircraft is moving in two opposite directions causing the radar returns to either decrease or increase in wavelength determined on
whether or not the target is moving towards or away from the radar. It is this Doppler Shift that allows radars to classify targets as incoming or outgoing.

\[ \text{(Doppler Shift Resolution)} \]

\[ \text{Figure 2.7 Spectra of received signal showing Doppler shift.} \]

(Cite Ref 1)

In Figure 2.7, one can see the difference in frequency of a closing or receding target based on the frequency shift. The frequency shift is given by Equation 6.6.

\[ \text{Figure 2.8 Target 1 generates zero Doppler. Target 2 generates maximum Doppler.} \]

Target 3 is in between.

(Cite Ref 1)
The frequency shift from the previous figure can be used to determine what angle the target is travelling in relation to the fixed location of the radar. It is such trigonometry that BMD radars use to get an early, rough estimate of trajectory.

The primary task of BMD radar systems have to accomplish is to continuously scan a specified volume in space searching for Ballistic Missile launches. Once detection is established, target information such as range, angular position and possibly target velocity are extracted by the radar signal and data processors. Depending on the radar design and antenna, different search patterns can be adopted. In the 2-D fan case, the beam width is wide enough in elevation to cover the desired search volume along that coordinate; however, it has to be steered in azimuth. In a stacked beam search pattern, the beam has to be steered in azimuth and elevation. This latter kind of search pattern is normally employed by phased array radars. Search volumes are normally specified by a search solid angle in steradians.

2.2 Noise

As we will see from the radar equation later, the receiver Signal to Noise Ratio (SNR) is inversely proportional to the radar losses. Hence, any increase in radar losses causes a drop in the SNR, thus decreasing the probability of detection, as it is a function of the SNR. Often, the principal difference between a good radar design and a poor radar design is the radar losses. Radar losses include ohmic (resistance) losses and statistical losses.
Ohmic losses are defined as “The voltage drop across the antennae during passage of current due to the internal resistance of the antennae” while statistical losses are losses not associated with resistance within the radar.

\[
\text{SNR} = \frac{\text{Received Signal Energy}}{\text{Noise Energy}}
\]

**Figure 2.9 Signal to Noise**

(Cite Ref 15)

In Figure 2.9, a graphic representation of a radar return is pictured in blue when compared to the noise in red. The return is easily identifiable in this figure but not so much in the next figure.
In Figure 2.10, we see a few more returns. From left to right, we first see a false return which is indicative of anything from a flock of birds to an unexpected surge in voltage within the radar itself. Next, we see the tell-tale return of a target followed by what is called a missed target. It is labeled such because the return is being drowned out by the background noise and it is below the detection threshold.

Since I just mentioned noise, let’s briefly discuss it. Signal noise is an internal source of random variations in the signal, which is generated by all electronic components. Noise typically appears as random variations superimposed on the desired echo signal received in the radar receiver. The lower the return of the desired signal, the more difficult it is to discern it from the background noise. The Signal to Noise Ratio is a measure of the noise received by a receiver compared to the intensity of the reflected
radar wave from the target. In nearly all cases, this ratio needs to be minimized as early detection gives more time to engage a target.

Noise is also generated by external sources, most importantly the natural thermal radiation of the environment surrounding the target of interest. In modern radar systems, the internal noise is typically about equal to or lower than the external noise. An exception is if the radar is aimed upwards at clear sky, where the scene is so "cold" that it generates very little thermal noise. There is an appealing intuitive interpretation of this relationship in a radar. Matched filtering allows the radar to compress the entire energy received from a target into a single bin. Once on the surface, it would appear then that within a fixed interval of time one could obtain perfect, error free, detection. To do this one simply compresses all energy into an infinitesimal time slice. What limits this approach in the real world is that, while time is arbitrarily divisible, current is not. The quanta of electrical energy is an electron, and so the best one can do is match filter all energy into a single electron. Since the electron is moving at a certain temperature (Plank Spectrum), this noise source cannot be further eroded.

Transmit and receive losses occur between the radar transmitter and antenna input port, and between the antenna output port and the receiver front end, respectively. Such losses are often called plumbing losses. Typically, plumbing losses are on the order of 1 to 2 dB.

As the radar scans across a target, the antenna gain in the direction of the target is less than maximum, as defined by the antenna's radiation pattern. The loss in SNR due to not having maximum antenna gain on the target at all times is called the antenna pattern loss. Once an antenna has been selected for a given radar, the amount of antenna pattern
loss can be mathematically computed. (For additional information on this, Cite Ref 1 Section 1.8 Radar Losses) If the antenna scanning rate is so fast that the gain on receive is not the same as on transmit, additional scan loss has to be calculated and added to the beam shape loss. Scan loss can be computed in a similar fashion to beam shape loss. Phased array radars are often prime candidates for both beam shape and scan losses.

When the number of integrated returned noise pulses is larger than the target returned pulses, a drop in the SNR occurs. This is called collapsing loss. When target returns are displayed in one coordinate, such as range, noise sources from azimuth cells adjacent to the actual target return converge in the target vicinity and cause a drop in the SNR. This is illustrated in the following figure.

![Figure 2.11 Illustration of collapsing loss.](Cite Ref 1)

In Figure 2.11, noise sources in cells 1, 2, 4, and 5 converge to increase the noise level in cell 3 thus making it harder to detect the target in cell 4.
Figure 2.12 Radar Block Diagram

(Cite Ref 15)
3. Derivation of the Radar Range Equation

Assume that the electromagnetic waves propagate under ideal conditions, without dispersion, interference, etc. and if high-frequency energy is emitted by an isotropic source, then the energy will propagate uniformly in all directions. Areas with the same power density therefore form spheres around the radiator with the same amount of energy spreading out on a spherical surface at some radius. That means the power density on the surface of a sphere is inversely proportional to the square of the radius of the sphere. Hence, the inverse square law is introduced. Common applications of the inverse square law include: gravity, sound, radiation, light and electric fields.

![Figure 3.1 Inverse Square Law](image)

(Cite Ref. 29)
\[ I = \frac{S \cdot W}{4\pi r^2 \cdot m^2} \quad \text{Equation 3.1} \]

\( I \) is the non-directional power density and \( S \) is the source strength.

Now say, we don’t care about what’s going on in every direction, we are only interested in one specific area. If the power radiated is focused in one direction, then this results in an increase of the power density in the direction of the radiation. This effect is called antenna gain which is talked about more extensively in later pages. This gain is obtained by directional radiation of the power. Thus, the directional power density is given by,

\[ I_{\text{directional}} = I \times G \quad \text{Equation 3.2} \]

Where \( I_{\text{directional}} \) is the directional power density and \( G \) is the antennae gain.

Some antennas are highly directional; that is, more energy is propagated in certain directions than in others. The ratio between the amounts of energy propagated in these directions compared to the energy that would be propagated if the antenna were not directional is known as its gain.

However much we would like them to be, radar antennas aren’t “partially radiating” isotropic radiators. Radar antennas must have a small beam width and an antenna gain with common examples being parabolic dish antennas and phased array antennas.

Thus far, we have only discussed electromagnetic radiation travelling outwards. However, in order to detect anything, we must also discuss the radiation returning from...
the object we wish to detect. Since target detection isn't only dependent on the power density at the target position but also on how much power is reflected in the direction of the radar. In order to determine the useful reflected power, it is necessary to know the radar cross section. This quantity depends on several factors but we can say that a bigger area reflects more power than a smaller area. Small planes obviously have larger radar cross sections than birds but this cross section is also highly dependent on design, materials, weather, etc.

With that in mind we can say: The reflected power, $P_r$, at the radar depends on the power density $S_u$, the antenna gain $G$, and the variable radar cross section $\sigma$.

$$P_{\text{reflected}} = \frac{SG\sigma}{4\pi r^2} \text{ in Watts} \tag{3.3}$$

The power received from the target by the radar is dependent on the returning power density and the surface area of the radar that is capable of transmitting and/or receiving which is called the effective antenna aperture.

$$P_{\text{received}} = S_{\text{received}} \times A_e \text{ in Watts} \tag{3.4}$$

Where $P_{\text{received}}$ is the power at the receiver, $S_{\text{received}}$ is the power density received and $A_e$ is the effective antenna aperture.

The effective antenna aperture arises from the fact that all antennas suffer from a myriad of losses and therefore, the received power at the antenna is not equal to the input
power. As a rule, the efficiency of the antenna is around 0.6 to 0.7 (Efficiency coefficient $K_a$).

Applied to the geometric antenna area, the effective antenna aperture is:

$$A_e = \text{Radar Dish Area} \times \text{Efficiency} \quad \text{Equation 3.5}$$

The last piece of the puzzle is the equation for antenna gain in terms of the radiation wavelength and antenna efficiency.

$$G = \frac{4\pi A_e}{\lambda^2} \quad \text{Equation 3.5}$$

Now, since the target is reflecting a signal back to the receiver, it can be said the target is a transmitter sending out its own EM wave given by,

$$S_{\text{received}} = \frac{P_{\text{reflected}}}{4\pi r^2} \quad \text{Equation 3.6}$$

Now, Equation 3.6 becomes,

$$P_{\text{received}} = \frac{P_{\text{reflected}}}{4\pi r^2} \times A_e \quad \text{Equation 3.7}$$

$$P_{\text{received}} = \frac{SG\sigma}{4\pi r^2} \times A_e = \frac{SG\sigma}{(4\pi)^2} \times A_e \quad \text{Equation 3.8}$$

Lastly, if Equation 3.5 is solved for $A_e$,

$$P_{\text{received}} = \frac{SG\sigma}{(4\pi r^2)^2} \times \frac{G\lambda^2}{4\pi} = \frac{SG^2\lambda^2\sigma}{(4\pi)^3 r^4} \quad \text{Equation 3.9}$$
If we make the connection that \( S = P_{\text{transmit}} \) and rearrange to solve for \( r \), we get the classic radar range equation.

\[
r = \frac{4P_{\text{transmit}}G^2\lambda^2\sigma}{(4\pi)^2P_{\text{received}}} \tag{Equation 3.9}
\]

On a side note, we realize that if we receive just enough of a signal back to detect above the noise, this represents the minimum \( P_{\text{received}} \) and since \( r \) and \( P_{\text{received}} \) are inversely related, that corresponds to the maximum detection range \( r_{\text{max}} \). Thus far, we have neglected losses but since \( (P_{\text{received}})_{\text{min}} \) represents such a small received signal, they must be included here.

\[
r_{\text{max}} = \frac{4P_{\text{transmit}}G^2\lambda^2\sigma}{(4\pi)^2(P_{\text{received}})_{\text{min}} \times L} \tag{Equation 3.10}
\]

For a different approach on the derivation of the Radar Range Equation, see;

M. C. Budge, Jr, “Radar Range Equation”, ©2005

http://www.ece.uah.edu/courses/material/EE619/RadarRangeEquation(2).doc

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4. Detection

4.1 Radar Targets

A target is said to be detected when the return wave voltage is above the background noise level. The case when the noise subtracts from the signal, while a target is present, to make the signal smaller than the radar threshold is called a miss. Radar designers seek to maximize the probability of detection for a given probability of false alarm. The probability of false alarm is defined as the probability that a sample of the return voltage will exceed the threshold voltage when noise alone is present in the radar. The probability of detection is the probability that a sample of the signal will exceed the threshold voltage in the case of noise plus signal.
Figure 4.1 Basic Target Detection Test

(Cite Ref 16)

Figure 4.1 is a graphical representation of the probability densities of the noise, false alarm, detection and signal to noise ratio. The math behind this will be given in a few more pages. (Probability Density Function).

The Threshold Voltage \( (V_T) \) is tied to the Probability of False Alarm \( (P_{fa}) \) and the number of integrated pulses \( (n_P) \) and can be calculated indirectly during non-coherent integration by using,

\[
P_{fa} = 1 - \Gamma \left( \frac{V_T}{\sqrt{n_P}}, n_P - 1 \right)
\]

Equation 4.1
Then, using the incomplete gamma function,

\[
\Gamma_l \left( \frac{V_T}{\sqrt{n_p}}, n_p - 1 \right) = \frac{V_T}{\sqrt{n_p}}^{(n_p-1) - 1} e^{-\frac{V_T}{\sqrt{n_p}}} \int_0^{\infty} \frac{\Gamma^{(n_p-1) - 1}}{((n_p-1) - 1)!} \, d\gamma
\]

Equation 4.2

It is worth noting here that when \( \frac{V_T}{\sqrt{n_p}} = 0 \), \( P_{fa} = 1 \) meaning that the probability of a false alarm is 100%. The inverse is true when \( \frac{V_T}{\sqrt{n_p}} = \infty \), \( P_{fa} = 0 \) which means, if we have an infinitely high Threshold Voltage, we will never see a false alarm or anything else for that matter.

The threshold value can then be approximated by the recursive formula used in the Newton-Raphson method. (Cite Ref. 1)

\[
V_{T, m} = V_{T, m-1} - \frac{G(V_{T, m-1})}{G'(V_{T, m-1})}; \quad m = 1, 2, 3, \ldots
\]

Equation 4.3

Where \( G \) and \( G' \) are,

\[
G(V_{T, m}) = (0.5)^{n_p/n_{fa}} - \Gamma_l(V_T, n_p)
\]

Equation 4.4

\[
G'(V_{T, m}) = -\frac{e^{-V_T} V_T^{n_p-1}}{(n_p-1)!}
\]

Equation 4.5

Which finally lands us at the Threshold Voltage for \( m = 1 \),

\[
V_{T, 0} = n_p - \sqrt{n_p} + 2.3 \sqrt{-\log P_{fa}} \left( \sqrt{-\log P_{fa}} + \sqrt{n_p} - 1 \right)
\]

Equation 4.6
We can see the relationship between the threshold voltage and the Probability of False Alarm in Equation 4.1,

\[ V_T = \sqrt{2\Psi^2 \ln \left( \frac{1}{P_{fa}} \right)} \]  \hspace{1cm} \text{Equation 4.7}

Where \( \Psi^2 \) is the Variance and \( P_{fa} \) is the Probability of False Alarm. Assuming normally distributed noise,

\[ P_{fa} = \int_{V_T}^{\infty} \frac{r^2}{2\Psi^2} e^{-\left(\frac{r^2}{2\Psi^2}\right)} dr = e^{\left(\frac{V_T^2}{2\Psi^2}\right)} \]  \hspace{1cm} \text{Equation 4.8}

So if you know either the Probability of False Alarm or the Threshold Voltage, you can find the other unknown.

Now that I have mentioned the probability of false detection, let us talk about items that may induce this false detection. After all, we want to know exactly what we are looking at, be it a person lost or a bullet. Debris refers to echoes returned from targets which are uninteresting to the radar operators. This debris may be coincidental or intentional. Such debris includes natural objects such as ground, sea, precipitation, sand storms, birds, atmospheric effects. Debris may also be returned from man-made objects such as buildings and, intentionally by radar countermeasures such a countermeasures and penetration aids.
Figure 4.2 Clutter Doppler Spectra

Figure 4.2 gives a quick visual of how different types of natural clutter can show up on a radar return signal. Notice how relatively dim the target return is compared to the clutter. If the target is a small boat, it could easily be drowned out by the radar noise created by rough seas.

Some debris may also be caused by a long radar waveguide between the radar transceiver and the antenna. Debris is considered a passive interference source, since it only appears in response to radar signals sent by the radar.

Debris is detected and neutralized in several ways. Debris tends to appear static between radar scans; on subsequent scan echoes, desirable targets will appear to move, and all stationary echoes can be eliminated. Sea debris can be reduced by using
horizontal polarization, while rain is reduced with circular polarization. Other methods attempt to increase the signal to noise ratio.

The most effective debris reduction technique is Pulse-Doppler Radar. Doppler separates debris from aircraft and spacecraft using a frequency spectrum, so individual signals can be separated from multiple reflectors located in the same volume using velocity differences. This requires a coherent transmitter in order to accomplish this. Another technique is a moving target indicator that subtracts the received signal from two successive pulses using phase to reduce signals from slow moving objects. This can be adapted for systems that lack a coherent transmitter, such as time-domain pulse-amplitude radar.

Constant False Alarm Rate is a method that relies on debris returns far outnumbering echoes from targets of interest. The receiver's gain is automatically adjusted to maintain a constant level of overall visible debris. While this does not help detect targets masked by stronger surrounding debris, it does help to distinguish strong target sources. In the past, radar False Alarm Rate was electronically controlled and affected the gain of the entire radar receiver. As radars evolved, False Alarm Rate became computer-software controlled and affected the gain with greater granularity in specific detection cells.

In general, the probability of detection for a target can be computed from the following equations.

$$P_D = \int_{V_T}^{\infty} \frac{r}{\sqrt{\pi}} I_o \left( \frac{rA}{\sqrt{r^2+A^2}} \right) e^{-\left( \frac{r^2+A^2}{2}\right)} \, dr$$

Equation 4.9
Where \( A \) is the Amplitude, \( P_D \) is the Probability of Detection and \( I_0 \) is the modified Bessel Function of Zero Order. A plot of this distribution can be found in Figure 4.1.

\[
I_0(\beta) = \frac{1}{2\pi} \int_0^{2\pi} e^{\beta \cos \theta} d\theta \quad \text{Equation 4.10}
\]

If I assume that the radar signal is a sinusoidal wave with amplitude \( A \), then the power is \( \frac{A^2}{2} \).

Now if I use the \( \text{SNR} = \frac{A^2}{2\psi^2} \) and \( \frac{\psi^2}{2} = \ln \left( \frac{1}{P_{fa}} \right) \), Equation 4.9 becomes,

\[
P_D = \int_0^{\infty} \frac{r}{2\psi^2 \ln \left( \frac{1}{P_{fa}} \right)} I_0 \left( \frac{rA}{\psi^2} \right) e^{-\left( \frac{r^2 + A^2}{2\psi^2} \right)} \, dr = Q \left[ \sqrt{\frac{A^2}{\psi^2}}, \sqrt{2 \ln \left( \frac{1}{P_{fa}} \right)} \right] \quad \text{Equation 4.11}
\]

\[
Q[\alpha, \beta] = \int_{\beta}^{\infty} \beta I_0(\alpha \beta) e^{-\left( \frac{\beta^2 + \alpha^2}{2} \right)} \, d\beta \quad \text{Equation 4.12}
\]

Where Equation 4.3 is called the Marcum’s Q-Function.

Here it is noteworthy to make the distinction that \( \sqrt{\frac{A^2}{\psi^2}} = 2 \times \text{SNR} \).

Now if I assume \( P_{fa} \) is small and \( P_D \) is relatively large making the threshold large also, Equation 4.11 then becomes,

\[
P_D \propto F \left( \frac{A}{\psi} - \sqrt{2 \ln \left( \frac{1}{P_{fa}} \right)} \right) \quad \text{Equation 4.13}
\]

Where \( F(x) \) is given by,

\[
F(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\frac{\xi^2}{2}} \, d\xi \quad \text{Equation 4.14}
\]
Equation 4.13 now becomes,

\[ P_D \approx 0.5 \cdot \text{erfc}\left(\sqrt{-\ln P_{fa}} - \sqrt{\text{SNR} + 0.5}\right) \]  

Where \text{erfc} is the complimentary error function given by,

\[ \text{erfc}(z) = 1 - \frac{2}{\sqrt{\pi}} \int_0^z e^{-v^2} dv \]  

Since there is no closed form to this solution, numerical methods are used to calculate \( P_D \). Tables have been computed and are the most widely used way of find the \( P_D \). (Cite Ref 1)

So far, the probability of detection assumed a constant target cross section (i.e. the target is not rolling, tumbling or maneuvering in any way but is static). This work was first analyzed by Marcum (Cite Ref 18). Swerling extended Marcum's work to four distinct cases that account for variations in the target cross section. These cases have come to be known as Swerling models. They are: Swerling I, Swerling II, Swerling III, and Swerling IV respectively. The constant RCS case analyzed by Marcum is widely known as Swerling 0 or equivalently Swerling V. Target radar cross section fluctuation lowers the probability of detection, or equivalently reduces the SNR. Swelling I targets have constant amplitude over one antenna scan; however, a Swerling I target amplitude varies independently from scan to scan according to a Chi-square probability density function with two degrees of freedom. The amplitude of Swerling II targets fluctuates independently from pulse to pulse according to a Chi-square probability density function with two degrees of freedom. Target fluctuation associated with a Swerling III model is similar to Swerling I, except in this case the target power fluctuates independently from
pulse to pulse according to a Chi-square probability density function with four degrees of freedom. Finally, the fluctuation of Swerling IV targets is from pulse to pulse according to a Chi-square probability density function with four degrees of freedom. Swerling showed that the statistics associated with Swerling I and II models apply to targets consisting of many small radar scattering objects of comparable RCS values such as a group of similarly sized and shaped objects, while the statistics associated with Swerling III and IV models apply to targets consisting of one large RCS scatterer and many small equal RCS scatterers such as a re-entry vehicle with countermeasures and penetration aids. (Cite Ref 18)

<table>
<thead>
<tr>
<th>Nature of Scattering</th>
<th>RCS Model</th>
<th>Fluctuation Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Similar amplitudes</td>
<td>Exponential (Chi-Squared DOF=2)</td>
<td>Slow Fluctuation</td>
</tr>
<tr>
<td></td>
<td>$p(\sigma) = \frac{1}{\sigma} \exp\left(-\frac{\sigma}{\bar{\sigma}}\right)$</td>
<td>&quot;Scan-to-Scan&quot;</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Fast Fluctuation</td>
</tr>
<tr>
<td></td>
<td></td>
<td>&quot;Pulse-to-Pulse&quot;</td>
</tr>
<tr>
<td>One scatterer much</td>
<td>(Chi-Squared DOF=4)</td>
<td>Swerling I</td>
</tr>
<tr>
<td>Larger than others</td>
<td></td>
<td>Swerling II</td>
</tr>
<tr>
<td></td>
<td>$p(\sigma) = \frac{4\sigma}{\bar{\sigma}^2} \exp\left(-\frac{2\sigma}{\bar{\sigma}}\right)$</td>
<td>Swerling III</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Swerling IV</td>
</tr>
</tbody>
</table>

$\bar{\sigma} = \text{Average RCS (m}^2\text{)}$

**Figure 4.3 Swerling RCS Target Models**

(Cite Ref 16)
<table>
<thead>
<tr>
<th>Nature of Scattering</th>
<th>Amplitude Model</th>
<th>Fluctuation Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Similar amplitudes</td>
<td>Rayleigh</td>
<td>Slow Fluctuation “Scan-to-Scan”</td>
</tr>
<tr>
<td></td>
<td>$p(a) = \frac{2a}{\sigma} \exp\left(-\frac{a^2}{\sigma}\right)$</td>
<td>Swerling I</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Fast Fluctuation “Pulse-to-Pulse”</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Swerling II</td>
</tr>
<tr>
<td>One scatterer much Larger than others</td>
<td>Central Rayleigh, DOF=4</td>
<td>Swerling III</td>
</tr>
<tr>
<td></td>
<td>$p(a) = \frac{8a^3}{\sigma^2} \exp\left(-\frac{2a^2}{\sigma}\right)$</td>
<td></td>
</tr>
</tbody>
</table>

$\overline{\sigma} = \text{Average RCS (m}^2\text{)}$

**Figure 4.4 Swerling Amplitude Target Models**

(Cite Ref 16)

Figures 4.3 and 4.4 define the different Swerling RCS and Amplitude Models in a graphical representation that is more intuitive and easily understood.
The cases I and III apply for search radars. The fluctuation loss depends on the probability of detection (PD) and is shown in Figure 1. There is a fluctuation gain for a PD < 30%. This is because of the statistically changing of the magnitude excels small signal-to-noise ratios.
Figure 4.6 is a treasure trove of information. With higher band radars such as x-band, an observer can see very small details about an object of interest. Notice that on the top bandwidth, a trained observer can pick out the nosetip/re-entry vehicle, the 1\textsuperscript{st} and 2\textsuperscript{nd} stages of the rocket and the aft portion of the rocket where the fins and exhaust is located. This level of detail has more consequences other than just identifying different portions of the missile. Computer algorithms can also use the time between the voltage returns to make a rough estimate to the size of each specific part of the missile. Once the size is estimated and this information is compared to other types of intelligence, a person can
make an educated guess as to the type of missile on the radar. Once a certain type is identified, this can tell the defender what sort of payload is aboard the missile, be it a small, unitary payload such as carried on a Scud-C, or a Maneuverable Re-entry Vehicle or MaRV for short or the more dangerous Multiply Targetable Re-entry Vehicles (MiRVs). Also, the type of missile can give information to the observer about the missiles ability to possibly carry countermeasures (Cms) or penetration aids (PenAids), all of which will show up on radar once released. These CMs and PenAids are designed to confound and distract radars from tracking the real threat that is the RV.

Figure 4.7 Sample Radar Cross Section of a notional RV

(Cite Ref 15)
Notice that when a radar is looking nearly along the RV's trajectory that the radar return is minimal. This is due to the radar wave between deflected away from radar A so that radar A sees a minimal return. A scenario like this could cause the RV to be masked in the background noise. However, radar B is in a much better position as the RV is presenting a broadside to this radar.

In general, there are three main categories of missile parts that create RCS values. The first is structural such as the missile body and control surfaces. The second is propulsion such as inlets, exhaust, etc. The third is avionics like receivers, antennas, etc.

Figure 4.8 RCS of the Johnson Generic Aircraft Model

(Cite Ref 16)
In Figure 4.8 above, one can instantly tell that the target is symmetric on two sides and even without the aspect angles on the x-axis, one could still discern the nosetip of the target.

The main point to take away here is that accurate estimation of target signatures should draw upon all available tools.

<table>
<thead>
<tr>
<th></th>
<th>Square meters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small, single engine aircraft</td>
<td>1</td>
</tr>
<tr>
<td>Four passenger jet</td>
<td>2</td>
</tr>
<tr>
<td>Large fighter</td>
<td>6</td>
</tr>
<tr>
<td>Medium jet airliner</td>
<td>40</td>
</tr>
<tr>
<td>Jumbo jet</td>
<td>100</td>
</tr>
<tr>
<td>Helicopter</td>
<td>3</td>
</tr>
<tr>
<td>Small open boat</td>
<td>0.02</td>
</tr>
<tr>
<td>Small pleasure boat (20-30 ft)</td>
<td>2</td>
</tr>
<tr>
<td>Cabin cruiser (40-50 ft)</td>
<td>10</td>
</tr>
<tr>
<td>Ship(5,000 tons displacement, L Band)</td>
<td>10,000</td>
</tr>
<tr>
<td>Automobile / Small truck</td>
<td>100 - 200</td>
</tr>
<tr>
<td>Bicycle</td>
<td>2</td>
</tr>
<tr>
<td>Man</td>
<td>1</td>
</tr>
<tr>
<td>Birds</td>
<td>$10^{-2} - 10^{-3}$</td>
</tr>
<tr>
<td>Insects / Bullets</td>
<td>$10^{-4} - 10^{-5}$</td>
</tr>
</tbody>
</table>

Figure 4.9 Some Examples of Radar Cross Sections
Figure 4.10 Propagation Effects on Radar Waves

(Cite Ref 15)

Figure 4.10 gives a few examples as to what may effects a radar wave may encounter as it travels through a medium.

The fluctuation loss can be viewed as the amount of additional SNR required to compensate for the SNR loss due to target fluctuation, given a specific Probability of Detection value.

The cumulative probability of detection refers to detecting the target at least once by the time it is at some range R. More precisely, consider a target closing on a scanning radar, where the target is illuminated only during a scan. As the target gets closer to the radar, its probability of detection increases since the SNR is increased. Suppose that the probability of detection during the nth frame is $P_d$; then, the cumulative probability of detecting the target at least once during the nth frame is given by,

$$P_c = 1 - \prod_{i=1}^{n} (1 - P_d)$$

Equation 4.17
So far I have discussed ways to improve our radars, but I need to also briefly mention ways to decrease a radars ability to detect and track targets. Knowing how this is done will allow us to overcome similar challenges in the future.

### 4.2 Radar Jamming

Radar jamming refers to radio frequency signals originating from sources outside the radar, transmitting in the radar’s frequency and thereby masking targets of interest. Jamming may be intentional, as with an electronic warfare tactic, or unintentional, as with friendly forces operating equipment that transmits using the same frequency range. Jamming is considered an active interference source, since it is initiated by elements outside the radar and in general unrelated to the radar signals.

Jamming is problematic to radar since the jamming signal only needs to travel from the jammer to the radar receiver whereas the radar echoes travel from the radar to the target and back to the radar and are therefore significantly reduced in power by the time they return to the radar receiver. Jammers therefore can be much less powerful than
their jammed radars and still effectively mask targets along the line of sight from the jammer to the radar called main lobe jamming. Jammers have an added effect of affecting radars along other lines of sight through the radar receiver's side lobes intuitively called side lobe jamming.

Main lobe jamming can generally only be reduced by narrowing the main lobe solid angle and cannot fully be eliminated when directly facing a jammer which uses the same frequency and polarization as the radar. Side lobe jamming can be overcome by reducing receiving side lobes in the radar antenna design and by using an omnidirectional antenna to detect and disregard non-main lobe signals. Other anti-jamming techniques are frequency hopping and polarization.

Since we are at the end of this section and we already mentioned probability density functions, we need to provide a little math to top off the subject.

According to the Swerling models the RCS of a reflecting object based on the chi-square probability density function with specific degrees of freedom. These models are of particular importance in theoretical radar technology.

The Chi-squared Probability Density Function with $2N$ degrees of Freedom can be written in the form,

$$f(\sigma) = \frac{N}{(N-1)!} \left( \frac{N \sigma}{\bar{\sigma}} \right)^{N-1} e^{-\frac{N \sigma}{\bar{\sigma}}}$$

Equation 4.18

Where $\bar{\sigma}$ is the average Radar Cross Section value.

In the case of $N = 1$, Equation 4.10 becomes,
Swerling I and II targets RCS are modeled where the RCS varies according to a Chi-squared probability density function with two degrees of freedom \(N = 1\). This applies to a target that is made up of many independent scatterers of roughly equal areas.

\[
f(\sigma) = \frac{1}{\sigma} e^{-\frac{\sigma}{2}} \text{ for } \sigma \geq 0 \quad \text{Equation 4.19}
\]

Figure 4.12 clearly shows that as the RCS value increases, \(f(\sigma)\) decrease to near 0 very rapidly.

In the case of \(N = 2\),

\[
f(\sigma) = \frac{4\sigma}{\sigma^2} e^{-\frac{4\sigma}{\sigma^2}} \text{ for } \sigma \geq 0 \quad \text{Equation 4.20}
\]

Swerling III and IV targets RCS can be modeled because the RCS varies according to a Chi-squared probability density function with four degrees of freedom \(N\)
This PDF approximates an object with one large scattering surface (such as a missile booster stage) with several other small scattering surfaces (such as separation debris). The RCS is constant through a single scan just as in Swerling I.

Figure 4.13 shows that for some value of $\bar{\sigma}$, there exists a maximum value for $f(\sigma)$.

The Chi-Squared PDF’s are used to assess detection performance of RCS fluctuation effects. The probability of detection for a fluctuating target ($\sigma$ varies) can be computed from the following conditional probability density function equations.

$$f\left(\frac{z}{\sigma}\right) = \left(\frac{2z}{\sigma^2}\right)^{\frac{n_p-1}{2}} e^{-\frac{n_p \sigma^2}{2z^2}} I_{n_p-1}\left(\sqrt{\frac{2n_p \sigma^2}{z^2}}\right)$$

Equation 4.21
Where $z$ is the result of non-coherent integration of $n_p$ radar pulses.

$$z = \sum_{n=1}^{n_p} \frac{r_n^2}{2\sigma^2}$$

Equation 4.22

Using the relation $f(z, \sigma) = f\left(\frac{z}{\sigma}\right) f(\sigma)$,

$$f(z) = \int_{\sigma_T}^{\infty} f(z, \sigma)f(\sigma) \, d\sigma = P_D$$

Equation 4.23

Performing the above integration will lead one to get the incomplete Gamma Function.

So far, we have assumed that the radar equation had a constant target RCS and did not account for integration loss. In this next section, a more comprehensive form of the radar equation is introduced.

The radar range equation connects target properties such as RCS, radar characteristics such as power and antenna size, the distance between the radar and the target and the properties of the medium such as atmospheric attenuation.
5. Signal to Noise Ratio and the Radar Range Equation

The above figure is a graphical illustration of how radar determines the range to a target. The reflected pulse will have a different frequency from the original pulse and it is this difference that allows radar to give an estimate of the targets speed. The targets range is computed from the amount of time it takes the pulse to travel to the target and back.
The power density from an isotropic antenna can be written as, \( \frac{P_T}{4\pi R^2} \) where \( P_T \) is the peak transmit power, \( R \) is the range from the radar to the target and the power density from the directive antenna is \( \frac{P_T G_T}{4\pi R^2} \) where \( G_T \) is the transmit gain which can also be written as,

\[
Gain = \frac{4\pi A_e}{\lambda^2}
\]

Equation 5.1

where \( \lambda \) is of course, the wavelength of the radiation.

Now the power of the reflected radar wave at the target's location can be written as \( \frac{P_T G_T}{4\pi R^2} \sigma \) where \( \sigma \) is the RCS in \( m^2 \). Once the reflection reaches the radar, its power is now \( \frac{P_T G_T}{4\pi R^2} \frac{\sigma}{R^2} \) meaning that the power density diminishes as a \( \frac{1}{R^2} \) value.
Figure 5.3 Return Signal Power

Figure 5.3 is a broken down extract from the derivation of the Radar Range Equation introduced in Section 3.

I can now compute the power received by the antenna itself as,

\[ P_r = \frac{P_T G_T \sigma A_e}{4\pi R^2} \times \frac{1}{4\pi R^2} \]

Equation 5.2

where \( A_e \) is the effective surface area of the antenna.

So, now that I have a term for the power of the reflected wave from the target, what about the background noise? That can be calculated using,

\[ N = k \times T_s \times B_n \]

Equation 5.3

where \( k \) is Boltzmann's constant, \( T_s \) is the system noise temperature in Kelvin and \( B_n \) is the noise bandwidth of the receiver.
If I write the total system losses as $L$, then our signal to noise ratio becomes,

$$\text{SNR} = \frac{P_T G_T G_R \lambda^2 \sigma}{(4\pi)^3 R^4 k T_s B_n L}$$

Equation 5.4

where,

$$T_s = T_a + T_r + L_r T_e$$

Equation 5.5

with $T_a$ defined as the contribution from the antenna, $T_r$ is the RF components contribution, $L_r$ is the loss of input from the RF components and $T_e$ is the temperature of the receiver.

The above SNR radar equation is useful when the targets position (i.e. range, altitude, etc.) is known meaning the radar is tracking the target. This maybe a result of a different radar cueing the observers radar as where to look for the target.
However, for a target whose position is not known, I must include additional parameters.

$$\text{SNR} = \frac{P_{av} t_s A_o \sigma}{4n \Omega R^4 k T_s L}$$

Equation 5.6

where is $\Omega$ is the solid angle that is being searched, $P_{av}$ is the average power and $t_s$ is the scan time.
Figure 5.5 Search Example

(Cite Ref 16)

If I rearrange the SNR equation to solve for $P_{av}$ in watts, I can determine the power required to detect a target at a given range.

$$P_{av} = \frac{4\pi R^4 k T_s L(SNR)}{t_s A_e \sigma}$$

Equation 5.7

Equation 5.6 is independent of wavelength, highly dependent on R and is a linear function of the other variables. So you ask, why do I need to know these parameters? Well, by inspection, one can see that making $P_{av}$ and $A_e$ larger, improves the radars detection and while decreasing the wavelength will also have this effect. On the other end, in Equation 5.7, increasing the range or the SNR degrades the radars abilities significantly.
6. Resolution

As one would suspect, the range at which a target is acquired greatly affects the resolution that the radar has of the target at that range. The closer the target, the more detail the radar can obtain and vice versa.

The range from the radar to the target can easily be computed using the simple equation,

\[ R = \frac{c\Delta t}{2} \]

Equation 6.1

Where \( c \) is the speed of light and \( \Delta t \) is the time delay between the transmitted pulse and the received echo from the target.

![Transmitted and received pulses.](image)

**Figure 6.1 Transmitted and received pulses.**

(Cite Ref 1)

Note from the Figure 6.1 that \( R_2 \) can be calculated using,

\[ R = \frac{c(t+\Delta t)}{2} \]

Equation 6.2
Now that I can easily determine a target’s rough range, I can determine the resolution with which I expect to see this target. In order to do this, I first must know a few items of interest about our radar. First, I need to know the minimum range the radar was designed to operate at and second, I need to know the maximum range. Now, I need to divide this range between the maximum and minimum into “bins”. Each bin needs to be of sufficient distance to completely resolve the target in range. Hence, the number of bins for each radar is directly tied to its resolution capability.

Say, for example, that I have two incoming ICBM’s and I need to know the range between these two missiles so I can allocate the proper amount of time for a response to both missiles. It is quite easy to do this as you take the difference of ranges between the two targets using,

\[ \Delta R = R_2 - R_1 = \frac{c(t_2 - t_1)}{2} = \frac{c \delta t}{2} \]  
\[ \Delta R_{\text{min}} = \frac{c \tau}{2} \]

Now suppose that the two targets were launched at the same time and are flying almost identical paths, how much \( \Delta R \) is needed to be able to resolve two targets from one? In order to know this, I need to know the pulse width, \( \tau \), of our radar pulses. In order to distinctly see two targets, \( \Delta R \) has to be greater than or equal to \( \frac{c \tau}{2} = \frac{c}{2B} \) where \( B \) is the radar bandwidth.

\[ B = \frac{1}{\tau} \]
The goal of any radar designer is to minimize $\Delta R$ as to increase the number of bins and increase the resolution of the radar. As one can see, in order to have high resolution at long ranges, the radar must use a very short pulse which in turn causes significant problems with transmitted energy and bandwidth.

So far, range resolution has been the only type of resolution discussed. This means that I am talking about two stationary targets in relation to the radar. However, few targets will remain stationary so I must also include a Doppler resolution.

Earlier in this paper, *(Doppler_Shift_Figure)*, I discussed the Doppler Shift of the radar pulse frequency due to the velocity of the moving target. Now, we’ll expand on that concept.

Recall that the Doppler shift is given by,

$$f_D = \frac{2v}{\lambda} = \frac{2vf_0}{c}$$  \hspace{1cm} \text{Equation 6.6}$$

Where $v$ is the targets radial velocity, $\lambda$ is the wavelength of the radar pulse, $f_0$ is the radar frequency and $c$ is the speed of light.

If I start with target one having a spectrum given by,

$$\psi(f) = \int_{-\infty}^{\infty} \Psi(t)e^{-i2\pi ft}dt$$  \hspace{1cm} \text{Equation 6.7}$$

And I say that the second targets spectrum can be given by

$$\psi(f) = \Phi(2\pi f - 2\pi f_0),$$ then in order to distinguish between these two targets at the same range, I must use the integral squared.
I can now define the Complex Frequency Correlation Function as,
\[ \chi_t(f_B) = \int_{-\infty}^{\infty} \Phi^*(2\pi f)\Phi(2\pi f - 2\pi f_B)df = \int_{-\infty}^{\infty} |\varphi(t)|^2 e^{i2\pi f_B t}dt \]

Equation 6.9

Now I can compute the Doppler Resolution constant \( \Delta f_d \) as,
\[ \Delta f_d = \frac{\int_{-\infty}^{\infty} |\chi_t(f_B)|^2df}{\chi_t^2(0)} = \frac{1}{\tau} \]

Equation 6.10

Since I now have the Doppler Resolution and we know from section 2 in the text that velocity influences the doppler shift of the frequency, we can define the velocity resolution to be,
\[ \Delta v = \frac{c\Delta f_d}{2f_o} = \frac{c}{2f_o \tau} \]

Equation 6.11

It is useful to discuss some other interesting features of modern radars before proceeding further. One is the concept of the "range bin", which allows automated range tracking of targets and the ability to pick targets more easily out of noise which is briefly discussed later in this paper.

Range bins are a scheme in which a digital radar uses a set of range gates to chop up the return trace into segments and sum the value for each segment into an associated memory "bin". The radar system can inspect the bins to see where the target is along the
trace and track its range, but it can also sum up values from trace to trace relative to an average level. Since noise will tend to fluctuate around the average level as we will see later, the contents of most of the range bins will remain at the average level. The radar system can keep track of the average noise level in the range bins and adjust the noise threshold accordingly which is known as "constant false alarm rate (CFAR)". If there is a return signal in a range bin, it will tend to add up over time, making it easier to pick out of the noise. If the signal shows up in two or more adjacent bins, the radar can also interpolate between the two to get a more accurate range estimate.

This is shown in Figure 6.2 below, which displays the summing of eight return traces in a set of eight range bins. Digitizing a waveform means it is resolved into a discrete set of values or steps; this diagram uses the simplifying assumption that the values of each trace are very close to the noise level and only have values of a single step above, at, or below the average signal value level, which is shown in gray. Of course, in reality they may be several steps above or below the average signal value.
Another modern improvement is known as pulse compression where there is a tradeoff in determining pulse width. A short pulse gives better range accuracy, but it also means less energy dumped out to sense a target. Pulse Doppler makes the matter worse because interpreting the Doppler shift from a short pulse is harder than interpreting the shift from a long pulse, and so a short pulse gives poorer velocity resolution. However, in situations where we are only concerned with location, pulse compression would be a great choice. (Cite Ref. 30)
In the simplest form, it amounts to generating a pulse as a frequency-modulated ramp or chirp, rising from a low frequency to a high frequency or vice versa. This increases the energy of the pulse and permits much less ambiguity in Doppler interpretation. Essentially, pulse compression trades bandwidth for pulse length, and pulse compression schemes are rated by a "compression factor" given by: (Cite Ref. 30)

\[
\text{Compression Factor} = \text{Chirp Range} \times \text{Pulse Duration} \tag{6.12}
\]

There are also "coded" schemes for pulse compression that involve shifting parts of the pulse in phase such as a simple sine wave, going through cycles. A normal sine wave will go through identical cycles in sequence, varying from positive to negative through each cycle. Now suppose that every third cycle, the sine wave is inverted in polarity, varying from negative to positive instead of positive to negative, or in other words shifted 180 degrees in phase: (Cite Ref. 30)
In any case, by Fourier analysis, the abrupt transition from a "+' cycle to a "-' cycle involves a very wide spectrum of Fourier components, and so by the seemingly simple change of inverting one cycle of polarity results in a compressed high-bandwidth pulse generated from a relatively low-bandwidth signal. Of course, the received return pulse is processed by summing the echoes obtained from the three pulses, with the third cycle returned to normal polarity in the summation. (Cite Ref. 30)

One last interesting phenomenon about a radar beam is that it can be reflected by the ground. The combination of the direct and re-reflected ground echo changes the transmitting and receiving patterns of the antenna. This is substantial in the VHF range and decreases with increasing frequency. For the detection of targets at low heights, a reflection at the Earth's surface is necessary. This is possible only if the ripples of the area within the first Fresnel Zone do not exceed the value $0.001 \, R$ meaning that within a radius of 1000 m no obstacle may be larger than 1 m.
A Fresnel zone is one of a theoretically infinite number of a concentric ellipsoids of revolution which define volumes in the radiation pattern of a usually circular aperture. Fresnel zones result from diffraction by the circular aperture. Radio waves will travel in a straight line from the transmitter to the receiver normally, but if there are obstacles near the path, the radio waves reflecting off of those objects may arrive out of phase with the signals that travel directly and reduce the power of the received signal. This effect is a reason of Fading in radio communication. On the other hand, the reflection can enhance the power of the received signal if the reflection and the direct signals arrive in phase.

This ground reflection depends on the type of soil, its dampness and how rough the soil is. The surface roughness diffuses radar energy in all directions with the diffusion getting worse with rougher soils. This diffused energy reduces the amount of energy reflected in the specular direction and thus, the ground plane enhancement becomes less significant with rougher grounds. Vegetation is especially responsible for diffusion of radar energy. The reflection co-efficient for ground reflections depends principally on the dielectric constant $e$ and the conductivity $\sigma$. It is different for vertical and horizontal polarization.

\[ P_r \]

\[ P_i \]

\[ \theta \]

\[ \text{Ground} \]

\[ \text{Specular Reflection} \]

**Figure 6.5 Reflection Geometry**

With the resulting reflection coefficients are given by the equations below:
\[ p_h = \frac{\sin \theta - \sqrt{\varepsilon_r - \cos^2 \theta}}{\sin \theta + \sqrt{\varepsilon_r - \cos^2 \theta}} \quad \text{Equation 6.13} \]

\[ p_v = \frac{(\varepsilon_r)\sin \theta - \sqrt{\varepsilon_r - \cos^2 \theta}}{(\varepsilon_r)\sin \theta + \sqrt{\varepsilon_r - \cos^2 \theta}} \quad \text{Equation 6.14} \]

Where \( \varepsilon_r \) is the relative permeability of the soil, and the subscripts \( h \) and \( v \) referring to horizontal and vertical polarizations with \( \theta \) being the grazing angle.

In general, the reflection co-efficient is a complex number and the reflected signal will therefore differ in both amplitude and phase. Making substitutions for complex permittivity and complex permeability gives the complex reflection coefficients:

<table>
<thead>
<tr>
<th>Surface</th>
<th>Conductivity (Siemens)</th>
<th>Relative Dielectric Constant</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dry Ground</td>
<td>0.001</td>
<td>4-7</td>
</tr>
<tr>
<td>Average Ground</td>
<td>0.005</td>
<td>15</td>
</tr>
<tr>
<td>Wet Ground</td>
<td>0.02</td>
<td>25-30</td>
</tr>
<tr>
<td>Sea Water</td>
<td>5</td>
<td>81</td>
</tr>
<tr>
<td>Fresh Water</td>
<td>0.01</td>
<td>81</td>
</tr>
</tbody>
</table>

**Figure 6.6 Some Typical Reflectivity Values**

(Cite Ref 21)

The reflections from rough surfaces are termed non-specular reflections. In general, the resulting reflection from a real surface will be a combination of specular and non-specular reflections as shown in Figure 6.7.
The relative smoothness of surface depends on the wavelength and the angle of incidence.

\[
\Delta \phi = \frac{4\pi d \sin \theta}{\lambda}
\]

Equation 6.15
If, $d < \frac{\lambda}{8 \sin \theta}$ then the surface is considered smooth and will reflect the incident radar wave with minimal scattering of the wave. If we define a rough surface as

$$\Delta \phi > \frac{\pi}{2}$$

then,

$$d \geq \frac{\lambda}{8 \sin \theta}$$  \hspace{1cm} \text{Equation 6.16}

For example, if a search radar we are using radiates at an angle of 30 degrees compared to the horizontal with a wavelength of 3.34 cm, a rough surface has a depth of 8.5 mm while at 5 degrees it is 4.88 cm. This means that for someone lost in the woods, the rough terrain (bushes, trees, dead logs on the ground) will scatter the radar beam a great deal. However, if we mount our search radar on an unmanned aerial vehicle (UAV) or other aircraft and we radiate towards the ground at higher angles, then we can compensate somewhat for the surface depth changes.
7. Classification

The ability to determine a particular class to which a target belongs among the many possible classes is known as target classification. Target classification generally requires a greater signal to noise ratio than target detection. The many small returns comprising a target might be more important for classification than the few large returns that are more important for detection. Furthermore, the ability to perform non-cooperative target classification depends on the amount of information collected and processed by the system.

There are two main aspects to target classification. The first is to isolate the target returns from the clutter echoes such as by filtering and to extract the features that can help distinguish the class of the target. The second aspect is related to the method used for performing the decision as to which class or target type the feature data belongs such as RV-like and non-RV like. When target classification is achieved through automatic computation it is usually referred to as Automatic Target Recognition.

In Automatic Target Recognition, the classification task requires complex techniques and there are a number of approaches that can be used. For example, in a model based technique, a model of the target is made by CAD and Electro-Magnetic simulations. This enables many simulated versions to be compared with the target signature to be classified. However, this is a computationally intensive technique. Alternatively, in a template matching based technique, many real versions of the target signatures cataloged at a large number of geometries are stored in a database and
subsequently compared with the target detected in order to assign it to a class. Consequently a very large database is needed. Further, if the target is altered in some way such as an added tank may carry some additional fuel or warheads, then the templates may no longer represent the modified signature and the classification can fail. Finally, pattern based techniques exploit features extracted from the input signature. These might include peak amplitudes and their locations in a High Resolution Radar (HRR) profile or Inverse Synthetic Aperture Radar (ISAR) image. These then are used to make a multidimensional feature vector which can be compared with the stored feature vectors from previous measurements of known targets in order to perform classification. This technique is less costly in terms of computation and it is consistent with a netted radar framework as a number of perspectives of the same object are available.

The decision, which is usually made automatically by the computer, may be performed with two different data input types. The first is one dimensional target classification by HRR profiles. The HRR profile can be thought of as representing the projection of the apparent target scattering centers onto the range axis. Hence the HRR profile is a one dimensional feature vector. The second data input is the two-dimensional ISAR image. This perhaps lends itself to being processed by standard classifying procedures such as the method of moments, classification of scattering centers or the projection classification where the projection of the pixels intensities are expressed as a combination of two orthogonal vectors to make a two-dimensional feature.

Traditional classification procedures, that normally process a time sequence of returns collected from a single aspect of the target, are often characterized by poor performance. Because of their crucial fields of application in both military and civilian
worlds, a higher confidence of correct classification is required. Networks of co-operating radar systems offer a possible solution for improving classification performance as a consequence of the richer data set that can be gleaned from multiple perspectives. With more perspectives, an approaching target is seen from different directions. As a consequence, the target’s reflectivity function can be extended over the monostatic case to include a spatial dimension. In this way, more information is being input to make the classification decision by the greater number of radars. There are a number of ways in which the network configuration can be exploited. The simplest is to treat the individual radar systems as independent and I simply have a number of reflectivity functions to attempt classification rather than the simple one provided by the monostatic case. Alternatively every radar station could cooperate with the others during the data collection phase. If collected coherently, the data be may reassembled using the principles of tomography to provide a single, more complete three-dimensional image. This has a higher level of information content and hence has the potential for better classification if the image can be correctly formed. The multi-perspective classifiers may use other information such as the angle between the lines of sight.
8. Discrimination

Just as good target discrimination is vital in radar weather forecasting; the same principles will apply to any type of target you wish. After all, a radar is worthless if you don’t know what you’re looking at. Discrimination can be implemented in a variety of ways including but not limited to: Radial Velocity Discrimination, Differentiation and Moving Target Indicator.

In many circumstances, it is beneficial to know both the range and the radial velocity of the target. Since the radial velocity is the range rate, a measurement of the radial velocity can be used to predict the target's range in the near future, thus allowing operators to predict future locations of the target. This has huge implications for weather radar as well as civilian air traffic control. Radial velocity discrimination can also be used to eliminate unnecessary targets from the display such as sea clutter or buildings.

One very simple way to measure radial velocity is for the system to simply measure the range at fixed intervals and compute the rate of change between the measurements. For example, if a target is at 1500 m for the first measurement and at 1492 m for the next measurement made 1 sec later, the range rate is \(-8\) m/s. Light detection and ranging (LIDAR) systems use this method. Accuracy is improved by taking several quick measurements and computing the average rate of change. The intervals cannot be chosen to be too small however, since the target must be able to change range during the measurement interval. Also, the devices that use this technology such as law enforcement speed radars can only work if the target is moving nearly parallel to the LIDAR beam. If
the target is travelling perpendicular to the LIDAR beam or at an angle, the LIDAR will give an incorrect reading. LIDAR is also severely hampered by weather effects such as dense fog, heavy rain and snow. Law enforcement LIDAR is also restricted to very short ranges on the order of several hundred meters or so.

 Whatever the case and circumstances, we need a way to discern between different objects and different types of objects at a distance. The science of doing just that is called discrimination.
9. Tracking, Trajectory Analysis and Prediction

Tracking measures the targets coordinates and provides data which may be used to determine the targets trajectory. All or part of the targets range, elevation, azimuth angle and Doppler frequency shift may be used to build a trajectory and predict the future position of the target. Nearly all radars have the ability to track so it’s just a question of how long, to what degree of fidelity and how reliable is the track/position. Even though many types of data can be used in tracking a target, it is the range/velocity tracking and angle tracking that is normally considered as tracking. However, since I have already distinguished between range/velocity tracking and angle tracking, it is also important to differentiate between continuous, single target tracking and multiple target track while scan radars. The former supplies continuous target data on just one target while the later supplies sampled data on one or more targets.

Before a radar can track a target, it must first acquire a target that the observer wants to follow. To do this, some radars operate in a search mode with predesigned search “fences” as well as using special search patterns such as helical, cluster and spiral patterns to name a few. These fences are locations in space where a radar (we’ll call it radar A for now) is focusing its attention in anticipation of finding a target of opportunity. This search fence may be hastily built because another radar (we’ll call it radar B) has detected an object that will be entering radar A’s range at a specific point. Radar B
leading radar A into a search pattern is called cueing. Instead of using its limited resources searching a huge amount of space, radar A can spend all of its power focusing on a relatively small sector and as a result, produce very detailed target parameters and a reliable target trajectory.

One drawback of continuous tracking is that while the radar is tracking one specific target, it can gather no information about additional targets entering the battle space. For the purposes of this paper, when I say tracking radar, I am referring to continuous tracking as target parameters and trajectory is what I need.

Angle tracking is concerned with generating continuous measurements of the target's angular position in the azimuth and elevation. The accuracy of early generation angle tracking radars depended heavily on the size of the pencil beam employed. Most modern radar systems achieve very fine angular measurements by utilizing monopulse tracking techniques.

Tracking radars use the angular deviation of the target from the antenna main axis within the beam to generate an error signal. The resultant error signal describes how much the target has deviated from the beam main axis. Then, the beam position is continuously changed in an attempt to produce a zero error signal. If the radar beam is normal to the target so that there is maximum gain, then the target angular position would be the same as that of the beam. In practice, this is rarely the case. In order to be able to quickly change the beam position, the error signal needs to be a linear function of the deviation angle. It can be shown that this condition requires the beam's axis to be
squared by some angle called the squint angle off the main axis. This angle will be shown in Figure 9.3.

Sequential lobing is one of the first tracking techniques that utilized by the early generation of radar systems. Sequential lobing is often referred to as lobe switching or sequential switching. It has a tracking accuracy that is limited by the pencil beam-width used and by the noise caused by either mechanical or electronic switching mechanisms. However, it is very simple to implement. The pencil beam used in sequential lobing must be symmetrical in azimuth and elevation beam-widths.

Figure 9.1 Lobe Switching
(Cite Ref 15)

In Figure 9.1, the two lobes from the radar are shown in yellow while the resulting "summed" signal in blue. If the aircraft were to move slightly up, the signal strength in the lower lobe would drop off rapidly, while growing rapidly in the upper lobe. The operator would then aim the antenna upward to compensate, until the two returns were equal again.
Tracking is achieved by continuously switching the pencil beam between two pre-determined symmetrical positions around the antenna's Line of Sight (LOS) axis. Hence, the name sequential lobing is adopted. The LOS is called the radar tracking axis, as illustrated in the below figure.

![Diagram showing sequential lobing](image)

(a) Target is located on track axis, (b) Target is off track axis.

**Figure 9.2 Sequential Lobing**

(Cite Ref 1)

As the beam is switched between the two positions, the radar measures the returned signal levels. The difference between the two measured signal levels is used to compute the angular error signal. For example, when the target is tracked on the tracking axis, as the case in Figure 9.2 (top), the voltage difference is zero. However, when the target is off the tracking axis, as in Figure 9.2 (bottom), a nonzero error signal is
produced. The sign of the voltage difference determines the direction in which the antenna must be moved. Keep in mind, the goal here is to make the voltage difference be equal to zero.

In order to obtain the angular error in the orthogonal coordinate, two more switching positions are required for that coordinate. Thus, tracking in two coordinates can be accomplished by using a cluster of four antennas, two for each coordinate or by a cluster of five antennas. In the latter case, the middle antenna is used to transmit, while the other four are used to receive.

Figure 9.3 shows a simplified conical scan radar system with the squint angle discussed previously. The envelope detector is used to extract the return signal amplitude and the Automatic Gain Control (AGC) tries to hold the receiver output to a constant value. Since the AGC operates on large time constants, it can hold the average signal level constant and still preserve the signal rapid scan variation. It follows that the tracking error signals (azimuth and elevation) are functions of the target's RCS; they are functions of its angular position off the main beam axis.

In order to illustrate how conical scan tracking is achieved, I will first consider the case as the antenna rotates around the tracking axis, all target returns have the same amplitude meaning they have a zero error signal. Thus, no further action is required.
In the case of Figure 9.3, the antenna is continuously rotated at an offset angle, or has a feed that is rotated about the antenna's main axis. Figure 9.2 shows a typical conical scan beam. The angle between the antenna's LOS and the rotation axis is the squint angle. The antenna's beam position is continuously changed so that the target will always be on the tracking axis.

Range is measured by estimating the round-trip delay of the transmitted pulses. The process of continuously estimating the range of a moving target is known as range tracking. Since the range to a moving target is changing with time, the range tracker must be constantly adjusted to keep the target locked in range. This can be accomplished using a split gate system, where two range gates comprised of an early and late gate are utilized.

The early gate opens at the anticipated starting time of the radar echo and lasts for half its duration. The late gate opens at the center and closes at the end of the echo signal. For this purpose, good estimates of the echo duration and the pulse center time must be reported to the range tracker so that the early and late gates can be placed properly at the
start and center times of the expected echo. This reporting process is widely known as the "designation process." The early gate produces positive voltage output while the late gate produces negative voltage output. The outputs of the early and late gates are subtracted, and the difference signal is fed into an integrator to generate an error signal. If both gates are placed properly in time, the integrator output will be equal to zero. Alternatively, when the gates are not timed properly, the integrator output is not zero, which gives an indication that the gates must be moved in time, left or right depending on the sign of the integrator output.

Radar systems sample each target once per scan interval, and use sophisticated smoothing and prediction filters to estimate the target parameters between scans. To this end, the Kalman filter and the Alpha-Beta-Gamma filter are commonly used. Once a particular target is detected, the radar may transmit up to a few pulses to verify the target parameters, before it establishes a track file for that target. Target position, velocity, and acceleration comprise the major components of the data maintained by a track file.

Modern radar systems are designed to perform multi-function operations, such as detection, tracking, and discrimination. With the aid of sophisticated computer systems, multi-function radars are capable of simultaneously tracking many targets. In this case, each target is sampled once, mainly range and angular position during a scan interval. Then, by using smoothing and prediction techniques, future samples can be estimated. Radar systems that can perform multi-tasking and multi-target tracking are known as Track-While Scan radars discussed previously Track_While_Scan.

Once a TWS radar detects a new target, it initiates a separate track file for that detection. This ensures that sequential detections from that target are processed together
to estimate the target's future parameters. Position, velocity, and acceleration comprise
the main components of the track file. Typically, at least one other confirmation detection
in order to verify detection is required before the track file is established.

Unlike single target tracking systems, TWS radars must decide whether each
detection belongs to a new target or belongs to a target that has been detected in earlier
scans and in order to accomplish this task, TWS radar systems utilize correlation and
association algorithms. In the correlation process, each new detection is correlated with
all previous detections in order to avoid establishing redundant tracks. If a certain
detection correlates with more than one track, then a pre-determined set of association
rules is exercised so that the detection is assigned to the proper track.

TWS radars have several limitations including: the sample rate is low, being the
search scan (frame) rate (this rate may be fairly high in sector scanning radars using
electromechanical or electronic scan). The correlation gates must be wide enough to
encompass target maneuver and errors in reporting detection, and hence, as a result of
low sample rate, they may expand enough to accept reports on other targets or on un-
canceled clutter, resulting in multiple split tracks or loss of track on the desired target.
The signal to noise energy ratio is limited to that received on a scan past the target, and
hence the inherent accuracy of the individual reports is low, compared with trackers
which dedicate more time to an individual target. The position errors on a given scan are
inherently greater, because of target scintillation, than with on axis tracking radar using
such techniques as mono-pulse. The vulnerability to angle jamming techniques is greater
than with dedicated trackers, both because of sensitivity to amplitude modulation and
lower signal to noise energy ratio resulting from spreading of radar energy into many beam positions.

Choosing a suitable tracking coordinate system is the first problem a TWS radar has to confront. It is desirable that a fixed reference of an inertial coordinate system be adopted. The radar measurements consist of target range, velocity, azimuth angle, and elevation angle. The TWS system places a gate around the target position and attempts to track the signal within this gate. The gate dimensions are normally azimuth, elevation, and range. Because of the uncertainty with the exact target position during the initial detections, a gate has to be large enough so that targets do not move appreciably from scan to scan; more precisely, targets must stay within the gate boundary during successive scans. After the target has been observed for several scans the size of the gate is reduced considerably as the track fidelity is greatly improved.

Gating is used to decide whether an observation is assigned to an existing track file, or to a new track file such as for a new detection. Gating algorithms are normally based on computing a statistical error distance between a measured and an estimated radar observation. For each track file, an upper bound for this error distance is normally set. If the computed difference for a certain radar observation is less than the maximum error distance of a given track file, then the observation is assigned to that track.

All observations that have an error distance less than the maximum distance of a given track are said to correlate with that track. For each observation that does not correlate with any existing tracks, a new track file is established accordingly. Since new measurements are compared to all existing track files, a track file may then correlate with no observations or with one or more observations. The correlation between observations
and all existing track files is identified using a correlation matrix. Rows of the correlation matrix represent radar observations, while columns represent track files. In cases where several observations correlate with more than one track file, a set of predetermined association rules can be utilized so that a single observation is assigned to a single track file.

Any respectable paper that discusses radar tracking and trajectory needs to briefly touch on the Kalman Filter so in the next few paragraphs, I will introduce that concept. I am leaving it up to the reader to find more information if they wish to.

The role of the Kalman Filter is to take the current known state such as position, heading, speed and possibly acceleration of the target and predict the new state of the target at the time of the most recent radar measurement. In making this prediction, it also updates its estimate of its own uncertainty derived from inherent errors found in this prediction. It then forms a weighted average of this prediction of state and the latest measurement of state, taking account of the known measurement errors of the radar and its own uncertainty in the target motion models. Finally, it updates its uncertainty of the state estimate. A key assumption in the mathematics of the Kalman Filter is that the measurement equations (the relationship between the radar measurements and the target state) and the state equations (the equations for predicting a future state based on the current state) are linear. There exists a non-linear version of the Kalman Filter such as Reference 17.

The Kalman filter is a recursive estimator. This means that only the estimated state from the previous time step and the current measurement are needed to compute the estimate for the current state. In contrast to batch estimation techniques, no history of
observations and/or estimates is required. When performing the actual calculations for the filter, the state estimate and covariances are coded into matrices to handle the multiple dimensions involved in a single set of calculations. This allows for representation of linear relationships between different state variables such as position, velocity, and acceleration in any of the transition models or covariances.

The Kalman Filter assumes that the measurement errors of the radar, and the errors in its target motion model, and the errors in its state estimate are all zero-mean Gaussian distributed. This means that all of these sources of errors can be represented by a covariance matrix. The mathematics of the Kalman Filter is therefore concerned with propagating these covariance matrices and using them to form the weighted sum of prediction and measurement.

The α-β tracker is a fixed gain formulation of the Kalman filter and is still widely used because it is easy to implement and performs well in general. The α-β Tracker equations for range tracking are defined below,

Smoothing:

\[
R'_{n} = R'_{pn} + \alpha(R_n - R'_p) \\
\]

Equation 9.1

\[
V'_{n} = V'_{pn} + \frac{\beta}{t_{sample}}(R_n - R'_p) \\
\]

Equation 9.2

Prediction:

\[
R'_{p(n+1)} = R'_{n} + V'_{n} \times t_{sample} \\
\]

Equation 9.3
\[ V'_{p(n+1)} = V'_n \]  

Equation 9.4

Where: \( = R'_n \) is the Smoothed Estimate of Range, \( V'_n \) is the Smoothed Estimate of the Range Rate, \( R_n \) is the Measured Range, \( R'_{p(n+1)} \) is the Predicted range after T seconds, \( V'_{p(n+1)} \) is the Predicted Range Rate after T seconds, \( R'_{pn} \) Predicted range at the Measurement Time, \( V'_{pn} \) is the Predicted Velocity at the Measurement Time, \( t_{sample} \) is the Sample Time and \( \alpha \) and \( \beta \) are Smoothing Constants.

In situations where the target motion conforms well to the underlying model, there is a tendency of the Kalman Filter to become "over confident" of its own predictions and to start to ignore the radar measurements. If the target then maneuvers, the filter will fail to follow the maneuver. It is therefore common practice when implementing the filter to arbitrarily increase the magnitude of the state estimate covariance matrix slightly at each update to prevent this.

Since we touched upon target motion, we can discuss Moving Target Indicator (MTI) which begins with the sampling of two pulses. This starts immediately after the radar transmission ends and sampling continues until the next transmit pulse begins. If an object is moving in the location corresponding to both samples, then the signal reflected from the object will survive this process, otherwise the two samples will cancel and very little signal will remain if all objects are stationary at that distance. In order for MTI to work, the initial phase of both transmit pulses must be sampled and the 180 degree phase rotation must be adjusted to achieve signal cancellation on stationary objects.
Minimum detectable velocity (MDV) is a fundamental consideration for the design, implementation, and exploitation of moving-target indication (MTI) radar imaging modes. All single-phase-center air-to-ground radars are characterized by an MDV, or a minimum radial velocity below which motion of a discrete non-stationary target is indistinguishable from the relative motion between the platform and the ground. Targets with radial velocities less than MDV are typically overwhelmed by endo-clutter ground returns, and are thus not generally detectable. Targets with radial velocities greater than MDV typically produce distinct returns falling outside of the endo-clutter ground returns, and are thus generally discernible using straightforward detection algorithms.

Target velocity in MTI radar imaging modes is usually measured with respect to the scene center. The scene center has a range rate that is dictated by the platform velocity and imaging geometry:

\[
\frac{dr_s}{dt} = v \cos \theta \cos \varphi \quad \text{Equation 9.5}
\]
Hence, a zero-velocity target is a target whose absolute radial velocity is identical to that of the stationary scene center. Note that a zero-velocity target may actually be non-stationary; it will have a radial range rate that is equal to that of the scene center, but it may have a large tangential velocity component.

A pure broadside-looking imaging geometry, as depicted in Figure 9.4, is defined as any geometry in which squint angle $\psi$ is $90^\circ$ or $270^\circ$. It follows that the scene center in any broadside geometry has an absolute radial velocity of 0. However, any point in the leading direction of the projected beam such as point $A$ in Figure 9.5 has a negative instantaneous radial velocity, because the range between the platform and the point is decreasing. Similarly, any point in the trailing direction of the projected beam such as point $B$ has a positive radial velocity. Points $A$ and $B$ respectively attain approximately the minimum and maximum range rates of any illuminated, range-gated ground points in

Figure 9.4 Arbitrary geometry for GMTI returns

(Cite Ref 25)
a broadside-looking geometry. All imaged ground points will thus have relative range rates depending on their relative position between points $A$ and $B$.

**Figure 9.5 Broadside-looking geometry.**

(Cite Ref 25)

Point $A$ represents the leading edge of the beam, which attains the greatest negative range rate; point $B$ represents the trailing edge of the beam, which attains the greatest positive range rate.

A pure forward-looking imaging geometry, as depicted in Figure 9.6, is defined as a geometry in which squint angle $\psi$ is 0 degrees. This leads to a different MDV than that obtained in a broadside imaging geometry.
Figure 9.6 Forward-looking geometry

(Cite Ref 25)

Point $C$ represents the trailing edge of the range gate, which attains the greatest negative relative range rate (with respect to the scene center); point $D$ represents the leading edge of the range gate, which attains the greatest positive relative range rate (with respect to the scene center).

\[
\frac{dr_0}{dt} = v \cos \theta
\]  

Equation 9.6

We also need to find a way to calculate the velocity resolution for our radar. Otherwise, how can we be sure we found the lost hiker if we can’t resolve the hiker’s movements from say the wind blowing tree limbs or animals walking in the area being searched?

In the next few pages, one possible way to detect a target return that is below the radar threshold and obscured by background noise will be discussed.
In Figure 9.7, I see the results of 5 sets of radar scans over an arbitrary space. Notice that the noise is masking an unseen target return. The return is so small that it is impossible to attribute this return to anything but noise instead of establishing it as a legitimate target. The problem I face here is how to separate the background noise from the return of the target.

In order to separate the noise from the target return, I calculated the moving average of the 5 sets of noise and plotted those results in Figure 9.8. Since the average signal from the target is zero at all times except when a return is received, there should be a single sharp peak at that point in time.
Figure 9.8 Moving Average of Noise Reveals the Target Return

Figure 9.8 shows the moving average in blue, the average target return signal and the same detection threshold as seen in Figure 10.1. Now that the noise level from 5 separate scan sets has been averaged over time, the target return is much easier to pick out. What if the target return is very weak and continuously tracked?
Figure 9.9 Continuously Tracked Target With Returns Lower Than Noise

In Figure 9.9, a target enters the search sector of the radar and lingers for some period of time. However, the target is very far away and thus, has a low radar return that would be easily masked by noise like we saw in Figure 9.7. After all, by using a moving average of the noise, the resulting plot is cleaned up. Notice though that the target return is still lower than some of the noise peaks but more importantly, pay attention to how the noise is fairly “uniform” with time (i.e. does not favor any particular value). Now that we know what the noise should look like, we can easily spot the abnormal signal that is absent for the first half of the scan time but makes an abrupt appearance. This strange signal then maintains a fairly constant level for the rest of the scan time. Albeit, this is not
enough information to classify the strange signal as a target but it should raise red flags and warrant further scrutiny. In reality, as time passes, the target return level will change based on the range to the target and the target returns strength. However, this change will not discontinuous and should still be identifiable.

Figure 9.10 Continuously Changing Target With Returns Lower Than Noise

Figure 9.10 shows a target whose return strength is changing with time. Again, the noise is fairly “uniform” but more importantly, the noise has very abrupt jumps while our target return signal is more constant. While the signal from the target may look like the rise in background noise attributed with sunrise through sunset, it all depends on the length of time discussed. Typically, a target will be in the view of the radar for a matter of minutes, much too short a time to see a rise like what is seen here. As discussed in
Figure 9.9, this return signal is not enough to classify this as a target but should still raise suspicion and warrant further scrutiny.

Figures 9.7 through 9.10 have shown a way with which to detect a possible target below the threshold of a radar but this method would require advanced algorithms or a trained individual to physically inspect the radar return plots.

There are a myriad of other ways to better resolve a target hidden within background noise. The two I am discussing here are Moving Target Indicator (MTI) and Pulse Doppler Processing, both of which are subsections of Doppler processing.

First, MTI Filtering operates across pulse repetition intervals for a fixed range sample with the purpose of detecting moving targets which will be of use for us in our R&D radars in the next section.

MTI filter is designed to reject the dominant DC clutter return and pass target Doppler frequencies and thus, detects moving targets. The principle of operation is fairly simple and straightforward. It first assumes clutter is stationary from pulse to pulse and then subtracts the previous pulse from the current pulse. Any moving targets will have phase difference and will not cancel in the subtraction. The major drawback of this kind of filter is that it requires transmitter stability from one pulse to the next which plays right into our first design challenge in the next chapter. Also, since the digital filter is periodic with the sample pulse repetition frequency, nulls at zero Doppler also appear at multiples of the PRF. Thusly, targets moving at speeds that result in a Doppler equal to a multiple of the PRF will be attenuated. These speeds are called blind speeds.

\[
v_{\text{blind}} = \frac{c}{2f_c} \times \text{PRF}
\]

Equation 9.7
To overcome this, Staggering PRF’s allow an increase in the first blind speed without degrading unambiguous range. PRF staggering can be implemented using two different approaches. 1) Pulse-to-Pulse Stagger or 2) Dwell-to-Dwell Stagger. MTI filtering uses pulse-to-pulse staggering which increases Doppler coverage in a single dwell while the non-uniformly sampled sequence prevents coherent Doppler filtering. This type of approach is used mostly with low PRF modes. Pulse Doppler Processing uses dwell-to-dwell stagger with multiple dwells required to resolve ambiguities. This action preserves coherent Doppler filtering.

Rather than a single filter being applied to the pulse repetition interval as in MTI, Pulse Doppler Processing generates a complete spectral analysis of the PRI data for each range bin. Spectral analysis is then performed with a Fast Fourier Transform of the PRI data. The Fast Fourier Transform is equivalent to a bank of filters across the Doppler frequencies. Filter response of an unweighted FFT exhibits the typical 13 dB peak side lobe level. However, weighting of PRI samples before the FFT will reduce the side lobes but has the unfortunate effect of broadening the filter bandwidth because wider bandwidths allows more noise in Range/Doppler sample which reduces our Signal to Noise Ratio which in turn reduces our detection range.

Dwell-to-Dwell Stagger uses multiple PRF’s to resolve blind speeds. The advantages include clutter can be canceled with coherent MTI and transmitter stability is not as critical compared to pulse-to-pulse staggering. However, the disadvantages are significant in that it Consumes radar resources and causes an increased timeline for radar
signal processing. Something highly undesirable for our second scenario in the next chapter.
II. Research and Design

10. Tying Everything Together Using a Model

Before I jump straight from the background information into the simulation and results, I think it is prudent to discuss the link between the two. Mainly by explaining the connections and assumptions made using a model.

10.1 Assumptions

I made several assumptions in order to simplify the model and resulting simulation. Some are given in this section and some in section 10.2. These include:

a) There are no strenuous circumstances influencing our radar system such as inclement weather, atmospheric effects, jamming, etc.

b) The Side Lobes and Back Lobes do not return significant amounts of unwanted data and play a small role overall. Furthermore, the antennae pattern of the system can be represented by Figure 10.1.1.
The main beam (or main lobe) is the region around the direction of maximum radiation (usually the region that is within 3 dB of the peak of the main beam). The main beam in Figure 10.1.1 is northbound.

The sidelobes are smaller beams that are away from the main beam. These sidelobes are usually radiation in undesired directions which can never be completely eliminated. The sidelobe level (or sidelobe ratio) is an important parameter used to characterize radiation patterns. It is the maximum value of the sidelobes away from the main beam.
main beam and is expressed in Decibels. One sidelobe is called backlobe. This is the portion of radiation pattern that is directed opposing the main beam direction.

c) The antennae pattern is based on the cosecant squared formula.

10.2 Connections and Explanations

Figure 10.2.1 Ground Based Antenna Searching for Elevated Targets

Figure 10.2.1 shows some of the possible problems with ground based radars when clutter (represented by the tree) is brought into the environment. This has the effect of degrading radar performance and limiting detection range. As I could not find a satisfactory explanation, equation or argument for how much clutter degrades performance, I am assuming that the environment I intend to simulate, if void of large clutter that could obscure a target with similar RCS to that of a person. Trying to model clutter in the system would be an undertaking beyond the scope of this thesis as different kinds of clutter each have a unique way they influence radar waves and not every type of clutter will be represented in any one environment. Basically, there are too many possible combinations of clutter/environment to sufficiently model with the current setup. This is why I am making the assumption that clutter will not play a major role in my results.
However, in a real world field environment, the effects clutter has on the system will be of consequence. Ground clutter can be avoided if the radar is positioned below the target and is looking at some angle above the horizontal as depicted in Figure 10.2.1.

![Elevated Antenna Searching for Targets on the Ground or Originating from the Ground](image)

**Figure 10.2.2 Elevated Antenna Searching for Targets on the Ground or Originating from the Ground**

Figure 10.2.2 shows how one can avoid ground clutter by raising your antenna above the clutter level. This can be accomplished either by raising the transmitter/receiver up on a stand or by mounting the system to an airborne object such as an Unmanned Aerial Vehicle or Helicopter. The second scenario presented in Chapter 11 has this setup and thus, avoids much of the clutter degradation that is possible in the first scenario of Chapter 11. By raising the antenna or mounting it to an aerial vehicle, the sidelobes will return data at large ranges that could be confused with an actual target signal return. A workaround for this is to position the antenna in a slight depression so that the sidelobes will strike the ground and/or clutter relatively close to the radar. A final way, which has been discussed previously, is to use Doppler Processing to remove much of the ground clutter. This is the approach chosen to be modeled and incorporated into the later simulation.
Since we wish to detect a target signal in any future simulation, we first need to understand what variables we need to model and understand why we are them.

**Figure 10.2.3 Radar Equation and Detection Process Model**

(Cite Ref 16)

Figure 10.2.3 shows the hierarchical structure between processes and variable that we will need to include in the simulation later.

Some of the distinguishing characteristics of this model are that it is static (does not model in time) with parts being deterministic (repeatable) with other parts being stochastic (uses pseudo random numbers to create a random result). The static aspect of this model and resulting simulation is that the radar system is not modeled to run over a time period but instead takes a “snapshot” in time. Parts of the model and simulation are deterministic because the user can determine the inputs and resulting outputs are
calculated along a “linear” pathway while other parts are stochastic and rely on random numbers to generate results. However, the entire model and simulation are continuous in that the program integrates from 0 meters up to the maximum range that the user defines.

Now since a model is an abstraction that emphasizes certain dynamics of the system, I have chosen to proceed with the dynamics shown in Figure 10.2.3.

First, let’s conceptually model the problem we have chosen to analyze. I believe a good starting point would be to take a closer look at each input used in the Radar Equation. Our radar parameters must be chosen in such a way as to reflect physical reality while staying within our limits.

10.2.1 Radar Parameters

1) Our transmitter power needs to be fairly low in order for the system to be carried in a backpack or to be mounted on a helicopter. Here, we make our first assumption and that is the system will be very limited as to the resources available for use.

2) Our next assumption is that the antenna gain can be described by the Equation 3.5

3) As for our frequency, well this requires some thought. Using Figure 10.2.4, we can narrow down the frequency ranges we desire based on the wavelength of our radar wave.
Since one of our prospective targets is similar in size and composition to a human, we want a wavelength on the order of fractions of a meter. The exact wavelength is highly dependent upon the composition of the target so let’s not worry with what our target is made of. The two bandwidths I chose to use reflect the nature of both classifying a target and using discrimination to determine features of the target. The first wavelength I have chosen is 0.2 meters (L-Band) which will allow us to classify the target. The second wavelength is 0.027 meters (X-Band) which allows us to see more detail on the target such as size, appendages, surface features, etc.
4) Next, our pulsewidth needs to be fairly small so that we don’t spend a lot of time transmitting when we need to be listening. This minimizes the problems of blind zones.

5) I believe the Pulse Compression Waveform will be best suited to this task as it extends the range as your bandwidth increases. Also, let’s improve our model by training Coherent Pulses together will give better range and Doppler resolution. Due to this, the results listed later in the paper will tend to lean more on Coherent Techniques versus Non-Coherent techniques.

10.2.2 Target Characteristics and Radar Cross Section

The reflectivity of a target depends upon the size, shape, aspect, and dielectric properties of that target. It includes not only the effects of reflection but also scattering and diffraction. Reflectivity does have a few limitations. Anomalous propagation can contaminate some of the data and since the Earth's surface is curved, the radar beam is continually climbing in altitude as it travels further from the transmitter. This can cause overshooting of distant targets resulting in missing data.

Now for determining the Radar Cross Section of a person, animal, insect, etc., one must take into account the target’s ability to reflect radar signals in the direction of the radar receiver. Radar Cross Section is a measure of the ratio of backscatter power per steradian in the direction of the radar coming from the target to the power density that is intercepted by the target. The conceptual definition of RCS includes the fact that not all of the radiated energy falls on the target. A target’s RCS is most easily visualized as the product of three factors:
\[
\sigma = \text{Projected cross section} \times \text{Reflectivity} \times \text{Directivity}
\]

with Reflectivity being the percent of intercepted power reradiated (scattered) by the target and Directivity being the ratio of the power scattered back in the radar's direction to the power that would have been backscattered had the scattering been uniform in all directions (i.e. isotropically). The RCS of a target can be viewed as a comparison of the strength of the reflected signal from a target to the reflected signal from a perfectly smooth sphere of cross sectional area of 1 m².

For some of the Physics behind the RCS, I found that a dielectric property of \( \varepsilon_r = 50 \) was the closest I could find for the dielectric properties of skin. This resulted in a \( \sigma = 1 \text{ S/m} \) for the body dielectric material, which is close to the skin dielectric properties.
From Figure 10.2.5 and previous discussions, the RCS of a person in ambiguous depending on the circumstances, it is difficult to pin down an exact value of the RCS of a person. However, many results tend to have the RCS of a person anywhere from 0.15 square meters up to 1 square meter.

The backscatter value of the two targets to be used in the simulation later in this paper was approximated from the previous discussion and using the following rule of thumb categories developed by NASA.

- **Very high backscatter** (above -5dB)
  - Terrain slopes towards radar
  - Radar looking almost straight down
- Very rough surface
- Flooded forest
- Man-made objects

- **High backscatter** (-10 to 0dB)
  - Rough surface
  - Dense vegetation, e.g. forest

- **Moderate backscatter** (-20 to -10dB)
  - Moderately rough surface
  - Medium level of vegetation, e.g. shrubs, agricultural crops

- **Low backscatter** (below -20dB)
  - Terrain slopes away from radar
  - Smooth surface, e.g. water, road surface
  - Very dry material, e.g. sand in an arid desert

Now that we are armed with this information, let us make another assumption.
Let’s assume that we have a smooth surface that does not diffuse our radar wave a great deal and that ground clutter is kept to a minimum. This way, we can focus more on the Physics of the system and not that of the environment. In other words, the system is operating under a non-strenuous environment such as no degrading atmospheric effects like hard precipitation, sandstorms, etcetera and ground clutter plays a minor role.

The intensity of the backscattered energy that has the same polarization as the radar's receiving antenna is used to define the target RCS. When a target is illuminated by RF energy, it acts like an antenna, and will have near and far fields. Waves reflected and
measured in the near field are, in general, spherical. Alternatively, in the far field the wavefronts are decomposed into a linear combination of plane waves. (Cite Ref 1)

Now assume the target re-radiates the radar energy equally in all directions and that the RCS will be monostatic (the incident and scattered angles are similar). Thus, the total target RCS can be calculated using,

\[
\sigma_t = \frac{1}{4\pi} \int_0^{2\pi} \int_0^\pi \sigma(\theta_s, \phi_s) \sin \theta_s \, d\theta \, d\phi_s
\]

The amount of backscattered waves from a target is proportional to the ratio of the target extent (size) to the wavelength, \( \lambda \), of the incident waves. In fact, the radar will not be able to detect targets much smaller than its operating wave-length. (Cite Ref 1)

<table>
<thead>
<tr>
<th>Scatterer</th>
<th>Aspect</th>
<th>Radar cross section</th>
<th>Definition of symbols</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sphere ................</td>
<td>Axial</td>
<td>( \sigma = \pi a^2 )</td>
<td>( a = ) radius</td>
</tr>
<tr>
<td>Cone ..................</td>
<td>Axial</td>
<td>( \sigma = \frac{\lambda^2}{16\pi} \tan^4 \theta_h )</td>
<td>( \theta_h = ) cone half angle</td>
</tr>
<tr>
<td>Paraboloid ...........</td>
<td>Axial</td>
<td>( \sigma = 4\pi z_o^2 )</td>
<td>( 2z_o = ) apex radius of curvature</td>
</tr>
<tr>
<td>Prolate spheroid ......</td>
<td>Axial</td>
<td>( \sigma = \frac{\pi b_o^4}{a_o^2} )</td>
<td>( a_o = ) semimajor axis</td>
</tr>
<tr>
<td>Ogive ..................</td>
<td>Axial</td>
<td>( \sigma = \frac{\lambda^2}{16\pi} \tan^4 \theta_h )</td>
<td>( b_o = ) semiminor axis</td>
</tr>
<tr>
<td>Circular plate ........</td>
<td>Incidence at angle ( \theta ) to normal</td>
<td>( \sigma = \pi a^2 \cot^2 \theta \frac{J_2(\frac{4\pi a}{\lambda} \sin \theta)}{\frac{4\pi A^2}{\lambda^2}} )</td>
<td>( \theta_h = ) half angle of target noise cone</td>
</tr>
<tr>
<td>Large flat plate of arbitrary shape</td>
<td>Normal</td>
<td>( \sigma = \frac{4\pi A^2}{\lambda^2} )</td>
<td>( a = ) radius of plate</td>
</tr>
<tr>
<td>Circular cylinder ....</td>
<td>Incidence at angle ( \theta ) to broadside</td>
<td>( \sigma = \frac{a\lambda \cos \theta \sin^4 (kL \sin \theta)}{2\pi \sin^2 \theta} )</td>
<td>( a = ) radius ( L = ) cylinder length</td>
</tr>
</tbody>
</table>

Figure 10.2.6 Formulas for RCS

(Cite Ref 5)
With RCS examined, we assume that the Aspect Angle allows the radar to see the maximum Radar Cross Section of the target.

The RCS chosen for that of a person was pulled from the IEEE Paper on “Measurement of the Radar Cross Section of a Man” by Schultz, Floyd V., Burgener, R. C. and King, S. Their findings for different frequencies indicated that the best choice for the RCS of a person was about 1 square meter.

10.2.3 Range

The range on our radar needs to be carefully examined because the strength of the return signal is inversely proportional to the range to the fourth power. Based on the possible uses such as those listed below and circumstances that this radar may be used in, I believe that modeling out to 10 kilometers will be sufficient for the human RCS and 2.5 kilometers in the case of the bullet.

- Common uses include;
  - Conservation by tracking animal movements and patterns
  - Automated search for lost or escaped persons
  - Early warning of attack for military outposts
  - Detection and Tracking of small sized targets such as animals, birds, insects, etc.

10.2.4 Properties of the Propagation Medium
Since we intend to use our radar to detect terrestrial objects, our propagation medium will be the atmosphere. This means we have to worry about atmospheric effects such as attenuation, precipitation, refraction and interference. This is not all the possible effects but they representative of the problems we face. In some cases, we will need to also worry about Rayleigh Scattering.
Figure 10.2.7 Rayleigh Scattering Particle Size vs. Wavelength

(Cite Ref 33)

With our wavelengths being 0.2 meters and 0.027 meters represented by the two vertical lines to the right hand side of Figure 10.2.7, we can determine what types of airborne objects might degrade the radars ability to perform it’s tasks. Armed with this
information, we can say that as long as we continue to assume a non-strenuous environment (not raining or hailing), atmospheric scattering is negligible.

![Specific Attenuation for Atmospheric Oxygen and Water Vapour](image)

**Figure 10.2.8 Specific Attenuation for Atmospheric Oxygen and Water Vapor**

(Cite Ref 34)

Due to the short ranges I intend to emulate, attenuation and refraction will not be of huge concern. With a wavelength of 0.027 meters, we get a frequency of 11.1 GHz which puts us right near one of the attenuation peaks for Oxygen. However, even at this peak, we are losing approximately 2 dB per kilometer for oxygen and water vapor combined. With a wavelength of 0.2 meters (1.5 GHz frequency), atmospheric attenuation is nearly non-existent. With the second scenario, that of detecting the bullet, our frequency is roughly 60 GHz which puts us right in the middle of the largest spike for atmospheric Oxygen attenuation. Before we panic though, take into account the nature of
the target. A bullet is extremely small and by extension, has a very small RCS. This in turn results in a low signal return thus severely limiting our detection range to fairly close to the radar aperture. Due to these reasons, I chose to not include attenuation in my upcoming simulation. A possible work around to avoiding attenuation is simply choosing a different frequency to operate at.

With the arguments presented in sections 10.2.1 through 10.2.4, we have a sufficient model for the Radar Equation that outputs the SNR. Keep in mind that the Radar Equation is only part of the overall picture. We must also consider the detection process itself.

### 10.2.5 Target Fluctuation Statistics

Since we are using Coherent techniques for our waveform, this limits our target categories. Before, we could choose a target in any of the four Swerling classes but now, we are limited to only Swerling I and Swerling III target categories. This is because Swerling II and Swerling IV target return behave similarly to noise due to their fast fluctuation (fluctuating from pulse to pulse). Before we continue, let’s narrow our target type down to one category. Since most points on a target with a similar RCS to that of a person will most likely be returning dissimilar RCS and amplitudes, we can make the assumption that the only Swerling Target category we will need will be that of the Swerling III.

For this model, targets with similar RCS to that of a person would move fairly slow and would fit the Swerling I category while targets such as a bullet, if spin stabilized
by rifling within the weapon, would also fit into a Swerling I category making Coherent Integration possible for both targets. However, if the bullet is tumbling such as a mini-ball from a musket, then the RCS would change rapidly, perhaps from pulse to pulse, and as a result would be a Swerling II Target and require Non-Coherent Integration.

10.2.6 Noise Statistics

When using Coherent Integration techniques, the signals and the noise is added together and integrated before the detection check is performed. These noise signals grow and shrink but by assuming the noise is zero-mean and Gaussian, the noise signals will cancel when the integration is performed. Thus, we do not need to worry about integrating noise over several pulses thanks to this assumption.

10.2.7 Detection Threshold

For the purposes of this model, I assume the Detection Threshold is a fixed value over all ranges as will be shown in later figures. However, a Detection Threshold in general can either be fixed over time of vary depending on circumstances. If one raises the threshold, they lower the probability of false alarms but at the same time, they will dramatically reduce the range at which detection of a legitimate target is possible. If one lowers it too much, false alarms will run rampant in the system rendering the radar little more than an expensive paperweight. In this thesis, I choose to calculate the threshold using the recursive formula used in the Newton-Raphson method seen below.
\[ V_{T,m} = V_{T,m-1} - \frac{G(V_{T,m-1})}{G'(V_{T,m-1})} \quad ; \quad m = 1, 2, 3, \ldots \]  

Equation 10.2.7.1

Where,

\[ G(V_{T,m}) = (0.5)^{n_p/n_p} - \Gamma(I(V_T, n_p)) \]  

Equation 10.2.7.2

\[ G'(V_{T,m}) = -\frac{e^{-V_T} V_T^{n_p-1}}{(n_p-1)!} \]  

Equation 10.2.7.3

\[ V_{T,0} = n_p - \sqrt[n_p]{n_p} + 2.3 \sqrt[n_p]{-\log P_{fa}} \left( \sqrt[n_p]{-\log P_{fa}} + \sqrt[n_p]{n_p} - 1 \right) \]  

Equation 10.2.7.4

The Threshold Voltage is thus based off of the Number of Pulses to be Integrated which is itself a function of scan time, the Probability of False Alarm, the False Alarm Number and the previous Threshold Voltage. If there is no previous Threshold Voltage, then \( m = 1 \) and we only need the Number of Pulses to be Integrated and the Probability of False Alarm.

With the modeling description and arguments in this chapter, the end result is the Probability of Detection and the Probability of False Alarm, both of which will be calculated by the simulation that follows.

### 10.3 Coherent and Non-Coherent Integration
To make a long explanation short, Coherent and Non-Coherent Integrators differ in that, Coherent Integrators become very cumbersome, costly and inefficient when the number of pulses to be integrated is very large where as Non-Coherent Integrators do not. Also, the design and placement of the integrator hardware within the radar differs for the two types of integrators.

![Figure 10.3.1 Coherent Integrator Placement](image)

In Coherent Integration, the integrator is placed before the amplitude detector and as a result, the phase information is not lost. The placement of the coherent integrator and the way the integrator functions can lead to large, costly, unwieldy and impractical integrators if the radar has a slow scan time resulting in large amounts of time spent radiating a target. Some of this can be overcome but limiting the number of stored pulses in the memory of the integrator places an upper limit to the number of pulses we can use to detect a target.

In analog processors, the integration (summation, accumulation) is accomplished by filters while it is accomplished by Fast Fourier Transformations (FFTs) in digital signal processors. The noise on each pulse is zero-mean and Gaussian and the noise
samples from each pulse are uncorrelated. Thus the noise out of the coherent integrator has the same statistical properties of the noise out of the matched filter from the IF-amplifier.

The IF amplifier has the capability to vary both the bandpass and the gain of a receiver. After conversion to the intermediate frequency, the signal is amplified in several IF-amplifier stages. Most of the gain of the receiver is developed in the IF amplifier stages. The overall bandwidth of the receiver is often determined by the bandwidth of the IF stages. Gain must be variable to provide a constant voltage output for input signals of different amplitudes.

We assume that the signal level at the input to the coherent integrator is constant from pulse to pulse. This is indicative of a Swerling 0, Swerling 1 or a Swerling 3 target. These signal levels will be added in the integrator. The specific amplitude over the $n_p$ pulses integrated by the coherent integrator is governed by the probability density function for the specific target type. So we can essentially consider the output of the coherent integrator as the return from a single pulse whose SNR is $n_p$ times the SNR provided by the standard radar range equation.

Coherent integration offers no benefit for Swerling 2 and Swerling 4 targets. This stems from the fact that the signal of these targets is not constant from pulse to pulse but, instead, behaves like noise and is constantly changing.

Coherent integration gain is the effect obtained by increasing the length of time during which a coherent signal is observed. In order to obtain coherent integration gain, the signal must be coherent.
In coherent integration, a perfect integrator is used to integrate some number of pulses in order to improve the SNR. Rapid target fluctuation, instability in the radar oscillator or propagation changes can all lead to coherent integration losses which is undesirable as we are trying to minimize loss. Due to the changing aspects of the target and environment, coherent integration is unsuitable for use over a large number of pulses. The resulting improved SNR can be calculated according to the process shown in Figure 10.3.2.

**Figure 10.3.2 Non-Coherent Integrator Placement**

Notice in Figure 10.3.2 that the Non-Coherent Integrator is located after the amplitude detector. In doing so, we lose the phase information about our radar wave but we gain the ability to integrate any number of radar return pulses we wish, thus overcoming the upper limit we had to introduce on the Coherent Integrator. In solving one problem, we introduce new problems though. Non-Coherent Integrators have their own host of problems including but not limited to; loss due to integration and inefficiency at low \( n_p \). Non-coherent integration gain is the effect obtained by averaging together signal estimates taken during successive time slices, each having the same, fixed length.

The third way to extend a radars’ detection range is to use ground bounce returns to amplify your gain. Using this technique under ideal circumstances can nearly double your detection range. Ground Bounce is the reflection off the mismatch from the earth/air interface and ground foliage, and refraction into the earth, reduced beam
distortion (ground focusing/defocusing), dissipation and signal attenuation, signal distortion, etc.

In many environments, ground reflection may not be possible due to vegetation, large boulders, structures and other objects that prevent the radar beam from cleanly reflecting off the surface. Due to this, we will not include ground bounce in any of our calculations.

Whatever the case, the important fact to be discussed is the detection threshold of the radar. This facet of a radar can usually be optimized (i.e. changed) depending on the situation. Hence, one can tune a radar’s detection threshold lower, giving us a better signal to noise ratio. Yet, by lowering the detection threshold of the radar, one is inviting false detections. This would be a nightmare as to many false detections would cause distrust in the radars ability to perform the search and negate all the work we have done here.

10.4 Refraction

When analyzing radar data, it is often helpful to know the height of the radar beam at any range. Assuming a standard atmosphere where moisture, temperature, and pressure all decrease with altitude, the height of a radar beam can be calculated using an equation developed by the National Weather Service. With a standard atmosphere, radar waves are refracted downward which causes fault in the distance and height measurements.
If we approximate the refractive index of air to be about 1.0, then we can calculate the Beam Centerline Height.

\[ Beam \ Centerline \ Height = \left( Slant \ Range \times \sin \theta_e \right) + \left( \frac{Slant \ Range^2}{2} \times refractive \ index \times R_{\ Earth} \right) \]

In Table 10.4.1, we see various Beam Centerline Heights based on the slant range and Elevation Angle. Figure 10.4.1 gives a graphical representation of this table so that one can approximate the Beam Centerline Height at various Elevation Angles.

<table>
<thead>
<tr>
<th>Slant Range 1</th>
<th>Slant Range 2</th>
<th>Elevation Angle</th>
<th>Beam Centerline Height 1</th>
<th>Beam Centerline Height 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 Km</td>
<td>2 Km</td>
<td>0 Degrees</td>
<td>&lt; 45 m</td>
<td>&lt; 20 m</td>
</tr>
<tr>
<td>5 Km</td>
<td>2 Km</td>
<td>2.5 Degrees</td>
<td>~ 210 m</td>
<td>~ 85 m</td>
</tr>
<tr>
<td>5 Km</td>
<td>2 Km</td>
<td>5 Degrees</td>
<td>~ 450 m</td>
<td>~ 170 m</td>
</tr>
<tr>
<td>5 Km</td>
<td>2 Km</td>
<td>15 Degrees</td>
<td>~1260 m</td>
<td>~ 520 m</td>
</tr>
</tbody>
</table>

**Table 10.4.1 Beam Centerline Height**
Figure 10.4.1 Beam Centerline Height as a Function of Radar Elevation Angle

The most important result from Figure 10.4.1 is that as the elevation angle increase beyond 5 degrees, one can approximate the predicted Beam Centerline Height as nearly linear. These values were calculated using parameters of the WSR – 88D radar system. Other types of radars will have slightly different results but not significantly enough to ignore the above approximation.

Of the three atmospheric variables that influence refraction (temperature, moisture, and pressure), moisture – or more specifically, water vapor – has the greatest effect on refraction. Temperature has the next greatest effects on refraction, followed by pressure. The simple rule of thumb for moisture is “more moisture means more refraction”.

Moisture and temperature can (and frequently do) work together to significantly alter refraction. The simple rule of thumb for temperature is “higher temperatures means less refraction”.

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Although pressure is one of the meteorological elements that influence refraction, its effects are small. Pressure variations alone provide no significant change in refraction.

From the arguments and results in this section, we should model the Elevation Angle as a very small angle on the order of fractions of a degree. However, we cannot ignore larger elevation angles because they contain important results and could be applicable in a multitude of situations.
11. Simulation and Scenarios

1. Searching for a Man Sized Target

<table>
<thead>
<tr>
<th>Variable Name</th>
<th>Variable Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Speed of Light</td>
<td>299790000 m/s</td>
</tr>
<tr>
<td>Boltzmann’s Constant</td>
<td>1.38E-23 joule/K</td>
</tr>
<tr>
<td>Peak Power</td>
<td>100 w</td>
</tr>
<tr>
<td>Frequency</td>
<td>11,100,000,000 Hz</td>
</tr>
<tr>
<td>Radar Diameter</td>
<td>0.5 m</td>
</tr>
<tr>
<td>Effective Aperture Area</td>
<td>0.196349541 m²</td>
</tr>
<tr>
<td>Wavelength</td>
<td>0.027008108 m</td>
</tr>
<tr>
<td>Gain</td>
<td>3382.606</td>
</tr>
<tr>
<td>Gain in dB</td>
<td>35.29251 dB</td>
</tr>
<tr>
<td>Range Resolution</td>
<td>5 m</td>
</tr>
<tr>
<td>Pulse Width</td>
<td>0.0000002 s</td>
</tr>
<tr>
<td>Bandwidth</td>
<td>5,000,000 Hz</td>
</tr>
<tr>
<td>Parameter</td>
<td>Value</td>
</tr>
<tr>
<td>-----------------------------------------------</td>
<td>----------------</td>
</tr>
<tr>
<td>Target Radar Cross Section</td>
<td>0.8 m²</td>
</tr>
<tr>
<td>Noise Temperature</td>
<td>290 K</td>
</tr>
<tr>
<td>Noise Figure</td>
<td>8 dB</td>
</tr>
<tr>
<td>Radar Loss</td>
<td>10 dB</td>
</tr>
<tr>
<td>Maximum Detection Range</td>
<td>10000 m</td>
</tr>
<tr>
<td>SNR Sensitivity</td>
<td>15 dB</td>
</tr>
<tr>
<td>Scan Time</td>
<td>2 s</td>
</tr>
<tr>
<td>Range Sensitivity</td>
<td>10 m</td>
</tr>
<tr>
<td>Azimuth Angle</td>
<td>90 degrees</td>
</tr>
<tr>
<td>Elevation Angle</td>
<td>5 degrees</td>
</tr>
<tr>
<td>Omega</td>
<td>0.137077</td>
</tr>
<tr>
<td>Radar 3dB Beamwidth</td>
<td>0.06752 degrees</td>
</tr>
<tr>
<td>Scan Rate</td>
<td>45 degrees/s</td>
</tr>
<tr>
<td>Time on Target</td>
<td>0.0015 s</td>
</tr>
<tr>
<td>Number of Pulses to be Integrated</td>
<td>15</td>
</tr>
<tr>
<td>Background Noise Figure</td>
<td>2E-14</td>
</tr>
<tr>
<td>Pulse Repetition Frequency</td>
<td>10000 Hz</td>
</tr>
<tr>
<td>Pulse Repetition Interval</td>
<td>0.0001 s</td>
</tr>
<tr>
<td>Unambiguous Range</td>
<td>14989.5 m</td>
</tr>
<tr>
<td>Time of False Alarm</td>
<td>600 s</td>
</tr>
<tr>
<td>---------------------------</td>
<td>-------</td>
</tr>
<tr>
<td>Probability of False Alarm</td>
<td>5E-09</td>
</tr>
<tr>
<td>False Alarm Number</td>
<td>2.08E+09</td>
</tr>
<tr>
<td>Probability of Detection</td>
<td>0.9001</td>
</tr>
<tr>
<td>Threshold Voltage</td>
<td>48.63171 w</td>
</tr>
<tr>
<td>Threshold in dB</td>
<td>15 dBw</td>
</tr>
<tr>
<td>Total Loss Due to NCI</td>
<td>2 dB</td>
</tr>
</tbody>
</table>

**Table 11.1.1 Inputs for the Simulation**

Something that I have neglected until now is a discussion of antenna efficiency. I'll briefly touch on it since the radar efficiency is used to calculate gain. Good radars typically have an efficiency of about 1 meaning that the physical surface area of the antenna is equal to its effective surface area; however, efficiencies as low as 0.7 are also very common. For the purposes of the following simulations, it is assumed that the radar efficiency is 1.

For our first research and simulation, suppose we wish to build a radar that can search, and detect a person in a range of environments while we ourselves are stationary. (Picture a search for an escaped criminal or a lost hiker in a blizzard). We wish to know the targets range and speed relative to our location at any point in time. We know from earlier that a man has a RCS of approximately $1 \text{ m}^2$ so let’s choose something slightly smaller and assume the wavelength of our radar is just under 3 cm. Finally, we want a
range resolution of less than 5m in order to successfully discern between objects at a distance.

I’ll chose a pulse width of $\tau \approx 2 \times 10^{-7}$ seconds because as you make the pulsewidth smaller, you limit yourself to how much energy you can transmit with each pulse which in turn, results in less reflected energy from a target. In short, very small pulsewidths make detection incredibly difficult. Also, we want a larger bandwidth for the simple fact that extra bandwidth allows for more information to be gathered hence why I chose the frequency $f = 5 \times 10^6$ Hz.

Now since this is a search radar, it needs to be very lightweight and portable which translates to small size and low power requirements. Let’s design the radar such that the surface area of our radar is about half a meter in diameter. Since we also have a need for low power consumption, we need an expression for the power used per pulse. Assuming we have a peak power supply of 100 watts, the single pulse energy is,

$$E = P_t \tau = \frac{(4\pi)^3 k T_r FR^2 SNR}{G^2 \lambda^2 \sigma} = 100 \text{ watts} \times (2 \times 10^{-7} \text{ seconds}) = 2 \times 10^{-5} \text{ joules.}$$

and the gain $G$ is given in Equation 6.1,

$$G = \frac{4\pi A_s}{\lambda^2} = \frac{4\pi (\pi \times 0.25^2)}{(0.027)^2} \frac{m^2}{m^2} = 3382.606$$

With the gain in decibels being,

$$G_{dB} = 10 \times \log_{10}(3382.606) = 35.3 \text{ dB}$$

Using Equation 6.3, we can calculate the average noise power of the environment that the radar is receiving.
Now that we are beginning to design our radar, we need to decide what detection range we want to use. For the purposes of this exercise, I have chosen a maximum range of 10,000 meters as the maximum desired detection range. If we assume the target of interest can be found anywhere from 1 meter to 10,000 meters away, we can begin calculating the radar signal return strength. Before we do that, we must also include data about the “dead zone” close to the radar.

When I refer to the “dead zone” of a radar, I am referring to the set of ranges where the radar receiver is not operating due to a pulse being transmitted. During the transmitting phase the radar cannot receive: the radar receiver is switched off using an electronic switch, called duplexer. The minimal measuring range, $R_{min}$ also called the blind range, is the minimum distance which the target must have to be detected. Any targets at a range equivalent to or less than the pulse width from the radar, will not be detected.

The percentage of time the radar spends in an active, transmit mode is the duty cycle and can be calculated as,

$$Duty\ Cycle = \frac{\text{pulse width}}{PRI} = \frac{2 \times 10^{-7}}{0.0001} = 0.002\%$$

The blind range can be calculated using,

$$R_{min} = \frac{c \times (\text{pulse width} + t_{\text{recovery}})}{2}$$

$$N = k \times T_s \times B_n = \left( 1.381 \times 10^{-23} \frac{1}{K} \right) (290 K)(5 \times 10^6 \text{Hz}) = 2.002 \times 10^{-14} \text{ Watts}$$

$$N_{dB} = 10 \times \log_{10}(1.244 \times 10^{-15}) \approx -140 \text{ dBw}$$
Where $t_{\text{recovery}}$ is the time needed for the monostatic (transmitter is the same as the receiver) radar to switch from transmit to receive mode.

The time required for recovery is determined by the time taken by the radar switch to de-ionize after each transmitted pulse. It is usually defined as the time required for the receiver to return to within 6 dB of normal sensitivity after the end of the transmitter pulse but some manufacturers use the time required for the sensitivity to return to within 3 dB of normal sensitivity. Recovery time is a factor that limits the minimum range of a radar because the radar receiver is unable to receive until the switch is de-ionized. In various radars, the recovery time may differ from less than 1 microsecond to about 20 microseconds. For the purposes of this research, let’s assume the Engineers want a recovery time of 2 microseconds.

$$R_{\text{min}} = \frac{(2.9979 \times 10^8 \text{ m/s}) \times (2 \times 10^{-7} \text{s} + 2 \times 10^{-6} \text{s})}{2} \approx 330 \text{ m} \approx 1080 \text{ feet}$$

It’s safe to say that any target that close to the radar can be easily found using other means. For a 1 microsecond recovery time, the minimum detection range is about 180 meters (590 feet). The strength of the return radar signal from the target can be calculated using Equation 6.2.

$$P_r = \frac{P_t}{4\pi R^2} G \frac{1}{4\pi R^2} \sigma A_g = \frac{(100 \text{ watts}) (3382.605) (0.8\text{m}^2) (\pi \times 0.25^2)}{16\pi^2 R^4} = 336.47 \times \frac{1}{R^4}$$

It’s evident that the power received is heavily dependent upon the range from the transmitter to the hiker. It’s impractical to use large, heavy radars for a task that requires constant movement so we must now find a way to minimize the radar loss while maximizing the gain and the SNR.
We will first assume Coherent Integration as Coherent and Non-Coherent give slightly different results for a series of key variables. Using the Signal to Noise Equation from Figure 6.2 and plugging in the variables from Table 12.1, we can calculate the base SNR of a person sized target out to 10,000 meters.

![Figure 11.1(a) Plot of the Base SNR](image)

In Figure 11.1, the return signal is very strong for short ranges, which is expected because of the $\frac{1}{R^4}$ dependence of the SNR. At these short ranges, the radar would have no problem detecting a man but we want more than that, we want to detect them kilometers away which as you can see, the SNR drops off substantially at higher ranges. If we convert the RCS of the human target from square meters to dBsm, we get approximately -0.97 dBsm. So with just the base SNR, we could detect a person at about
5300 meters away or about 3.29 miles away. However, we must worry about the threshold voltage. If we choose to keep this voltage constant, we must accept the fact that the number and frequency of false alarms may cause us some heart ache. All of this is of course not taking into account attenuation, clutter, interference and a series of other complicating factors, all which reduce the detection range. In cases where silence is paramount such as during military special operations, 5300 meters gives the team plenty of warning time to either move away or set up an ambush. For targets larger than a man such as vehicles, this range would be much further but for now, let’s get back to our person. There are multitudes of techniques to increase this range, far too many to name here, but several can be found in Reference 1.
As you can see in Figure 11.1(b), the SNR doesn’t change very dramatically between 50 to 150 watts of peak transmit power. Of course, 150 watts will increase our detection range but would it be viable if we are trying to maximize maneuverability and minimize cost and size? That’s the key question.
From Figure 11.1(c), one can see that increasing the wavelength results in a shortening of the detection range. As always, designing a competent radar system requires compromise. If we make the wavelength too small, the radar wave may pass through the target but to overcome that by increasing the wavelength, we shorten the range at which we can detect a target. The Designer must decide on how much range reduction is worth overcoming the possibility that the wave might pass through the target and not be reflected.
The measure of performance that is usually used to characterize search radars is called the Power-Aperture Product which we see in Figure 11.2a above. The Power Aperture Product is just that, the average power multiplied by the effective antenna area of the radar. The primary way to improve radar performance is to increase the average radiated power and/or the antenna aperture area. For example, search radar (which we are interested in here) performance scales with the power-aperture product $P_{av} \times A$ with track radar scaling in proportion to $P_{av} \times A^2$. By either increasing the radiated power or by increasing the aperture size, we increase our ability to detect the target. This could leave one to say, making a low power radar that satisfies most of our requirements set forth is fairly easy. Just increase the aperture area to very large sizes. Doing so would...
severely limit its portability, increase its cost immensely and be self-defeating for the types of situations we are analyzing.

**Figure 11.2(b) PAP in $\times \ m^2$ versus Range in Km**

Figure 11.2b is a different view of the Power Aperture Product as Figure 11.2a was in decibels. As long as the PAP of the target (Black solid line) is below that of the PAP for the range sensitivity, the radar return signal form the target will be sufficiently large to be detected by our theoretical design. As the solid black line surpasses the solid blue line, we begin to have to significantly worry about false alarms and detection among noise.
One of the main ways to extend a radar’s detection range is to use coherent integration techniques. With coherent integration, we insert a coherent integrator (signal processor) between the matched filter and amplitude detector. This signal processor samples the return from each transmitted pulse at a spacing equal to the range resolution of the radar set and adds the returns from $n_p$ pulses.

![SNR After Integration]

**Figure 11.3 SNR after Coherent Integration**

If we coherently integrate using,

$$(SNR)_{imp} = n_p (SNR)_0$$

where $(SNR)_{imp}$ is the improved SNR, $n_p$ is the number of pulses integrated and $(SNR)_0$ is the original SNR, we get what we see in Figure 11.3. At first glance, one may not
notice much of a difference but in fact, the base SNR we saw in Figure 11.1 has been improved here. If our person still has a -0.97 dBsm return, we can now detect him up to about 5300 meters or about 3.29 miles under ideal circumstances.

The number of pulses we can use to integrate is dependent upon the amount of time spent radiating the target and the PRF. For Figure 11.3, $n_p = 15$.

$$15^{1/4} = 1.968$$

$$2700m \times 1.968 \approx 5314m$$

It would seem that our job is done because the search radar we designed can already do what we want it to do but this is far from the case. We still have to cover all of the bases before declaring this system is ready.
Figure 11.4 Pulsewidth versus SNR Required for Three Different Detection Ranges

If Figure 11.4 above, you see pulse width needed for the minimum required SNR at 3 separate ranges. Keep in mind, the smaller the pulse width, the smaller the amount of energy contained within the pulse so it makes sense that small pulse widths can only detect targets up close. The minimum is at 0.01 meters, well below the blind range as we calculated in the beginning) while the median range being 5,000 meters and the maximum range at 10,000 meters. The median (red dashed) and the maximum (blue dotted) lines are much closer together because the SNR rate of change from Figures 11.1 and 11.3 declines less rapidly than it does from the minimum range to the median range.
In Figure 11.5(a), we see the Power Aperture Product for three different values of the RCS. (0.08 m², 0.8 m² and 8 m²). This graph reinforces what one would expect, detecting a bird (0.08 m²) would be much harder than detecting a school bus (8 m²). What this means is if we designed this radar as say a bird avoidance radar for a plane and we used it to detect a school bus instead, we could decrease the aperture size or the power required and still retain detection capabilities.
Figure 11.5(b) PAP in $W \times m^2$ versus Detection Range in $Km$

Figure 11.5(b) is simply Figure 11.5a converted from dB back in $W \times m^2$. In practice, the power aperture product is widely used to categorize the radar’s ability to fulfill its search mission. Normally, a power aperture product is computed to meet a predetermined SNR and radar cross section for a given search volume defined by $\Omega$. 

$$P_{av} \times A_g = \frac{SNR (16R^4kT_o FLN)}{\sigma_{tsc}}$$

The PAP is often used to calculate the aperture area or average power when the other term as well as the SNR is known. This can be very useful in finding the correct
aperture size if one knows what SNR they want to be able to detect and have limited power resources.

![Radar Average Power versus Power Aperture Product](image)

**Figure 11.6 Radar Average Power versus Aperture Size**

The three data lines above represent the Power Aperture Product using the variables discussed in Table 11.1. As we can see, the PAP rate of decline decreases as the aperture size increases even while the average power decreases. This might lead one to make the claim that an infinitely large aperture would require infinitely small average power input which is technically true but is impractical by far. We want to minimize size and power requirements, thus maximizing flexibility and maneuverability. This means we have to reach a compromise on a size and power that we feel comfortable with.
Thus far we have only discussed Coherent Integration but that’s only one way to increase the SNR and by extension, the detection range. The next figure will begin to introduce Non-Coherent Integration.

**Figure 11.7 SNR Improvement using Coherent and Non-Coherent Integration**

There are several items to notice in Figure 11.7. First, notice the x-scale is now a log scale. Next is the realization that Coherent Integration alone gives more of an SNR improvement than Non-Coherent does regardless of how many pulses you integrate and how much time you spend radiating the target. The Y-axis (SNR – dB) represents the amount of dB improvement attributed to integrating the corresponding number of pulses on the x-axis.
Results such as those presented in Figure 11.7 may lead one to ask, why use Non-Coherent Integration at all, which would be a fair question to ask. To answer that, we have to leave the world of theoretical design and enter the world of engineering.

Now let’s compare, side by side, the SNR before and after Coherent Integration.

![Figure 11.8 SNR Before and After Coherent Integration](image)

**Figure 11.8 SNR Before and After Coherent Integration**

In Figure 11.8, the blue solid line represents the -0.97 dBsm of the person we are trying to detect, the solid, red line is the SNR of said person before integration and the dashed, black line is after integration. The results of the SNR after integration being nearly twice as good as before integration are presented here graphically.

As stated previously, the Non-Coherent Integration results in a lower detection range which for our case, is undesired and unwarranted since we are integrating only 15 pulses, but I will include it for the purpose of being thorough in this research. If instead
of integrating 15 pulses, we integrate 15,000 pulses, the detection range would still be smaller but a Coherent Integrator would no longer be the best choice for reasons previously presented.

![SNR Before and After Integration](image)

**Figure 11.9 SNR Before and After Coherent Integration**

Figure 11.9 above is a zoomed in view of Figure 11.8. Here we can more clearly make out the value at which the SNR crosses the threshold value.
In Figure 11.10(a), we continue our analysis of using a larger wavelength for our system. In doing so, we are giving ourselves a range of values that will allow us to see where we can and cannot compromise. It also gives us the ability to better understand and predict outcomes with wavelengths between 0.027 meters and 0.2 meters. With a wavelength of 0.2 meters, we can detect the target out to about 3 kilometers. This translates to about a 1400 meter loss in detection range ability just by increasing the wavelength by a factor of 10. This can be overcome by increasing the peak transmit power and/or the aperture size or even by slowing our scan rate to allow more pulses to be integrated.
Figure 11.10(b) SNR Before and After Coherent Integration at a Wavelength of 0.2 meters, 150 watts peak transmit power and a slightly larger aperture size

Since there is a possibility that a 0.027 meter wavelength might be too small and we know that we get about a 1400 meter loss if we increase the wavelength without altering anything else, we wish to reach a compromise. In Figure 11.10(b), the peak transmit power has been increased to 150 watts (the maximum in Figures 11.1(b and c)), the aperture size is now 0.75 meters, the scan time is 3 seconds and the scan rate has been slowed from 45 degrees per second to 30 degrees per second. In making these few changes, we have recovered the ability to detect a target to about 3900 meters. We are still about 300 meters shy of where we were with a wavelength of 0.027 meters but now we have alleviated some of the concern about the wave passing through the target.
Figure 11.10(c) SNR Before and After Coherent Integration With a 1 Degree Elevation Angle and Wavelength of 0.027 Meters

If we decrease the elevation angle from 5 degrees to 1 degree, the range at which the SNR crosses the Threshold increases by about 100 meters. This is not a huge change but the reasoning for doing this is to ensure that the radar beam will have a smaller chance of going over the head of the target at about 4 kilometers.
Figure 11.10(d) SNR Before and After Coherent Integration With a 1 Degree Elevation Angle

<table>
<thead>
<tr>
<th>Wavelength</th>
<th>0.2 Meters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak Transmit Power</td>
<td>150 Watts</td>
</tr>
<tr>
<td>Aperture Size</td>
<td>0.75 Meters</td>
</tr>
<tr>
<td>Number of Pulses Integrated</td>
<td>111</td>
</tr>
<tr>
<td>Scan Time</td>
<td>3 Seconds</td>
</tr>
<tr>
<td>Scan Rate</td>
<td>30 Degrees per Second</td>
</tr>
</tbody>
</table>

Table 11.1.2 List of Inputs for Figure 11.10(e)
Figure 11.10(e) is the same as Figure 11.10(d) except for the change to the parameters listed in Table 11.2. As with Figure 11.10(d), the change in Elevation Angle did very little to affect our detection range. The reason I left the Elevation Angle at 1 degree is to compensate for the atmospheric attenuation which tends to cause an Electro-Magnetic Wave to “bend” downwards. If I had lowered the Elevation Angle to 0 degrees, the wave would eventually be bent downward into the ground and possibly limited my maximum detection range unnecessarily. One possible work around that is not considered in this paper is to have an antenna with an adjustable Elevation Angle that depends on your desired maximum range.
Figure 11.11 Improvement Factor versus the Number of Non-Coherently Integrated Pulses for Four Different Probability of Detection and Four Different False Alarm Numbers

The improvement factor along the y-axis of Figure 11.11 is the value corresponding to the number of pulses Non-Coherently Integrated is added to the base SNR. As expected, if we design our radar so that it detects the person 50% of the time \((pd = .5)\), the false alarm number \((nfa)\) is relatively small. If we design our radar such that we nearly always detect our person, we also run the risk of a very high false alarm number.
One way to filter out some of these false alarms is actually straightforward. If we assume that the lost hiker is moving, then the radar return from the hiker will have some Doppler shift while the surrounding landscape will not. Writing a simple algorithm to filter out the non-doppler shifted returns would correct for some of the false returns.

The false alarm number is determined by the probability of false alarm and by extension, the time of false alarm.

\[ nfa = \frac{-\ln 2}{\ln(1 - pfa)} \]

Where the \( pfa \) is determined by,

\[ pfa = \frac{1}{\text{Bandwidth} \times tfa} \]
Figure 11.12 Integration Loss versus the Number of NCI Pulses

In Figure 11.12, we see a very important result for Non-Coherent Integration. As the number of non-coherently integrated pulses increase, the loss due to integration increases steeply until about \( n_p = 11 \) and then assumes a nearly linear increase on a log scale. If we were to integrate a hundred thousand pulses, we will have to incorporate a fairly significant loss due to integration. Without lowering the noise figure or increasing the gain, this integration loss could place an upper limit to the number of non-coherently integrated pulses that we can afford to integrate.
Figure 11.13 Clutter RCS as a function of range

Figure 11.13 represents the clutter RCS we might see from our radar returns. Clutter RCS can be defined as the equivalent radar cross section attributed to reflections from a clutter area \( (A_c) \) and is calculated using,

\[
\sigma_c = \sigma^0 \times A_c
\]

Where \( \sigma^0 \) is referred to as the clutter scattering coefficient. (Cite Ref 1)

This clutter is due to the fact that the radar we are theoretically building, has a very low grazing angle (angle from the ground surface to the main axis of the beam. The three main factors that contribute to the clutter seen in Figure 11.13 are; low grazing angle, surface roughness (discussed later in this section) and the radar wavelength. The
wavelength and scattering coefficient are inversely proportional to one another, i.e. smaller wavelengths results in higher scattering coefficients. The received RCS from the clutter is calculated by adding the main beam clutter RCS and the sidelobe clutter RCS such as seen in Figure 11.14. The crest of the clutter RCS at very low ranges (barely visible here but more visible in the next example) is created by the return from the sidelobes discussed in the next few pages.

![Figure 11.14 Ground Based Radar Clutter detected by the Main Beam and Sidelobe](image)

The airplane in Figure 11.14 represents a target but it is fair to place any type of target in the figure and get similar results. For the case of a person standing in an open field or desert, we would still have main beam clutter and sidelobe clutter to worry about.
Figure 11.15 Detailed side and top view of ground based radar clutter.

In Figure 11.15, $\theta_E \equiv 3$ dB Elevation angle, $R$ is the slant range and $\theta_A \equiv 3$ dB Azimuth angle. The above figure breaks up Figure 11.14 into a more detailed look so that we may be able to see the origins and locations of variables used with the Matlab code to produce this simulation and research.
Figure 11.16 CNR as a function of range

The Clutter to Noise Ratio (CNR) is calculated identically to the SNR with the only difference being that instead of using the target RCS, we use the clutter RCS from Figure 11.13. As expected, upon simulation, the CNR is much lower than the SNR as we’ll see in Figure 11.17. However, any nearby clutter will impede the radar’s ability to detect distant targets.
Figure 11.17 Combined Plots of SNR after Integration and CNR

I’ve included Figure 11.17 simply for comparison reasons and to reinforce a comment made earlier that the CNR would be vastly lower than the SNR at all ranges.
Figure 11.18 SNR After NCI

We already saw the results for CI earlier in this example but here are the SNR results for NCI. As you can see, the SNR drops below the -0.97 dB level mush sooner than it did with CI.
Figure 11.19(a) SNR Before and After NCI with Sigma and Threshold Voltage Also Plotted

As we can see in Figure 11.19, NCI does not offer us much in the way of SNR improvement. It’s nowhere near what we would want for a system such as this.
Figure 11.19(b) SNR Before and After NCI with a Wavelength of 0.2 meters, 150 watts peak transmit power, 0.75 meter aperture size and 3 second scan time

When comparing Figures 11.19(a) and (b), one can immediately tell that by using the longer wavelength, we reduce our detection range which is consistent with what we saw with Coherent Integration techniques. Here, we lose about 600 meters off of our detection range even after we try and compensate by increasing the transmit power, aperture size and scan time. With these two results, one could conclude that at least theoretically, Non-Coherent Integration is not the preferred method here.
Figure 11.20 Randomly Generated Target Return

In Figure 11.18, I have generated a target with a random location in the azimuthal search of the radar as well as the SNR for a random range. The point of this figure is to show that with a single plot and setting up radar dish so that 0 degrees represents looking due North, we can determine the range and bearing to the random target.

The frequency information below correlates to Figure 11.18. It is the random Doppler Frequency shift of ground clutter (trees, bushes, leaves, etc.) due to wind as well as the target moving at a random speed, heading in a random direction. By locking onto and tracking the target, we can build a profile of his travel path thus giving us the direction he is walking. The speed is slightly more math intensive.
Our last venture is to calculate the range and direction the target is travelling. To do this, we need the SNR of the target at a random range for simulation purposes and the Doppler shift of said target.

The average signal returned from the target that was randomly generated using ranges between 1 and 10,000 meters is -48.48 dB. Now that we have the signal return value, we can work backwards to determine the range that the target was detected at.

\[
SNR = \frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 kt_e BFL R^4} \rightarrow R = 4 \sqrt{\frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 kt_e BFL (SNR)}}
\]

\[
R = 4 \sqrt{\frac{(pt+g^2+\lambda^2+sigma)}{(4+pi)^3(1.381+10^{-23})+te+b+(nf+loss)+average\_return\_signal}} \approx 4 \sqrt{77.68 \text{ w dB}^2 \text{ m}^4} \approx 6324 \text{ meters}
\]

The Doppler Shift in Frequency due to the person walking at 1 m/s is 3.23Hz.

If we know the speed of the person, we can calculate the direction they are walking relative to the radar. Recall that 0 degrees is due North.

\[
Walking \ Direction = \cos^{-1} \left( \frac{bf \times c}{f \times v} \right) = \cos^{-1} \left( \frac{3.23 \times 1.9979 \times 10^8}{11.1 \times 10^8 \times 1} \right) = 85 \ degrees
\]

If we do not know the speed and direction, we can track the target for a short time and as the range changes over time, we can calculate his/her direction. With the direction, we would be able to get the speed and with that, we could tell if the target is running, walking or stationary.
Now to get rid of clutter, we would simply look for items with Doppler shifts outside what a person could produce, from being stationary to sprinting and remove those items. Removing those would get rid of returns from birds, fast ground animals like deer, foxes, etc. To get rid of trees, bushes, leaves, etc., we would compare the returns with what would be generated by wind blowing through the foliage and remove those returns. With both sets of clutter and false positives removed, we would have a much easier time picking out the true target.
2. Detection of Small Arms Fire from an In-Flight Helicopter

<table>
<thead>
<tr>
<th>Variable Name</th>
<th>Variable Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Speed of Light</td>
<td>(2.9979 \times 10^8) m/s</td>
</tr>
<tr>
<td>Boltzmann's Constant</td>
<td>(1.38 \times 10^{-23}) joule/K</td>
</tr>
<tr>
<td>Peak Power</td>
<td>1000 w</td>
</tr>
<tr>
<td>Frequency</td>
<td>(6 \times 10^{10}) Hz</td>
</tr>
<tr>
<td>Radar Diameter</td>
<td>1 m</td>
</tr>
<tr>
<td>Effective Apperture Area</td>
<td>0.785398 m²</td>
</tr>
<tr>
<td>Wavelength</td>
<td>0.004997 m</td>
</tr>
<tr>
<td>Gain</td>
<td>395337.5</td>
</tr>
<tr>
<td>Gain in dB</td>
<td>55.96968 dB</td>
</tr>
<tr>
<td>Range Resolution</td>
<td>5 m</td>
</tr>
<tr>
<td>Pulse Width</td>
<td>(2 \times 10^{-8}) s</td>
</tr>
<tr>
<td>Bandwidth</td>
<td>50000000 Hz</td>
</tr>
<tr>
<td>Target Radar Cross Section</td>
<td>(2.49 \times 10^{-5}) m²</td>
</tr>
<tr>
<td>Noise Temperature</td>
<td>290 K</td>
</tr>
<tr>
<td>Noise Figure</td>
<td>10 dB</td>
</tr>
<tr>
<td><strong>Radar Loss</strong></td>
<td>15 dB</td>
</tr>
<tr>
<td>-------------------------</td>
<td>----------------</td>
</tr>
<tr>
<td><strong>Maximum Detection Range</strong></td>
<td>2500 m</td>
</tr>
<tr>
<td><strong>SNR Sensitivity</strong></td>
<td>5 dB</td>
</tr>
<tr>
<td><strong>Scan Time</strong></td>
<td>2 s</td>
</tr>
<tr>
<td><strong>Range Sensitivity</strong></td>
<td>0.05 m</td>
</tr>
<tr>
<td><strong>Azimuth Angle</strong></td>
<td>360 degrees</td>
</tr>
<tr>
<td><strong>Elevation Angle</strong></td>
<td>30 degrees</td>
</tr>
<tr>
<td><strong>Omega</strong></td>
<td>3.289843</td>
</tr>
<tr>
<td><strong>Radar 3dB Beamwidth</strong></td>
<td>0.006246 degrees</td>
</tr>
<tr>
<td><strong>Scan Rate</strong></td>
<td>15 degrees/s</td>
</tr>
<tr>
<td><strong>Time on Target</strong></td>
<td>0.000416 s</td>
</tr>
<tr>
<td><strong>Number of Pulses to be Integrated</strong></td>
<td>17</td>
</tr>
<tr>
<td><strong>Background Noise Figure</strong></td>
<td>2E-13</td>
</tr>
<tr>
<td><strong>Pulse Repetition Frequency</strong></td>
<td>40000 Hz</td>
</tr>
<tr>
<td><strong>Pulse Repetition Interval</strong></td>
<td>0.000025 s</td>
</tr>
<tr>
<td><strong>Unambiguous Range</strong></td>
<td>3747.375 m</td>
</tr>
<tr>
<td><strong>Time of False Alarm</strong></td>
<td>600 s</td>
</tr>
<tr>
<td><strong>Probability of False Alarm</strong></td>
<td>5.67E-10</td>
</tr>
<tr>
<td><strong>False Alarm Number</strong></td>
<td>2.08E+10</td>
</tr>
</tbody>
</table>
Table 11.2.1 Inputs for the Simulation

<table>
<thead>
<tr>
<th>Probability of Detection</th>
<th>0.825795</th>
</tr>
</thead>
<tbody>
<tr>
<td>Threshold Voltage</td>
<td>55.1172 w</td>
</tr>
<tr>
<td>Threshold in dB</td>
<td>17.41244 dBw</td>
</tr>
<tr>
<td>Total Loss Due to NCI</td>
<td>12.30934416 dB</td>
</tr>
</tbody>
</table>

Since the introduction of the helicopter into a battlefield, they have been targeted by ground troops with small arms fire. Generally, this fire isn’t enough to severely damage a modern helicopter but occasionally, it is enough to bring one down. Improvements in helicopter capabilities and tactical applications have led to the development of an array of anti-helicopter weapons systems. Helicopter designers must take into account the requirement for the helicopter to survive and continue to operate as an effective combat platform in a hostile environment. Inevitably, helicopters will be drawn into close combat and will be engaged by anti-aircraft weapons systems. Shooting down a slow flying helicopter is often used as a propaganda victory for terrorists and more often than not, the crew is killed in the process. All modern jets have some sort of radar to help detect enemy anti-aircraft weapons, but most helicopters lack any sort of radar system for detecting said weapons, especially small arms fire. The detection of small arms fire, fired towards a helicopter would allow the crew to either return fire or rapidly fly out of harm’s way. Such a system can and will save lives and untold amounts of money, not to mention take away even this smallest of victories for terrorists.

Any system designed to detect small arms fire that is mounted on a helicopter needs to be lightweight, low power, have excellent range and azimuth/elevation
resolution to detect the source of the threat. Helicopters are relatively weakly powered, especially if they are fully loaded with fuel, troops and equipment. So with these few guidelines, let’s see if we can design a system that will fulfill our goals of lightweight, accurate and low power.

Figure 11.2.1 Notional Radar Antenna Pattern on a Helicopter

(Cite Ref 26)

Figure 11.2.1 provides a graphical representation of a radar system with a 360 degree field of view. However, instead of placing 4 radar devices on a chopper, let’s place one small, rotating dish that can be attached to the under-carriage of the helicopter.
Now that we have an idea of how we want to protect the helicopter, we need to discuss what we are protecting against. Since the majority of small arms fire used against NATO helicopters is the 7.62mm round or larger, we’ll focus our radar design on trying to detect the 7.62mm round. If we can detect such a small target, then we can detect the larger caliber weapons at further ranges.

Before we can begin our calculation, we need to know the RCS of a 7.62mm round.
Figure 11.2.3 7.62mm Bullet and Case Dimensions.

(Note: All dimensions are in millimeters)
(Cite Ref 27)

We now have our dimensions so we can calculate the RCS. Since we are only concerned with the bullet itself, I’m approximating it as a doubly curved surface where the RCS can be calculated using,

\[ \sigma = \pi \times r_1 \times r_2 \]

Where \( r_1 \) and \( r_2 \) are the the principle radii of curvature of the body at the specular point. I’m choosing \( r_1 = 1\text{mm} \) and \( r_2 = 7.92\text{mm} \). With this, we can get an RCS for the bullet to be,

\[ \sigma = \pi \times 0.001m \times 0.00792m \approx 2.488 \times 10^{-5}m^2 \]
The RCS of the bullet, if it were travelling at some arbitrary angle and not directly at the helicopter, would be larger than the above value; thus making it easier to spot at more distant ranges.

Also, let’s assume the bullet has a velocity of 800 m/s. We’ll need this later to determine the Doppler Shift of the radar pulse. I’m also assuming that our helicopter’s top speed is 200 km/hr ≈ 56 m/s.

Now that we have the RCS of the bullet, we need to know how much power a helicopter can devote to the radar. I’m choosing to use the UH-60L model Blackhawk helicopter for this radar which has dual turbines producing about 1410 kW each of power. Let’s assume out of the total power of 2820 kW, we can use just 1 kW for the radar. After all, the electronics still need to function and we are trying to avoid adding heavy batteries or generators to power our radar.

We wish to know the bullets range, velocity and direction relative to the helicopters location at any point in time. Let’s design our radar so that we get 1 pulse per second from a 60 GHz V-band radar with a wavelength of 0.5 cm. Further assume that there is only slight noise in the background of about 2 dBsm with a receiver loss of 3dB. Finally, we need a range resolution of less than 5m in order to successfully locate the origin of the bullet (i.e. the enemy soldier).

Our energy per pulse is given by Equation 11.1.

\[ E = P_t \tau = 1000W \times 2 \times 10^{-7} = 2 \times 10^{-4} \text{Joules} \]

With the gain being calculated as,
Now let’s calculate the power density at some range \( R \) away from the helicopter.

The strength of the return radar signal from the bullet can be calculated using Equation 5.2.

\[
P_r = \frac{P_t}{4\pi R^2} G \frac{1}{4\pi R^2} \sigma A_g = \frac{(1000 \text{ watts})(55.97 \text{ dB})(2.488 \times 10^{-5} m^2)(0.7854 m^2)}{16\pi^2 R^4}
\]

I’m going to limit the maximum detection range to 2500 meters for our helicopter and in doing so, the role of our radar is simply to pinpoint where the small arms fire is originating from.

Using Equation 6.3 again, we can calculate the average noise power of the environment that the radar is receiving.

\[
N = k \times T_s \times B_n = \left( 1.381 \times 10^{-23} \frac{J}{K} \right) (290K)(50 \times 10^6 Hz) \approx 2.002 \times 10^{-13} \text{ Watts}
\]

\[
N_{dB} = 10 \times \log_{10}(2.002 \times 10^{-13}) = -126.99 \text{ dBw}
\]

Using Equation 6.4 to calculate the SNR with a loss of 15 dB and the transmit and receive gains the same,

\[
SNR = \frac{P_r G_r G_s \sigma^2 \lambda^2}{(4\pi)^5 R^4 k T_s B_n L} = \frac{1000W(55.97 dB)^2(0.0049965m)^3(2.488 \times 10^{-5} m^2)}{(4\pi)^5 R^4 (1.381 \times 10^{-23} \frac{J}{K})(290K)(50 \times 10^6 Hz)(15 dB)} = \frac{3225.37 m^4}{R^4}
\]
In Figure 11.2.4, we see the SNR for three different values of the peak transmit power; 500 watts, 1000 watts and 1500 watts. As we can see, there is not a huge difference between all three values at any range. This tells us that we would need to increase the power dramatically to achieve detection of a bullet at ranges exceeding 1500 meters. This should be overkill as small arms rapidly lose accuracy beyond 1 km.

If we compare the SNR to the background noise, we would need to travel quiet a distance of before the SNR would drop below that of the background noise though, the threshold voltage for our radar will limit how far we can detect before false alarms become a problem.
**Figure 11.2.5 PAP of the Bullet**

In Figure 11.2.5, we see a disadvantage in that, since the target has a very low radar cross section, we require a very high search power-aperture product to cover the required volume in space that we wish to detect a target in. As the line in Figure 11.2.5 increases with increasing range, this means that for such a small target such as a bullet, we must radically increase the PAP in order to maintain detection at longer distances. As a result, either the effective aperture area must be increased which for our case results in a heavier load for the aircraft or we must increase the power supplied to the aircraft. Since neither option is desired, we must find other ways to increase the detection range of the radar.
Figure 11.2.6 PAP in $W \times m^2$

Figure 11.2.6 may provide a clearer picture from the discussion of Figure 11.2.5. Here, as range increases, the PAP must also dramatically increase if we wish to maintain our detection capabilities to further ranges. Either the power input or the aperture size must be increased to compensate for the ever shrinking target return signal.
Figure 11.2.7 SNR After CI Integration

As we saw in the previous design section, Figure 11.2.7 contains the SNR from the target after coherent integration.

Using coherent integration from Equation 11.2, we can get the SNR to be,

$$(SNR)_{imp} = 17^{1/4}(SNR)_0 = 2.0305(SNR)_0$$

Thus our radar is twice as powerful when using CI when compared to the unimproved SNR but keep in mind, a twice as powerful does not correlate with twice the improvement for the detection range.
Figure 11.2.8 Pulsewidths required for minimum, median and maximum range detection

What Figure 11.2.8 helps us decide is what pulse width we would want to use for a certain minimum required SNR from the target. Keep in mind that a smaller pulse width also carries less energy and choosing a pulse width that is very small, could lead to complications.
Figure 11.2.9 PAP in $dB$ versus Detection Range for Multiple RCS Values

As the PAP curve flattens out for larger distances, this tells us that in order to detect this target at very large ranges, we would need either a very large receiver or a very high power source. At some point, we need to compromise on detection range versus PAP or we run the risk of developing a very expensive, impractical radar for our purpose of detecting small arms fire onboard a helicopter. I plotted the 3 values for the RCS as indicators. Sigma is the RCS of the bullet flying straight towards the helicopter, $10 \times$ sigma is representative of a bullet flying perpendicular (presenting it’s broadside) to the radar and $\frac{\sigma}{10}$ as an indicator of bullets of a smaller caliber such as 5.56 mm or smaller.
Figure 11.2.10 PAP in $W \times m^2$ versus Detection Range for Multiple RCS Values

At first glance, this appear to closely mimic earlier results but this PAP plot is after CI has been used and it is also for 3 separate RCS values. As before, as the range increases, the PAP must also increase in order to keep the level of detection we wish to maintain.
**Figure 11.2.11 Power versus Aperture Size with PAP Data Line**

In Figure 11.2.11, we see three different RCS values and the corresponding PAP for each. As the aperture size increases, we can decrease the average input power and maintain a sufficient detection level (indicated by the data lines) that allows us to still detect the bullet in flight.
Figure 11.2.12 CI versus NCI Integration Improvement

Figure 11.2.12 shows us that for fewer integrated pulses, both coherent and non-coherent, we won’t see a large increase in the SNR. However, any increase that results in a large detection range will be welcomed news.
Figure 11.2.13 Clutter RCS versus Slant Range

Figure 11.2.13 shows use the clutter RCS versus range and as we would expect, the clutter RCS drops off dramatically for large ranges. What’s new here is the dip and hump we see at very close ranges. This hump occurs at the grazing angle that corresponds to the space between the main beam and the first sidelobe. However, we wouldn’t necessarily have to worry about this region because it may very well be within the blind zone of our radar.

\[ R_{\min} = \frac{c \times (\text{pulse width} + t_{\text{recovery}})}{2} \]

Where \( t_{\text{recovery}} \) is the time needed for the monostatic (transmitter is the same as the receiver) radar to switch from transmit to receive mode.
The time required for recovery is determined by the time taken by the radar switch to de-ionize after each transmitted pulse. It is usually defined as the time required for the receiver to return to within 6 dB of normal sensitivity after the end of the transmitter pulse but some manufacturers use the time required for the sensitivity to return to within 3 dB of normal sensitivity. Recovery time is a factor that limits the minimum range of a radar because the radar receiver is unable to receive until the switch is de-ionized. In various radars, the recovery time may differ from less than 1 microsecond to about 20 microseconds. For the purposes of this research, let’s assume the Engineers want a recovery time of 2 microseconds.

\[ R_{\text{min}} = \frac{(2.9979 \times 10^8 \text{ m/s}) \times (2 \times 10^{-8} \text{s} + 2 \times 10^{-8} \text{s})}{2} \approx 303 \text{ m} \approx 993 \text{ feet} \]

It’s safe to say that any target that close to the radar can be easily found using other means. For a 1 microsecond recovery time, the minimum detection range is about 153 meters (501 feet).
Figure 11.2.14 CNR versus Slant Range

Figure 11.2.14 shows us that the CNR will be much less than the SNR for nearly all ranges, this is anticipated as it was with the previous design data.
Figure 11.2.15 SNR After CI Integration

Figure 11.2.15 shows several important bits of information. First, is the SNR before and after CI is used. Next is the background noise strength in blue and finally, the radar threshold voltage converted to dB. Notice that the SNR of the target is only above the threshold voltage of the radar for a very short range, less than a kilometer. In order to extend the range, we must lower our threshold voltage but in doing so, we could possibly introduce false alarms into our system. As it usually is, we must compromise on what level of false alarms we are comfortable with before we can determine how far away we want our maximum detection range to be. However, we have a saving grace with this. Typically, a person firing small arms at a helicopter is not going to just shoot a single bullet, but rather will fire multiple shots. Because of this fairly accurate assumption, we
can lower our Threshold Voltage without too much of a worry. We could introduce a filter that would disregard individual false alarms.

![SNR After Integration](image)

**Figure 11.2.16 SNR After NCI Integration**

The results shown in Figure 11.2.16 clearly show that NCI produced a less improved SNR when compared to CI but this is expected. Figure 11.2.16 is included in the event that cost and simplicity is placed above efficiency. Even though the results are less than we would want, NCI in this case may be required. Since the bullet is so small and moving fairly rapidly, our pulse width may have to be very small which in turn would mean we would need to integrate a large number of pulses. This is why NCI is included here. Unlike the slow moving human sized target in the previous research data, the bullet will require many more resources to accurately detect. These resources come at
a price and here, the price may be that we have to use NCI for the results we want. Once detected, tracking a bullet is a simple exercise since bullets fly in straight path until they contact another object. Also, over short time scales, the velocity of the bullet will not vary by a significant amount, thusly, making our task easier.

![SNR NCI with and without the integration](image)

Figure 11.2.17 SNR After NCI Integration
With our bullet scenario here, we must entertain the idea that we may not have the desired probability of detection we want all the time. Figure 11.2.18 is here to give us a range of improvement factors for a given range within a specific probability of detection and false alarm number. With this graph, we can quickly interpolate what improvement we can expect for our SNR within the range of detection probabilities and number of pulses integrated.
Since we are integrating multiple pulses together, we must pay careful attention to the integration loss. Figure 11.2.19 shows that as we integrate more pulses in an attempt to detect and track our bullet, we must include higher integration losses. This result leads to the idea of diminishing returns and the possibility that integrating very high numbers of pulses may lead to “less bang for our buck” where it is not worth integrating more pulses when compared to the limited improvement.

We still need to determine how much of a Doppler shift we will see from incoming small arms fire.

The average signal returned from the target is -61.78 dB using the 800 m/s velocity of the bullet and the forward speed of 56 m/s of the helicopter.
Now that we have the signal return value, we can work backwards to determine the range that the target was detected at.

\[
SNR = \frac{P_tG^2\sigma^2}{kT_eBLR^4} \rightarrow R = 4\sqrt{\frac{P_tG^2\lambda^2\sigma}{(4\pi)^3kT_eBLR(SNR)}}
\]

\[
R = 4\sqrt{\frac{(ptg^2\lambda^2\sigma)}{(4\pi)^33(1.381\times10^{-23})\times12(b\times(f\times loss)\times average\_return\_signal)}} \approx 4\sqrt{\frac{0.00195 \text{ w dB m}^4}{3.955 \times 10^{-14} \text{ w dB}^2}} \approx 471 \text{ meters}
\]

The Doppler Shift in Frequency due to the bullet is 58594.86 Hz.

\[
\text{Bullet Orignation Direction} = \cos^{-1}\left(\frac{\Delta f \times c}{f \times \nu}\right) = \cos^{-1}\left(\frac{58594.86 \times 2.9979 \times 10^8}{60 \times 10^9 \times 856}\right) = 70 \text{ degrees}
\]

To get rid of the clutter, we can follow the same guidelines set in the previous example except this time, we have to make sure to compensate for the velocity of the helicopter.

Since we can fairly easily remove unwanted clutter and movement from our returns, we can begin to compare the residual returns with a database. In the case of the bullet, the velocity of nearly all types of ammunition is known and by searching this database for 7.62 mm data, we can compare the radar return Doppler Shift values to the data and if it matches within a certain percentage, say 80% up to 120% of the value in the table, the radar flags that as a detection and alerts the crew.
12. Terminology

Beam Shape Loss – Loss incurred as a result of the target not being located in the center of the beam.

Chirp – Ramping up or down of a pulse compression wave.

Classification – Identifying the target object as a certain class of target.

Detection Threshold – Lowest possible signal strength that the radar can confirm a legitimate target return.

Discrimination – Resolving closely spaced objects into individual, distinct targets or debris

Doppler Shift – The change in frequency of a radar return compared to the transmitted signal due to the velocity of the target and/or the radar.

Ground Bounce – Refers to the reflected radar signal off the surface of the earth.

Noise Figure – It is a measure of degradation of the signal to noise ratio for a given bandwidth caused by components in the RF signal chain. It is the increase in noise power.
of a device from the input to the output that is greater that the signal gain. In effect, it is the amount of decrease of the signal-to-noise ratio.

Probability of Detection – cumulative probability of detecting the target at least once by the time it is at some range R.

Radar Cross Section – The ratio of the radar signal power back-scattered per solid angle in the direction of the receiver to the power transmitted to the target.

Radar Jamming – radio frequency signals originating from sources outside the radar, transmitting in the radar’s frequency and thereby masking targets of interest.

Radar Mile – A time interval of 12.36 microseconds which equals the amount of time for radio frequency energy to travel out from a radar to a target and back to the radar.

Radar Range Equation – The radar range equation is a basic relationship which permits the calculation of received echo signal strength, if certain parameters of the radar transmitter, antenna, propagation path, and target are known.

Range Cell – In a radar, a range cell is the smallest range increment the radar is capable of detecting. If a radar has a range resolution of 50 yards and a total range of 30 nautical miles (60,000 yds), there are: \(60000/50 = 1200\) range cells.
Range Gate – A movable gate used to select radar echoes from a very short-range interval. A gate voltage used to select radar echoes from a very short range interval.

Range Resolution – How well the radar can distinguish between different objects along the line of sight.

Ranging – The measurement of the distance to a remote object (target), from a known observation or reference point.

Scan Loss – Loss of antennae gain as the array is scanned. Greater scan angle results in greater scan loss.

Signal to Noise Ratio – Ratio of the intensity of background noise when compared to that of the targets average radar return signal.

Target Scene – Refers to the region of space immediately surrounding the RV/missile where multiple objects may be present such as in Figure 8.1.

Velocity Resolution – How well the radar can distinguish between objects with different velocities.
### 13. Acronyms

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{\text{Scatter}}$</td>
<td>Surface Area of a Scatterer</td>
</tr>
<tr>
<td>CAD</td>
<td>Computer Aided Design</td>
</tr>
<tr>
<td>CI</td>
<td>Coherent Integration</td>
</tr>
<tr>
<td>$\text{cm}$</td>
<td>Centimeter</td>
</tr>
<tr>
<td>CMs</td>
<td>Counter- Measures</td>
</tr>
<tr>
<td>CNR</td>
<td>Clutter Noise Ratio</td>
</tr>
<tr>
<td>dB</td>
<td>Decibel</td>
</tr>
<tr>
<td>$\text{dBsm}$</td>
<td>Decibel Per Square Meter</td>
</tr>
<tr>
<td>dBW</td>
<td>Decibel Watt</td>
</tr>
<tr>
<td>deg</td>
<td>Degree</td>
</tr>
<tr>
<td>DOF</td>
<td>Degree of Freedom</td>
</tr>
<tr>
<td>ECCM</td>
<td>Electronic Counter-Countermeasures</td>
</tr>
<tr>
<td>EWR</td>
<td>Early Warning Radar</td>
</tr>
<tr>
<td>FFT</td>
<td>Free Fourier Transform</td>
</tr>
<tr>
<td>$\text{ft}$</td>
<td>Foot</td>
</tr>
<tr>
<td>GHz</td>
<td>Giga-Hertz</td>
</tr>
<tr>
<td>HF</td>
<td>High Frequency</td>
</tr>
<tr>
<td>HRR</td>
<td>High Resolution Radar</td>
</tr>
<tr>
<td>Hz</td>
<td>Hertz</td>
</tr>
<tr>
<td>ISAR</td>
<td>Inverse Synthetic Aperture Radar</td>
</tr>
<tr>
<td>j</td>
<td>Joules</td>
</tr>
<tr>
<td>K</td>
<td>Kelvin</td>
</tr>
<tr>
<td>k</td>
<td>Boltzmann’s Constant</td>
</tr>
<tr>
<td>ln</td>
<td>Natural Logarithm</td>
</tr>
<tr>
<td>$m$</td>
<td>meters</td>
</tr>
<tr>
<td>$\text{m/s}$</td>
<td>Meters Per Second</td>
</tr>
<tr>
<td>MaRV</td>
<td>Maneuverable Re-entry Vehicle</td>
</tr>
<tr>
<td>MiRV</td>
<td>Multiple Independently Targetable Re-entry Vehicle</td>
</tr>
<tr>
<td>MMW</td>
<td>Milli-Meter Wave</td>
</tr>
<tr>
<td>NCI</td>
<td>Non-Coherent Integration</td>
</tr>
<tr>
<td>PAP</td>
<td>Power Aperture Product</td>
</tr>
<tr>
<td>PenAids</td>
<td>Penetration Aids</td>
</tr>
<tr>
<td>(PFD)</td>
<td>Power Flux Density</td>
</tr>
<tr>
<td>(PFD)$_{\text{Scatter}}$</td>
<td>Scattered Power Flux Density</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Definition</td>
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<tr>
<td>PRI</td>
<td>Pulse Repetition Interval</td>
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<tr>
<td>PRF</td>
<td>Pulse Repetition Frequency</td>
</tr>
<tr>
<td>RADAR</td>
<td>Radio Detection and Ranging</td>
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<tr>
<td>RCS</td>
<td>Radar Cross Section</td>
</tr>
<tr>
<td>RF</td>
<td>Radio Frequency</td>
</tr>
<tr>
<td>RV</td>
<td>Re-entry Vehicle</td>
</tr>
<tr>
<td>SBIRS</td>
<td>Space Based Infrared System</td>
</tr>
<tr>
<td>SNR</td>
<td>Signal to Noise Ratio</td>
</tr>
<tr>
<td>tgt</td>
<td>Target</td>
</tr>
<tr>
<td>UHF</td>
<td>Ultra High Frequency</td>
</tr>
<tr>
<td>VHF</td>
<td>Very High Frequency</td>
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</tbody>
</table>
14. Works Cited / References


3. Patent 16556 Verfahren zur Bestimmung der Entfernung von metallischen Gegenständen (Schiffen o. dgl.), deren Gegenwart durch das Verfahren nach festgestellt wird.

4. Introduction To Radar Systems, Massachusetts Institute of Technology Lincoln Laboratory 2012


13. xa.yimg.com/kq/groups/22180149/1606172469/name/Lecture+8-Radar.ppt


17. Wan, Eric A. and van der Merwe, Rudolph, "The Unscented Kalman Filter for Nonlinear Estimation", Oregon Graduate Institute of Science & Technology


26. 7.62 mm Bullet Dimensions,


28. General Inverse Square Law, http://hyperphysics.phy-astr.gsu.edu/hbase/forces/isq.html


30. Search Radar Range Equation,
    http://www.ece.uah.edu/courses/material/EE619/SearchRRE(3).doc

31. Yamada, Naoyuiki, “Radar Cross Section For a Pedestrian in the 76 GHz Band”, Toyota CRDL Volume 39


33. Atmospheric Scattering,
