

POINCARÉ DISC: HEAR THE WHOLE MIND SING

Laura J. Moss, John T. Shrontz, Dr. Claudio Morales (Advisor)
Department of Mathematical Sciences

Henri Poincaré legitimized the hyperbolic plane by providing scholars with a map; it is one thing to know something exists, but quite another to understand it.



Fig. 1. French mathematician Henri Jules Poicaré, 1854 - 1912

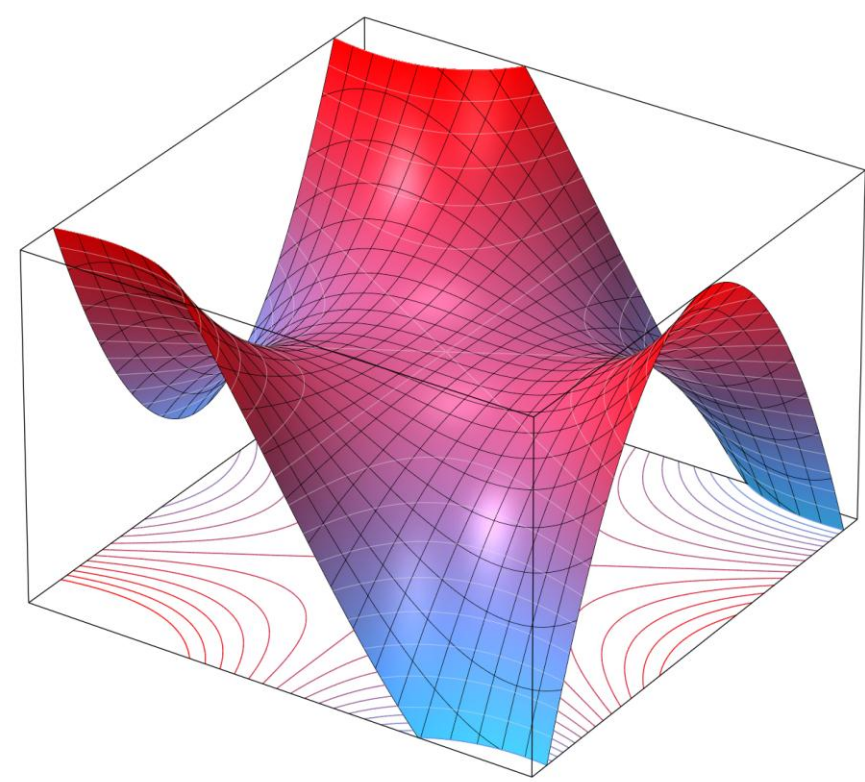


Fig. 3 Saddle shape with negative curvature

Mapping is understanding. By drawing a picture of this uncharted territory Poincare opened the door for others to share his insights.



Fig. 5 Map of spherical earth onto a plane.



Fig. 6 Equilateral triangles covering the plane.

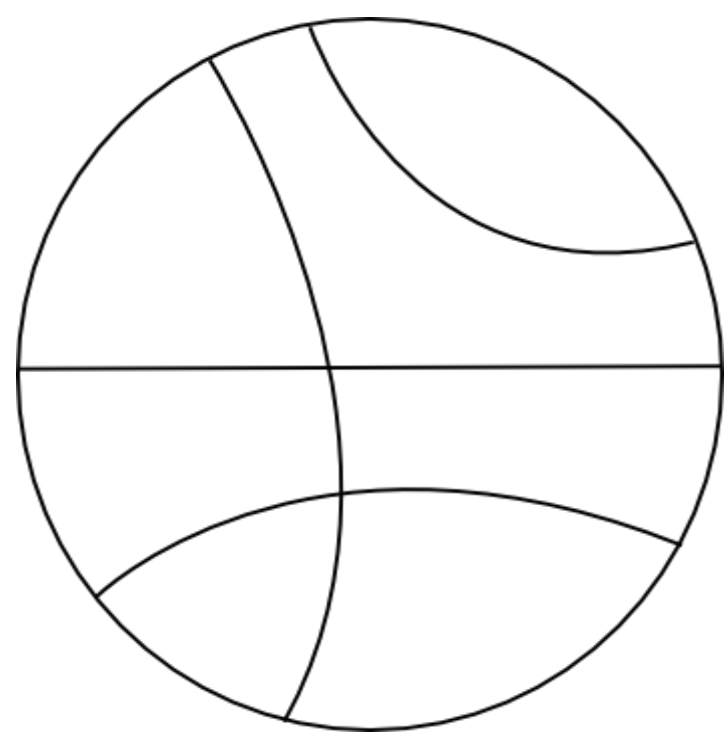


Figure 2: Hyperbolic lines in the disc model

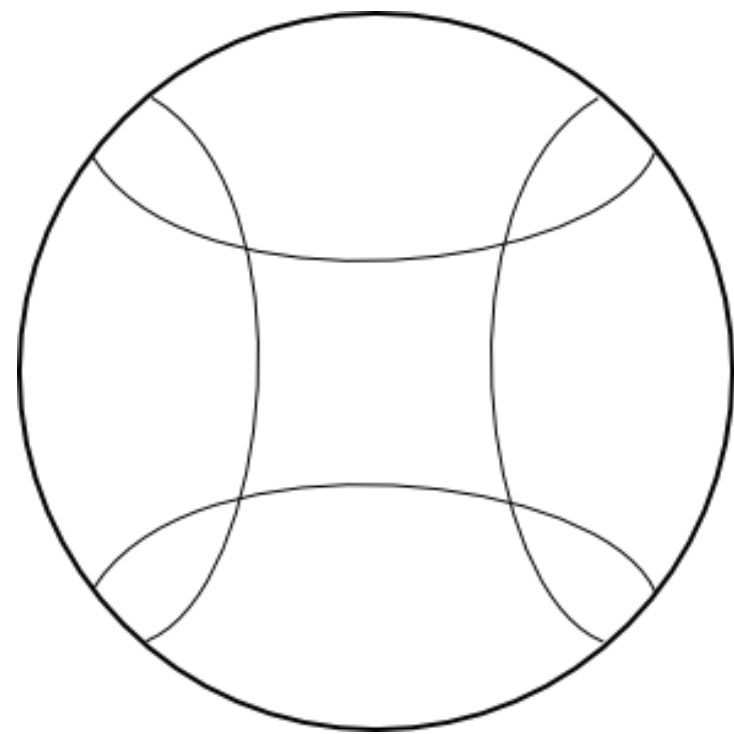


Fig 4. Hyperbolic polygon

Distance Formula

$$d(A, B) = \ln \left| \frac{AQ}{BQ} \times \frac{BP}{AP} \right|$$

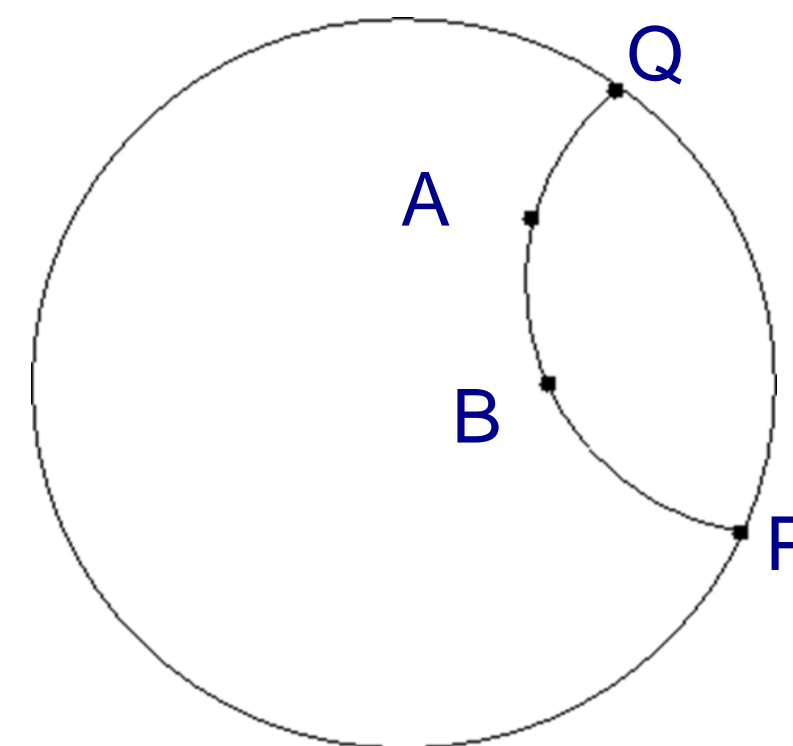


Fig. 7 Measuring hyperbolic distance using four points on a line

Poincare's genius was using perspective to map negative curvature in the Euclidean plane already so familiar to us.

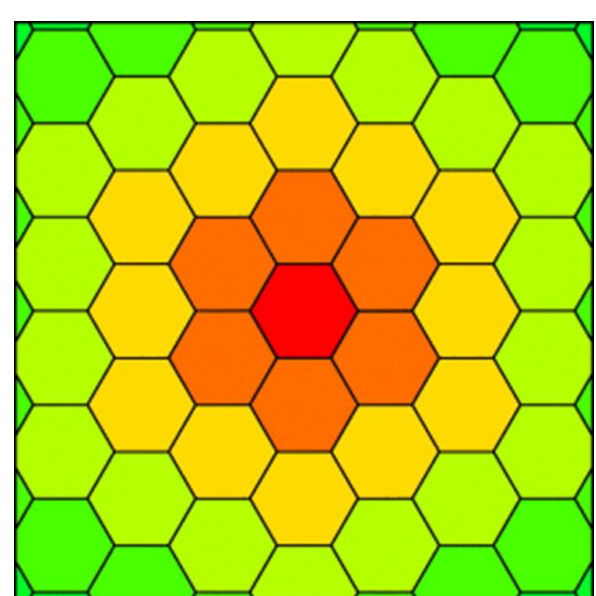


Fig. 8 Tiling of the Euclidean plane.

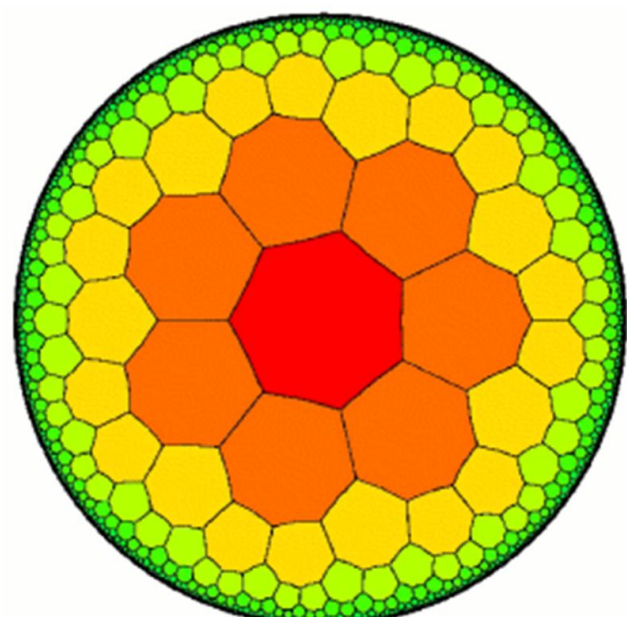


Fig. 9 Regular tiling of the hyperbolic plane using the Poincaré disc

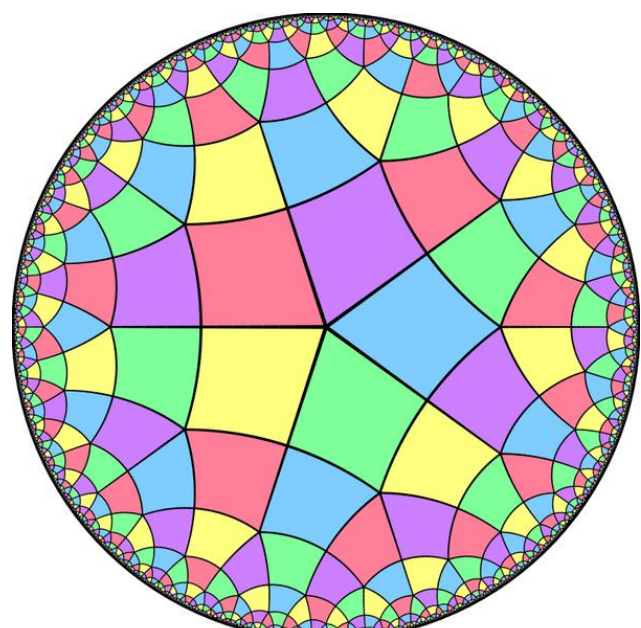


Fig. 10 Irregular tiling of the disc model

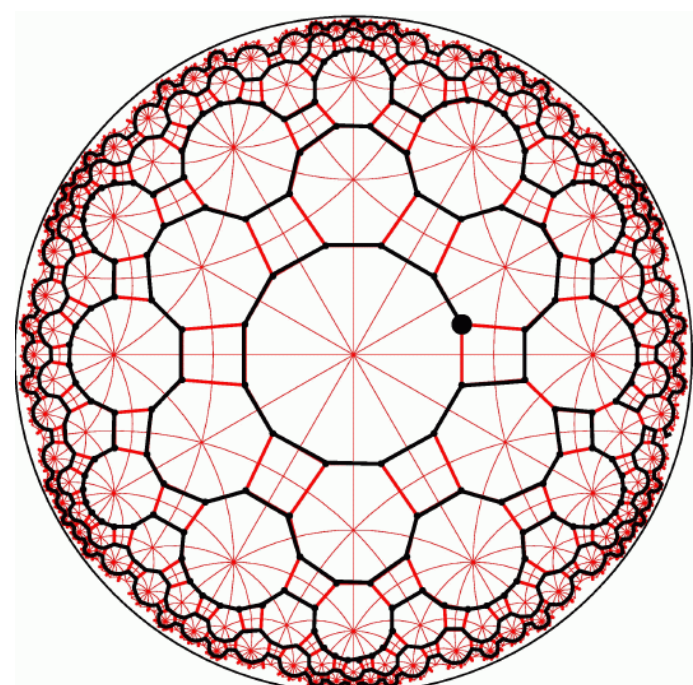


Fig. 11 Hamiltonian path over a red triangle tessellation & Cayley graph

Mathematics sings with you feel it in your whole brain.[†] My goal is to reach out to local area schools & share my insights so that they, too, can sing.



Fig. 12 Lettuce leaves with negative curvature

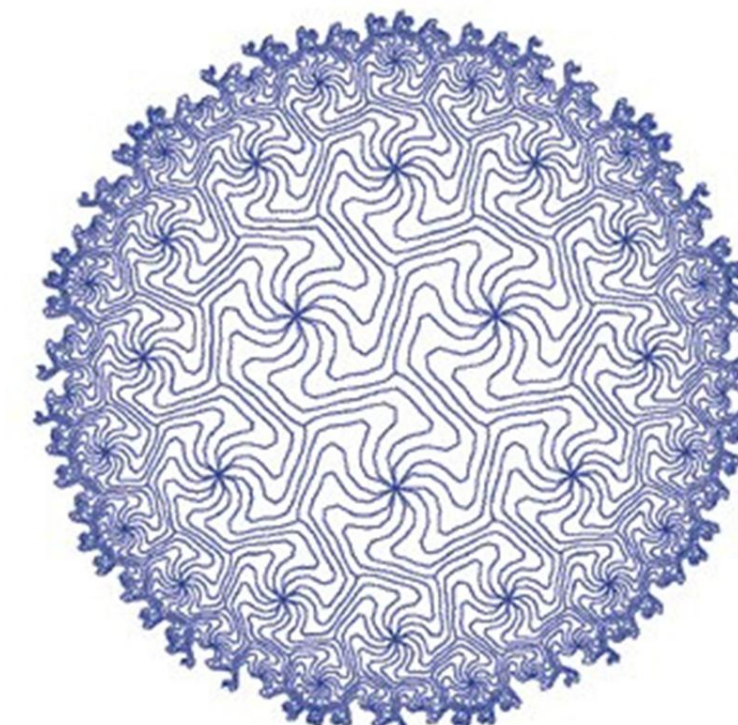


Fig. 13 Quarks, symmetry and cold electrons