1-1-2020

Optimal Control and Numerical Bifurcation Analysis of Interactive Sterile and Wild Mosquitoes Populations

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Summer 2020 Research and Creative for Undergraduate Students (RCEU) Proposal

Optimal Control and Numerical Bifurcation Analysis of Interactive Sterile and Wild Mosquitoes Populations

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Project Summary:
Mosquito-borne diseases, such as malaria, dengue fever, yellow fever, West Nile virus, etc. are considerable public health concerns worldwide. These diseases are transmitted between humans by blood-feeding mosquitoes. According to the World Health Organization (WHO), approximately one to three million people die of malaria every year, the vast majority of which are pregnant women and children who live mostly in Africa and in South America. Since there is not vaccine to control the mosquito-borne diseases, all efforts are directed to avoid the proliferation of the mosquito population. Chemicals have been and are still extensively used all over the world to control wild mosquito populations. However, in the long run mosquitoes can develop resistance to chemical products. Besides, the WHO only allows a limited number of insecticides in view of polluting disasters. One of the most promising and non-polluting methods is the Sterile Insect Technique/Technology.

The sterile insect technique (SIT) is a method of biological control in which the natural reproductive process of mosquitoes is disrupted. Utilizing radical or other chemical or physical methods, male mosquitoes are genetically modified to be sterile which are incapable of producing offspring despite being sexually active. These sterile male mosquitoes are then released into the environment to mate with the wild mosquitoes that are present in the environment. A wild female that mates with a sterile male will either not reproduce, or produce eggs, but the eggs will not hatch. Repeated releases of genetically modified mosquitoes or releasing a significantly large number of sterile mosquitoes may eventually wipe out a wild mosquito population, although it is often more useful to consider controlling the population rather than eradicating it.

The SIT brings new and effective weapons to reduce or eradicate wild mosquitoes. Nevertheless, the interactive dynamics of wild and sterile mosquitoes are such complex that investigations and assessments of the impact of releasing sterile mosquitoes into field are very challenging. To assist effectiveness of the SIT and provide useful guidance for good strategies of releasing sterile mosquitoes, one of the proposers (Ai) and his collaborators have formulated mathematical models of differential equations to study the interactive dynamics of wild and sterile mosquitoes. One important term in these models characterizes the release rate of sterile mosquitoes. We incorporated three different releasing strategies, namely, the constant release rate, the release rate proportional to wild mosquito population, and the proportional release rate with saturation. We

found the threshold release value for each releasing strategy and showed that mild mosquitos can be eliminated if the release rate is bigger than the corresponding threshold value. Using the sizes of these threshold values we made comparisons of economic advantage and disadvantage among these releasing strategies.

In this research project, we propose to use the sterile mosquitos release rate as a control function in our differential equations models and formulate the problem as an optimal control problem. Namely, the problem is to find the optimal release rate that minimize the "difference" of wild mosquito population from a given target value in a fixed period. The mathematical questions to face are: (i) Establish existence and/or uniqueness of an optimal solution. (ii) Derive necessary and/or sufficient optimality conditions. (iii) Construct algorithms of the numerical approximation to the optimal solution. Furthermore, in our differential equations models, we have a parameter describing the population dependent death rate for wild mosquito population. We propose to use this parameter as the bifurcation parameter and perform the numerical bifurcation analysis, the goal being to investigate the dependence of the threshold release value on the problem parameters. The solutions from this project are expected to provide insights to understand the local and global dynamics of the models and the effectiveness of the SIT.

Student Prerequisites:

The perspective student should have good background in calculus, differential equations, numerical analysis. The student should have high motivation for the proposed research project.

Student Duties:

The student will be working about 40 hours per week during 10 weeks in summer 2020. Under the supervision of faculty mentors, the student is expected to learn additional necessary materials supporting this research, review scientific papers in the literature to understand the status of the proposed problems, initiate and develop the methodology to tackle the problems, write algorithms and perform numerical simulations.

Mentor Supervision and Interaction:

The faculty mentors will be introducing the background to the student during the first two weeks. After that, meeting with the mentors will be set up on a weekly basis, reporting the progress, discussing any problems encountered and developing further strategies and techniques to tackle the problems. However, the student is encouraged to see the mentors for advice and suggestions whenever needed. At the end of the program, the student will make both oral and written presentations to the mentors on his/her research results. Publishable results will be written in LaTex and sent to appropriate journals for publication.