

Analysis of a Generalized Discrete Periodic Model for the Spread of *Wolbachia* in a Mosquito Population

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Overview and Impact

Wolbachia bacteria in mosquitoes keeps them from spreading certain harmful diseases and sterilizes males. The spread of *Wolbachia* has been widely studied, and here we performed analysis of a generalized discrete model (below) for *Wolbachia* spread, with the intention of making the model useable by researchers seeking to perform field releases of *Wolbachia* infected mosquitoes. We sought conditions to guarantee the existence of periodic solutions to the model: $x_{n+1} = F_n(x_n)$. These solutions are the attractors for the model; they determine the global dynamics.



$$F_n(x_n) = \frac{(1 - \mu)(1 - s_f)(1 + m_n)(x_n + f_n)}{s_h x_n^2 - (s_f + s_h + m_n(s_f - s_h))x_n + 1 + (1 - s_f)f_n + (1 - s_h)m_n + (1 - s_f)f_n m_n}$$

Methods

- Guarantee at least one periodic solution by common interval (Fig 1)
- Guarantee maximum number of periodic solutions by conditions (Fig 2)
- Results were obtained from model, and verified numerically

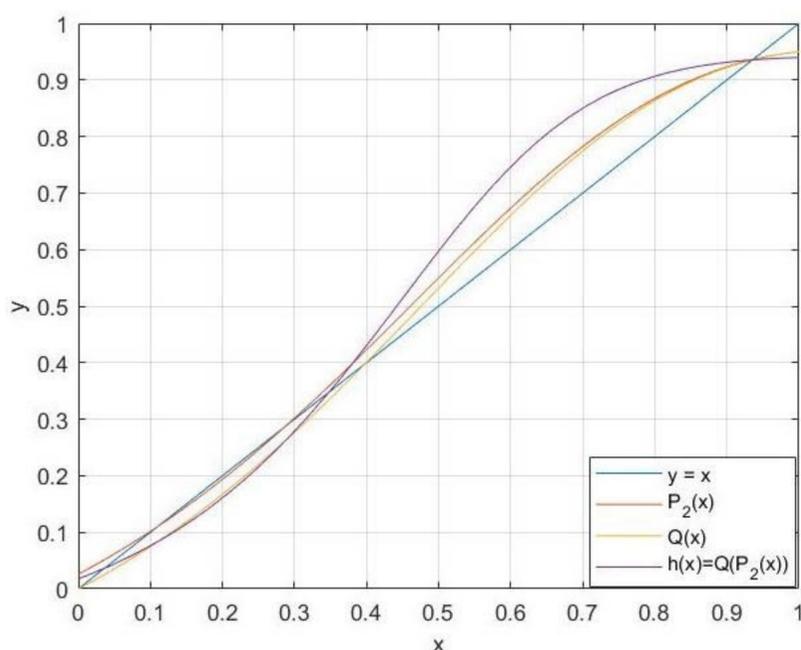


Fig 1: Graph for the second case where $\mu \neq 0$

Results

We were able to find conditions to guarantee a minimum of one periodic solution, and conditions to guarantee the maximum number of periodic solutions. The latter also provide an interval on which the solution exists, assuming the conditions are met.

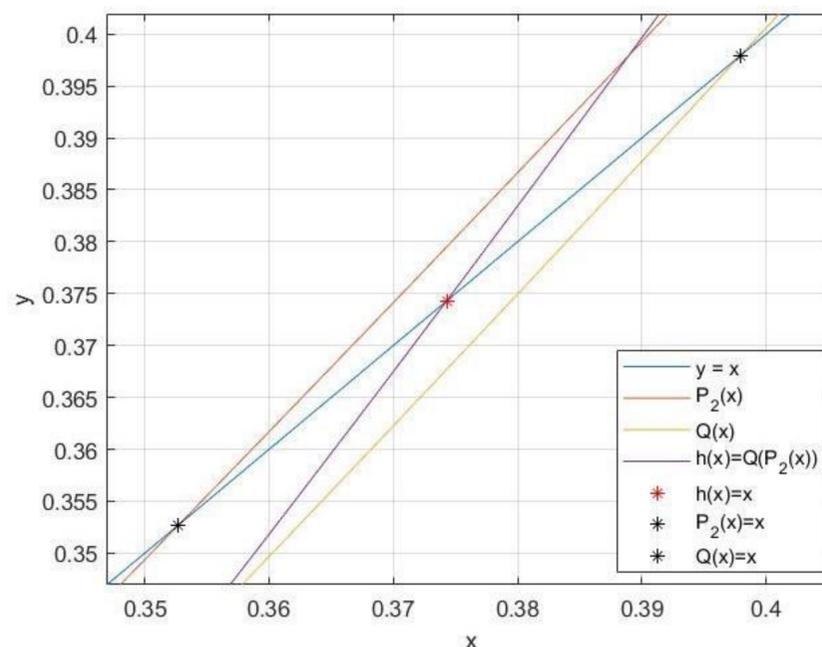


Fig 2: Zoomed in graph for the second case where $\mu \neq 0$

References

1. B. Zheng, J. Li, and J. Yu, "One Discrete Dynamical Model on *Wolbachia* Infection Frequency in Mosquito Populations." *Science China Mathematics*, Feb. 02, 2021.

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