A Mosquito Population Model with Four Positive Equilibrium Points And Sterile Insect Release

Maxwell Fox, Dr. Shangbing Ai and Dr. Mark Pekker, Department of Mathematical Sciences

Introduction

Sterile Insect Technique (SIT) uses lab-created, sterilized insects to disrupt reproduction for insect populations, including disease spreading mosquitoes. Mathematical models have been useful in understanding the impact of varying sterile insect release rates. Let \( w(t) \) and \( g(t) \) be wild and sterile mosquito populations at time \( t > 0 \). Then the dynamics can be described by the system

\[
\begin{align*}
\frac{dw}{dt} &= \frac{w^2}{1 + w + g} - (\mu_1 + \xi_1(w + g))w \\
\frac{dg}{dt} &= \frac{bw}{1 + w} - (\mu_2 + \xi_2(w + g))g
\end{align*}
\]  

(1)

It was previously shown that if \( \mu_1 = \mu_2, \xi_1 = \xi_2 \), then there are at most two positive equilibrium points (EPs)[1]. Others argued that this held for any positive set of coefficients[2]. However, we can provide some counterexamples to this. For instance, it can be shown that if

\[
\mu_1 = 0.01, \mu_2 = 0.5, \xi_1 = \xi_2 = 0.04, b = 5
\]  

(2)

then system (1) has 4 positive EPs. These points, along with their stability, are displayed in Figures 1 through 4.

Key Findings

Using (2) as a starting point, continuation of equilibrium curves using \( \mu_1 \) as a parameter was performed with the MATLAB package CL_MATCONTL to locate bifurcation points. Most importantly, this analysis suggests a saddle-node bifurcation occurs when \( \mu_1 \approx 0.01391547 \). If \( \mu_1 \) is varied greater than this, then system (1) has no more than two EPs.

Impact/Conclusions

Knowledge of the equilibria is essential to understanding the population dynamics. Thus, knowing all possible EPs and their stabilities will help researchers in predicting the dynamics of mosquito populations when using this release rate for SIT population control.

References


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