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Dark Matter Presence in Galaxy Clusters

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DARK MATTER PRESENCE IN GALAXY CLUSTERS

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ABSTRACT

Many astronomers have speculated about the existence of dark matter in galaxy clusters, groups of galaxies that are in relatively close proximity to each other, and it is now presumed that dark matter exists not only just inside galaxies, but in between the galaxies as well. Dark matter is defined as matter in which emitted radiation has yet to be detected, and has only been perceived because of its gravitational effects on other galactic objects. In this research, I have checked for the presence of dark matter by using two observational means for determining the mass of a galaxy cluster. The first was determined by taking luminosity measurements of visible matter in the clusters and then using a mass and luminosity relationship to determine the mass. The other equation we use to calculate the mass is derived by solving for mass in the Hydrostatic Equilibrium Equation. I used data from X-ray satellites, specifically NASAs *Chandra* X-Ray Telescope. Based on the results from the data gathered from these sources and the two mass equations, I have been able to make a logical conclusion regarding the presence of dark matter in galaxy clusters. There were two possible results that I could have expected; either the mass from the luminosity equation is equal to the mass from the Hydrostatic Equilibrium equation, implying that there is no dark matter present or vice versa, from which I could deduce that there exists dark matter in galaxy clusters.

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1. Introduction

In this research I have attempted to determine the presence of dark matter in the universe, specifically in galaxy clusters. Dark matter is matter which scientists suppose to be in the universe but has yet to be physically detected. There are several astrophysical phenomena which have lead to this supposition of the existence of dark matter including gravitational lensing (the bending of light from a distant source around a massive object), flat rotational curves when plotting the density of matter in galaxies versus the distance from the center of the galaxy, and in the difference between calculated vales of the total mass and the gas mass in a galaxy cluster. The cosmological model assumes that the universe is homogeneous and isotropic. The model is described by the following parameters: the Hubble constant, H; the mass density parameter Ω_M (which is the ratio of the density, ρ_M and the critical density ρ_{cr}; Ω_b represents the baryonic matter alone); and the energy density parameter Ω_Λ. The *Wilkinson Microwave Anisotropy Probe* (WMAP) provides evidence for a flat universe, that is a universe where $\Omega = \Omega_\Lambda + \Omega_M \simeq 1$ (Komatsu et al. 2008). Based on others' research we now believe that the universe is comprised of dark energy ($\Omega_\Lambda \simeq 0.7$), dark matter ($\Omega_M \simeq 0.3$), and baryonic matter ($\Omega_b \simeq 0.03$). This proposal addresses the measurement of Ω_M using X-ray observations of galaxy clusters.

Galaxy clusters are good indicators of the dynamics of the universe as a whole because they are the largest gravitationally bound structures in the universe and are formed early in the history of the universe. Therefore, using data collected from galaxy clusters, it is possible to get approximate measurements for the parameters of the cosmological model, in particular dark matter, Ω_M.

The research that I have been performing began with the formulation of the theory needed to measure the total mass (M_{tot}) and the gas mass (M_{gas}) of galaxy clusters, and then I applied it to the X-ray data of clusters observed by *Chandra.* Since I used X-ray observations, I was able to measure the mass of a galaxy cluster via the X-ray surface brightness of the gas, and the total mass of the galaxy cluster using the hydrostatic equilibrium assumption. Once the equations for mass had been formulated, I was then able to determine Ω_M from the equation, $M_{gas}/M_{tot} = \Omega_b/\Omega_M$, since Ω_b is known and easily found. I focused mainly on the comparison of the two mass values, first using typical parameters of galaxy clusters and then determining actual values for four specific galaxy clusters, Abell 2204, MKW 4s, AWM 5, and NGC 6329 Cluster.

2. Background Physics and Methods of Analysis

2.1. Hydrostatic Equilibrium Mass Equation

I began by formulating an equation of mass using the Hydrostatic Equilibrium equation, P_i=-P_g, where P_i is the internal thermal pressure of the object and P_g is the pressure due to gravity that is acting on the object. Using the ideal gas law PV=NkT, I found an equation for P_i. This equation is P_i=nkT, where n is the number of molecules per volume. Then I found a relation for P_g using the equation $P_g=\frac{F_g}{A}$, where A is the surface area and F_g is the gravitational force acting on the cluster. Substituting in an equation $F_g = \frac{GMm_p}{r^2}$, I then determined the equation, $P_g = \frac{GMm_p}{Ar^2}$, which assumes a Hydrogen gas setting $m_H \approx m_p$. In order to determine the mass equation I had to calculate the differential of the internal and gravitational pressures using Hydrostatic Equilibrium

$$\frac{1}{n_g(r)}\frac{dP_g}{dr} = -\frac{GM(r)}{r^2} \tag{1}$$

I determined that the differential gravitational pressure is

$$dP_g \cdot A = dF_g = \frac{GM(r)dm}{r^2} = \frac{GM(r)n_g(r)m_pAdr}{r^2} \tag{2}$$

where I found the relation for the differential mass, dm, to be

$$dm = n \cdot m_p \times A \cdot dr. \tag{3}$$

And the differential internal pressure was calculated as

$$dP_i = nkdT + Tkdn. \tag{4}$$

Now substituting all of these equations into the Hydrostatic Equation, I solved for mass

$$M_{tot}(r) = \frac{-kr^2}{Gnm_p}\left[T\frac{dn}{dr} + n\frac{dT}{dr}\right] \tag{5}$$

which can be used with information about the distribution of density and temperature of the hot gas to estimate the total mass. (LaRoque et al. 2006)

2.2. Surface Brightness and Mass Equation

Using the surface brightness of galaxies I was able to find another equation for the mass, the mass of the gas in the galaxy cluster. To solve for this mass I started with a radial profile of density equation

$$n_e(r) = n_{eo}\left(1 + \frac{r^2}{r_c^2}\right)^{-3\beta/2} \tag{6}$$

and a surface brightness equation

$$S_x = \frac{1}{4\pi(1+z)^4}\int n_e^2 \Lambda_e dl. \tag{7}$$

where Λ_e is the emissivity of the plasma. Combining these two equations, I found that

$$S_x = A\int_{L(b)} n_e^2(r)dl, \tag{8}$$

where S_x is the surface brightness, $A = \frac{\Lambda_e}{4\pi(1+z)^4}$ and is constant, $n_e(r)$ is the density at a radius r, and $L(b)$ is the line of sight at impact parameter b. Solving the integration (see Appendix A), the surface brightness equation reduces to

$$S_x(\theta) = S_{x_0}\left(1+\frac{\theta^2}{\theta_c^2}\right)^{(1-6\beta)/2} \tag{9}$$

where S_{x_0} is the central value of the surface brightness at $\theta = b = 0$. (Birkinshaw et al. 1991)

A simple formula used to find the gas mass is

$$M_{gas} = \int n_e m_p \cdot dV \tag{10}$$

assuming hydrogen gas, with $n_p = n_e$. Using this simple formula as a model I found the gas mass from the surface brightness and density equations. By pulling all of the constants out of the integral and setting them equal to A I got the general formula for the gas mass to be

$$M_{gas}(r) = A\int n_e(r)4\pi r^2 dr = A\int r^2\left(1+\frac{r^2}{r_c^2}\right)^{-\alpha} dr \tag{11}$$

where α is equal to $3\beta/2$, according to Equation (7). Now using the angular radius instead of the physical radius, where $r = \theta \cdot D_A$, this equations becomes

$$M_{gas}(r) = A\cdot D_A^3\int_0^{r/D_A} \theta^2\left(1+\frac{\theta^2}{\theta_c^2}\right)^{-3\beta/2} d\theta \tag{12}$$

where $A = 4\pi\mu_e n_{e0} m_p$ and μ_e is the mean molecular weight of the electrons. For a Hydrogen plasma $\mu_e = 1$.

This equation for mass is the actual mass of the gaseous matter in the galaxy cluster, and the mass found in Section 2.1 is the total mass, the mass of the gaseous matter in addition to any matter that may be in close proximity to the gas. I then wanted to determine the relationship between the two masses, for generic typical values of the density and temperature of the gas.

3. Calculations of Gas and Total Mass for β Model

3.1. Model Parameters

When calculating the mass of the galaxy cluster due to the two different equations there are a few parameters that will be used to find typical results to which we can compare future results from collected data.

For actual measurements of the mass due to the composition of the cluster and the total mass due to hydrostatic equilibrium, I used the following typical values for the models of Equation (6) and Equation (12)

$$\begin{cases} \alpha = 1 (or \beta = \frac{2}{3}) \\ r_c = 100 kpc \\ n_{e_0} = 10^{-3} \frac{atoms}{cm^3} \\ T = 10^8 K \end{cases} \tag{13}$$

3.2. Calculating M$_{tot}$

To calculate the Total Mass I used Equation (6), first finding the differentiation of the density with respect to r

$$J(r) = \frac{dn}{dr} = -2n_{eo}\frac{r}{{r_c}^2}\left(1+\frac{r^2}{r_c^2}\right)^{-2\alpha} \tag{14}$$

and then the equation for the total mass became

$$M_{tot}(r) = \frac{-kr^2}{Gnm_p}\left[T\frac{dn}{dr}\right] = \frac{-kr^2}{Gnm_p}T \cdot J(r) \tag{15}$$

in which T is assumed constant.

3.3. Calculating M$_{gas}$

Solving the integration in Equation (12), I found a general equation for M$_{gas}$

$$I(r) = \int_0^r r^2\left(1+\frac{r^2}{r_c^2}\right)^{-\alpha} dr = r_c^3\left[\frac{r}{r_c} - \arctan\left(\frac{r}{r_c}\right)\right] \tag{16}$$

using the parameters in listed in Equation (14). Thu the equation for the gas mass is

$$M_{gas}(r) = A \int_0^r r^2 \left(1 + \frac{r^2}{r_c^2}\right)^{-\alpha} dr = A \cdot I(r) \tag{17}$$

3.4. Results

I used Eq. (16) and Eq. (17) and the parameters chosen in (14) in order to calculate the mass profiles. The results are shown in Figure 1, Figure 2, Figure 3, and Figure 4.

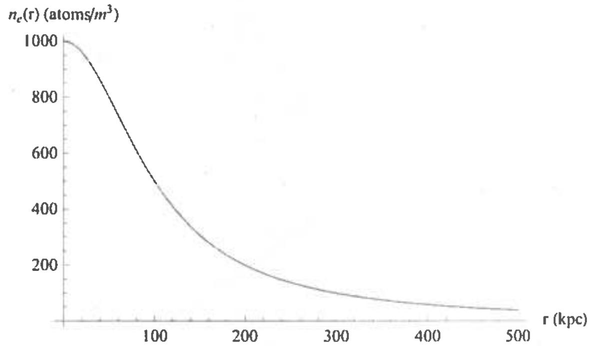

Fig. 1.— Graph of density as a function of radius using the specific set of parameters.

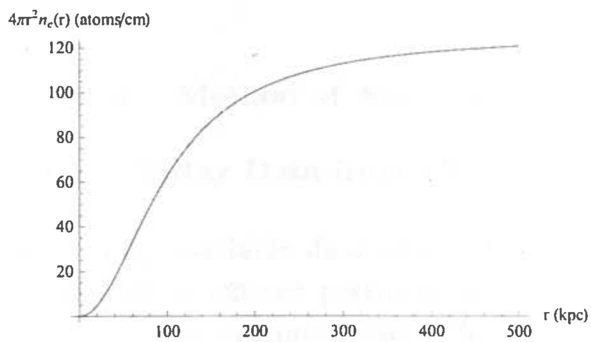

Fig. 2.— Graph of the surface area density as a function of radius using the specific set of parameters.

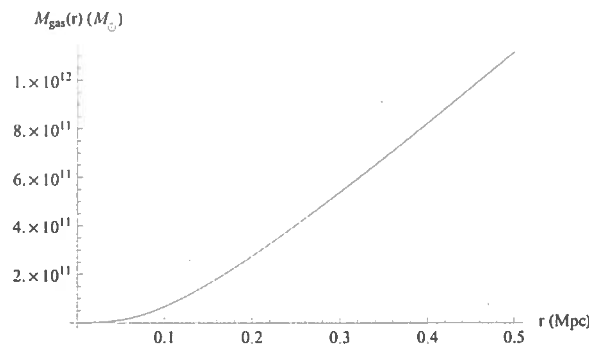

Fig. 3.— Graph of the mass as a function of radius, M_{gas}(r) using the specific set of parameters.

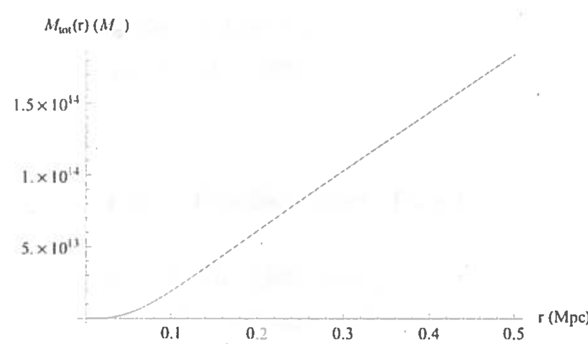

Fig. 4.— Graph of the mass as a function of radius, M_{tot}(r) using the specific set of parameters.

4. Method of Analysis

4.1. X-Ray Data from *Chandra*

For this project, I used readily available data collected from the *Chandra* X-ray Telescope. From this data, I was able to extract pertinent information, including density, β, surface brightness, core radius, and the maximum radius from which data was collected, for the different galaxy clusters being examined. I was able to obtain this information by using a *Markov chain Monte Carlo* program. (Bonamente et al. 2004) Using these values for a galaxy cluster, I was able to find a numerical value for the mass of the baryonic, or visible, matter in the cluster.

These two masses are valuable for determining the ratio of Ω_b/Ω_M by means of the gas fraction of a galaxy cluster,

$$f_{gas} = \frac{M_{gas}}{M_{tot}} \propto \frac{\Omega_b}{\Omega_M} \tag{18}$$

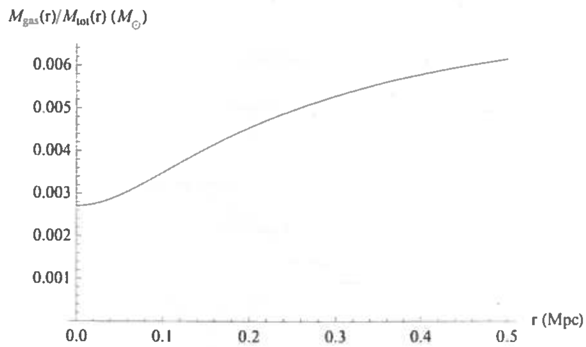

Fig. 5.— Graph of Gas Mass/Total Mass

and thus a more accurate estimate of the value of Ω_M. In fact, Ω_b has been accurately measured by other means. (Tytler et al. 1999)

4.2. Preliminary Results

I analyzed one galaxy cluster Abell 2204 using values determined from data obtained from a *Chandra* X-ray image of the cluster and using the β model to determine the best fit for these values. The results of my analysis are shown in Figure 6 and Table 1.

Table 1: Preliminary Results for Abell 2204

Cluster	S_{x0}	r_c	β
Abell 2204	370±8.0	5.16±0.1	0.483±0.002

Figure 6 shows the radial profile of the surface brightness for cluster Abell 2204 that I obtained using the *Markov chain Monte Carlo* software developed by Bonamente et al. (2004). The fit is not satisfactory at large radii, where the model does not fit the data.

Also, I obtained the radial profile of the surface brightness of cluster Abell 1835. The long *Chandra* exposure detects photons out to 500", which corresponds to $\sim$ 2.5 Mpc.

4.3. Use of More Accurate Models for *Chandra* Data

As I stated, I began by fitting the *Chandra* data of cluster Abell 2204 to the basic β model for the surface brightness. However, when performing this data analysis, I noticed that this model proved to not be as accurate as I would have liked. Notice that at large

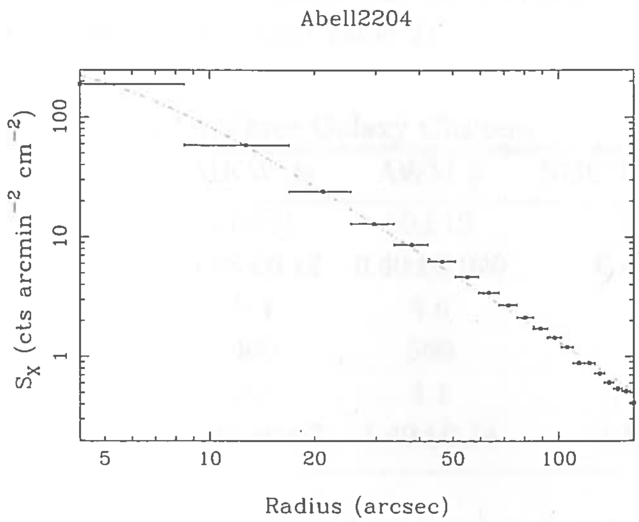

Fig. 6.— Radial Profiles of Surface Brightness for Abell 2204

radii especially, the surface brightness profile deviates substantially from β model (Figure 6). Therefore the surface brightness of a galaxy cluster is not well described by the β model. In order to account for these deviations, I found two potentially more accurate models for surface brightness. The first is the density model (Vikhlinin et al. 2006):

$$n_e^2(r) = n_0^2 \frac{(r/r_c)^{-\alpha}}{1+(r^2/r_c^2)^{3\beta-\alpha/2}} \frac{1}{(1+r^\gamma/r_s^\gamma)^{\epsilon/\gamma}} + \frac{n_{02}^2}{(1+r^2/r_{c2}^2)^{3\beta_2}}. \tag{19}$$

And the other model is the pressure model (Nagai et al. 2007):

$$P_e(r) = \frac{P_{e,i}}{(r/r_p)^c[1+(r/r_p)^a]^{(b-c)/a}}. \tag{20}$$

These models can be used with Equation (15) and Equation (17) to model the surface brightness, and to determine M_{gas} and M_{tot} (recall that the total mass depends on the gradient of the pressure).

5. Application of Hydrostatic Equilibrium Equation Models

5.1. Calculation of Masses

After I had analyzed *Chandra* data, I then decided to qualify my equations by checking the research of *Diffuse X-ray Emissions in Three Poor Clusters of Galaxies* performed by

M. Dahlem and I. Thiering. In their research they determined the variables I needed for my calculations for three galaxy clusters. (See Table 2)

Table 2: Spatial Fitting Results for Three Galaxy Clusters

Parameter	MKW 4s	AWM 5	NGC 6329 Cluster
r_c (kpc)	17±11	60±19	31±16
β	0.38±0.18	0.40±0.020	0.44±0.20
n_{e_0} (10^{-3}cm^{-3})	5.4	4.6	6.2
r_{dl} (kpc)	400	500	600
M_{grav} ($\times 10^{13} M_{\odot}$)	2.8	4.1	4.1
kT (keV)	1.35±0.07	1.49±0.14	1.12±0.07

Please note that in this table $r_{dl} = r_{max}$ (maximum radius observed) and $M_{grav} = M_{tot}$.

To more easily determine the total mass and the gas mass values I wrote a computer program in C language (See Appendix B). When I ran the calculations for the values determined by Dahlem and Thiering, my results were off by a negligible factor. (See Table 3) Therefore, I am confident that the equations that I derived are valid and yield values that are comparable to the values determined by other scientists.

Table 3: Mass Values for Three Galaxy Clusters

Parameter	MKW 4s	AWM 5	NGC 6329 Cluster
M_{tot} ($\times 10^{13} M_{\odot}$)	2.278	3.267	3.267
M_{gas} ($\times 10^{11} M_{\odot}$)	2.085	2.442	24.42
M_{gas}/M_{tot}	0.009	0.036	0.075

5.2. Interpretation of Results

Therefore by determining the total and gas masses for the galaxy cluster, I was able to determine the percentage of dark matter present in these galaxy clusters using $M_{gas}/M_{tot} = \Omega_b/\Omega_M$. The ratio of M_{gas}/M_{tot} for each of the galaxy clusters is show in Table 3. This shows that for cluster MKW 4s the baryonic matter comprises only 1% of the total mass, therefore 99% of the matter is undetectable by X-rays and thus is considered dark matter. Likewise, galaxy cluster AWM 5 is 96% dark matter and cluster NGC 6329 is 93% dark matter.

6. Conclusion

In this research I performed theoretical calculations of M_{tot} and M_{gas} based on the Hydrostatic Equilibrium model. I determined two analytical equations that could easily be solved to determine the total mass and the mass due to the gaseous matter in the cluster. Using typical parameters, I showed that, as expected, the total mass was indeed larger than the gas mass, implying the existence of dark matter within the cluster. I also did a fitting of *Chandra* X-ray data using a *Markov chain Monte Carlo* program to determine the most accurate values for characteristics of the galaxy cluster. These values were critical to determine the masses for actual galaxy clusters, for these characteristics are variables in the mass equations. I also qualified my research by applying the Hydrostatic model to the data of Dahlem and Thiering, and ultimately showed that clusters are indeed composed of $90 - 99\%$ dark matter.

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Appendix A
Derivation of the Surface Brightness Equation

The derivation of Eq. (10) begins with solving the integration in Eq. (9)

$$S_x = A\int_{L(b)} n_e^2(r)dl \tag{21}$$

(insert figure here) From Figure 8, we can see that $r^2 = l^2 + b^2$, therefore $l = \sqrt{r^2 - b^2}$ and $dl = \frac{1}{2}(r^2 - b^2)^{-1/2}2rdr$, so Eq. (9) becomes

$$S_x = An_{e_0}^2 \int_0^\infty \left(1 + \frac{r^2}{r_c^2}\right)^{-6\beta/2} (r^2 - b^2)^{-1/2} r dr \tag{22}$$

when substituting in Eq. (7) for n_{e_0}. Then I did a change of variables using

$$\begin{cases} \theta_c = \frac{r_c}{D} \\ \frac{b}{D} = \sin\theta \\ \frac{l}{D} = \xi \end{cases} \tag{23}$$

using the convention of $\sin\theta \approx \theta$ (for small angles), which yields

$$S_x \propto \int_0^\infty \left(1 + \frac{\theta^2 D^2 + \xi^2 D^2}{\theta_c^2 D^2}\right)^{-6\beta/2} d\xi = \int_0^\infty \left(1 + \frac{\theta^2 + \xi^2}{\theta_c^2}\right)^{-6\beta/2} d\xi \tag{24}$$

therefore

$$S_x = \int_0^\infty \left(1 + \frac{\theta^2}{\theta_c^2} + \frac{\xi^2}{\theta_c^2}\right)^{-6\beta/2} d\xi \tag{25}$$

then I used the following integral formula to solve the integration

$$\int_0^\infty (p + qm^2)^{-(n+1)} m^{(\mu-1)} dm = \frac{1}{\nu p^{(n+1)}} \left(\frac{p}{q}\right)^{\mu/\nu} \frac{\Gamma(\mu/\nu)\Gamma(1 + m - \mu/\nu)}{\Gamma(1+m)} \tag{26}$$

where $p = 1 + \frac{\theta^2}{\theta_c^2}, q = \frac{1}{\theta_c^2}, \nu = 2, n + 1 = 6\beta/2, \mu = 1, andm = \xi$. Substituting these variables into Eq. (24)

$$S_x \propto \frac{1}{2\left(1 + \frac{\theta^2}{\theta_c^2}\right)^{6\beta/2}} \cdot \left(\left(1 + \frac{\theta^2}{\theta_c^2}\right)\theta_c^2\right)^{1/2} \cdot \frac{\Gamma(1/2)\Gamma(6\beta/2 - 1/2)}{\Gamma(6\beta/2)} \tag{27}$$

thus

$$S_x \propto \left(1 + \frac{\theta^2}{\theta_c^2}\right)^{-6\beta/2 + 1/2} \tag{28}$$

Hence, we have Eq. (10)

$$S_x(\theta) = S_{x0}\left(1 + \frac{\theta^2}{\theta_c^2}\right)^{(1-6\beta)/2} \tag{29}$$

which includes the constants that I had pulled out of the integral.

Appendix B
C Program Code for Determining the Masses of Galaxy Clusters

```
#include <stdio.h>
#include <math.h>
#include <stdlib.h>
#include <string.h>


#define M_SUN 1.989e33 /* solar mass */
#define KPC_2_CM 3.086e21 /* convert from kpc to cm */
#define KEV_2_ERG 1.6e-9 /* convert from keV to erg */
#define M_P 1.6726e-24 /* mass of proton */
#define G 6.673e-8 /* gravitational constant */
#define PI 3.14159 /* pi */


double calc_Mtot();
double calc_Mgas();
double calc_Ratio();


main()
{

double beta,rc,kT,mu_tot,mu_e,n,rmax;
double mTot;
double mGas;
double ratio;


beta=0.40;
rc=60.0; //kpc
kT=1.49; //keV
mu_tot=0.6; //
mu_e=1.15; //
n=0.0046; //atoms/cm^3
rmax=500.0;// kpc


mTot=calc_Mtot(beta,rc,kT,mu_tot,rmax);
```

```
mGas=calc_Mgas(beta,rc,mu_e,n,rmax);
ratio=calc_Ratio(mTot,mGas);


printf("Total mass is %4.3e",mTot);
printf("Gas mass is %4.3e",mGas);
printf("Ratio of Gas mass to the Total mass is %4.3e",ratio);
}

double calc_Mtot(beta,rc,kT,mu_tot,rmax)
double beta, rc, kT, mu_tot, rmax;
{
double Mtot;
// Line below contains the total mass for the beta model
Mtot = (3.0 * beta * kT * KPC_2_CM * KEV_2_ERG * pow(rmax, 3))/(G * mu_tot * M_P *
(pow(rc, 2) + pow(rmax, 2)) * M_SUN);
return(Mtot);
}


double calc_Mgas(beta,rc,mu_e,n,rmax)
double beta, rc, mu_e, n, rmax;
{
double Mgas;
// Line below contains the gas mass for the beta model
Mgas = 4.0*PI*mu_e*n*M_P*pow(rc*KPC_2_CM, 3.0)*((rmax/rc)-atan((rmax/rc)))/M_SUN;
return(Mgas);
}

double calc_Ratio(Mtot,Mgas)
double Mtot, Mgas;
{
double ratio;
// Line below contains the ratio of gas mass to total mass
ratio=Mgas/Mtot;
return(ratio);
}
```