Explaining/Modeling of Unexpected Congestion on Local Freeways

Susanna Manning

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Explaining/Modeling of Unexpected Congestions on Local Freeways

by

Susanna L. Manning

An Honors Capstone
Submitted in partial fulfillment of the requirements
For the Honors Diploma
To

The Honors College

of

The University of Alabama in Huntsville

April 22, 2019

Honors Capstone Director: Dr. Boris Kunin

Susanna Manning 4/23/19

Student Date

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Date

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04/23/19 5/1/19
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Abstract

Major traffic congestion happens on freeways at times for no apparent reason. This means the speed of cars drop to almost zero, creating a standing train of cars, when there are no accidents, crashes, or entering and exiting traffic on the freeway causing the congestion. This phenomenon is specifically observed locally in Huntsville, AL on weekdays at the traffic ramp exiting Memorial Parkway and entering I-565 Westbound around 8am and the stretch of I-565 Westbound between Wall-Triana and Greenbrier Road between 3pm and 5pm. This project seeks to model these phenomenon on a ‘microscopic’ level (car by car). A Mathematica program is built to model simplified versions of the real-world phenomenon, and screenshots from Google Maps with traffic are analyzed to assess the model’s accuracy.
**Introduction**

Major traffic congestion happens on freeways at times for no apparent reason. This means the speed of cars drop to almost zero, creating an almost standing train, while there are no accidents, crashes. Even when there is traffic entering or exiting the freeway, the nominal ramp speeds are much higher than the ultimate crawling speeds of traffic. This phenomenon is often called a phantom traffic jam, and is a current open topic of research.

There are three main approaches to traffic flow analysis: cellular models, microscopic models, and macroscopic models. Cellular models consider partitioning the roadway into cells of a set width with each cell containing an average traffic density. With each time step, the flux of vehicles travelling from one cell to the next is measures. These models are used due to their relative simplicity to code and resemblance of general traffic jamming behavior. Microscopic models seek to describe the behavior of individual vehicles. These models consider a some number of cars on the road and calculate the position, velocity, and acceleration of each vehicle for every prescribed time step. Macroscopic models describe aggregate quantities of traffic flow and lay the framework for the concepts of shock waves of traffic instability that occur. These models consist of equations for traffic variables – density, velocity, flow rate – and model traffic as if it is a liquid instead of individual cars. [Seibold]

An MIT research team studied this phenomenon with a macroscopic model using equations that model traffic similar to the equations that model fluid flow – not on a car by car basis. They found that phantom traffic jams are caused by travelling waves of localized high traffic density. Their studies showed that these travelling waves matched the concept of detonation waves, which are stable structures that travel with constant velocity along the road. [Flynn]
The University of Nagoya in Japan released experimental video footage of this phenomenon on a circular track. The experiment displays a microscopic model that begins with 22 various cars spaced equidistant apart along a circular track. As the cars begin driving and trying to maintain a constant speed, pockets of high density form as the cars run up on one and another, resulting in phantom traffic jams. [Scientist]

The present project explores the phantom traffic jam phenomenon by means of a microscopic model with the idea that exaggerated adjustments made by drivers in response to the car in front of them causes a pile-up. All cars travel on a one lane road, obey all traffic laws, and move in a deterministic and predictable manner. Cars also do not enter or exit the flow. For our model, we consider two situations that have been observed near Huntsville, AL where phantom traffic jams emerge: cars leaving an exit ramp from a highway and cars travelling on a straight highway with no exits nearby. The final model describes both cases.

In the case of the exit ramp, cars approach the ramp at 50 mph and enter the ramp at 25 mph. This logically should result in every car slowing appropriately and exiting at 25 mph with no congestion, but it often results in a line of standing cars waiting to exit the ramp. Our model suggests the phantom jam could be produced due to overcompensation in the slowing of successive cars, resulting to speeds at nearly zero.

In the case of the straight highway, cars are considered to travel at 70 mph with no prescribed slowdowns. This situation should logically result in all cars continuing to travel 70 mph with no congestion, but regularly in our area of interest a phantom traffic jam occurs. Our model suggests this is due to small fluctuations in the speed of drivers (a difference of 5 mph) during high density traffic.
For both cases, we considered a simplified approach allowing for discontinuous velocities for ease of analysis. We then focused on analyzing the effect of different traffic flow parameters: initial distance between cars, length of time the first car slows, and difference between the initial car speed and the first car’s final slow speed. Our simplified model showed results of a phantom traffic jam happening in both cases for certain parameter thresholds.

This report will consist of our algorithm descriptions, results of the model and parameter analysis, discussion of results, list of references, conclusion, and appendices with the code and traffic data.
Algorithm Description

The cars in this model move on a one lane road, obey all traffic laws, and move in a deterministic and predictable manner. Cars are analyzed in pairs, so for ease of description, we call the front car in a pair, A, and the back car in the pair, B. All cars begin equidistant apart with the comfortable space between each pair determined relative to the speed of A. We define this comfortable space as \( L(u) = 15 \left( \frac{u}{10} \right) + 15 \) where \( u \) is the velocity in miles per hour of A and \( L(u) \) is a number of feet. This is simply the commonly suggested safety norm of leaving one car length between A and B per every 10 mph. We supposed a car length is a constant 15 feet, so we added that value to account for the entire distance between the midpoints of successive cars. We name the distance \( L(u) \) trailing behind its corresponding car – measured from the midpoint of the car - the plume of that car.

In this project, we explored two types of algorithms. In our simplest algorithm, velocity is discontinuous and we do not consider acceleration. This causes sudden changes in velocity throughout the model. In our second algorithm, we consider discontinuous acceleration along with discontinuous velocity. While sudden changes in velocity still occur, they are less drastic than the first model. Both proposed models test the hypothesis that phantom traffic jams are caused by overreaction of drivers to expected or miniscule slowdowns that happen in front of them.

Discontinuous Velocity (Instantaneous Infinite Accelerations)

We suppose that all cars begin travelling at the same speed \( U \) with some distance \( cL(U) \) separating each pair, where \( 0 < c < 1 \). We use this \( c \) because our analysis of start distance parameters showed pile-ups only occurred when cars began at a more dense state, meaning when each car started somewhere inside the plume of the car before it. We then suppose that first car’s velocity instantaneously changes to some \( u < U \). This causes the distance between the first and
second car to decrease with time. We suppose the distance decreases until it reaches \( \frac{1}{2} L(u) \). At that point, the second car’s velocity instantaneously changes to \( \tilde{u} < u \) such that \( L(\tilde{u}) = \frac{1}{2} L(u) \). This causes the distance between the first and second car to increase until the distance reaches \( L(u) \). At that point, the second car’s velocity instantaneously changes to \( u \). This procedure is prescribed to every pair of cars as it travels along the road, such that each car reacts to the car directly ahead of it.

In the setting of the ramp, cars slow to the exit speed but then eventually speed back up to the speed limit of the new road. In the setting of the slight slowdown on the straight freeway, the car that initially slowed by a minimal amount will speed back up to the original speed after some time. Because of this, we assume in our model that the first car returns to the initial velocity of \( U \) after some prescribed time.

Figure 1 depicts a flow chart of the overall algorithm in terms of analyzing a pair of cars, where car A is directly in front of car B. We let \( u_A \) and \( u_B \) denote the velocities of A and B, respectively. For each time increment, the following is done for each pair of cars in the model. \( u_A \) and \( u_B \) are compared to determine where the pair is in the algorithm. If \( u_B > u_A \), steps 1 through 8 from Figure 1 are prescribed. This means we check if B has reached the midpoint of the plume of A. If \( u_A > u_B \), steps 6 through 8 from Figure 1 are prescribed. This means we check if B has reached the far end of the plume of A. If \( u_B = u_A \), the cars continue to travel at this velocity, and only position is updated for each car. Then the position and velocities of the two cars after that time increment are updated.

We leave initial velocities, the coefficient for the initial distances, the amount of time the first car remains slow, and the number of cars in the model as parameters to test various situations. This allowed us to test this model for both the cases of the ramp and the straight highway.
Figure 1: Discontinuous Velocity No Acceleration Algorithm Flow Chart. Each velocity referenced in the chart is to be understood as “current velocity” (i.e. velocity at that instant).

Mildly Discontinuous Velocity and Finite Discontinuous Acceleration

Again, we suppose that all cars begin travelling at the same velocity $U$ with some distance $cL(U)$ separating each pair, where $0 < c < 1$, and that the first car’s velocity instantaneously changes to some $u_A < U$. (In this description, $u_A$ will refer to the velocity of A at the moment the step of the algorithm is occurring. This accounts for $u_A$ changing throughout the process.) We then calculate the acceleration of the second car, call it $a_B$, such that the second car will be travelling a velocity of $u_A$ when it reaches a distance of $\frac{1}{2}L(u_A)$. This acceleration remains
constant until the distance between the two cars is $\frac{1}{2}L(u_A)$. At that point, the second car’s velocity instantaneously changes to $u_B = u_A$. We let $a_B$ continue to remain constant until the distance between the two cars is $\frac{3}{4}L(u_A)$. At that point, we calculate $a_B$ such that the second car will be travelling a velocity of $u_A$. When the distance between the cars reaches $L(u_A)$, the second car’s velocity instantaneously changes to $u_A$. This procedure is prescribed to every pair of cars as it travels along the road, such that each car reacts to the car directly ahead of it.

Similar to the discontinuous velocity no acceleration model, we assume that the car which caused the initial slow down eventually returns to the initial velocity of $U$ after some prescribed time.

Figure 2 depicts a flow chart of the overall algorithm in terms of analyzing a pair of cars, where car A is directly in front of car B. We let $u_A, a_A, u_B$ and $u_B$ denote the velocities and accelerations of A and B, respectively. For each time increment, the following is done for each pair of cars in the model when B is in the plume of A. $u_A$ and $u_B$ are compared to determine where the pair is in the algorithm. If $u_A < u_B$, we check:

1) If this is the first time $u_A < u_B$ after $u_A$ had been greater than $u_B$ for the time increments prior
2) If $u_A < u_B$ for the time increments prior

If 1 is true, we calculate $a_B$ such that when B reaches the midpoint of A’s plume, $u_B = u_A$. We start at step 3 in Figure 2. If 2 is true, an appropriate $a_B$ has been calculated during a prior time increment and we continue to use that value. We start at step 4 in Figure 2. We then continue through step 12.

If $u_A > u_B$, we check:
1) If this is the first time $u_A > u_B$ after $u_A$ had been greater than $u_B$ for the time increments prior

2) If $u_A > u_B$ for the time increments prior

If 1 is true, we let $a_B$ remain the same until B reaches $\frac{3}{4}$ of A’s plume. We wait until then to calculate $a_B$ such that $u_B = u_A$ when B reaches the far end of As plume. So, we start at step 7 in Figure 2. If 2 is true, we check if B has just reached $\frac{3}{4}$ of A’s plume. If so, we calculate $a_B$ such that $u_B = u_A$ when B reaches the far end of As plume. So we would start at step 8. If that point has not yet been reached (start at step 7) or has been passed (start at step 10), an appropriate $a_B$ has been calculated during a prior time increment and we continue to use that value. We then continue through step 12.

If $u_B = u_A$, we check if B is at the far end of As plume. If so, the cars continue to travel at this velocity, and only position is updated for each car. If not, we start at step 6 in Figure 2.
Figure 2: Discontinuous Velocity and Discontinuous Acceleration Algorithm Flow Chart. Each velocity referenced in the chart is to be understood as “current velocity” (i.e. velocity at that instant).
Results

Due to time constraints, we were only able to create a working code for the algorithm of the model with discontinuous velocity with no finite acceleration. The program was written in Mathematica and the complete code is in Appendix A. This model produced phantom traffic jams in both settings for certain parameter values (see below).

Table 1 shows a summary of traffic jam lengths for 200 cars with time intervals at $\frac{1}{10}$ of a second. For the highway measurements, cars started at 70 mph and the first car slowed to 65 mph. For the ramp measurements, cars started at 50 mph and the first car slowed to 25 mph. Each simulation ran the traffic for 4 minutes and output data every 30 seconds. This chart displays the exploration of the relationship between the time that the first car remains slow and the initial separation distance of the cars. This graph uses these specific parameter values as an illustration. Other parameter values with similar relationships to one another produce similar results. For both the ramp and the highways settings, the coefficient for the initial separation distance was tested for many values in the range of 0.50 to 1.00 (0.55, 0.60, 0.65, 0.70, 0.75, 0.80, 0.85, 0.90, 0.95, 0.96, 0.97, 0.98, 0.99, 1). Also, for both settings the number of seconds the first car travelled slowly was tested for all whole second values between 1 and 10.

In both settings, we found that shrinking the initial separation of the cars and keeping all other parameters the same results in faster accumulation of cars in the phantom traffic jam. This is depicted by columns 2, 3, and 4 in Table 1 for the highway and columns 6 and 7 for the ramp.

We also noticed that the less time the first car remained slow, the smaller the initial distance between cars needed to be. This was much more noticeable for the highway situation than for the ramp. We noticed that for this parameter, phantom traffic jams occurred only when the amount of time the first car went slower was not small. When cars exit a ramp, they maintain a slower speed.
for a longer time than when a car dips slightly in speed on a highway. Because of this, we focused more on how small the amount of time that a car wavering in speed on the highway needed to be in order to cause a phantom traffic jam instead of on the ramp. Column 2 and 5 in Table 1 demonstrate this relationship. Cars on the highway started a distance of $0.98L(70)$ apart. When the first car went slow for only one second, no traffic jam occurred. However when the first car went slow for ten seconds, a traffic jam did slowly accumulate.

<table>
<thead>
<tr>
<th>Time (minutes)</th>
<th>Highway</th>
<th>Ramp</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Start Distance : 0.98$L(70)$</td>
<td>Start Distance : 0.85$L(50)$</td>
</tr>
<tr>
<td></td>
<td>Time First Car Slows: 1 second</td>
<td>Time First Car Slows: 1 second</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1/2</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>12</td>
</tr>
<tr>
<td>3/2</td>
<td>0</td>
<td>18</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>25</td>
</tr>
<tr>
<td>5/2</td>
<td>0</td>
<td>32</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>38</td>
</tr>
<tr>
<td>7/2</td>
<td>0</td>
<td>NA*</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>NA*</td>
</tr>
</tbody>
</table>

*NA* represents the times that cars beyond the 200 original cars would be piling up. Since we were measuring how many of the original 200 cars would be piled up, the numbers entered in these slots would deceptively decrease. See Index for visual graphs.

*Table 1: Comparison of parameter adjustments*

Table 2 displays another example of how both situations react under the same model. For both, the initial start distance between cars ($0.85L(u)$ where $u$ is the appropriate start speed), number of cars (500), time the first car travels slower (10 seconds), and the time interval length ($\frac{1}{10}$ second) remained the same in both cases. The only differences were that for the ramp, cars started at 50mph and the first car slowed to 25mph, and for the highway, cars started at 70mph
and the first car slowed to 65mph. We can easily see that a drastic traffic jam occurs in less than 10 minutes for each case. The traffic jam on the ramp occurs more rapidly, possibly from the larger changes in velocity.

<table>
<thead>
<tr>
<th>Time (minutes)</th>
<th>Highway</th>
<th>Ramp</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>12</td>
<td>17</td>
</tr>
<tr>
<td>2</td>
<td>25</td>
<td>34</td>
</tr>
<tr>
<td>3</td>
<td>38</td>
<td>52</td>
</tr>
<tr>
<td>4</td>
<td>51</td>
<td>69</td>
</tr>
<tr>
<td>5</td>
<td>65</td>
<td>86</td>
</tr>
<tr>
<td>6</td>
<td>78</td>
<td>103</td>
</tr>
<tr>
<td>7</td>
<td>91</td>
<td>120</td>
</tr>
<tr>
<td>8</td>
<td>104</td>
<td>137</td>
</tr>
</tbody>
</table>

Table 2: Comparison of 500 cars with the ramp and highway scenario

Appendix B contains the graphs for each of the described situations. Each output consists of a position and velocity graph. The position graph lists the cars on the x-axis and their location on the road on the y-axis, measured in feet. Initially, the first car begins at the position of y = 0 on the road and all cars travel in the positive direction. The velocity graph also lists the cars on the x-axis. The y-axis describes the velocity of each car in feet per second. Each group of graphs is preceded by a description of the parameters used during their generation.
Discussion

Our results showed that our hypothesis of exaggerated driver responses to a sudden, even minor, slow down does cause a phantom traffic jam for certain parameters with our discontinuous velocity model. It was interesting to notice that cars needed to be closer than the standard, conventional comfortable distance measurement \((L(u))\) from each other for this to occur in our model. Our analysis of cars separated by exactly \(L(u)\) never resulted in a true traffic jam, regardless of the prescribed distance that cars needed to be separated by in order to decrease velocity. This seems to suggest that there exists a density threshold that guarantees phantom traffic jams for this model with respect to certain parameters. The fact that a relationship exists between the amount of time the first car travelled at a slower speed and the initial distance between cars could be helpful to further analyze that density threshold.

Further Research

There are many routes to explore from this point. With this crude discontinuous velocity and no acceleration model, a more thorough analysis of the relationship between initial density of the cars and the amount of time that the first car travels slowly would help us understand the interaction of the cars in the model and what the density threshold needs to be to cause phantom traffic jams in various real world situations. For example, if we know the average speed and density of cars on a stretch of road, we may be able to more accurately predict how slow a car can travel before creating congestion.

We could also analyze how changing the place in a car’s plume where the car behind instantaneously switches velocity affects the creation of congestion. We did analyze this when cars began a distance of \(L(u)\) apart by analyzing what happens when the car behind changes to a slower
velocity at $\frac{1}{n}L(u)$ from the car ahead. However, we did not consider this when cars begin in a more dense state.

Another opportunity for further research is writing the program for discontinuous acceleration and mildly discontinuous velocity model and explore its predictions. We suspect that the same phenomenon of congestion will occur, but there will be less drastic jumps in velocity resulting in a more realistic model. If the phantom traffic jams still occur, we could then compare the relationships noticed in the new model with the discontinuous velocity and no finite acceleration model, such as the initial density and change in speed of the first car. If the congestion no longer occurs, we could analyze why the addition of acceleration changed the initial results so drastically.

Real world data and observation of the phenomenon would help determine the accuracy of the models and approximate realistic parameters - such as initial density, velocity differences, and amount of time to create congestion. Since we know the two exact spots that this type of congestion occurs, we gathered data by taking screenshots from the traffic layer on Google Maps in attempt to get an idea of where, when, and how this congestion begins and propagates (see Appendix C). The maps were helpful, but we were not able to see the congestion occur in real time nor see the individual cars and their responses to the traffic. Somehow obtaining aerial video of the congestion from start to end would be most beneficial for further study.
Conclusion

Our hypothesis of drivers’ exaggerated responses to the sudden, yet small, change in speed of a car ahead resulted in a simple model simulating the phenomenon of a phantom traffic jam. Future enhancement of this model could provide more insight into what is actually happening on the road that causes frustration to many drivers. This hypothesis is just one of many approaches explored on the topic. Further research on this topic in general could result in suggestions to alter certain roadways or speed limits prone to this phenomenon in order to decrease the chance of phantom traffic jams.
References


Appendix A

Code for discontinuous velocity and no acceleration model

Clear["Global`*"]
startspeed = 70; (* Initial speed of cars in mph *)
finalspeed = 65; (* Speed that the first car slows to in mph *)

FPStoMPH = 3600 / 5280; (* conversion factor from feet per second to mph *)
MPHtoFPS = 5280 / 3600; (* conversion factor from mph to feet per second *)

start = MPHtoFPS * startspeed; (* Initial speed of cars in fps *)
end = MPHtoFPS * finalspeed; (* Speed that the first car slows to in fps *)

L::usage = 
"L[u] gives the comfortable distance between cars in feet based on the speed in
miles per hour of the car in front";
L[u_] := ((1.5 u FPSstoMPH) + 15);

checkX::usage =
"checkX[t, x, u] gives the new location of a car based on the time interval
length, the car's current location, and the car's current velocity";
checkX[t_, x_, u_] := x + u t;

startdist = (.85) L[start]; (*Initial distance between all cars*)

(*Parameters*)
timeSlow = 10; (* Amount of time the first car travels the slower speed *)
numCars = 500; (* Number of cars *)
frac = 10; (* Denominator of the fraction of a second *)
t = 1 / frac; (* Length of each time tick *)
umDeltaTs = min * 60 * frac; (* Number of time ticks *)
lfrac = 1/2; (* Fraction of the plume a car must reach before changing velocity *)
min = 10; (* Number of minutes to simulate traffic flow *)
(* Lists *)
X = {0}; (* Position *)
U = {end}; (* Velocity *)

(* Initialize lists *)
For[i = 2, i ≤ numCars, i++,
  AppendTo[X, (X[[i - 1]] - startdist)];
  AppendTo[U, start];
];

(* Output *)
Print[ListPlot[X, Axes → True, PlotLabel → "Location on Road", AxesLabel → {Cars, Location},
  LabelStyle → Directive[Black, Bold]]];
Print[ListPlot[U, Axes → True, PlotLabel → "Velocity of Cars", AxesLabel → {Cars, Velocity},
  LabelStyle → Directive[Black, Bold]]];

(* Begin loop *)
For[i = 1, i ≤ numDeltaTs, i++,
  Xtemp = checkX[t, X[[1]], U[[1]]]; (* Check position and velocity of first car *)
  Utemp = U[[1]];
  For[j = 2, j ≤ numCars, j++,
    X = ReplacePart[X, (j - 1) → Xtemp];
    (* Update position array for each pair of cars (Call them A and B) *)
    X = ReplacePart[X, j → checkX[t, X[[j]], U[[j]]]];]
  (* Adjust car to right behind the other car if overlap occurs *)
  If[X[[j - 1]] < X[[j]],
    X = ReplacePart[X, j → (X[[j - 1]] - 15)]
  ];
  (* Temp variables *)
  Xtemp2 = X[[j]];
  Utemp2 = U[[j]];
  If[U[[j]] > Utemp, (* If B is faster than A *)
    If[Abs[X[[j]] - X[[j - 1]]] ≤ lufrac*L[Utemp],
      (* Check B's relation to half plume of A*)
      newU = u /. Solve[L[u] = lufrac*L[Utemp], u][[1]]; U = ReplacePart[U, j → newU]);
  ];
];
If[U[[j]] < Utemp, (* If B is slower than A *)
    If[Abs[X[[j]] - X[[j - 1]]]] > L[Utemp], (* Check B's relation to full plume of A *)
        U = ReplacePart[U, j -> Utemp]; (* B changed velocity to uA*)
        (*else*)
        U = ReplacePart[U, j -> U[[j]]]; (* B maintains same velocity*)
    ];
]

(* Don't allow negative velocities *)
If[U[[j]] < 0,
    U = ReplacePart[U, j -> 0]
];

Xtemp = Xtemp2;
Utemp = Utemp2;
]

If[1/100 == timeSlow,
    U = ReplacePart[U, 1 -> start] (* Return first car to original speed *)
];

(* Output *)
If[Mod[i, (60*frac)] == 0,
    Print[" ";
    Print[ListPlot[X, Axes -> True, PlotLabel -> "Location on Road",
        AxesLabel -> {Cars, Location}, LabelStyle -> Directive[Black, Bold]]];
    Print[ListPlot[U, Axes -> True, PlotLabel -> "Velocity of Cars", AxesLabel -> {Cars, Velocity},
        LabelStyle -> Directive[Black, Bold]]];
    Print["Number of cars piling: ", numCars = Count[U, end] - Count[U, start]];
];
Appendix B

Graphs of Simulations
This appendix contains examples of the Mathematica graphs obtained from running the model. There is one example of a simulation of a phantom traffic jam happening in each of the situations: the ramp and the highway. The time in the simulation corresponding to the plot data is listed below each graph.

Number of Cars: 500
First Car’s Slow Velocity: 25 mph
Time Interval: .1 seconds
Time until First Car Speeds Back Up: 10 seconds
Initial Distance Apart: .85\(L(50)\)
Initial Common Velocity: 50 mph
Simulation Total Time: 10 minutes

Time: 0 minutes
Time: 2 minutes
Number of cars piling: 69

Time: 4 minutes
Number of cars piling: 86

Time: 5 minutes
Time: 6 mintues
Number of cars piling: 137

Time: 8 minutes
Number of Cars: 500

Time Interval: .1 seconds

Initial Distance Apart: \(0.85L(70)\)

Initial Common Velocity: 70 mph

First Car’s Slow Velocity: 65 mph

Time until First Car Speeds Back Up: 10 second

Simulation Total Time: 10 minutes

Time: 0 minutes
Time: 2 minutes
Number of cars piling: 51

Time: 4 minutes
Number of cars piling: 91

Time: 7 minutes
Appendix C

Maps
Google Map images of phantom traffic jam on straight highway