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Wideband delay for Solvable Chaos

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Abstract

We show the development of a wideband delay engineered using the closed-form analytic expression of a solvable chaotic oscillator. This delay serves as a crucial component for the development of a matched filter receiver that optimally detects chaotic waveforms in the presence of noise.

Background

Chaos is an unpredictable, noise-like behavior that generally precludes closed-form solutions. The lack of these solutions requires engineers to use empirical characterizations in lieu of theory to guide designs. Recently, simple optimal detection hardware has been identified for a special class of solvable chaotic systems [1]. One primary advantage to solvable chaos when compared to most chaotic systems is the use of theoretical expressions that guide designs [2] [3]. Here, we use the analytic spectral expression for second order solvable chaotic waveforms to develop a wideband filter delay in support of a matched filter receiver.

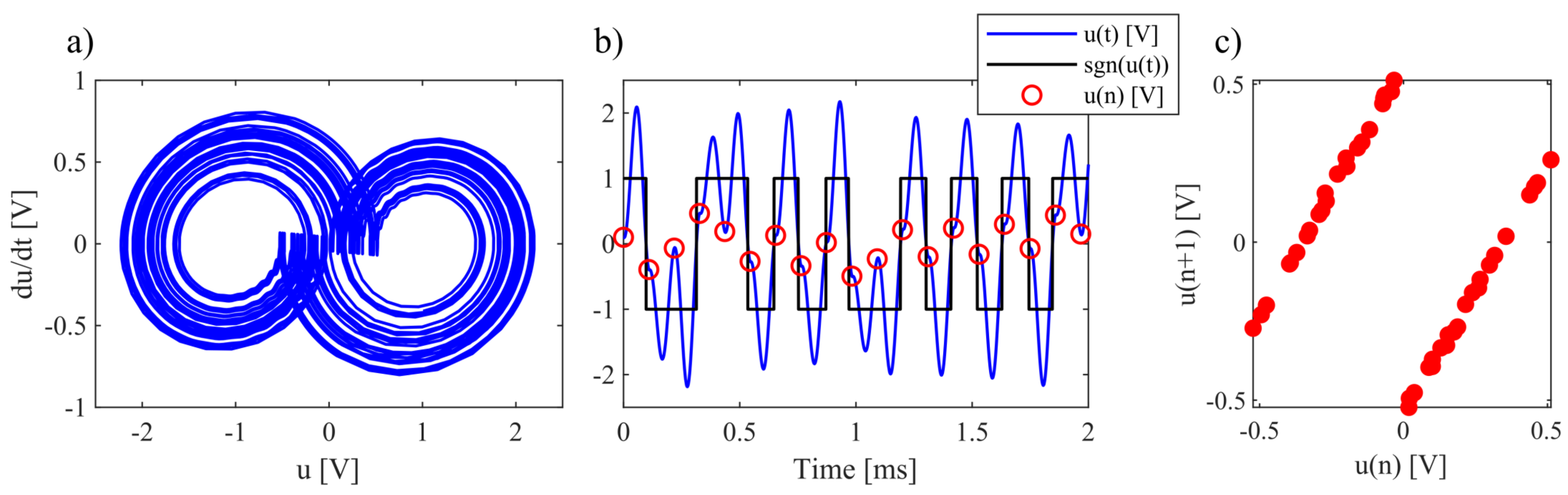


Figure 1. a) Phase portrait showing the chaotic attractor for a solvable chaotic system. b) SPICE time-series simulation of chaotic system. c) Return map for solvable chaotic system.

Hardware Design & Experimentation

The closed-form solution for these chaotic waveforms can be written in a form where a random symbol sequence weights a uniform, fixed basis function. The spectrum of this basis function allows for the design of supporting hardware with specifications informed from theory rather than observation. The focus of this work is a passive, wideband Bessel filter with maximally flat group delay. We show cascaded delay cells engineered to keep a predetermined amount of harmonics in the pass-band. Delay cells were designed to keep f_0, $2f_0$, $3f_0$ and $4f_0$ of the chaotic waveform's spectral content and the corresponding delay outputs are compared to theoretical expectations.

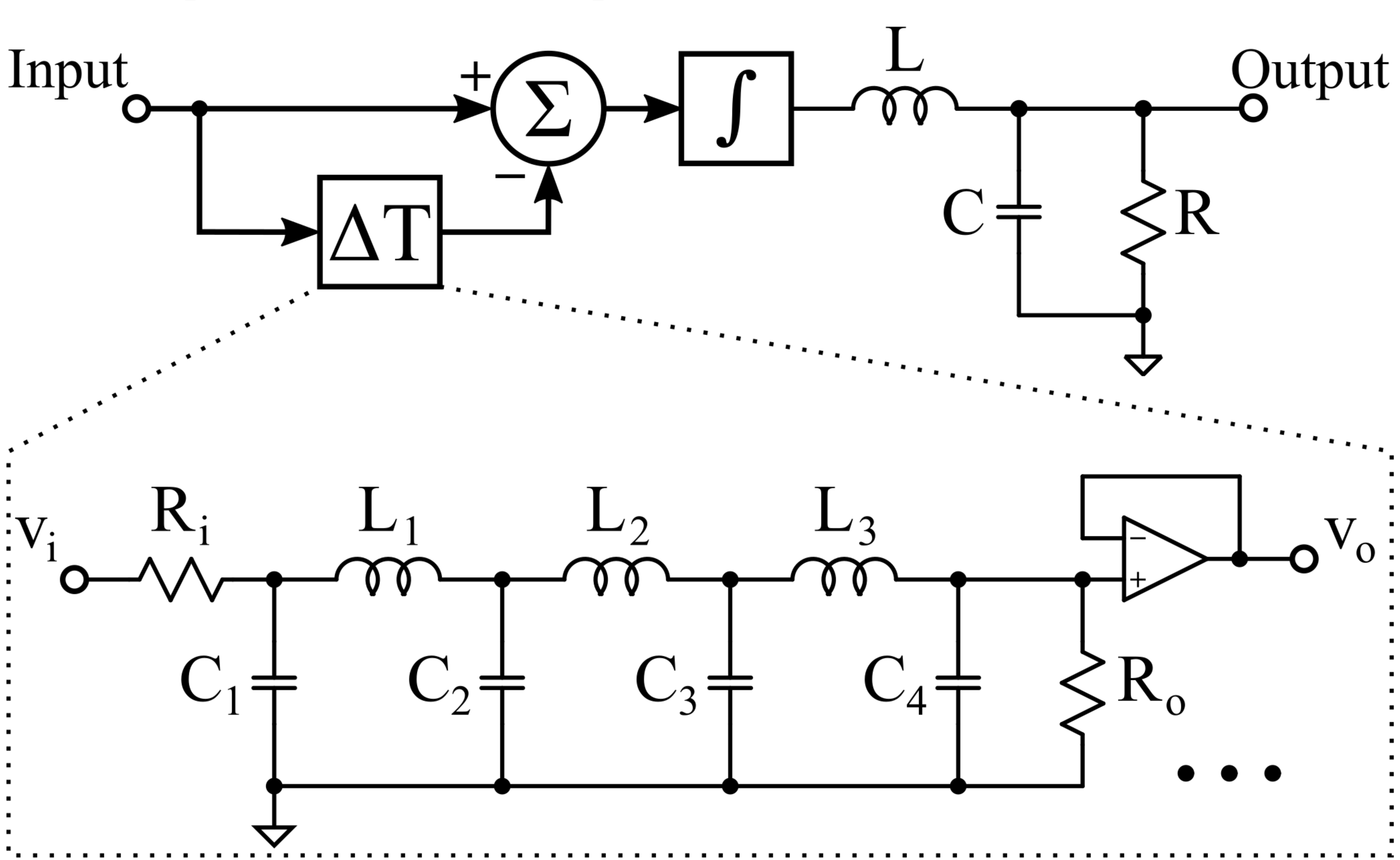
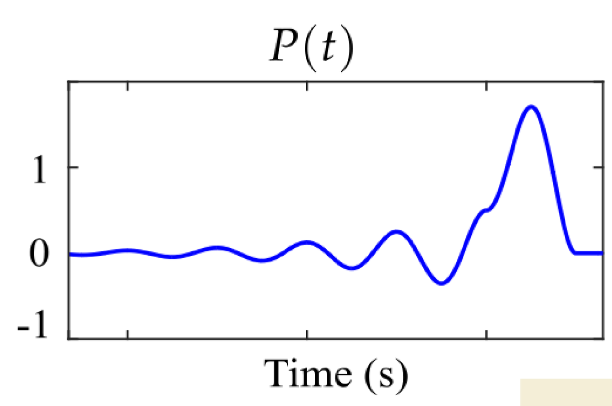


Figure 2. Function block diagram of a matched filter for chaos with exploded view of a 7th order Bessel filter delay cell.

Design Advantages:

- Different from the active component designs in the past,
- All passive design, thus relatively low power consumption,
- Precise cut-off frequency and delay.



$$P(t) = \begin{cases} (1-e^{-\beta T})e^{\beta t}[\cos(\omega t) - \frac{\beta}{\omega}\sin(\omega t)] & t \le 0 \\ 1 - e^{\beta(t-T)}[\cos(\omega t) - \frac{\beta}{\omega}\sin(\omega t)] & 0 \le t \le T \\ 0 & t \ge T \end{cases}$$

$$\mathcal{P}(\omega) = \frac{1-e^{-j\omega T}}{j\omega} \cdot \frac{\beta^2 + \omega_o^2}{(\beta - j\omega)^2 + \omega_o^2}$$

Figure 3. Top) Basis function for solvable chaos. Bottom) Fourier transform of the basis function which allows for analytic filter design.

Results

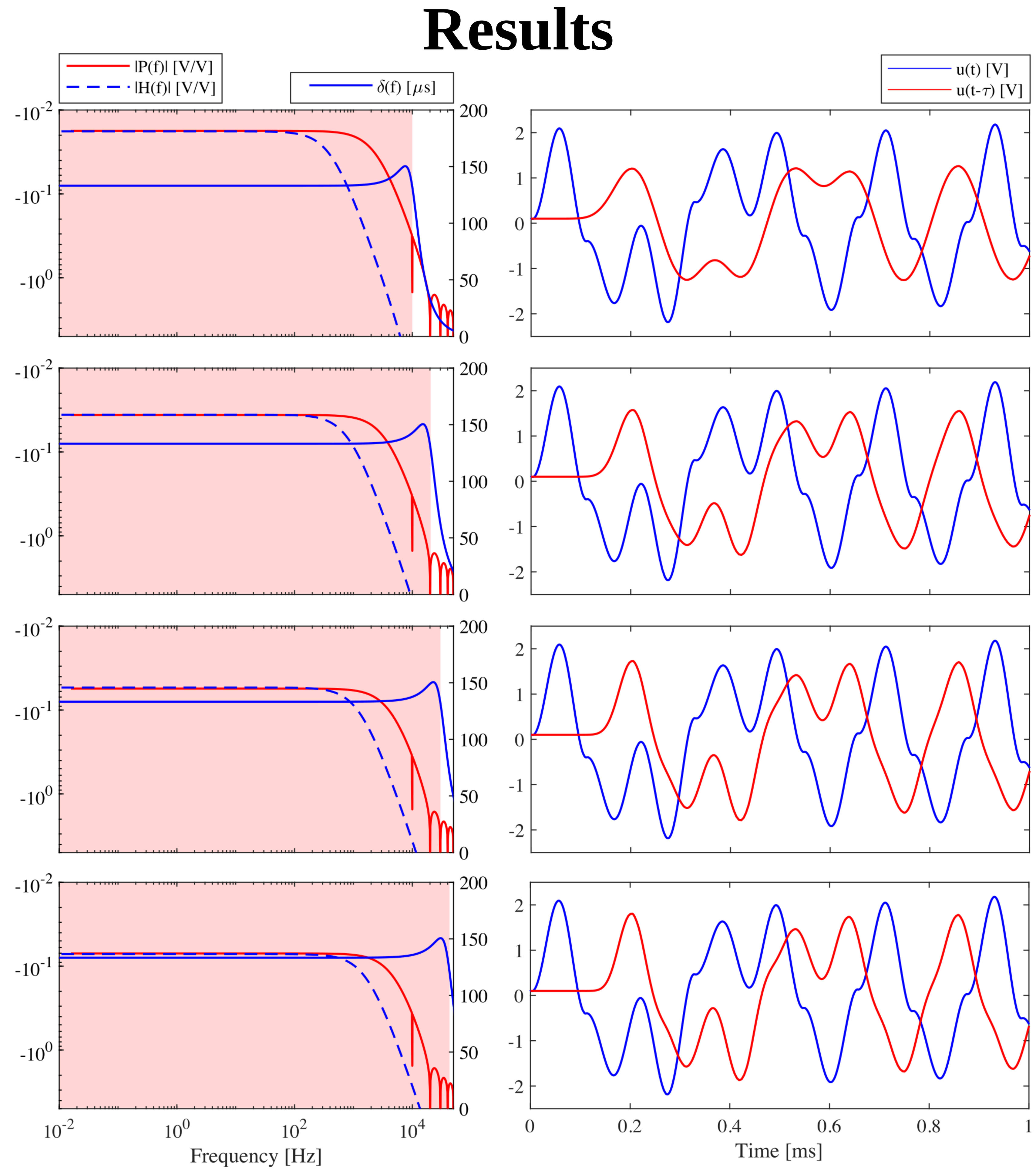


Figure 4. Left) Frequency domain plot showing the theoretical frequency content of a solvable chaotic oscillator and delay bandwidths engineered to keep increasing frequency content. Right) Corresponding time-series data.

Conclusion and Impact

We have illustrated a tradeoff space of delay bandwidth informed by theoretically expected frequency domain representations of solvable chaos. This work highlights a use case where an analytic expression unique to a special class of solvable chaotic systems may be used to inform a practical engineering design. Here, we note that delays of bandwidth $3f_0$ and $4f_0$ preserve key return map features.

- ➢ Chaos is a suitable non-linear random source for engineering applications.
- ➢ We highlight important tools for non-linear systems.
- ➢ Analytic solution provides engineers with new tools (not explored so far) for design.
- ➢ Broadens the application space for solvable chaos.

References

[1] Corron, N. J., Blakely, J. N., & Stahl, M. T. (2010). A matched filter for chaos. *Chaos: An Interdisciplinary Journal of Nonlinear Science*, *20*(2), 023123.

[2] Beal, A. N., Cohen, S. D., & Syed, T. M. (2020). Generating and Detecting Solvable Chaos at Radio Frequencies with Consideration to Multi-User Ranging. Sensors, 20(3), 774.

[3] Cohen, S. D., & Gauthier, D. J. (2012). A pseudo-matched filter for chaos. Chaos: An Interdisciplinary Journal of Nonlinear Science, 22(3), 033148.