Dynamic wavefront capability for anti-jam electronics and controlled reception pattern antenna

Gregory Reynolds

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DYNAMIC WAVEFRONT CAPABILITY FOR ANTI-JAM ELECTRONICS AND CONTROLLED RECEPTION PATTERN ANTENNA

by

GREGORY REYNOLDS

A DISSERTATION

Submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in The Department of Electrical and Computer Engineering to The School of Graduate Studies of The University of Alabama in Huntsville

HUNTSVILLE, ALABAMA

2018
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DISSERTATION APPROVAL FORM

Submitted by Gregory Reynolds in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Electrical Engineering and accepted on behalf of the Faculty of the School of Graduate Studies by the dissertation committee.

We, the undersigned members of the Graduate Faculty of The University of Alabama in Huntsville, certify that we have advised and/or supervised the candidate of the work described in this dissertation. We further certify that we have reviewed the dissertation manuscript and approve it in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Electrical Engineering.

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ABSTRACT

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Title Dynamic Wavefront Capability for Anti-Jam Electronics and Controlled Reception Pattern Antenna

The need to test and evaluate adaptive nulling technology for Global Navigation Satellite System (GNSS) receivers is increasing commensurately with the demand for additional protection against GNSS interference. Because the cost and time associated with approval to conduct optimum outdoor testing is often prohibitive and safety of flight concerns arise during integration testing as well, affordable and accurate means of simulated testing are desired. This dissertation investigates the theory and implementation of Radio Frequency (RF) wavefront simulation that eliminates expensive RF front-end electronics, as well as other active components that limit dynamic range, add exogenous noise, and restrict maximum power capabilities. The method employs mechanical tap-delay lines as the core component of the wavefront simulation that provides many benefits in calibration, cost, and precision RF phase delay. The novel theory presented in this dissertation quantifies a truth estimate for the discrete mechanical tap-delay performance through an a priori mapping of the per-channel phase error, while minimizing the hardware limitations.

A main focal point of the research and theory developed is to answer the question, ”what is truth?” The accuracy and repeatability of hardware undergoing
evaluation is compared to truth to determine the usefulness of the solution. Therefore, it is critical to understand what truth is in a simulation environment. However, there is not a straightforward answer when there are known errors affecting the ability of the simulation to create a valid RF wavefront environment. A proposed solution to define a truth estimate that applies novel analysis techniques to understand the effect of per-element RF phase error at a system level is presented. The concept, which is described in detail in the dissertation, defines a discrepancy between a strict truth estimate using a Steiner tree model, and a loose truth estimate using the well-known MUltiple SIgnal Classification (MUSIC) algorithm. A valid region exists when the discrepancy remains within the observability of the system under test. In essence, this method bounds the error and serves to define the valid regions for the RF wavefront environment. An enhanced tap selection method is also described that increases the available angle of arrival possibilities that fall within the criteria for being valid. This creates an enhanced capability that uses theory and concepts from engineering and mathematics to overcome hardware limitations.

Abstract Approval: Committee Chair

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I would like to thank my wife, for her endless love and support. This has been a long road, and I could not have done it without her. To my parents, for instilling in me the drive to do great things. I will not forget how much they have taught me over the years about hard work and perseverance. To my children, you are a large source of my motivation, and have helped me balance my work time and play time.

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For Rebecca
CHAPTER 1

INTRODUCTION

Men of action are favored by the Goddess of good luck.
—George S. Clayson

The world today has become driven by digital data. Computers, cell phones, automated farming, shipping, and many more technologies that enhance the productivity of our culture. It is not commonly known that these systems are all dependent on the same underlying technology to efficiently function time synchronized. The communication system that drives this is the Global Positioning System (GPS). The GPS infrastructure provides a means for civilian and military users to have very accurate location and timing solutions at very little cost for the user. The main burden of cost is incurred by the U.S. Air Force who maintains a majority of the system’s infrastructure. GPS provides single meter position accuracy for the precise position system (PPS), less than 10 meter accuracy [1] for the standard positioning service (SPS), and on the order of 10 nanosecond timing accuracy. The satellite based navigation system is very well designed and the current coverage provided by the system is global. The satellites are at an altitude of around 20,000 km, with a orbit time of 11 hours 58 minutes.
GPS is very robust and a very well designed communication system, but it incurs the same hindrances that all wireless communication channels must face, interference. There are many types of interference sources that can affect the functionality of a GPS receiver’s ability to track and use the RF signals. The three main types of interference signals are self-induced, accidental, and intentional. The self-induced interference may be a spectrally matched signal from multi-path. Accidental interference includes unintended signals like spurious emissions from another RF source, either narrow-band or broadband. The intended interference could come from a trucker trying to evade employee monitoring or anyone trying to purposely disrupt GPS operations for any reason. In each of these cases, the GPS signal can be degraded, denied or deceived. There are, however, many researchers, businesses, and government entities developing mitigation techniques to aid the GPS receivers in overcoming these undesired interference signals, and for the purposes of this research it is not relevant where the interference originated, but that it exists.

One of the main mitigation techniques being developed is RF nulling technology using Anti-Jam (AJ) electronics and corresponding controlled reception pattern antenna (CRPA). These devices use a method of comparing relative channel phase delay through an eigen decomposition of the covariance matrix to determine from which direction a signal has originated. The accuracy required for this type of phase-coherent system is very difficult to accomplish in a simulation environment. Every picosecond of timing error in the generation hardware at the primary GPS frequency is equivalent to approximately 0.6° of per-channel phase error in the simulated RF signal. To test the most advanced GPS AJ technology, up to eight RF signals must be
created and maintain a phase-coherent accuracy of only a few picoseconds for hours worth of testing, over varying temperature and locations, and with multiple other constraints on setup and calibration for the simulation hardware. This constraint on the phase coherency of the RF signals that comprise the wavefront simulation are a key component to understanding and characterizing a successful RF environment.

This dissertation looks at the simulation technologies that have been developed to test nulling capabilities. It discusses the theory behind the simulation technology and the system under test, as well as develops the theory and analysis techniques that drastically decrease the cost-to-performance ratio of testing capability for wavefront simulation, and satisfies some gaps left by currently available systems.

A main focal point of the research and theory developed is to answer the question, "what is truth?" The accuracy and repeatability of hardware undergoing evaluation is compared to truth to determine the usefulness of the solution. Therefore, it is critical to understand what truth is in a simulation environment. However, there is not a straightforward answer when there are known errors effecting the ability of the simulation to create a valid RF wavefront environment. A proposed solution to define truth that applies novel analysis techniques to understand the effect of per-element RF phase error at a system level is presented. The concept, which is described in detail in the dissertation, defines a discrepancy between a strict truth estimate using a specific implementation of the Steiner tree model, and a loose truth estimate using the well-known MUSIC algorithm. A valid region exists when the discrepancy remains within the observability of the system under test. In essence, this method bounds the error and serves to define the valid regions for the RF wavefront environment.
Some of the other main contributions of this dissertation are in the development of an analysis technique that provides a system level understanding of wavefront simulation theory and a method to create a more accurate representation of the RF wavefront environment for adaptive nulling technology. Specifically, the analytical technique provides a mathematical and graphical description of the per-channel phase coherency effects on the signal representation and AJ electronic interpretation. The analysis technique is then used to expand the understanding of the discrete effects of the communication channel and used to construct a more accurate mapping of the per-channel phase error for an enhanced evaluation capability. The discrepancy calculation that relates the phase errors to the bound is derived and applied.

These advances in theory will then be transitioned to a hardware configuration that utilizes the engineering and mathematical principles to overcome technological difficulties and reduce cost while increasing modularity of wavefront simulation capability. The systematic analysis technique and its contribution to error evaluation are what make this architecture a novel implementation of a wavefront simulation capability. More detailed technical descriptions of the novel concepts will be summarized in Section 4.12.

This dissertation covers the underlying communication theory behind GPS, channel simulations for wavefront applications, and will cover many aspects of the defined problem with testing the devices in laboratory environments and the analysis methods that have been developed to better evaluate the potential interference solutions of AJ electronics. Chapter 2 covers the theory behind the GPS signal structure, derives the equations for the interference calculations and the effect that certain types
of interference signals have on the GPS signal, the basics of some nulling techniques that are being designed to overcome the interference, and a literature survey of the channel simulation techniques that are being developed by others. Chapter 3 covers the theory of an RF wavefront, RF channel model, discrete analysis techniques, and reviews the well known MUSIC algorithm that is widely used for single source Angle of Arrival (AoA) calculations using a phased array antenna. Chapter 4 covers the theory and development of a novel analysis technique that defines the wavefront accuracy on a system level, the derivation of the stability region for the simulation, and a statistical method of using the derived error bound to more accurately measure the effectiveness of AJ and CRPA technology. Chapter 5 provides a look at the theory from Chapter 4 applied in hardware and shows the relevance of the theory and analysis method developed.
CHAPTER 2

BACKGROUND, MOTIVATION, AND LITERATURE SURVEY

The underlying theory that defines a communication system and related research are described in the background section. This section will define the GPS architecture, several integration techniques, and basic AJ nulling technology. This will set the stage and shape the decisions made for various wavefront simulation design considerations. The signal power levels necessary to test the upper limits of the nulling capability will also be derived to provide the relevant power considerations for the system.

2.1 GPS Signal Structure

GPS is a constellation of military satellites that broadcast data for use in positioning, navigation, and timing. The system is funded by the United States Government and is offered as a free service for civilian and commercial use. Because of the nature of the GPS signal and the distance of the wave propagation from the satellite to the earth of approximately 20,000 km, the power level of a GPS signal is around -130 dBm ($10^{-13} mW$) as it reaches the surface of the earth. GPS receivers can track this extremely low power level because the communication system employs a
coding technique where the RF signal is spread in frequency using Gold codes by the transmitting satellite and de-spread by the receiver using phase coherent correlators. However, because of the incredibly low power level received in near-earth use, any small amount of undesired interference signals in the GPS frequency bands can cause errors in the reception of the GPS signals.

Many commercial enterprises have become dependent on GPS signals to provide position, navigation, and timing (PNT) solutions. The United States military also uses GPS in land, air, and marine vehicles for the same PNT solutions. These are the reasons why it is important to protect the integrity of these signals and why so much research is going into developing better interference mitigation solutions.

GPS receivers are very susceptible to interference due to their low received power. The intent of describing the GPS signal structure is to define the noise floor, signal power levels, interference properties of various signal structures, and to help define the overall needs that will flow into requirements for a wavefront simulation implemented in hardware.

There are three types of GPS signal structures that are discussed, and the corresponding interference effects are derived. They are the civilian coarse acquisition (C/A) code, the encrypted P(Y) code, and the new military code (M-Code).

The civilian code is created using a Direct Sequence Spread Spectrum (DSSS) modulation and the transmitted signal is made up of three components. The three components are the RF carrier frequency at 1575.42 MHz, the spreading waveform at a frequency of 1.023 MHz, and the navigation data message at a rate of 50 Hz. Each satellite uses a unique Gold code sequence to spread the corresponding satellite’s
Figure 2.1: GPS signal structure using Gold codes as the Coarse Acquisition pseudo-random noise codes to orthogonalize the signals.

The Gold code is 1023 bits long and is transmitted at a rate of 1.023 MHz. This rate is often referred to as the chipping rate and denoted by $R_c$. This means that each iteration of Gold code accounts for 1 ms of time. Each satellite is given a unique Gold code that is known by the receiving system for de-spreading. The data is sent at a rate of 50 Hz, which means that there are 20 iterations of the gold code sequence for each data bit sent. This allows the receiver to integrate over 20 ms to increase the power level of the received signal to a usable level. A graphical representation of the GPS signal structure is shown in Figure 2.1.

The constellation of GPS satellites all transmit at the same primary frequencies. The DSSS architecture with differing Gold codes for each satellite is what allows all the signals to be received at the same time on the same frequency. This modulation technique is referred to as code division multiple access (CDMA). This type of waveform has many important features for satellite communication systems including the ability to receive very low power and the ability to use the phase changes in the signal for precise ranging.
The encrypted P(Y) code used by military systems is very similar to C/A, but uses a much longer spreading sequence that cannot be directly acquired through typical correlation routines. This signal modulation also has an extra step beyond the civilian code that encrypts the navigation data and requires the receiver to have access to special crypto-keys for demodulation. The civilian C/A code and the military P(Y) code both use a Bi-Phase Shift Keying (BPSK) structure. Because of this, the signals can be combined and sent simultaneously from the same satellite with the C/A code on the in-phase portion and the P(Y) code on the quadrature portion of a QPSK signal structure as shown in Figure 2.2 and Equation 2.1.

As described in Misra [2], the navigation data, D(t), and the Gold code, C(t), are combined using a modulo 2 sum. The resulting signal is then modulated to the carrier frequency. This signal comprises the civil (C/A) code for GPS and is entirely on the in-phase component. The encrypted military signal is created using a similar procedure with a much longer code consisting of approximately $10^{14}$ chips. This code repeats once each week, as opposed to the 1 ms repetition time of the C/A code. The encrypted GPS signal is sent on the quadrature component, which allows the two
signals to be demodulated at the receiver without interference from the other. An example of the GPS signal transmitted from the $i^{th}$ satellite can be represented as

$$S_{L1}^{(i)}(t) = \sqrt{2P_C}D(t)C_{C/A}(t)\cos(2\pi f_{L1}t + \theta_{L1}) + \sqrt{2P_Y}D(t)C_{P(Y)}(t)\sin(2\pi f_{L1}t + \theta_{L1})$$  \hspace{1cm} (2.1)

$$S_{L2}^{(i)}(t) = \sqrt{2P_Y}D(t)C_{P(Y)}(t)\sin(2\pi f_{L2}t + \theta_{L2})$$  \hspace{1cm} (2.2)

where $P$ represents the signal power, $C_{C/A}$ and $C_{P(Y)}$ represent the Pseudo-Random Noise (PRN) code sequences, $D(t)$ represents the navigation data, and the carrier signal $f(t)$ has been represented by the sin and cosine waves. A similar formula is used to create the data at the L2 frequency, but only the encrypted $P(Y)$ signal on the quadrature phase is broadcast at that frequency as shown in Equation 2.2.

Acquisition is accomplished by searching over time and frequency in bins using the known Gold codes assigned to each satellite. One technique is to use a circular correlation routine that sweeps over code and frequency bins until a peak is found.

Because of the desire to create a simulation environment that accurately reflects the real world, the effects of external interference on acquisition, carrier tracking, and demodulation of GPS are described in detail. The effective carrier to noise relationship $C_S/N_0$, is defined [3] as

$$\left(\frac{C_S}{N_0}\right)_{eff} = \frac{1}{\frac{1}{C_S/N_0} + \frac{C_l/C_S}{QR_e}}$$  \hspace{1cm} (2.3)
where $C_S/N_0$ is the carrier to noise ratio of the unjammed portion of the received signal. The jamming to received signal ratio is $C_l/C_S$. $Q$ is the jamming resistance quality, which will be derived for various signal interference types, and $R_c$ is the chipping rate.

For the case of narrowband interference, $Q$ is defined [3] as

$$Q = \frac{1}{R_c S_S(f_l)}$$  \hspace{1cm} (2.4)

The term narrow-band is used for interference signals with much less bandwidth than the desired signal, or very narrow such as continuous wave (CW) signals. If the interference from the narrow-band source is at the exact center frequency of the received BPSK signal, then $S_S(f) = 1/R_c$ and $Q$ becomes

$$Q = \frac{1}{R_c/R_c} = 1.$$  \hspace{1cm} (2.5)

The $Q$ value for the narrow-band interference case increases when the signal is not precisely at the center frequency of the received signal. For the new Military (M-Code) Binary Offset Carrier (BOC) signal, $Q$ can range from $1.9 \leq Q \leq 2.5$.

The defined $Q$ value for a random sequence, spectrally matched interference is

$$Q = \frac{1}{R_c \int_{-\infty}^{\infty} [S_S(f)]^2 df}.$$  \hspace{1cm} (2.6)
When $S_S$ represents the BPSK C/A code, the equation can be reduced to

$$Q = \frac{1}{R_c \int_{-\infty}^{\infty} T_c^2 \text{sinc}^4(\pi f T_c) df} = 1.5 \quad (2.7)$$

For the case of the M-Code BOC signal, Equation 2.6 can range from $3 \leq Q \leq 4.5$.

In these equations the higher Q values relate to better ability of the GPS receiver to mitigate the interference naturally due to the inherent properties of the designed communication channel. The derived Q values can be used, in the fundamental Equation 2.3, with the known signal powers needed to overtake a tracking GPS receiver, and the resulting baseline numbers can be used to develop power requirements for an interference simulation capability.

The generic interference to signal ratio needed for a spectrally matched BPSK noise source to deny a GPS receiver tracking based on the chipping rate and data rate is shown in Equation 2.8. The specific interference to signal ratio needed for a spectrally matched BPSK noise source to deny a GPS receiver tracking C/A code is shown in Equation 2.9. It is shown that the level is approximately 43 dB J/S. This means the power level of the interference signal only has to be -87 dBm ($2e^{-9}$mW) at the receive antenna to knock off a tracking GPS receiver as shown in Equation 2.10.

$$\text{BPSK } J/S = 10 \log \left( \frac{\text{chipping rate}}{\text{data rate}} \right) \approx \text{Value dB} \quad (2.8)$$

$$\text{BPSK } J/S = 10 \log \left( \frac{1.023M}{50Hz} \right) \approx 43dB \quad (2.9)$$

$$\text{GPS Signal Strength} - \text{Interference Signal Strength} = -130dBm - (-43dBm) = -87dBm \quad (2.10)$$
As previously discussed, the chipping rate for the encrypted P(Y) code is 10.23 MHz. The extra protection this increased rate provides the receiver in terms of power in the noise signal that can be handled by the GPS receiver before losing a signal that is already being tracked is shown in Equation 2.11. This value is 10 dB higher than the C/A value due to the increased chipping rate.

$$\text{BPSK } J/S = 10 \log \left( \frac{10.23 M}{50Hz} \right) \approx 53dB \quad (2.11)$$

$$\text{GPS Signal Strength} - \text{Interference Signal Strength} = -130dBm - (-53dBm) = -77dBm \quad (2.12)$$

These derived power levels will be used in conjunction with the improvement that is expected from a nulling system to calculate the absolute power above the noise floor that is required to test the upper end of the navigation system.

2.2 Antenna Pattern

A typical GPS system uses a single right-handed circularly polarized (RHCP) patch antenna. These antennas are designed to have maximum gain in the direction of the incoming GPS signals, which is typically up, and low gain in all other directions to reduce the effects of multipath and other undesired interference signals. An antenna is comprised of both a gain and phase pattern that defines how the incoming signal will be affected depending on where it is arriving spatially versus the antenna attitude. This antenna pattern information is combined in the manifold data.
To overcome any undesired signals in the GPS spectrum, there are many companies developing AJ electronics paired with controlled reception pattern antenna (CRPA) including Rockwell Collins, Mayflower, Toyon, L3, Raytheon, Cobham, and many more. The CRPA incorporates multiple miniature patch antenna elements into a single antenna system as shown in Figure 2.4, with a typical number of elements for GPS applications between 2 and 8. The AJ and CRPA combinations are capable of creating RF nulls in the direction of an undesired signal while still allowing GPS signals from other portions of the sky to pass through.

The principle theory behind the nulling of undesired signals using multiple receive antenna elements is shown in Figure 2.5. As the undesired signal approaches
the receive antenna there is a unique phase offset and corresponding time delay in
the reception of the signal by each antenna element. The combination of delay values
are used by the AJ electronics to estimate the direction of the incoming signal and
place a null in the corresponding direction.

Outdoor testing of AJ and CRPA systems is one of the optimum methods
for testing systems, but is difficult to obtain approval to broadcast signals in the
GPS frequency ranges. Therefore, rapid test and evaluation of electronics created to
overcome these signals must often times be carried out in a simulation environment.
However, simulations of interference signals without an accurate wavefront model do
not adequately test the AJ electronics and CRPA since the AJ hardware does not
know which direction to create the null, and thus no real understanding of the system
effectiveness is gained.

The incorporation of AJ hardware has been shown to suppress undesired sig-
nals by up to 50 dB or more based on size, weight, and power of the system, and can
theoretically handle as many as $n - 1$ spatially separated noise sources, where $n$ is
the number of antenna elements in the antenna array.

2.3 Channel Simulation Stages

For this analysis and simulation environment, the wavefront channel simulation
is defined in three sections. The first section is the generation of the signal itself.
The second section models the macro effects to the signal, which includes path loss,
channel noise, large Doppler effects, etc. The third section models the micro effects of
antenna motion, inter-element Doppler, fine resolution time differences, and antenna
characteristics including phase and gain pattern effects. A graphical representation of the three simulation stages is shown in Figure 2.6.

2.4 Basics of Nulling Technology

The basic premise of RF nulling is fairly simple: use the benefit of multiple antenna elements to create a null in the direction of undesired signals. This will allow the receiver to still track the other satellites available. Because of the various orbital
paths of the satellites, there is normally good spacial diversity on the satellites in view at any given time.

The most basic form of nulling is accomplished by creating weights based on the received signals from a CRPA to create an adaptive null in the direction of an undesired signal. This is effectively changing the antenna gain pattern in real time to only provide gain to portions of the sky with low power GPS signals. The interference signal information is mapped from the antenna through a covariance matrix in the AJ electronics. Understanding the way these weights are created is best accomplished by discussing the the relevant channel characteristics and the formation of the covariance matrix. The output data relating the received signal as seen by each of the antenna elements is defined in a covariance matrix that is an $n \times n$ matrix, where $n$ is the number of elements in the antenna array. The values on the diagonal are comprised of real numbers and represent the amplitude information of the received signal at each element. The off-diagonal components are complex numbers that relate the information from each element to each of the other elements in amplitude and phase.
as depicted in Figure 2.7. The information in this covariance matrix provides the necessary details about the incoming signal to create the weights and corresponding nulls to optimize the controllable antenna pattern.

The covariance matrix is constructed as shown in Equation 2.13.

\[
\begin{bmatrix}
\text{Cov}[X(t_1)X(t_1)] & \text{Cov}[X(t_1)X(t_2)] & \ldots & \text{Cov}[X(t_1)X(t_n)] \\
\text{Cov}[X(t_2)X(t_1)] & \ldots & \ldots & \text{Cov}[X(t_2)X(t_n)] \\
\vdots & \ddots & \vdots & \vdots \\
\text{Cov}[X(t_n)X(t_1)] & \ldots & \text{Cov}[X(t_n)X(t_n)]
\end{bmatrix}
\]

(2.13)

The amplitude and phase of each element can be solved for by the relationships shown in Equation 2.14 through Equation 2.16

\[
\text{Cov}[X(t_1)X(t_2)] = a - bi
\]

(2.14)
\[
\text{Mag}\{\text{Cov}[X(t_1)X(t_2)]\} = \sqrt{a^2 + b^2} \quad (2.15)
\]

\[
\text{Phase}\{\text{Cov}[X(t_1)X(t_2)]\} = -\tan^{-1}\left(\frac{b}{a}\right) \quad (2.16)
\]

The covariance matrix that is produced by the AJ is a self-adjoint (Hermitian) matrix. This is important to note since the output will be processed using an eigen decomposition. The eigenvalues for a hermitian matrix will always be real [4].

In the environment where this wavefront theory is being addressed, the relationship between the errors in the relative channel inputs that are mapped through the antenna elements are very similar to the errors that are present in the actual hardware due to inaccuracies in the antenna manifold data and RF front end.

In a situation where the incoming interference signal is arriving near zenith, all antenna elements are equidistant from the interference source. In this case, the exact or very small phase angle difference between the antenna elements means that the real component in the off-diagonal values of the covariance matrix are much greater than the imaginary component (real \(\gg\) imaginary). In a digital simulation with perfect manifold data and RF front end, the values would reduce to only the real portion. An example of a covariance matrix from a real hardware data collect with a zenith interference input is shown in Equation 2.17. The example covariance matrix for a 4 element antenna is taken from a hardware data collection that will be discussed in further detail in Chapter 5.
When the incoming signal has a 90° angle of arrival in elevation, meaning the signal is coplanar with the antenna elements there will be maximum phase delay for certain azimuth values. An example covariance matrix for this situation is shown in Equation 2.18. The much larger imaginary components can be seen in the matrix relating to a larger phase difference between elements.

\[
Cov = \begin{bmatrix}
1.0258 & 1.031e^{0.003i} & 1.028e^{0.001i} & 1.029e^{0.001i} \\
1.031e^{-0.003i} & 1.0335 & 1.0319e^{-0.004i} & 1.0324e^{-0.003i} \\
1.028e^{0.001i} & 1.0319e^{0.004i} & 1.0265 & 1.023e^{0.002i} \\
1.029e^{0.001i} & 1.033e^{0.003i} & 1.029e^{0.002i} & 1.029
\end{bmatrix}
\] (2.17)

The simplest form of weights can be created by inverting the covariance matrix to form a Weiner matrix [5]. Much more complicated methods exist utilizing both Space Time Adaptive Processing (STAP) [6] [7] and Space Frequency Adaptive Processing (SFAP) processes for nulling. These specific implementation methods will not be discussed in greater detail, since the intent is to study the wavefront validity.
2.5 Literature Survey

Tests take place every year to evaluate the effectiveness of standalone AJ and CRPA electronics, as well as analyzing the effects of integrating them into existing platforms that utilize GPS technology. One of the primary evaluation criteria is calculating the dB of attenuation the system can suppress while still tracking the desired GPS signals, and the effect this has on acquisition, track-ability, and reacquisition. Another critical measurement criteria is calculating the number of independent, spatially separated signals that can be suppressed by the electronics while still maintaining track of the GPS signals. There are other integration concerns as well, such as ensuring the quiescent time delay and time jitter of the RF signal through the AJ electronics, and the changing phase and gain pattern do not cause errors in the timing accuracy and corresponding position and velocity solutions.

A very popular approach to wavefront simulation is to combine the interference signal calculation and creation with a GPS/GNSS simulator. Hornbostel et al. [8] have developed a method of simulating an interference wavefront linked to a Spirent GNSS satellite constellation simulator. The multiple-output advanced signal test environment for receivers (MASTER) is comprised of two Spirent GSS7790 simulator capable of GPS and Galileo signals. The simulator was modified to provide the baseband I and Q digital information to the user with the capacity to add the interference signals prior to being converted up to the GNSS frequencies. An additional device called MATRIX has been added that handles the digital signal pro-
cessing of the digital signals to the correct antenna elements. The results obtained after calibration were reported to be $\pm 3^\circ$.

Thompson [9] has developed a unique approach to wavefront calibration by characterizing the antenna electronics under test using a vector network analyzer. The analyzer is used to measure the phase differences between the primary RF reference node and the other auxiliary nodes. The wavefront generator is embedded in the Spirent simulators output similar to the Hornbostel [8] approach. This setup however, requires the use of seven Spirent GSS7700 GPS simulators. The maximum jammer to signal (J/S) ratio capable in the system is reported to be around 60 dB. The signal set for the interference source can be set to broadband or continuous wave.

Lück et al. [10] have described another GNSS wavefront simulation using the NavX-NCS simulator manufactured by IFEN GmbH in an article titled “Antenna Array and Receiver Testing with a Multi-RF Output GNSS Simulator.” The constellation simulator allows the user to simulate up to four antenna elements simultaneously. The accuracy of the calibrated wavefront system is reported to be $\pm 0.5^\circ$ per-channel. The system described provides L band capabilities to produce “GPS, GLONASS, Galileo, BeiDou, QZSS, and SBAS” [10].

Another group that has contributed a substantial amount to navigation simulations is Navsys. Navsys is a company that has been dedicated to the simulation of GPS systems since the mid 90’s, has written several papers [11] [12] [13], and they have contributed numerous models, acquisition techniques, and algorithms to the field. Many of their algorithms are available as MATLAB toolboxes. They have developed and tested capabilities using hardware components as well. Numerous algo-
rithms have been developed in software defined radio GPS receivers and beamforming
techniques to test theoretical implementations of differing satellite power levels and
acquisition/tracking techniques.

The center for remote sensing, Inc. has created a multi-band GPS Wavefront
Simulator as described in [14]. This system utilizes custom RF front end technology
and has seven output ports. There was no information provided about the cost, phase
accuracy, or calibration procedures of the systems.

These systems have all made great strides to create accurate wavefront channel
simulations. An attempt to develop a more modular approach and the validation of
such a system is needed due to the dependence on GPS/GNSS simulators and the
concerning price, calibration, and initialization procedures.

Another path that groups are pursuing to evaluate AJ devices is using an
RF anechoic chamber as shown in Figure 2.8. These types of chambers are fitted
with special material that absorbs RF at certain frequencies and does not produce
reflections or noise in a designated quite zone. They also provide a Faraday cage type
of environment that allows a user to transmit at frequencies and power levels that
would not be legal in an open-air environment. In these situations the AJ devices
and antenna under test can be mounted in the quite zone and rotated through known
angles with very accurate positioner equipment. The standard use of these chambers
are to create accurate phase and gain patterns, but recent uses have been expanded
to include testing AJ electronics and CRPA, however, the use of live sky GPS in these
situations is not feasible for most situations. The environment also limits the motion
profiles the simulation can produce.
Figure 2.8: An anechoic chamber located on Redstone Arsenal, AL. This chamber is used for measuring antenna gain and phase patterns.

The typical accuracy of the positioner and corresponding AoA in these facilities is on the order of tenths of a degree. Other facilities have devised fairly complex setups to simulate certain RF environments like this facility in Germany [15] [16]. Many other labs around the United States have developed advanced anechoic chamber capabilities like those at Ft. Huachuca [17] and NSI-MI Technologies [18]. These are great assets to get truth data with very precise positioning to create models or excite a unit under test. One drawback of this setup is within the chamber the receiver lacks the ability to track live sky, and if a simulator is used it must be able to split out the individual satellites on tracks to simulate their motion through the sky. They also require a very large facility with over a million dollars of RF absorbing foam and precise positioner controller equipment.

Outdoor, or live sky testing, provides an optimum method to test AJ systems with all the pertinent hardware in the loop, and with surveyed locations for test as-
sets and interference sources with accurately calculated angles of arrival. However, outdoor testing becomes very expensive due to the complexity of getting approval to transmit in the GPS frequency band, and the number of safety concerns that are present from platforms operating in these GPS denied or degraded environments. This, along with time concerns for approval and test preparations are some of the driving motivations for the development of laboratory wavefront simulation capabilities.

The focus of this research has been to utilize commercial off the shelf products that comprise a scalable, open source, and relatively low cost solution to testing the full Navigation/AJ/CRPA systems in various laboratory and live sky scenarios, as well as applying engineering and mathematical techniques to increase understanding and capability in place of expensive hardware solutions.

The speed and efficiency of repeatable laboratory testing is a necessity in todays fast paced electronics world. The process from the FCC and other government organizations take 4 to 6 months for approval to transmit in the GPS bands for outdoor testing. Waiting 6 months to prepare for an outdoor test of hardware is not acceptable as a first step to system integration and limits the amount of data and scenarios that can be conducted. The repeatability of laboratory testing allows for better understanding of problems, and provides sufficient data sets through Monte Carlo testing for stochastic analysis of a system. Through simulation of random draws of known error sources, large amounts of data can be generated in a relatively short amount of time to ensure the integration of the AJ electronics are successful, and the desired suppression of the unwanted interference signals are meeting the goals of the
program. Accurate and repeatable laboratory testing is desired for safety concerns as well, since airworthiness is difficult to obtain in test cases that put assets and personnel at risk. This provides maximum confidence that when a live sky test does take place it will be successful, and the results will be predictable and reflect well on the system integrators and developers.

There are some very critical components needed in the channel simulation that are driving factors in the system design that have shaped the direction of this research. These factors are listed below in no particular order.

1. The ability to run as an independent entity (i.e. not requiring a GNSS simulator) or with a simulator as desired.

2. The ability to have diverse system architectures, or more specifically to be driven by various input sources and file types for profile and trajectory driven tests.

3. The ability to run in real-time with low latency capabilities (<1 ms latency).

4. The ability to use a known signal type or an unknown “black box” as the RF input from any signal generation device.

5. The ability to produce very high signal (noise) levels (90+ J/S for GPS) with minimal impacts to the simulation noise floor.

6. The ability to have reliable repeatability for statistical analysis of units under test (precision is more desirable than accuracy).

7. An efficient and cost effective way to analyze standalone and integrated performance of nulling capabilities.
8. The ability to be expandable in number of elements.

9. The ability to reuse hardware for various frequency ranges and bandwidths as GPS and GNSS frequencies are changing and adding to the spectrum.

10. Reducing the startup and calibration time of the system on a per-run basis.

11. The cost associated with procurement of hardware, licenses, and maintenance.

The architectures depicted in Figure 2.9 and Figure 2.10 represent a few of the implementation options that system integrators need to use the wavefront simulation capability to fully test AJ electronics.

The live sky architecture allows for the antenna to be in the system with the GPS receiver tracking real signals. This setup can be static or placed inside of a vehicle to induce motion. The wavefront system can be programmed with a relative interference location or an absolute interference location. The incoming AoA will move relative to the interference source as appropriate, as long as the system is fed the true attitude information from the host platform.

The setup using a GPS simulator allows the opportunity to use any method of producing GPS signals for simulation. The wavefront simulation is connected in a similar manner as the live sky, but the vehicle platform information is now being provided through a simulation host.

This research develops algorithms that define a wavefront simulation that utilizes a discrete phase delay capability using a mechanical tap delay architecture, implements and validates a low cost, time-synchronized, real-time channel simula-
Figure 2.9: This example is of a live sky system architecture. The CRPA is able to track live sky GPS signals, while the interference is injected through combiners and RF cables.

Figure 2.10: This example is of a laboratory system architecture. The GPS signals are simulated by an RF front end, while the interference is injected through combiners and RF cables.
tion application that minimizes the influence of HWIL artifacts through an analysis technique.

This dissertation builds upon a test capability that is being developed to provide the community with a cost-effective approach for evaluating AJ systems [19]. The test capability that is realized through hardware utilizes a core architecture with mechanical tap-delay lines [20] and flexible personal computer interface options. Its expandability provides a range of simulated radio frequency (RF) antenna-element channels and capabilities with variable cost. The theory and methods to advance the current understanding behind these concepts of wavefront simulation for GPS AJ and CRPA will be explored in the following chapters.
CHAPTER 3

THEORY OF A WAVEFRONT CHANNEL SIMULATION

3.1 Overview

A wavefront simulation expands upon the basic principles of a communication channel model. Creating a system that does not have limitations in a multi-element phased array sense begins with the definition of a single channel, then expands to \( n \) channels.

For the case of a single element model, the simulated RF system behaves like a standard communication channel. As discussed before, the system is broken into various components of the channel as stages that comprise the fundamental regions in a communication system. The stages are chosen to isolate the components within the simulation architecture that may be replaced by hardware or units under test if the user desires. Initially for the research, all of the stages will be theoretical models in a simulation environment.

An assumption is made that the signal generation is accurate to represent the desired interference source. This is a required assumption with a block box input. What then does it mean to have a valid channel model? For the purposes of this research, this does not mean that the model represents the system perfectly. It is
defined in this dissertation to be a communication channel that has been adequately characterized such that the errors in the model are well defined and the implications of the errors are known, so that the resulting tests on any system have more purpose.

3.2 Effects of a Discrete Channel Model

Just as in the development of linear systems theory, continuous channel models can be theorized and derived for communications systems. The effect of the discrete nature of simulating the various incoming communication signals is one of the main focuses of this dissertation, as well as investigating a method to mitigate errors caused by a lack of resolution in the discrete step size of the phase shift. It is important to understand how the discrete nature of the simulation changes the accuracy of the resulting RF wavefront. The main focus of this principle will be on the discrete nature of the signal’s phase angle. This will play into the phase accuracy, phase coherency between elements, and the resulting effect on the ability of the system under test to interpret the wavefront. A visual depiction of the discrete phase problem is shown in Figure 3.1. More in depth analysis on this topic will be presented in Section 4.7.

3.3 Source Distance and Incoming Angle Impacts

The Cartesian coordinates $x, y, z$ and the spherical coordinates $r, \theta, \phi$ are defined as shown in Figure 3.2, where $r$ is the source distance ($0 \leq r \leq \infty$), $\phi$ is the azimuth off the $x$ axis in a counter clockwise direction ($0 \leq \phi \leq 2\pi$), and $\theta$ is the elevation off zenith ($0 \leq \theta \leq \pi$).
Figure 3.1: Continuous versus discrete options for time delay of RF signals.

Figure 3.2: Cartesian and Spherical Coordinate System Definition

The spherical values are related to the Cartesian values by

\[ r = \sqrt{x^2 + y^2 + z^2} \]  
(3.1)

\[ \phi = \tan^{-1} \left( \frac{y}{x} \right) \]  
(3.2)
\[ \theta = \cos^{-1}\left(\frac{z}{r}\right) \] (3.3)

The following analysis goes through simulations moving the source location from 1 meter to 10 km from the antenna to view the relative AoA as seen by different elements on the antenna array. The reference element is kept constant throughout each of the test cases as well as the azimuth and elevation values. The effect of the perceived angle of arrival from each element is dependent on the antenna geometry as described in the manifold data. As the source distance is increased, there is a threshold at which the difference in perceived AoA between the elements is similar enough that it can be assumed to be the same. This value can be determined by the user since the location of the interference source should be known. However, the AoA will be used for all calculations in this research since very close interference sources could be used as well.

Figure 3.3 shows results from a scenario where the source is directly above the antenna plane moving in a vertical direction. The azimuth difference shows an initial separation of 120°. There is a major difference in the perceived azimuth well past 10 meters and the system does not pass the threshold of less than 0.1° in azimuth until the source is over 100 meters above the antenna.

Figure 3.4 shows the interference source on the same plane as the antenna moving away along the vector formed by elements 1 and 2. The azimuth difference shows an initial separation of a just under 5°. In this case the azimuth error converges to a similar value below the threshold of 0.1° by approximately 100 meters. The
Figure 3.3: Relative azimuth and elevation of the incoming signal as perceived by each element of the CRPA. The incoming signal is determined based off the reference element and is set to an azimuth of 0°, and an elevation of 0°. The source distance is varied from 1m to 10km.

Elevation values are always at 90° for all elements since they are coplanar with the source.

This analysis shows that the incoming angle of arrival as viewed by spatially separated elements will have an affect that relates to single digit degree errors in the perceived signal. This will effect the perceived power distribution of the incoming signal and would most likely be indistinguishable from errors by hardware contained on the system experiencing the interference.
3.4 Definition of a Controlled Reception Pattern Antenna for Analysis

The geometry of a theoretical seven-element antenna used for most of the evaluation in this research is shown in Figure 3.6. This antenna is used throughout most of the development and analysis for consistency, but the theory is expandable to a system with any number of elements and with any spatial geometry. This antenna geometry was chosen to incorporate off axis antenna elements, the use of more than 4 elements, and spacing based off the GPS primary L1 frequency of 1.57542 GHz.

The 7-element phased-array antenna provides an example antenna geometry where the signal is defined as a pointing vector with a relative phase offset, or time-bias, based on the separation of the elements and the reference elements azimuth
Figure 3.5: Relative azimuth and elevation of the incoming signal as perceived by each element of the CRPA. The incoming signal is determined based off the reference element and is set to an azimuth of 30°, and an elevation of 45°. The source distance is varied from 1m to 10km.

and elevation angle of arrival for the interference signal. Figure 3.7 represents the definition of derived per-element delays where \( \phi \) is azimuth relative to the x-axis in the x-y plane and \( \theta \) is the elevation off zenith, or boresight of antenna, relative to the +z-axis.

For the analysis presented in this paper, the receiving phased-array antenna is assumed to be in the far field of the undesired signal, however, the per-element angle of arrival is not necessarily equal as shown in Section 3.3. The near-field effects of an RF signal are complex and the desired simulation environment does not provide mechanization of complex, signal augmentation in the reactive near field, or the ability to manipulate the phase relationship between the E and H fields in the radiating
Figure 3.6: Example of a 7-Element antenna using miniature patch elements that is used for analysis.

Figure 3.7: Antenna Frame Definition

near field [21]. The calculations showing the approximate regions for these fields are shown in Equation 3.4 through Equation 3.7. $D$ is defined to be the largest dimension of the array, which was chosen to be approximately 0.12 meters. This far-field assumption also applies to relative-velocity effects between the individual elements and the transmit antenna, which might be negligible with benign attitude dynamics relative to the interference source [22].
Wavelength = \frac{c}{\text{Frequency}} = \frac{299,792,458}{1,575,420,000} \approx 0.1903 \text{m} \quad (3.4)

\text{Reactive Near Field} \leq 0.62 \times \sqrt{\frac{d^3}{\text{Wavelength}}} \approx 0.62 \times \sqrt{\frac{0.12^3}{0.1903}} \approx 0.059 \text{m} \quad (3.5)

\text{Radiating Near Field (Fresnel Region)} \leq \frac{2d^2}{\text{Wavelength}} \approx \frac{2(0.12)^2}{0.1903} \approx 0.15 \text{m} \quad (3.6)

\text{Far Field} \geq \frac{2d^2}{\text{Wavelength}} \approx \frac{2(0.12)^2}{0.1903} \approx 0.15 \text{m} \quad (3.7)

The per-element Doppler shift is typically negligible for center of gravity relative transmitter dynamics. In contrast, the rotational dynamics of the vehicle, projected on the unit-line-of-sight to the transmitter, are not negligible and are on the order of 100 Hz for nominal air speeds around 100 knots. The relationship between the relative rotational contribution to Doppler and the signal correlation determination is primarily a function of the integration period, or buffered correlated samples, of the AJ unit itself. For an integration time of 1 ms and element-one Doppler of 100 Hz, a pure complex signal correlation between element one and the center element would have an estimated error similar to 18 degrees delay shift from truth, when calculated with IQ simulated results. This empirically-derived delay-error relationship
is a function of the integration time and the distance between an element and the reference element.

3.5 MUSIC Algorithm Description

One of the most well known algorithms used for evaluation using a covariance system output is the MUSIC algorithm [23], which is used in direction finding using CRPA. Because of the utility using the covariance output from an AJ and the accuracy, it will be used as a comparison to the novel technique used to understand the simulation environment and to create a bound on the error in the simulation.

It is important to note that this is not a typical use of the MUSIC algorithm. The implementation will be more straightforward since the data being used to populate the covariance estimate will be the phase information directly from the output of the discrete tap delay line. This covariance estimate will not have noise, multipath, or multiple incoming signals to process.

The derivation of the MUSIC algorithm equations is based on the works of R. O. Schmidt [23] and the GIRD [24] report. Equation 3.8 defines a real sinusoid \( s(t) \), and Equation 3.9 defines a complex sinusoid.

\[
\alpha \cos (\omega t + \beta) = \frac{\alpha}{2} e^{j\beta} e^{j\omega t} + \frac{\alpha}{2} e^{-j\beta} e^{-j\omega t} = \rho_1 e^{j\omega t} + \rho_2 e^{-j\omega t} \quad (3.8)
\]

\[
s(t) = \alpha e^{j\beta} e^{j\omega t} = \rho e^{j\omega t} \quad (3.9)
\]
Based on the definition of the real and complex sinusoids, the delay of the sinusoid can now be shown as a phase shift of the signal $s(t - t_0)$, where $t_0$ represents a delay in the time arrival of the signal to the referenced $s(t)$ by $t_0$ seconds.

$$s(t - t_0) = e^{-j\omega t_0} \rho e^{j\omega t} = e^{-j\omega t_0} s(t) \quad (3.10)$$

If we consider a set of $N$ narrow-band noise sources

$$s_1(t) = \rho_1 e^{j\omega_1 t}, \ s_2(t) = \rho_2 e^{j\omega_2 t}, \cdots, \ s_N(t) = \rho_N e^{j\omega_N t} \quad (3.11)$$

If the assumption is made that all the frequencies are different and all amplitudes are uncorrelated, then we can say

$$E\{\rho_i \rho_j\} = \begin{cases} \sigma_i^2; & i = j \\ 0; & i \neq j \end{cases} \quad (3.12)$$

The delayed signal at element 1 from the reference element is described by

$$\tau_i = (i - 1)d \sin(\theta) \quad c \quad (3.13)$$

where $c = 299,792,458 m/s$ is the speed of light.

$$x_i(t) = e^{-j\omega \tau_i} s_1(t) = e^{-j\omega (i - 1)d \sin(\theta) / c} s(t) \quad (3.14)$$
where $a(\theta)$ is known as the steering vector. The steering vector can be considered a guess as to the true incoming angle of the signal, which will be used in the final equation of the algorithm. The process of creating steering vectors is chosen by the user in terms of desired accuracy. For very high accuracy of the MUSIC algorithm, the step size from one steering vector to the next might be very small, down in the tenths of a degree. However, it is easy to see how this would lead to a very processor intensive task and may not be worthwhile if the manifold data is not well known. Various methods of increasing the efficiency of the MUSIC algorithm have been developed, but since the contribution of this algorithm to the research is not time sensitive and accuracy is key, the unmodified MUSIC algorithm will be used.

The sources are assumed to be independent and uncorrelated as they reach the receiving antenna. The system can be represented by a series of vectors such as

$$x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ \vdots \\ x_N(t) \end{bmatrix} = \begin{bmatrix} 1 \\ e^{-j\omega \frac{d \sin \theta}{c}} \\ e^{-j\omega \frac{2d \sin \theta}{c}} \\ \vdots \\ e^{-j\omega \frac{(N-1)d \sin \theta}{c}} \end{bmatrix} s(t) = a(\theta)s(t) \tag{3.15}$$

$x(t)$ is the received signal vector with dimension $N \times 1$

$s(t)$ is the source signal vector with dimension $I \times 1$
\( n(t) \) is the noise vector with dimension \( N \times 1 \)

\[
R_s = E\{s(t)s^H(t)\} = \text{diag}\{\sigma_1^2, \sigma_2^2, \cdots, \sigma_i^2\} \tag{3.17}
\]

\[
R_x = E\{x(t)x^H(t)\} = AR_sA^H + \sigma_0^2 I \tag{3.18}
\]

Since the MUSIC algorithm is being used to characterize a known system where the inputs are controlled, these issues are not completely relevant but are worthy of mentioning so as to provide a complete understanding of the algorithm.

The matrix \( AR_sA^H \) is singular when \( N > I \) such that

\[
\det [AR_sA^H] = \det [R_x - \sigma_0^2 I] = 0 \tag{3.19}
\]

Equation 3.19 implies that the value \( \sigma_0^2 \) is an eigenvalue of \( R_x \). There should be \( N - I \) eigenvalues (\( \sigma_0^2 \)) of \( R_x \). Given that \( AR_sA^H \) and \( R_x \) are positive (non-negative) definite, there exists \( I \) eigenvalues (\( \sigma_i^2 \)) which satisfy

\[
\sigma_i^2 > \sigma_0^2 > 0 \tag{3.20}
\]

Define \( u_i \) to be the \( i^{th} \) eigenvector of \( R_x \) which corresponds to eigenvalue \( \sigma_i^2 \).

\[
R_xu_i = [AR_sA^H + \sigma_0^2 I]u_i = \sigma_i^2 u_i; \quad i = 1, 2, \cdots, N \tag{3.21}
\]
Equation 3.21 implies that

\[ \mathbf{A R_s A^H u_i = \begin{cases} (\sigma_i^2 - \sigma_0^2) u_i & i = 1, 2, \ldots, I \\ 0 & i = I + 1, \ldots, N \end{cases} } \]  

The N-dimensional vector space can now be partitioned into the signal subspace \( U_s \) and the noise subspace \( U_n \) as

\[ [U_s \ U_n] = \begin{bmatrix} u_1 & \cdots & u_I & u_{I+1} & \cdots & u_N \\ U_s: (\sigma_i^2 - \sigma_0^2) > 0 \text{ eigenvalues} & U_n: 0 \text{ eigenvalues} \end{bmatrix} \]  

The MUSIC algorithm provides information about the system by applying a user selected set of steering vectors. These number of steering vectors is based on the desired phase angle resolution of the output. This can become a very processor intensive action if the step size, or resolution, of the steering vector is very small and accurate, but can produce very detailed results. For the evaluations in this research, the step size of the steering vector is set to 0.01°. When Equation 3.26 is evaluated, there will be a peak at the value where \( \theta = \theta_i \). The implementation of a peak detection algorithm will provide the estimated angle of arrival value of the received signal sources.
\[ P(\theta) = \frac{1}{\text{a}^H(\theta)\text{U}_n} \]  

(3.26)

3.6 Current analysis techniques and the proposal of enhanced methods

The method of creating a per-element calibration is beneficial in the development of a statistical model of the phase coherent behavior achievable by a single communication channel. The model can include information relating the precision of a channel’s phase error and the repeatability as a unit of standard deviation. This reduces the effects caused by the delay errors, but what is left is the error that is caused by the discrete nature of the phase delay options. This per-element discrete effect does very little to help understand the observability of the system into the seemingly insignificant per-channel errors. In a desire to enhance the ability of simulation accuracy and system performance evaluation it is desirable to create a system level approach to evaluating the communication channel for a system/interference signal.

This system and the corresponding understanding of the relationship between the real world and the simulation stems from the fact that the communication system that we are addressing (GPS) was not ever meant to be a multiple input multiple output system by means of an antenna array. The simultaneous signal reception is accomplished using the DSSS theory as described before. The fact that a system is now attempting to add multiple receive antenna elements for a phase coherent signal creates a unique environment. The new method of analyzing the communication channel at a system level was first described in [25] and will be derived in detail in Chapter 4.
CHAPTER 4

ANALYTICAL DEVELOPMENT OF WAVEFRONT CHARACTERIZATION

4.1 Definition of AoA and the Region of Error

There is a desire to better understand and characterize the wavefront simulation, the phase coherent effects, and how all of the information is perceived by AJ electronics and CRPA. The discrete phase errors in the system are mapped into angle of arrival error regions in the spherical coordinate frame to accomplish this.

Based on the deterministic characteristics of the theoretical wavefront hardware, an ambiguity region defining the range of simulated AoAs between elements can be calculated for each desired AoA. This AoA uncertainty region defines an empirical region of hardware-in-the-loop artifacts tested against the given antenna geometry, and given the uncertainty we will derive how it quantitatively shows the limits of the signal representation. Analogously, in a CRPA design procedure, this relationship is similar to unmeasured manifold information or a discrete representation of the antenna and its influence on the resulting AoA estimate. Figure 4.1 depicts a visual representation of this potential error region.
Figure 4.1: AoA Illustrated Error Region

The error region is defined by the statistical model of errors in each tap-delay channel, and mapped through the simulated antenna geometry. Ideally, the resulting error region is reduced through a per-element calibration procedure which measures the average delay and $1\sigma$ variation in precision over dozens of iterations. Figure 4.2 illustrates theoretical error regions for a system with and without a per-channel calibration. The shapes are represented as circles, however, the shape, orientation, relationship and size for each error region at every AoA is dependent on the antenna geometry and step-size of the delay. The calibration procedure uses this as a criteria to determine the optimum region of the tap delay box. Specific examples of these
Figure 4.2: Theoretical description of the angle of arrival error region for calibrated and uncalibrated systems.

regions will be derived and discussed in more detail using the theoretical 7-element antenna and the calibration data from the labs tap-delay hardware.

4.2 Theory of Observations Applied to One and Two Dimensions

The theory of observations is applied in a novel manner to analyze the spatial construct of the phase ambiguity in the wavefront simulation. The underlying concept in the analysis technique is based on observations, overdetermined systems, and ambiguity in data. This is shown in one dimension as shown in Figure 4.3. If information about the location of an object is given to an observer at location x, the ambiguity in the information leaves multiple locations where the desired object could be located as indicated in Figure 4.3 by the blue dots.

The possible locations of the desired object can be reduced with additional information from another observer as shown in Figure 4.4. However, if each of the
observers has an error of 0.1 in their provided measurement information, the relative location of the object would still be known but with an uncertainty region relating to the value of the measurement error as shown in Figure 4.5.

Based on statistical probability of the actual location, the distribution of the object in the uncertainty region if both measurements are assumed to hold equal

Figure 4.3: Theory of observations in one dimension with one distinct measurement

Figure 4.4: Theory of observations in one dimension with two distinct measurements

Figure 4.5: Theory of observations in one dimension with two distinct measurements containing errors
weight would place the object at the center of the region. Therefore, the optimum location to use as a guess would be the center of the region defined by the inputs.

This same principle can be extended to two dimensions. The perfect case of three observers with information about the location of an object would provide an exact location of that object. If the provided information contains errors, there will become an uncertainty region as shown in Figure 4.6.

4.3 Properties of an Analysis Cone

When simulating a desired AoA, ideally there is a unique set of per-element delays relative to a reference point that can be applied to the RF communication channel. When the antenna electronics receive the signal, each element provides ambiguous information about the relative angle of the incoming signal. For the seven-element antenna, the reference point is set to the center element. The information from a single element provides a single phase delta that can be interpreted to an
Figure 4.7: The measured phase offset between the reference observer and a non-reference observer provides ambiguity about the incoming angle of arrival in the shape of a cone with the vertex at the reference observer and the cone center at the non-reference observer.

infinite number of AoAs forming a cone shape around the pointing vector from the reference point toward the element itself, as shown in Figure 4.7. Each of these cones, acting as the pseudo range equivalent in a GPS navigation solution, combines to form an ambiguity region and optimized angle of arrival solution. This means that based on a single observation, there is little known about the true AoA.

The number of elements and the delay reference point in the phased-array antenna determines the number of vertices that define the AoA region. The maximum number of vertices is $n - 1$, where $n$ is the number of antenna elements including the reference element. Typically, one of the $n$ antenna elements is chosen for a reference
point, but for pure simulation environments this restriction is not required. The minimum number of vertices is 3 for observability of azimuth and elevation. Similar to the GPS Dilution of Precision (DOP) analysis and position estimation [2], the likely AoA estimation region is highly influenced by the number of vertices, their geometric relationship and their respective delay error across the wavefront. Fewer vertices causes the channel errors to be more influential on the entire AoA error in the system.

The shape and number of vertices over each possible AoA provide valuable insight into the characteristics of the wavefront, as well as a visual representation of the observability the antenna will have into the azimuth and elevation accuracy based on the geometry of the antenna elements. Figure 4.8 shows an example where each individual channel has 5-ps standard deviation. In this figure, the cones are constructed from the mean value of the per-channel delay and given a $\pm 3\sigma$ bound based on the uncertainty of the delay for each element. The 6 red cones of the 7-element planar antenna intersect to form the black-blue angle-of-arrival error region. A higher resolution surface has been applied to the intersection point of the cones in order to show the boundary of each cone with more clarity. The planar array is symmetric over the xy-plane and the lower-hemispheres intersection region has been removed from this figure. The AoA error region is then reduced to just the intersections of the cones represented by colored vectors with the error region highlighted in green.
4.4 Observer Definition

The calibration method uses each antenna element as an observer viewing the incoming signal and relates that information to the antenna frame for analysis. As described, the antenna defines the reference element at the origin and the remaining six elements spaced $\frac{1}{4}$ wavelength in radial distance from the reference and spaced equally at an interval of $\frac{\pi}{3}$ along the circumference.

The phase of an incoming signal as seen by the $i^{th}$ element can be described by

$$\text{phase}_i = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \sin(\theta) \cos(\phi) \\ \sin(\theta) \sin(\phi) \\ \cos(\theta) \end{bmatrix}$$

(4.1)

where $\phi$ is the azimuth measured in a counter-clockwise direction from the positive x-axis in the x-y plane, and $\theta$ is the elevation measured from zenith.
The information from a single observer has ambiguity in the perception of the incoming signal since it cannot resolve the angle of incidence in three dimensions. The measured phase offset between each non-reference observer and the reference provides an infinite number of possible incoming incidence angles. The shape of every possible AoA, based on the phase delay from one input, forms a cone with the vertex at the origin and the opening of the cone passing directly through the element as shown in Figure 4.7.

The combination of the ambiguity cones from each observer-to-reference pair will form an overdetermined system that can be used to calculate the angle of the arriving signal. To develop this understanding a system of observation cones is constructed and the behavior of the perceived signal as the simulated angle of arrival is rotated around the upper hemisphere of the antenna plane is examined.

4.5 System Level Model of Incident Signal in 3-D

Relating the theory of observations and the analysis cone as described, there is sufficient information to begin constructing the mathematical model of the system based on these principles. A cone with a vertex at point \([0, 0, 0]\) and opening along the positive \(z\) axis can be described by

\[
x^2 + y^2 = \alpha z^2
\]  

(4.2)

where \(\alpha\) is the slope of the cone wall. The radius of the opening increases linearly as a function of the distance from the vertex [26] as shown in Equation 4.3. This
relation of the radius to vertex distance is relevant when choosing a distance for the observation sphere that will be described later.

\[ r = \alpha z \]  

(4.3)

\[ r \propto z \]  

(4.4)

The perceived incident angle as described by the observer is

\[ \phi = - \tan^{-1} \left( \frac{\tan y}{\tan x} \right) \]  

(4.5)

\[ \theta = \frac{\pi}{2} - \tan z \]  

(4.6)

where \( \theta \) defines the latitude and \( \phi \) defines the longitude of the incident wave.

To understand the inner-workings of the system level analysis, a sphere around the reference point in the observation matrix is defined on to which the information is projected. A sphere centered at point \([x_0, y_0, z_0]\) can be described by

\[ (x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = R^2 \]  

(4.7)

Since the reference point in this observer model has been centered at the origin, Equation 4.7 can be reduced to

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Since the sphere is a mapping of the measurement information and the distance from the reference does not effect the accurate representation of the information, a radius of \( R = 1 \) will be used to simplify the analysis and equations.

The equation of the area where the cone intersects the sphere if the vertex of the cone is co-located at the origin can be defined [27] as

\[
(z - z_0)^2 = \frac{x^2 + y^2}{\alpha} \tag{4.9}
\]

\[
r^2 = x^2 + y^2 + z^2 \tag{4.10}
\]

The combination of Equation 4.9 and Equation 4.10 yield

\[
\alpha(z - z_0)^2 + z^2 = r^2 \tag{4.11}
\]

\[
\alpha(z^2 - 2z_0z + z_0^2) + z^2 = r^2 \tag{4.12}
\]

\[
z^2(\alpha + 1) - 2\alpha z_0z + (z_0^2\alpha - r^2) = 0 \tag{4.13}
\]

The quadratic equation can be used to solve Equation 4.13 and produces

\[
z = \frac{\alpha z_0 \pm \sqrt{\alpha(r^2 - z_0^2) + r^2}}{\alpha^2 + 1} \tag{4.14}
\]
The intersection of the cone and sphere is planar and therefore forms a circle with a radius defined by

$$a = \sqrt{r^2 - z^2}$$  \hfill (4.15)

This circular shape at the cone-sphere intersection represents all the possible intersections between the incoming incident signal and the observation sphere.

Now that the cone, observation sphere and their intersections have been defined, the specific points of interest that lie on these surfaces need to be calculated.

A vector from the reference through an element $i$ can be described by
\[ \text{PointVec}_i = \begin{bmatrix} x \\ y \\ z \end{bmatrix}_i - \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{\text{ref}} \]  \hspace{1cm} (4.16)

The center of the circular region can be found by determining the location where the vector in Equation 4.16 pierces the observation sphere.

The simulated AoA is based on the reference element only. The vector representing the incident angle as seen by the \(i^{th}\) observation element can be described as

\[ \text{PointVecAoA}_i = \begin{bmatrix} r \sin(\theta) \cos(\phi) \\ r \sin(\theta) \sin(\phi) \\ r \cos(\theta) \end{bmatrix}_i - \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{\text{ref}}, \]  \hspace{1cm} (4.17)

where \(\phi\) is the angle of the incident wave from the \(x-y\) plane, and \(\theta\) is the elevation measured from the \(z\)-axis relative to the reference element.

The angle between two points on the surface of a sphere can be solved by the great circle distance equations [28] as depicted in Figure 4.9.

\[ \Delta \sigma = \cos^{-1}(\sin(\phi_1) \sin(\phi_2) + \cos(\phi_1) \cos(\phi_2) \cos(\Delta \theta)). \]  \hspace{1cm} (4.18)

The surface distance between the two points can then be calculated using the angle from Equation 4.18 and the radius of the sphere as
This process is then iterated through each observation point that is not co-
located at the reference point. The final result is the locations of the points on the
cone-sphere intersection that lie closest to a desired point that is being tested as the
AoA solution. With this information the system will be characterized using a Steiner
tree method. The specific Steiner tree implementation will be constructed to solve a
shortest path problem with a single Steiner point.

A Steiner tree problem is a method of solving a problem in combinatorial
mathematics for a set of interconnected points or nodes. Each of these nodes must
map to all the other nodes through direct branches that all intersect a single point.
The objective is to minimize the sum of the branches while requiring that all nodes
be accounted for in the solution.

A Steiner tree is created using the information provided by the cone-sphere
intersections. Since orthodromic navigation refers to navigation following a great
circle, the terminology for the Steiner tree on the surface of the observation sphere
will be called an orthodromic Steiner tree. The point through which every observation
input has a minimum orthodromic distance to the Steiner point is found. This point
describes the simulated angle of arrival that, by definition, uses all observations in
the solution. The descriptions of the simulated AoAs will be critical in correcting
errors in the system, as discussed in upcoming sections.
The orthodromic Steiner tree minimizes the surface distance between the observation cones. There is a one-to-one mapping of the surface distance to the angle (phase error) between elements, therefore, the Steiner point truly represents the least square solution of the per-channel phase errors using a combination of all six non-reference elements. A two-dimensional graphical representation of a Euclidean Steiner Tree created by various observation circles is shown in Figure 4.10. The constructed Euclidean Steiner tree is shown in Figure 4.11.

The process of determining the minimized orthodromic Steiner tree can be achieved by calculating the distance from the Steiner point, \( P \), to each of the cone-
sphere intersections. The closest point on the cone-sphere intersection is then calculated using a modified version of the great-circle navigation equations [28].

The point where the vector from the reference to the antenna element pierces the observation sphere can be calculated as

\[
\phi_c = \tan^{-1}\left(\frac{\text{ant}_y}{\text{ant}_x}\right) \\
\theta_c = \pi/2 - \text{ant}_z
\] (4.20) (4.21)

The definition of the desired values to construct the Steiner tree are

\[
C_n = (\theta_{cn}, \phi_{cn}) \quad (4.22)
\]

\[
D_n = (\theta_{dn}, \phi_{dn}) \quad (4.23)
\]

\[
P = (\theta_p, \phi_p) \quad (4.24)
\]

where the value \( n \) represents the element number of the non-reference observer. \( D \) represents the point on the cone-sphere intersection that lies closest to \( P \) and will become the defining endpoint of one of the Steiner branches.

The distance from all points on the cone-sphere intersection to \( C \) are the same, so any point can be used to calculate the surface distance between \( C \) and \( D \).

The distance from \( C \) to any point on the cone-sphere intersection can be calculated several ways, but since the angles used to calculate the value in this situation could be very small, the following implementation of the Vincenty formula [28] provides a
more accurate solution than other methods such as the spherical law of cosines. The
distance is calculated as

\[
\begin{align*}
\text{num}_1 &= (\cos(\theta_1) \sin(\phi_{01}))^2 \\
\text{num}_2 &= \cos(\theta_0) \sin(\theta_1) \\
\text{num}_3 &= (\sin(\theta_1) \cos(\theta_1) \cos(\phi_{01}))^2 \\
\text{den}_1 &= \sin(\theta_0) \sin(\theta_1) \\
\text{den}_2 &= \cos(\theta_0) \cos(\theta_1) \cos(\phi_{01})
\end{align*}
\]

(4.25)

where the inputs from Equation 4.25 are used to solve for \(\sigma_{CD}\) as

\[
\sigma_{CD} = \tan^{-1}\left(\frac{\sqrt{\text{num}_1 + \text{num}_2 - \text{num}_3}}{\text{den}_1 + \text{den}_2}\right)
\]

(4.26)

A great circle is created through points \(C\) and \(P\), and the distance \(\sigma_{CD}\) is used
to find a way-point on the great circle that provides the location of point \(D\). For all
calculations involving the arctan function of a quantity, the atan2 function is used to
preserve the correct quadrant of the output.

The difference in latitude between \(C\) and \(P\) is

\[
\phi_{02} = \phi_0 - \phi_2
\]

(4.27)

The angle \(\alpha_0\) is found by

\[
\alpha_0 = \tan^{-1}\left(\frac{\sin(\alpha_1) \cos(\theta_2)}{\sqrt{\cos^2(\alpha_2) + \sin^2(\alpha_1) \sin^2(\theta_1)}}\right)
\]

(4.28)
The value of $\theta_1$ is found by

$$\phi_1 = \tan^{-1}\left(\frac{\cos(\alpha_0) \sin(\sigma_{CD})}{\sqrt{(\cos^2(\sigma_{CD}) + \sin^2(\alpha_0) \sin^2(\sigma_{CD}))}}\right)$$

(4.29)

The value of $\phi_1$ can be found by

$$\phi_1 = \tan^{-1}\left(\frac{\sin(\alpha_0) \sin(\sigma_{CD})}{\cos(\sigma_{CD})}\right) + \phi_0$$

(4.30)

With point $D$ defined by $(\phi_1, \theta_1)$, the relationship between $D$ and $P$ is known. This vector from $D$ to $P$ forms one Steiner branch of our Steiner tree. This process is repeated for each observation point to create an array of Steiner tree nodes as shown in Equation 4.31.

$$D_m = \begin{bmatrix} (\phi_1, \theta_1)_{\text{Element 2}} \\ (\phi_1, \theta_1)_{\text{Element 3}} \\ \vdots \\ (\phi_1, \theta_1)_{\text{Element n}} \end{bmatrix}$$

(4.31)

The calculation of the orthodromic distance from $P$ to all $D_i$ nodes is

$$\text{Distance}_{PD_i} = \sqrt{\left(\sum_{i=2}^{n} \phi_i\right)^2 + \left(\sum_{i=2}^{n} \theta_i\right)^2}$$

(4.32)

Minimizing the distance in equation Equation 4.32 provides the location of the Steiner point that represents the least squared distance relating all observations and the AoA of the simulated signal. To find the minimum orthodromic Steiner tree
distance, the value of $P$ is chosen for each set of calculations in a semi-optimized fashion by creating a grid of test points around the desired angle of arrival within a marginal area. The algorithm steps used are as follows:

1. Set the initial guess as the desired AoA $(\phi_{\text{desired}}, \theta_{\text{desired}})$

2. Calculate the minimum orthodromic Steiner tree from -10 degrees to +10 degrees in azimuth and elevation with a step size of 1 degree, centered at $\phi_{\text{desired}}, \theta_{\text{desired}}$. Set the azimuth and elevation values corresponding the minimum value of $P$ to $\phi_{\text{resolution}1}, \theta_{\text{resolution}1}$.

3. Calculate the minimum orthodromic Steiner tree from -1 degree to +1 degree in azimuth and elevation with a step size of 0.1 degrees centered at $\phi_{\text{resolution}1}, \theta_{\text{resolution}1}$. Set the azimuth and elevation values corresponding the minimum value of $P$ to $\phi_{\text{resolution}1}, \theta_{\text{resolution}1}$.

4. Calculate the minimum orthodromic Steiner tree from -0.1 degree to +0.1 degree in azimuth and elevation with a step size of 0.01 degrees centered at $\phi_{\text{resolution}0.1}, \theta_{\text{resolution}0.1}$. Set the azimuth and elevation values corresponding the minimum value of $P$ to $\phi_{\text{resolution}0.1}, \theta_{\text{resolution}0.1}$.

5. The final minimum value with a resolution of 0.01 degrees is set as $\phi_{\text{min}}, \theta_{\text{min}}$.

The final orthodromic Steiner tree, and more specifically the Steiner point, describes the incidence angle from a system level that represents the least squared distance relating all observations and the AoA of the simulated signal. The Steiner point solution contains the contribution of all of the individual phase erred channel
values from each RF path and maps the information mathematically and graphically to an observation sphere.

4.6 System Analysis

This section looks at various angles of arrival incident waves using a digital simulation environment. A standard approach to error minimization is used to calculate a per-channel lookup table with calibrated data that is used to choose the corresponding phase delay that is closest to the necessary signal phase angle based on the delay calculation.

The system is analyzed by the cone analysis and Steiner point method of determining the simulated angle of arrival as well as the MUSIC algorithm with simulated perfect antenna and manifold data as shown in Figure 4.12. This concurrent path will provide insight into the system level understanding of the phase errors and how they are perceived by a system using a standard implementation of the MUSIC algorithm versus the new Cone analysis path.

The initial block represents the interference signal, and for this analysis the type of signal is not relevant. The second block represents the Hardware-n-the-loop (HWIL) artifacts, primarily referencing the phase coherency between the multiple RF channels for this analysis.

The Steiner tree and cone analysis path represents the method of evaluating the wavefront through an analytical approach to determining the system level performance of the multi-element delay simulation. This method calculates the AoA by incorporating each individual channel as a mandatory contribution to the AoA.
Figure 4.12: A digital simulation architecture for comparing AoA. The first path uses the cone theory and a Steiner tree implementation and the second path simulates a perfect AJ and CRPA and uses the MUSIC algorithm estimate.
The second path is used to represent a unit under test. The antenna manifold data is perfect in both gain and phase pattern to isolate the effects of the per-channel phase errors of the wavefront simulation. The AJ electronics are simulated by creating a covariance estimate using the known phase values for each element-to-reference pair. The covariance matrix is hermitian and is evaluated through an eigen decomposition. The MUSIC algorithm is used as a comparison to the Steiner point to provide a bound to the error allowed in the RF signal generation.

The output from each of these paths are compared for a specific AoA under investigation. Since antenna and AJ electronics do not have perfect observability, it is proposed that if the discrepancy between the Steiner point and the MUSIC algorithm is less than the observability of the AJ and antenna combination, then the resulting enhanced tap selections are valid and usable in the simulation environment. The value for the AoA observability of the incoming RF signal by the system under test must be determined by the user or provided by the vendor.

The covariance matrix relates each of the antenna elements phase and gain differences as compared to the reference element. The algorithms that calculate signal localization do various covariance analysis techniques by taking the eigen decomposition of the covariance output to rotate the system into the principal component frame. The formulation of the values that comprise the covariance matrix are created from the expected value of the signal based on the dwell time that processes the RF signal in the AJ hardware. The dwell time has impacts on the signal clarity and Doppler effects since each time period only considers a static view of the incoming signal. Also related to this dwell time is the vehicle dynamics. A larger dwell time has
better visibility into the signal, but is more susceptible to distortion if motion occurs
during the period. A large dwell time is possible with little to no adverse effect if
both the receiving and sending platforms are static since the Doppler would be zero.

\[
\text{Cov}_m = \begin{bmatrix}
cov[X(t_1)X(t_1)] & \ldots & cov[X(t_1)X(t_n)] \\
\vdots & \ddots & \vdots \\
cov[X(t_n)X(t_1)] & \ldots & cov[X(t_n)X(t_n)]
\end{bmatrix}
\] (4.33)

For the diagonal components we have the covariance of two identical signals
which reduces to

\[
cov[X(t_1)X(t_1)] = \text{var}(X) \equiv \sigma_X^2
\] (4.34)

The off diagonal components relate the phase and gain difference between the
corresponding elements, and will have a real and imaginary part as

\[
A + Bi = Re^{i\theta}
\] (4.35)

The MUSIC algorithm uses the eigen decomposition of the covariance matrix
to identify the approximate direction of the incoming signal. User defined steering
vectors, or guesses, are used to identify the angle from which the signal was received.
The accuracy of the MUSIC algorithm output is partly dependent on the resolution
of the angles that are used to create the steering vectors, so it will be set to the same
resolution as the Steiner point calculations. The MUSIC algorithm is considered to

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one of the optimum algorithms for doing covariance analysis. We define a set of steering vectors as shown in Equation 4.36.

\[
\alpha_k = \begin{bmatrix}
  j^{(2\pi)} & [\sin(\theta)\cos(\phi), \sin(\theta)\sin(\phi), \cos(\theta)] & \begin{bmatrix} x \\ y \\ z \end{bmatrix}
  \\
  e & \begin{bmatrix} x \\ y \\ z \end{bmatrix}
  \\
  \vdots
  \\
  j^{(2\pi)} & [\sin(\theta)\cos(\phi), \sin(\theta)\sin(\phi), \cos(\theta)] & \begin{bmatrix} x \\ y \\ z \end{bmatrix}
  \\
  e & \begin{bmatrix} x \\ y \\ z \end{bmatrix}
\end{bmatrix}
\]

The AJ covariance matrix can be written as an eigen decomposition

\[
\text{Cov} = Q\lambda Q^{-1},
\]

where the eigenvectors are equal to the columns of \(Q\), and the corresponding eigenvalues are the diagonal components of \(\lambda\). The eigenvectors provide a transformation from the antenna space to the principal component frame of the noise space.

The largest eigenvalue provides information about the signal with the most power. The problem of resolving how many possible interference signals are present
and the AoA region to search is reduced since the analysis is being completed on a system with a known single source input and approximate AoA. The largest eigenvalue indicates the corresponding eigenvector that is the closest to the incoming signal and defines the signal subspace and identify the principle component frame. The rest of the eigenvectors span the noise subspace.

The final $Q$ matrix that will be used for the AoA calculation is constructed by removing the eigenvector that corresponds to the largest eigenvalue. That vector is then replaced by a steering vector. The steering vector acts as a guess as to the direction of the incoming signal. The principle equation of the MUSIC algorithm, as shown in equation Equation 4.36, is then solved. This process is completed for each of the defined steering vectors.

$$M = \frac{\alpha_k^H a_k}{\alpha_k^H \hat{Q} \hat{Q}^H a_k}$$ (4.38)

The value of $\hat{Q}$ in equation Equation 4.36 represents the subset of eigenvectors making up the noise space. The denominator represents the dot product of the steering vector mapped to the noise space [29]. The value of $M$ is the largest when $\alpha_k$ is orthogonal to the noise space (aligned with the incoming interference). The determination of the maximum value of $M$, which is the solution of the simulated AoA, is desired. If an interference signal exists, the peak value of $M$ will provide the value of the direction of arrival matching the azimuth and elevation used to create the corresponding steering vector.
4.7 Evaluation of Discrete Step Sizes

The theory developed in this section is used to evaluate various levels of discrete step sizes and the effect on the wavefront accuracy. Simulations are run using several discrete step sizes for the tap delay line including 0.1ps, 1ps, 5ps, and 10ps. The analysis shows how the variation in phase accuracy affect the perceived AoA.

4.7.1 Discrete Tap Delay Effects

The concept of observation error that was described in 1 and 2 dimensions from Section 4.2 is expanded into three-dimensions to relate the phase and phase error of the RF channels to the desired AoA. Figure 4.13 relates the desired phase angle and possible phase angles from a single element-to-reference pair on an antenna. In this conceptual drawing there are infinite phase choices in the simulation that allow the system to construct a perfectly accurate representation of the RF signal. The information as viewed by one element-to-reference pair forms in the shape of a cone in three-dimensions as shown. This means that any vector along the cone would create the phase difference as seen by the antenna. All six element-to-reference pair cones intersect at an exact location in three-dimensional space when the system can provide perfect phase representation. This intersection point defines the AoA that is simulated by the wavefront system.

The concept is now used to represent a system with a discrete number of phase options. The system depicted in Figure 4.14 has discrete step sizes of 5ps to control the RF signals. When the desired time delay (phase angle) does not match
**Figure 4.13:** The fundamental concept of the per-channel phase error on the system level evaluation of the AoA. All measurements agree which provides an exact AoA.

**Figure 4.14:** The fundamental concept of the per-channel phase error on the system level evaluation of the AoA. Not all measurements agree which provides an ambiguous AoA region.

A possible discrete step, the closest time delay is chosen. This creates a delta in the desired versus simulated element-to-reference cones. A delta from each pair creates an ambiguity region when a perfect intersection is not created. The ambiguity represents the AoA error region formed by the per-element phase errors.

Figure 4.15 overlays the AoA region from the region formed by the cone intersections with the center of mass denoted with a red circle. The maximum value of
Figure 4.15: Output of the Cone region AoA using a center of mass calculation compared to the MUSIC algorithm output for AoA

The MUSIC algorithm is denoted with a green X. The cone analysis and the MUSIC algorithm have equal resolution to maintain a valid comparison.

The simulation in Figure 4.15 shows that the cones do in fact represent the same angle of arrival region as calculated by using the MUSIC algorithm path. The next step proceeds by looking at the effect on the system when discrete time steps are used for the tap delay options.

The accuracy of the simulation is dependent on the step size of the produced mesh field generated. Each data point in the mesh field represents an AoA in the simulation environment. The difference in the center of angles of interest cannot be expected to be more accurate than the resolution of the generated data.
Figure 4.16: Output of the Cone region AoA using a center of mass calculation compared to the MUSIC algorithm output for AoA, Step size of 0.1 ps

Figure 4.17: Output of the Cone region AoA using a center of mass calculation compared to the MUSIC algorithm output for AoA, Step size of 1 ps
Figure 4.18: Output of the Cone region AoA using a center of mass calculation compared to the MUSIC algorithm output for AoA, Step size of 5 ps

Figure 4.19: Output of the Cone region AoA using a center of mass calculation compared to the MUSIC algorithm output for AoA, Step size of 15 ps
The following section will apply these errors and the effects will be seen by the simulated unit under test as they relate through the covariance matrix.

4.7.2 System Level Evaluation With Discrete Steps

The analysis method shows the performance over azimuth and elevation for a certain set of input values. The phase angle errors due to the discrete values are shown with the corresponding effect on the AoA values that are capable of being simulated. The analysis shows three layers of information to convey the behavior of the signal in response to various tap delay step sizes. Layer one covers the upper hemisphere in azimuth and elevation in 5° degree steps. The second layer covers a ten degree pass in azimuth and elevation in 1° steps. The third and final layer covers a one degree pass in azimuth and elevation in 0.1° steps.

The plots are denoted with blue dots at each of the desired AoA azimuth and elevation shown. The red dots indicate the AoA created based on each channel being selected by the closest possible selection of tap offset. The resulting system level analysis is shown with the red dot. A quiver is drawn relating the position of each desired AoA to the system level point where the actual AoA is located. As the step size of the discrete channel level taps increases, these quivers are necessary and provide insight into the characteristics of the tap delay AoA generation mapping.

4.7.3 System Level Evaluation with Discrete Step Size of 0.1 ps

The first investigation of the system level analysis looks at the comparison of the system observed AoA in comparison to the desired. This value represents
Figure 4.20: Tap Delay with 0.1 ps Step Size. This is a progression of the effect of discrete step size of 0.1 ps on the system level evaluation of the Wavefront simulated signal.

the error in the system when no system level characterization is understood, or the Wavefront simulation is time varying and cannot be modeled accurately.

Every AoA has a unique incidence signal that can be accurately generated down to the 0.1 degree step size with an error of less than 0.02°, which can be seen in the third layer with the zoomed in azimuth and elevation profile.

4.7.4 System Level Evaluation with Discrete Step Size of 1 ps

The AoA signals created with a discrete step size of 1 ps are shown in figure Figure 4.21.
Figure 4.21: Tap Delay with 1.0 ps Step Size. This is a progression of the effect of discrete step size of 1 ps on the system level evaluation of the Wavefront simulated signal.

With 1 ps step size every desired AoA can has the created signal within 0.25° of the desired signal. This is slightly worse than the 0.1 ps step size in terms of system level accuracy. The channel level accuracy can be related back to the system level accuracy for AoA error in degrees of error over the upper hemisphere.

4.7.5 System Level Evaluation with Discrete Step Size of 5 ps

This reduction in perceived unit under test error is the critical outcome of this analysis technique and the application of this is what takes the tap delay architecture to the next higher level through analysis instead of additional hardware cost. The
**Figure 4.22:** Tap Delay with 5.0 ps Step Size. This is a progression of the effect of discrete step size of 5 ps on the system level evaluation of the Wavefront simulated signal.

Investigation into the three layers for a 5 ps resolution is shown in Figure 4.22. The artifacts and fewer AoA options by the wavefront simulator are apparent at this 5 ps discrete step size. The desired AoA options in the layer three image of Figure 4.22 show that all 100 desired phase coherency sets are mapped to only four different options. This knowledge and understanding can be readily applied to the output data from the unit under test to enhance the understanding and performance critique against the truly created signal versus the desired but unknown signal.
The main takeaway of this analysis is that the choice of AoA becomes much more limited with 5 ps discrete time steps vs the 1 ps step size, but the accuracy of the known simulated AoA remains just as accurate using the Steiner point calculations.

4.7.6 System Level Evaluation with Discrete Step Size of 10 ps

The same analysis is done as in the previous subsections but with 10 ps step size. The 10 ps step size is very limited and almost makes the system level simulation at this frequency range undesirable.

Figure 4.23: Tap Delay with 10.0 ps Step Size. This is a progression of the effect of discrete step size of 10 ps on the system level evaluation of the Wavefront simulated signal. Finish processing the data for Layer 1.
4.8 Source of Discrepancy Between Steiner Point and MUSIC

There is a difference in the Steiner point solution and the MUSIC algorithm solution near the plane of the antenna. This phenomena is present in the simulation with larger discrete phase steps. The MUSIC algorithm does not provide insight into the reason the errors are present. The cones analysis is used to show that in some situations, several pieces of information provided by multiple antenna elements are not being used in the solution. For a system under test, this is normal and the MUSIC algorithm does a good job of using the overdetermined system as best it can to create a solution. However, since we are the architect of the simulation and do not want to add any errors to the simulation that can be avoided it is prudent to investigate why this is happening and what can be done to fix it.

It is shown in the cone analysis that two of the cones do not intersect the other four cones to form a closed shape. When the cones form a closed shape, the solution is the center of mass which indicates the system is consistent and all RF channels are in agreement to produce a valid AoA.

What is the true AoA in this situation where the cones are not consistent? Is it the cone region or the MUSIC algorithm solution? This is where the Steiner tree and Steiner point provide additional information that will be used to understand and improve these situations.

Figure 4.24 shows the region near the antenna plane where the cones representing the 6 observations are shown. Due to the symmetrical nature of the antenna array and the fact that individual phase errors are not being added, only discrete
Figure 4.24: Example situation where two of the six cones do not intersect the other four due to lack of resolution in the per-channel phase selection steps, the cones representing the opposite antenna elements are equal. In this situation, four of the cones are combining to provide an AoA region that is consistent, and two other cones off to the left side do not intersect. Using a Steiner tree to calculate the AoA simulated as before produces a different results than the MUSIC algorithm due to the inconsistent data. This is where the ambiguity in the system is not desirable because during a live test, the path with perfect manifold information and perfect AJ electronics will be replaced with test hardware and the desire is to know the exact AoA that is simulated. Therefore, a method to correct for the error between the Steiner point, which only uses calibrated phase data, and the unknown accuracy of the test hardware is desired.
4.9 System Level Optimization using the Steiner Tree

Combining the knowledge gained about the system level characterization and effects of discrete step sizes, phase jitter in the tap selection, and manifold/RF front end errors, this information can now be used to optimize the Wavefront at the system level. This is done by making the choice to accept some errors in phase angle on a per element basis to create a more accurate understanding of the resulting AoA solution at the system level.

The other aspect of system level optimization is by using the knowledge of the created signal at the system level, as shown by the cone analysis to change the way the tap for a desired AoA is chosen. The concept is to show progressively how for a single AoA, various taps can be purposely given the wrong delay to simulate to counter errors in the other taps. This optimization would work similar to leveling a tripod where there would be an optimum setting for each tap to create the center of the cone region as close to the desired AoA as possible. Because of the changing intensity of the MUSIC algorithm around the peak, there must be a limit imposed on the maximum erred tap setting that can be chosen.

In a similar fashion, a process is developed using the Steiner tree information to manipulate the simulated AoA by using the respective tree nodes to optimize the accuracy of the desired AoA. This process stabilizes the system by ensuring that the Steiner point and MUSIC peak are equal, meaning that the system is not neglecting inputs, or all the observations are used to form the region around the angle of arrival.

The proposed steps to stabilize the system are as follows:
1. Calculate the Steiner point that minimizes the orthodromic Steiner tree distance.

2. Calculate the center of mass of the cone intersections.

3. If the Steiner point and center of mass are equal, the system is stable

4. Otherwise, calculate the vector from the center of mass to the Steiner tree center.

5. Find the Steiner tree branch with the most opposite vector.

6. Modify the tap selection of the corresponding observation to move the center.

7. Repeat until the Steiner point and the center of mass are equal.

The optimization algorithm compares the vector between the Steiner point and the center of mass to define the direction the Steiner point needs to move. The movement can be accomplished by comparing the vector to each Steiner tree branch vector. The implementation of the algorithm is shown in the following equations.

The Steiner tree branch vectors can be calculated as

\[ \mathbf{D} = [D_k - P]_{k=2:n} \]  

(4.39)

The Steiner point to center of mass, denoted M, can be calculated as

\[ \mathbf{O}_{opt} = M - P \]  

(4.40)
Figure 4.25: Definition of Steiner tree vectors and Steiner point to center of mass.

The vector that is in the most opposite direction of $O_{opt}$ can be found using the cosine similarity \[30\] as

$$\text{similarity} = \cos(\theta) = \frac{A \cdot B}{||A||_2 ||B||_2}$$

(4.41)

where $||A||_2$ is the Euclidean norm of $A$. The value $-1$ would indicate the vector was exactly opposite, so finding the minimum value will indicate the Steiner tree vector and corresponding tap delay channel that should be moved to the opposite side of the Steiner point.

The results show the ability of the algorithm to stabilize the angle of arrival region, and create a process to create a much more dense field of possible AoA by expanding the Steiner tree branches. This will require a bound of acceptable discrete error allowed because this effects the power perception of the MUSIC algorithm. It
would be best to keep this discrepancy between the Steiner point and the MUSIC output to less than the observability of the AJ electronics and CRPA under test.

The application of the orthodromic Steiner tree theory to evaluate a wavefront incident on a CRPA provides a system level understanding of the AoA. The application of the adaptive optimization of the Steiner point enhances the capability of the architecture by removing the uncertainty of the created AoA. The potential of this process ensures the created AoA is predictable and stable and could also be used to manipulate the discrete movement of the AoA.

4.10 Tap Selection Methodology

The selection of the taps is done in one of two ways. The standard approach calculates the desired phase angle for each element of the antenna array and selects the closest possible phase angle from a calibrated look-up table for the physical tap delay hardware. The enhanced method that is being proposed begins with the same taps chosen in the standard method, but allows for each tap to apply a known variation of ±2 taps, or approximately 10ps. The enhanced method creates a total of $5^6 = 15,625$ tap combinations per AoA. The value 5 represents the number of possible tap selections per channel, and the value 6 represents the number of channels (elements) that are not co-located at the reference point. The solution for each of the non-standard tap selections are compared through the digital architecture method shown in Figure 4.12. The number of valid solutions is reduced by limiting the discrepancy between the Steiner point and the MUSIC algorithm AoA to less than the observability of the AJ and antenna.
The enhanced tap selection technique applies a concept of selectable, or allowable, errors in the per-element phase angle to create a more predictable AoA region and relates mapped errors to antenna manifold errors. A more predictable AoA is produced by creating a known, but indistinguishable to the unit under test, error in the covariance matrix that is perceived as a change in average power.

If the antenna manifold data and observability of the system is $1^\circ$, then the resulting number AoA options expand because additional known error is being allowed. Some of the resulting values for different discrepancy values are shown in Figure 4.26 through Figure 4.29. The shape of the AoA values is primarily influenced by the antenna geometry.

The final selection of the discrepancy allowed between the Steiner point and the MUSIC solution is defined by the user based on the hardware under evaluation. This can be used to define the valid values based on the error that is acceptable, or
can alternatively be used to understand results that might lie outside of the desired error if they exist.

4.11 Performance Evaluation

The desired outcome of the software simulation runs are to show that the enhanced tap selection method correctly increases the AoA possibilities and the analysis
method is a valid process of determining AoA accuracy in a wavefront simulation. The results for the digital simulation architecture as shown in Figure 4.12 use calibration data that will be described in Chapter 5 to provide a method for comparing to hardware results.

A single AoA result of the digital simulation comparing the standard tap selection and the enhanced tap selection algorithm are shown in Figure 4.30. The data for this example shows that the AoA created using the standard tap selection method is over 5° away from the desired AoA. Additional tap combinations using the enhanced tap selection method provide more AoA possibilities, while still limiting the discrepancy to ≤ 2°. The new AoA option using the enhanced tap selection is approximately 0.6° from the desired AoA. This is an improvement of 2.57°, or 65% closer to the desired AoA value.

A profile over various azimuth and elevation values is used to create a diversity of data to compare the standard tap selection method with the enhanced method when
bounding the discrepancy. The azimuth was swept from 30° to 90°. The elevation profile was a sawtooth pattern moving between 45° and 90°. The profile plotted on the upper hemisphere is shown in Figure 4.31.

The results of the profile sweep relate information as shown in Figure 4.30 for each of the AoA values. For this simulation, the discrepancy was limited to 0.5° between the Steiner point and the MUSIC peak. The error in the standard tap selection is compared to the enhanced tap selection to evaluate the additional accuracy provided. The results show an average improvement of 87% in the ability of the enhanced tap selection to create a valid AoA as close to the desired angle.

**Figure 4.30:** Digital Simulation results using the standard tap selection algorithm and the enhanced tap selection method.
Figure 4.31: Profile of azimuth and elevation points for AoA evaluation.

Figure 4.32: Comparison of standard tap selection method versus enhanced tap selection method over a sweep profile.

as possible. The errors in the standard tap selection method are dominant as the elevation approaches 90°.
4.12 Summary of Contributions

The desire for any communication channel simulation is to construct and manipulate the RF signals in such a way that minimizes the hardware-in-the-loop artifacts, and ensures that what errors remain are known, understood, and the effect on the system being tested are acceptable. This ensures that the testing and integration of the evaluation hardware is worthwhile and relevant.

The novelty and contributions that were covered in Chapter 4 are an analytical approach to determining the system level performance of a multi-element delay simulation. This is accomplished by mathematical and graphical assimilation of the per-channel phase errors on a three dimensional observation sphere. The calculation of the least squared solution through the orthodromic Steiner tree implementation relates the impact of each individual phase error compared to a commonly used localization algorithm.

The application of the Steiner tree concept is used to recalculate the tap settings compared to the original per-channel selections to stabilize the system. The new tap selection, or per-element phase delay, is based on the Steiner branch vector directions compared to the movement of Steiner point in relation to the desired AoA.

Utilizing the analytical method of determining the effects of the phase errors on the AoA provides several significant benefits. The errors from the simulation environment are mapped through the covariance matrix to the unit under test in the same manner as the antenna manifold errors. This provides the user an opportunity to limit the phase error bound on hardware-in-the-loop parametric study requirements.
and AJ known capabilities. This limit would be unique for different systems. The design of the simulation can be set to accept these errors, or use them as an advantage in the process of manipulating the taps to ensure that the simulated AoA is known as accurately as possible.

The analysis technique applies a concept of selectable, or allowable, errors in the per-element phase angle selection to create a predictable AoA region and relates mapped errors to antenna manifold errors. A more predictable AoA is produced by creating a known, but indistinguishable to the unit under test, error in the average power. This is accomplished through the enhanced tap selection method.

A solution was defined to understand the system truth using the discrepancy between a strict truth estimate using a Steiner tree model, and a loose truth estimate using the well known MUSIC algorithm. This method used with the defined error bound describe the valid regions for the RF wavefront environment.
CHAPTER 5

HARDWARE IMPLEMENTATION OF WAVEFRONT AND ANALYSIS TECHNIQUES

This section will take the theory and analysis techniques that are described in Chapter 4 and provide simulations and data using physical hardware. This section will also go into topics not necessarily discussed in the theory sections about added noise, bit error rate (BER), power calibration, and other concerns that make this a fully understood and usable technology.

5.1 Overview of Hardware Architecture

The hardware architecture, rigid phased matched cables, isolators, splitters, tap delay boxes, FPGA controller, input signal options, were designed for use in a physical hardware-in-the-loop system. The hardware architecture for the tap delay line wavefront simulator is shown in Figure 5.1. The hardware inside the dotted box comprises the necessary tools to create a wavefront simulator, and has been built as a sealed 1U form factor box with a single RF input and 8 RF outputs. The tap delay lines are a product from Gigabaudics [31]. The tap delay hardware has been connected to the bulkhead connectors with rigid cables and phase calibrated so that
the zero delay options on the tap delay boxes are the same for each output. The signal input can be any waveform as prior discussions have indicated.

There is an assumption made when collecting the calibration data from the tap delay lines about the phase coherency at the first step for each tap selection. The 1U box is built by the JFW company with a certification that the rigid phase matched cables and all electronics provided for a phased matched output on each of the 8 channels when set to the step 1 (0° phase) setting is the same. Since the devices used to record the phase coherency between channels do not provide an absolute phase offset between the two input channels, this assumption was made to align the signals for the rest of the testing. Therefore, any errors in the step 1 setting will be a constant phase bias throughout the system during the calibration. This is however, system dependent and can be measured with the correct equipment and removed in physical hardware.
5.2 Pros and Cons of Design

This section will be an overview of the pros and cons of this design. For each of the cons, it will discuss how this lack of simulation capability will effect the results or what situations will not be tested with this type of architecture.

List of topics

- (pro) High power (>120dB J/S)
- (pro) High update rates (real time possibilities with 15kHz update rates)
- (pro) Modular designs allows for use in real-time HWIL and live sky settings
- (pro) Easily expandable for more elements
- (pro) Cost (relative to active hardware with similar phase coherent capability)
- (pro) Repeatability due to precise tap settings
- (con) Multiple sources are more difficult since each source requires a set of tap delay hardware

5.3 Noise Floor

GPS signals have a low power level when received, below the ambient perceived noise floor, as they reach the Earth. The received power level of an interference signal would only have to be -87 dBm, theoretically, to deny a tracking C/A code receiver as described. However, the AJ technology can provide benefits of 50 dB or more for the best systems. In order to fully test the potential of these AJ units, the capability
is needed for very high J/S levels in the simulation, which are not realizable by most active wavefront architectures. This is mainly due to their increase in noise floor and the limited dynamic range of the signal generator.

It is highly desirable to be able to create a system capable of producing interference signals far exceeding 100 dB J/S. To adequately test with these strong signals in the same dynamic, closed-loop environments necessary for system level integration requires the hardware to add negligible amounts to the noise floor. The theoretical J/S capable for the passive delay lines used, which are capable of input signals of +20 dB, is 150 dB J/S. With losses through cables and the necessary connectors, the maximum achievable J/S with a clean noise source is approximately 130dB J/S. This is an incredible noise level achievable that is not even possible with most approved outdoor testing events.

To show that the level of the noise floor is not considerably changed by the basic wavefront hardware, a simple test was conducted. Two identical receivers were set up to track a signal from a GNSS simulator. One of the systems received the signal straight from the simulator, while the other receiver’s data passed through the wavefront hardware. A noise source was used to slowly increase the noise floor. The CN0 levels of the received signals were observed and recorded. The hardware setup for this test is shown in Figure 5.2.

The data in Figure 5.2 shows that the CN0 levels are equal throughout the test. Testing of the noise floor and BER have shown the extraordinary benefits of this passive wavefront setup as the system can achieve very high J/S levels.
Figure 5.2: Noise Floor Setup

Figure 5.3 shows that the 3-dB drop occurs for the broadband (BB) noise source when it is equal to the $N_0$ estimated by the GPS receiver. This, along with the link budget from the block diagram, provides an estimate of the wavefront simulator’s noise floor (-195 dBW/Hz, approximately 11 dB above ambient noise floor).

5.4 Hardware Calibration

The GigaBaudics QPADL3 boxes provide passive mechanical delay with advertised 5ps resolution. These are passive RF devices with minimum inter-modulation distortion contribution [32] and fairly flat amplitude/phase response. The maximum
power input is +20dBm and the boxes add little contribution to the system-wide noise floor as shown. Together the tap-delay boxes provide a conservative jammer-to-noise ratio headroom of >120dB J/S. The tap-delay boxes will provide < 1° of per-channel phase error at L1 after calibration. Currently, JFW Industry provides an enclosed two-box 1U rack-mountable system with calibrated ±3° of path phase matching at 0 delay setting (JFW MODEL 50PDA-159).

The hardware is realized with varying levels of fidelity, and the full block diagram for the GigaBaudics simulator with 8-channel wavefront simulation is shown in Figure 5.4. Scaling the hardware may include eliminating the real-time channel simulator (RTCS), removing one of the 4-channel tap delay boxes and/or replacing the control computer FPGA interface with a simple static setting control of the GigaBaudics box(es).
A single 4-channel GigaBaudics mechanical delay box currently costs <\$6,000 and can be used as a stand-alone solution for simple wavefront dynamics. The estimated cost for the RF components and two configurations is provided in Figure 5.5.

The 4-channel configuration is shown below with the cabling for channel 1, element-1, of input to the AJ electronics. The CPU controls the tap delay setting through digital I/O ports, and drives the signal generated by the USRP. The current setup allows for a user to create signals at two different frequencies, which are combined with a 2-way combiner. The 8-way splitter takes the interference signal and feeds each necessary antenna element representation into the mechanical tap delay box. For the setup shown in Figure 5.6, a single RF line was connected to represent element 1. An additional RF cable is needed for each element of the antenna system under test, and more mechanical tap delays are needed if the number of antenna elements is greater than 4. The output cables from the mechanical tap delay box should be connected to the AJ unit under test, which is not shown. It is not necessary for all the cables to be phase matched since the calibration routine can handle small errors in delays. However, it is recommended that the hardware setup remain static after
calibration to maintain the optimized implementation. The rigid phased matched
cable architecture shown in Figure 5.1 is desired for a permanent setup.

Calibration procedures and techniques are discussed in the following section. In
addition to the hardware already mentioned, an Ettus/NI USRP is used for cali-
bration. The USRP will be placed on the output of the tap delay boxes and a BPSK
signal simulator will be used to provide a track-able phase signal from a two-channel
USRP recording. The USRP phase calibration is outlined in the following section.

5.5 Calibration Procedures

GigaBaudics advertises their tap-delay box as four delay channels, 5ps step
resolution, and ± 5ps of error. This error is repeatable and is demonstrated in the fol-
lowing section. The calibration procedure produces a mapping of the delay achievable
at each step to the delay desired, and results in a precise ($\approx \frac{1}{3}$ ps standard deviation)
and accurate (1024 achievable delays ranging from 0ps to 5115ps) command-able de-
lay. The achievable delay is the set of delays to which each channel can be set. The desired delay is the advertised range of 0ps to 5.112ns in 5ps steps. The group delay across all channels, representing the minimum delay, is estimated along with the individual absolute delays across each channel and its respective step delay command. The L1-band calibration is discussed and outlined. If multi-band calibration is required, such as in L1+L2 wavefront analysis, a user-defined cost function must be applied to weight the accuracy of delay across multiple bands.

The GigaBaudics QPADL3 provides precision delay capabilities along with minimum amplitude variation over the range of its capable delay. JFW Industries has taken two of these boxes and packaged them in a 1U rack mountable enclosure (50PDA-159 SMA) by request. In packaging the GigaBaudics boxes, JFW calibrated the 0-delay phase setting on each of the 8 channels to be aligned. In addition, JFW examined the L2 to L1 amplitude/phase response and VSWR of the channels. JFW’s provided channel specifics are shown in Figure 5.7.

The calibration procedure relies on the GigaBaudics box’s precision to provide a repeatable calibrated response for each channel. To examine the delay precision, the measured delay is statistically examined for repeatability over each channel and all 1024 steps of the individual channel. The process for measuring this delay is elaborated upon in the following paragraphs. Figure 5.8 shows the measured versus commanded delay for channel two. The precision of each individual commanded delay averages <1ps and most are \( \leq 0.5ps \). The top left plot represents the aggregate of all delay steps, and the top-right is a zoomed in version of 2500 ps commanded-delay step. The bottom right represents the histogram of the 2500ps commanded delay. Each
delay was commanded independently via software for a total of 40 measurements. A 0-delay was commanded prior to the 2500ps before each measurement. The bottom left shows the standard deviation for each commanded delay.

The repeatability of the channels allows for a full per-channel calibration procedure for mapping desired delay to commanded delay; the above $1\sigma$ statistics allow the user to assume $<65\%$ of time the commanded delay for most steps will be within 1ps of mean delay. The hardware setup allows the user to measure, with confidence, the ps-resolution of each channel and generate a lookup table for mapping commanded delay. This same equipment was used to collect the 40-measurements in Figure 5.8.
Several pieces of equipment are used to calibrate the tap delay boxes: a signal generator with frequency reference output, an Ettus/National Instruments USRP two-channel RF receiver, and the tap-delay boxes. The signal generator is used to create an RF signal with known structure and easily measurable relative delay. The USRP provides two channels of in-phase (I) and quadrature-phase (Q) recorded data samples for post-processing relative delay. Finally, the tap-delay boxes paired with a controller computer provide the mechanical delay steps which are being calibrated. Figure 5.9 illustrates the test setup block diagram and potential IQ illustration of recorded USRP output.

The equipment is configured to provide a measure of relative delay for each step on a per-channel basis. The signal generator is setup to provide a BPSK signal with 1.5MHz chipping rate and RF center frequency relative to the L1 band. NI/Ettus’s X310 model of USRP is setup to record IQ samples for two channels simultaneously. The USRP Hardware Driver (UHD) code allows for consistent phase alignment between channel 1 and channel 2 of the X310. The lab measured 1.2° 3σ
standard deviation over 200 UHD synchronization runs. The BPSK samples change phase and provide a resolution with 180 degrees modular ambiguity. Therefore, the 5ps steps are measured relative to channel 1 and the tap-delay channels are stepped to maintain an approximate 45° angle/delay relative to channel 1. The steps delay is estimated by the angle of the complex correlation coefficient relating the two channels as shown in Figure 5.9. Each channel is mapped to the commanded delay via a table after calibration.

The calibration table is validated by examining the delay error before and after application of the above process. Each channel’s per step delay error is plotted and the resulting histogram is shown. In an ideal case, with 5 ps resolution, the error for all delay steps should be within ±2.5 ps. The pre-calibration and post-calibration results are shown in Figure 5.10.
5.6 Performance Evaluation of Wavefront Simulation Using COTS AJ

The data from the live sky hardware test was collected over four discrete AoA periods. The data was recorded from a COTS AJ that provides the raw covariance estimate for processing. The cables and connectors used to connect the AJ under test with the wavefront simulator add unknown, but fixed delay bias, to each of the seven RF inputs. The desire is to have every phase angle in the covariance estimate equal to zero when the wavefront simulator is commanded to output a zenith AoA. This indicates that all of the elements are receiving the interference at the same time. This situation is used to calibrate out the added electrical lengths in the hardware prior to testing. The output from the uncalibrated covariance estimate is shown in Figure 5.11. It can be seen in the first section of data, from message count 1 to 220, that most of the angles are non-zero.

The added electrical length is calibrated out using the information collected during the zenith period. All seven taps are set to the same delay as the reference element during this time. The calibration is accomplished using a windowed sample of phase data from each of the covariance elements. The entire set of covariance
The uncalibrated covariance data erred electrical delay in the hardware setup. The data is then divided by the mean to remove the added phase error. The calibrated covariance angles after this calibration process are shown in Figure 5.12.

The hardware run was setup with an additional AoA region between the standard tap selection and the enhanced tap selection to ensure the regions were well defined in the data. The covariance estimate was then processed using the MUSIC algorithm to provide an AoA value as seen by the AJ unit. The results of the live
Figure 5.12: The calibrated covariance angles using a zenith period to remove electrical delay.

sky test comparing the standard tap selection versus the new tap selection algorithm are shown in Figure 5.13. The results show that the enhanced tap selection does provide a benefit in the delta between the desired AoA and the simulated AoA. The improvement in this scenario is not as profound as the software had predicted, but it still provided an improvement of 25% ($\approx 1^\circ$) closer to the desired AoA.
A few additional tests were run for other AoA values in the region between 75° and 90° elevation. The results for each AoA show that there are normally additional AoA options using the enhanced tap selection method with sufficiently small discrepancy values. In each situation the enhanced tap selection provides an equal to or better AoA option.

The benefit of the this method is an increase in the available AoA values while selecting the allowable error in the system based on the unit that is under evaluation. This increases the capability of a wavefront system using mechanical tap delay lines as described.
CHAPTER 6

CONCLUSION AND FUTURE WORK

6.1 Conclusion

The research presented in this dissertation investigated a simulation environment for the generation of a RF wavefront communication channel in the GPS frequency band. The design and test of CRPA antenna and anti-jam hardware systems require the use of a simulated RF wavefront, especially in frequency bands where broadcast of these signals is prohibited. The primary tool for the investigation was a digital simulation environment that allowed for practical examination of the developed communication theory.

The formulation of the problem being addressed began with the description of the communication theory that comprises GPS. This led to the understanding of the very low signal strength of the GPS signals at the reach the surface of the earth. Next, the development of adaptive nulling technology to mitigate the effect of undesired signals in the GPS frequency bands and how they used the information from the antenna through a covariance matrix was defined. Not all underlying nulling technologies are the same, but the manner in which the communication channel must be simulated is the same.
The phase errors for the RF channels are mapped through the covariance matrix, just as the manifold data is from the antenna. This relationship is beneficial to exploit and is used for creating a more accurately known AoA in the simulation.

The desired criteria for a simulation capability was presented in Section 2.5. The main constraints with most wavefront technology is the cost associated with phase coherent signal generation in active components. The modularity of such a design is also limited when there is a need for expanding number of channels in the simulation. The solution that was proposed and investigated consisted of an architecture where the single signal generation was phase delayed in discrete step sizes.

The chief contributions of this research lies in its calibration procedure, hardware simulation characterization, and the proposal of a bound to define the valid AoA regions using the defined discrepancy. These are developed to minimize the aggregate angle of arrival (AoA) error for the GPS L1 frequency, and to provide an analytical analysis method designed to evaluate the effect of the per-channel phase coherency at the system level.

An analytical method of determining the effects of the per-channel phase errors on the AoA is developed. The method provides a mathematical and graphical representation of the phase errors in 3 dimensions on an observation sphere. This method of presenting the phase coherent data provides an intuitive way to understand the effects of the discrete phase changes on the AoA.

A Steiner tree model was applied to relate the per-channel phase errors. The Steiner branches provide analytically derived information about the LSS and the observation points to allow for real-time calculation of system-level parameters to be
modeled in a HWIL. The concept is to recalculate the tap delay settings based on branch vector direction compared to desired movement of Steiner point to reduce the AoA errors. The analysis technique applies a concept of selectable (allowable) errors, or mitigation of hardware-in-the-loop artifacts, in typical per-element tap delay selection to create a predictable AoA region and relates mapped phase errors to antenna manifold errors (bounding the errors). A more predictable AoA can be simulated by creating a known, but indistinguishable to the unit under test, error in the average power.

The Steiner tree modeling of phase error is complimented by the use of the MUSIC algorithm. The MUSIC algorithm is implemented in a non-traditional way by providing insight into the deficiencies of the wavefront simulation. The information in the RF signal pertaining to the inter-element phase coherency is manipulated using manifold data from a perfect antenna, and by creating a simulated AJ output of a covariance estimate. The eigen decomposition of the covariance matrix and MUSIC algorithm provide a solution representing the AoA generated by a method that excels at solving overdetermined systems with inconsistent data.

The two methods of analyzing the simulated AoA are compared, and a discrepancy is defined. A discrepancy between the two AoA solutions that is less than the bound shows that the azimuth and elevation region is valid, and the simulation is capable of creating an accurate signal for the desired AoA. When the resolution in the phase is not sufficient, the discrepancy is larger than the bound, and the system is not capable of creating a valid AoA.
The proposed method of wavefront simulation analysis provides a mechanism to bound the error in the system. The bound should be modeled by the observability in the AJ electronics and CRPA under test. The discrepancy between the Steiner point and the MUSIC solution of AoA should remain within the bound to consider the region a valid for the wavefront. This analytical method and defined bound will ensure the phase manipulation of the RF signals created by the simulation environment are an accurate representation of the desired RF wavefront.

6.2 Future Work

The research and understanding of the principles in this dissertation are not a final solution to the modeling and simulation problem surrounding wavefront simulation environments. The theory developed within will be extended to analyze systems and as the technology advances, the simulation and understanding will need to adapt. Several additions to the wavefront simulation architecture will be studied for the impact to the truth in the system. These additions will include phase coherent per-channel attenuation, and mutual coupling effects between antenna elements. Additionally, the resulting environment will be evaluated for a situation when multiple location-independent sources are being simulated simultaneously.

Additional studies looking at the effect of the enhanced tap selection on the eigenvalue amplitudes and other aspects of interference tracking over profile motion will be done. The digital simulation environment that is used in this research will also be expanded to further investigate the effects of Doppler on the AJ solution. The
addition of the ESPIRIT algorithm as an input to the discrepancy calumniation will be performed.
REFERENCES


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