Ablation heat transfer characteristics of a polymer coolant medium for warm gas generator applications

Dakota J. Haring

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ABLATION HEAT TRANSFER CHARACTERISTICS OF A POLYMER COOLANT MEDIUM FOR WARM GAS GENERATOR APPLICATIONS

by

DAKOTA J. HARING

A DISSERTATION

Submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in
The Department of Mechanical and Aerospace Engineering to
The School of Graduate Studies of
The University of Alabama in Huntsville

HUNTSVILLE, ALABAMA
2020
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08/16/2020

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ABSTRACT

School of Graduate Studies
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Degree Doctor of Philosophy College/Dept. Engineering/Mechanical and Aerospace Engineering

Name of Candidate Dakota J. Haring

Title Ablation Heat Transfer Characteristics of a Polymer Coolant Medium for Warm Gas Generator Applications

Transporting high temperature solid propellant combustion gas through an ablating coolant tube is one method of lowering gas temperatures and providing clean, soot-free, pressurant for use in roll or divert attitude control systems. By utilizing the emerging technology of ablative warm gas generators, many of the challenges facing future space and kinetic kill vehicle designs may be mitigated. The data provided herein could assist in decreasing the mass, volume, and cost of warm gas generators capable of meeting the higher performance demands of the future.

The objective of this experimental research is to characterize the convective heat transfer correlations for an ablating, solid coolant subject to high-temperature, fuel-rich gases. An ammonium perchlorate/hydroxyl-terminated polybutadiene solid rocket propellant is used to generate hot gas that flows through a hollow polymethyl methacrylate coolant tube having initial diameters from 3.175 to 6.35 mm and an axial length of 2.54 cm. Experiments were conducted at pressure levels from 2,069 to 4,137 kPa, and mass flux levels in the bore from 101 to 531 kg/m²-s. A surface energy balance considering convection from
the gases and conduction into the ablating surface is coupled with the measured surface profiles to determine the heat flux, and convective heat transfer coefficient between the warm gas and the cooling material as a function of space and time. A normalized Nusselt number is used to observe the deviation of the flow from being fully developed. Additionally, the Nusselt number is compared to two other datasets found in literature that show a similar match to the dataset presented in this research. The data in this experiment indicates that the Nusselt number approaches the Dittus-Boelter equation for the higher Reynolds number experiments, which is also shown in previous research.

The surface of the coolant tube ablates at a non-uniform rate along the axis, which cannot be captured with a simple posttest analysis of the net coolant loss. This is due to the separation and reattachment of the developing momentum boundary layer. The developing momentum boundary layer influences the regression characteristics throughout the coolant tube. The real-time radiography measurements and novel data processing algorithm developed from this research provides time-dependent, two-dimensional ablation surface profiles to an accuracy of 0.10 mm, and error of 1.88% for the specimens used in the experiments. The ablative surface velocity ranges between 0.239 and 0.331 mm/s with an uncertainty of 20.8%, the ablative heat transfer coefficient ranged between 0.515 and 0.712 kW/m2-K with an uncertainty of 22.3%, and the Nusselt number for undeveloped flow ranges between 66 and 160 with an uncertainty of 21.4%. Thermochemical calculations show that the gases evolving from the ablative tube reduce the temperature of the combustion gas an average of 300 K with an uncertainty of 19.3%.
Abstract Approval:  Committee Chair

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<tr>
<td>( \phi )</td>
<td>Equivalence ratio</td>
</tr>
<tr>
<td>( \dot{m}_o )</td>
<td>Mass flow rate of oxidizer</td>
</tr>
<tr>
<td>( \dot{m}_f )</td>
<td>Mass flow rate of fuel</td>
</tr>
<tr>
<td>( l )</td>
<td>Instantaneous transmitted intensity</td>
</tr>
<tr>
<td>( I_0 )</td>
<td>Instantaneous intensity before transmission</td>
</tr>
<tr>
<td>( h_{\text{trans}} )</td>
<td>Length of transmission</td>
</tr>
<tr>
<td>( \mu_m )</td>
<td>Linear attenuation coefficient</td>
</tr>
<tr>
<td>( (\mu/\rho) )</td>
<td>Energy specific mass attenuation coefficient</td>
</tr>
<tr>
<td>( \rho_m )</td>
<td>Density of material that x-rays are passing through</td>
</tr>
<tr>
<td>( \bar{x} )</td>
<td>Output moving average signal</td>
</tr>
<tr>
<td>( T_w )</td>
<td>Moving average length</td>
</tr>
<tr>
<td>( x )</td>
<td>Input moving average signal</td>
</tr>
<tr>
<td>( z_t )</td>
<td>( t )th weighted moving average</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>Weighting constant</td>
</tr>
<tr>
<td>( f )</td>
<td>Probability density function</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>Standard deviation</td>
</tr>
<tr>
<td>( \mu )</td>
<td>Mean</td>
</tr>
<tr>
<td>( q_s )</td>
<td>Ablative heat flux potential</td>
</tr>
<tr>
<td>( \dot{m}_{f''} )</td>
<td>Mass flux of the fuel</td>
</tr>
<tr>
<td>( i_{fs} )</td>
<td>Latent heat of fusion</td>
</tr>
</tbody>
</table>
\( C_{p,f} \) Specific heat of the fuel
\( T_s \) Surface temperature
\( T_{melt} \) Ablative surface melting temperature
\( T_0 \) Initial temperature
\( \Delta h_{f,dec} \) Enthalpy of decomposition
\( \Delta h_{f,melt} \) Latent heat of melting
\( h_f \) Enthalpy of fuel
\( h_o \) Enthalpy of oxidizer
\( \dot{\tau}_{prop} \) Average regression rate of the propellant
\( a_o \) Burn rate law temperature coefficient
\( P_c \) Chamber pressure
\( n \) Pressure exponent or combustion index
\( \dot{m}_{prop} \) Mass flow rate of the high temperature combustion products
\( \rho_{prop} \) Density of the solid rocket propellant
\( A_b \) Burn surface area of the solid rocket propellant
\( \dot{m}'_{prop} \) Mass flux rate of the high temperature combustion products
\( A_{bore} \) Cross-sectional area of the ablative coolant tube internal bore
\( Re \) Reynolds number
\( D_{bore} \) Bore diameter of the ablative coolant tube
\( \mu_{prop} \) Viscosity of the high temperature combustion products
\( t_b \) Solid propellant burn time
\( L_{prop} \) Length of solid propellant

xxvii
\( E_{\text{right}} \)  Right edge pixel location
\( E_{\text{left}} \)  Left edge pixel location
\( \varphi \)  Scale factor
\( \overline{D}_{\text{known}} \)  Known initial drilled diameter
\( CV \)  Control volume
\( CS \)  Control surface
\( \partial t \)  Change in time
\( \rho \)  Material density
\( \partial V \)  Change in volume in control volume
\( V_{\text{rel}} \)  Relative velocity of particle
\( \partial A \)  Change in area over the control surface
\( V \)  Velocity of the mass in the control volume
\( P \)  Pressure at the control surface
\( \tau \)  Shear stress at the control surface
\( i \)  Enthalpy of the material
\( \dot{q}'' \)  Heat flux at a control surface
\( \dot{w}'' \)  Work flux at a control surface
\( V_{\text{coolant}} \)  Surface velocity of the ablative material
\( \Delta t \)  Intermediate timestep, 0.5s
\( \rho_{\text{coolant}} \)  Density of the ablative material
\( \dot{m}_{\text{coolant}}'' \)  Mass flux of the ablative surface
\( L_{\text{coolant}} \)  Length of the ablative coolant bore
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\dot{m}_{coolant}$</td>
<td>Mass flow rate of the ablative coolant material</td>
</tr>
<tr>
<td>$\dot{m}_{warmgas}$</td>
<td>Mass flow rate of the warm gas mixture</td>
</tr>
<tr>
<td>$\dot{m}_{warmgas}'$</td>
<td>Mass flux of the warm gas mixture</td>
</tr>
<tr>
<td>$\mu_{warmgas}$</td>
<td>Viscosity of the warm gas mixture</td>
</tr>
<tr>
<td>$Nu$</td>
<td>Nusselt number</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Coefficient of Nusselt number</td>
</tr>
<tr>
<td>$Pr$</td>
<td>Prandtl number of the warm gas mixture</td>
</tr>
<tr>
<td>$\dot{q}_{conv}$</td>
<td>Convective heat flux</td>
</tr>
<tr>
<td>$\dot{q}_{rad}$</td>
<td>Radiative heat flux</td>
</tr>
<tr>
<td>$C/P$</td>
<td>Coolant-to-propellant ratio</td>
</tr>
<tr>
<td>$T_{warmgas}$</td>
<td>Temperature of the warm gas mixture</td>
</tr>
<tr>
<td>$k_{warmgas}$</td>
<td>Thermal conductivity of the warm gas mixture</td>
</tr>
<tr>
<td>$Gz$</td>
<td>Graetz number</td>
</tr>
<tr>
<td>$\delta_{total}$</td>
<td>Total error</td>
</tr>
<tr>
<td>$\delta_{systematic}$</td>
<td>Systematic error</td>
</tr>
<tr>
<td>$\delta_{random}$</td>
<td>Random error</td>
</tr>
<tr>
<td>$UPC_i$</td>
<td>Uncertainty percent contribution</td>
</tr>
</tbody>
</table>
CHAPTER 1

INTRODUCTION

1.1 Solid Propellant Warm Gas Generator Fundamentals

For many years, Solid Propellant Warm Gas Generators (SPWGG) have been used to compactly generate high-pressure, low temperature gas for a variety of space, defense and commercial applications. A typical SPWGG consists of solid propellant grains within a combustion chamber which, upon ignition, rapidly react to generate gas-phase combustion products and particulate. The exothermic combustion process causes chamber pressurization resulting in hot gas flow through the downstream open end of the chamber. The hot gas flows from the chamber into a heat exchanger which reduces the temperature of the combustion gas before it is discharged through an exhaust port intended to disperse the cooled gas efficiently.

1.1.1 SPWGG Applications

A previous review conducted in 2004 by Bradford Engineering, divided SPWGG applications into six fundamental categories, each specific to their own characteristics [1]. These categories are: 1) propulsion systems, 2) inflatable structures, 3) pure gas generation, 4) mechanical actuation and tank pressurization, 5) power systems and, 6) thermal control systems. SPWGG propulsion systems are primarily utilized for Divert and Attitude Control Systems (DACS). DACS use impulse force typically through a nozzle to control the trajectory
(divert) and the attitude (pitch, yaw, and roll) of a vehicle flying inside or outside of the atmosphere [2]. DACS are used in the upper stages of space launch vehicles and anti-ballistic missile effectors [2]. A general illustration of the main building blocks of a solid propellant DACS is provided in Figure 1.1. The DACS shown in Figure 1.1 uses the hot gas from the solid propellant source for propulsion.

![Figure 1.1: Principle sketch of an S-DACS with 2 pairs of SGG and a common gas supply for DCS and ACS [3]](image)

DACS are also used in micro and nano satellites to enhance orbital sustainability and enable formation flying maneuvers of several satellites [3]. SPWGG inflatable systems are used to compactly store large amounts of potential pressure over long periods of time for solar sails, balloons, hypersonic re-entry systems, emergency inflatable rafts, and car airbags [4]. SPWGG pure gas generation systems are used to extinguish fire, purge systems, and pressurize space suits [5,6]. SPWGG mechanical actuation and tank pressurization systems are used in thrust vector control, deployable mechanisms, and stage and shroud separation [6]. SPWGG power systems are used to supply power to fuel cells, turbines, turbofans, and
combustion engines [7]. SPWGG thermal control systems are used for heating and cooling applications [1].

1.1.2 SPWGG Advantages

Conventional cold gas systems generally store pre-pressurized gas in a tank, because of the lower specific impulse and storage of the propellant as a gas, these systems have greater mass and volume cost than a SPWGG system. A comparison between a conventional cold gas system and a SPWGG system is given in Figure 1.2.

![Figure 1.2: Conventional cold gas system (left) and SPWGG system (right) [1]](image-url)
The main elements common to both systems are a propellant storage system, valves, a filter, and a set of nozzles. Conventional systems store gaseous nitrogen (GN2) in a pressurized tank which is released with a set of valves, filtered and then dispersed through a set of nozzles. The SPWGG system stores solid rocket propellant cartridges that, upon ignition, reacts to generate high temperature combust products that travel through a filter then is dispersed through a set of nozzles. Unlike the conventional system, the SPWGG system does not have pressure relief valves or fill and drain valve but does include a plenum.

There are multiple advantages of using SPWGG’s to generate high-pressure, low temperature gas in situ when compared to conventional cold gas storage methods. By storing the propellants chemically bonded in solid form, the system has no leakage, is unpressurized and can be stored for long periods of time without attendance [8]. A previous study conducted in 2007 showed that conventional micro and nano satellite propulsion systems are upwards of forty percent of the overall satellite mass [9]. SPWGG propulsion systems, when compared to conventional systems, have the potential to reduce the mass of the propulsion system, which will significantly reduce the overall mass and volume of a satellite. A study conducted in 2001 proposed that a decrease in satellite mass leads to a significant decrease in total satellite cost and vehicle bus cost due to functional and operational characteristics [10]. SPWGG propulsion systems minimize or eliminate the necessity of valve actuation required in conventional pressurized systems, allowing SPWGG systems to be less complex and more reliable [1]. Systems that do not cool the combustion gas generally require special heat resistant materials in their construction to avoid damage to the components. Heat resistant metals, such as columbium or niobium, are typically used in non-cooled designs. However, these metals are relatively heavy, scarce and expensive.
Carbon compounds have been used in constructing light-weight high temperature components, but their expense is relatively high.

### 1.1.3 Practical Considerations for SPWGG

A disadvantage of using solid propellants to generate high-pressure gas, when compared to conventional pressurized tank designs, is the high temperature associated with combustion of the solid propellant, which can be upwards of 4000 K, depending on the propellant formulation. Typically, the solid propellant combustion gas has a temperature much higher than the operable limits of most novel applications and materials [11]. Therefore, an efficient and reliable cooling mechanism is necessitated to reduce the temperature of the combustion gas before it can be used in operation. In this research an ablative cooling medium is used as the cooling mechanism. It is important to understand and quantify the spatial and temporal regression characteristics of ablative materials. Understanding the surface regression and heat transfer characteristics promotes design efficiency and model accuracy prior to incurring the cost of implementing an ablative thermal cooling system.

### 1.2 Objectives

The objective of the current study is to fully characterize the heat transfer attributes in terms of ablative, convective and radiative heat flux, an effective heat transfer coefficient, and temperature gradients as a function of time and location in a participating polymer coolant medium. Successful characterization of the ablative heat transfer model presented in this research will:
1. Promote design efficiency for ablative polymer coolant beds in SPWGG DACS,

2. Provide a detailed database of local heat transfer characteristics for a range of mass fluxes relevant to SPWGG DACS,

3. Test an ablative solid-solid hybrid coolant medium concept that will reduce mass, volume, and cost while increasing reliability in SPWGG DACS.

4. Provide a detailed procedure for obtaining time dependent, two-dimensional ablative surface profiles using real-time radiography and a newly developed edge detection algorithm.

1.3 Scope

The scope of this research includes conducting a series of experiments and developing an algorithm to track local time dependent surface regression of an ablating coolant medium.

The technical knowledge advanced in this research are:

1. A new real-time radiography regression measurement and digital processing algorithm.

2. A new physics-based ablative heat flux model applicable to these conditions, and

3. A new ablation heat transfer coefficient correlation.

During the experiments, high temperature and pressure solid rocket propellant combustion products will traverse through an ablative polymer coolant tube. Mass flux of the driving propellant through the ablative polymer coolant tube is varied by controlling the combustion pressure and geometry of the coolant medium. This will allow the heat transfer attributes to be characterized over a range of operating conditions. Real-time radiography
and digital edge detection algorithms developed from this research are utilized to non-destructively capture the change of local internal bore diameter as a function of time and axial location. The rate of change of the local internal bore diameter will be related to the solid mass deflagration rate of the coolant tube.

An ablative heat flux model is derived from the general conservation equations and a set of three control volumes to predict the local cooling potential that is available from the coolant tube along the axis as a function of time. The local cooling potential of the coolant tube will be related to the local regression rate and ablative material properties. Once the ablative heat flux boundary has been characterized, it can be used to solve for local gas temperature and composition.

General conservation equations applied to the fluid control volume will be used to predict the temperature of the combustion products as a function of space and time as it flows through the ablative polymer coolant tube. The cooling effectiveness of the ablative medium and geometry will be estimated. The local combustion gas temperature will then be related to a local ablation heat transfer coefficient that is established from this research.

A modification is made to the heat transfer coefficient to incorporate the effects of ablation in pipe flow. This requires solving for new coefficients and exponents of the Reynolds and Prandtl number, specific to the current experiments, which will be used to determine a Nusselt number of ablation and a heat transfer coefficient of ablation.
CHAPTER 2

REVIEW OF LITERATURE

This chapter presents a critical review of the topics related to the proposed ablative heat transfer model. This chapter will provide a review of the literature pertaining to common SPWGG cooling configurations, real-time radiography regression measurement techniques, and ablative heat transfer models as it applies to the characterization in this study.

2.1 SPWGG Gas Cooling Configurations

This dissertation is concerned with cooling high-temperature gases for thrust vector control applications. Therefore, a review of heat extraction is first completed. There are four primary approaches to extract heat from a high temperature gas in a SPWGG mechanism. These approaches are shell and tube heat exchanger, solid-liquid hybrid, solid-solid hybrid, and ablative solid-solid hybrid.

2.1.1 Shell and Tube Heat Exchanger

Conventionally, SPWGG’s reduce the temperature of high temperature gas-phase combustion products to an operable temperature using a liquid coolant [12]. When the coolant liquid and the high temperature gas-phase combustion products are not intended to be mixed, a heat exchanger is typically used. A heat exchanger uses a thermally conductive
wall, usually composed of a non-corrosive metal, to separate the combustion products from the coolant liquid. A common gas-liquid heat exchanger is a shell and tube heat exchanger (STHX), consisting of multiple metal tubes inside a sealed enclosure. The shell contains a flowing liquid coolant that envelopes the inner tubes. Heat is conducted through the tube walls and is transported via convection into the flowing coolant. The coolant flow can be parallel to, counter to, or across the hot gas flow in the tubes, depending on the cooling requirements. The heat exchanger effectiveness is determined by measurement or prediction of the inlet and exit temperatures of the coolant and gas. An illustration of a SPWGG STHX is given in Figure 2.1.

![Figure 2.1: SPWGG STHX Configuration [13]](image)

The shell material is typically insulated to mitigate heat transfer to the ambient [14]. The tube material is typically copper or brass to maximize the conductive heat transfer between the combustion products and the coolant liquid [14]. To maximize effectiveness, both the combustion product and coolant flow simultaneously through the heat exchanger, with as much coolant flowing as the design and supply allows.
2.1.1.1 Regenerative Cooling of Rocket Nozzles

Where cooling of high temperature combustion gases is impractical, regenerative cooling is used to directly cool the structures exposed to the high temperature combustion gases. This technique is utilized specifically in liquid rocket engines (LRE), where the liquid fuel or oxidizer traverses through channels that line the nozzle to prevent the nozzle from overheating and melting due to the high temperature combustion gases that flow through the nozzle [15]. An example of an LRE regenerative cooling nozzle is illustrated in Figure 2.2.

![Figure 2.2: Regenerative Cooling Nozzle Channels [15]](image)

The fuel or oxidizer that passes through the nozzle channels is initially at a low temperature, as it passes through the channels the temperature of the fuel or oxidizer increases as heat is extracted from the high temperature combustion gases.

2.1.2 Solid-Liquid or Gas Hybrid

A series of studies conducted in 2001 and 2011 explored injecting and mixing cool liquid or gas into the high temperature solid propellant gas-phase combustion products to reduce the temperature, a development coined as a solid-liquid hybrid warm gas generator...
A SLHCGG utilizes both solid and liquid components and proved to be an effective method to cool gas-phase combustion products [17]. The primary advantage of SLHCGG’s is that the coolant liquid and combustion products mix, allowing for large amounts of low enthalpy material to reduce temperature, unmitigated by the conduction and convection limitations typical of a STHX. An example of a SLHCGG is provided in Figure 2.3.

**Figure 2.3:** SLHCGG Configuration [16]

In Figure 2.3, solid propellant, upon ignition, is used to produce high temperature combustion products in a combustion chamber. The combustion products are then mixed with a compressed coolant gas in a mixing chamber. Finally, the cool combustion products after mixing with the compressed coolant gas are dispersed into the ambient using a diffuser. The absence of internal metal tubing allows the weight of SLHCGG to be reduced in comparison to conventional SPWGG STHX cooling methods. The disadvantage of SLHCGG’s are the increased design complexity and the necessity to store and maintain the pressurized coolant, which can increase the overall system mass, volume and cost.
2.1.3 Solid-Solid Hybrid

Solid-solid hybrid coolant gas generators (SSHCGG) are warm gas generators composed of a mixture of solid propellant and solid flame-retardant materials [18]. The flame-retardant material is chemically bound to the solid propellant to reduce flame temperature upon ignition of the solid propellant. The primary advantage of SSHCGG systems, when compared to SLHCGG, is the potential reduction in system mass and volume resulting from the absence of a pressurized liquid storage tank. A disadvantage of the SSHCGG system is that the flame temperature is reduced which causes a reduction in chamber pressure and can lead to decrease in specific impulse and thrust.

2.1.4 Ablative Solid-Solid Hybrid

Another method for extracting heat from the combustion gas is to use a downstream ablative coolant medium. An example is illustrated in Figure 2.4.

![Figure 2.4: Solid Propellant and Solid/Liquid Gas Generator Concept][6]
The main advantage of ablative coolant systems for SPWGG, when compared to other cooling systems, is the potential for reduced mass, volume, cost, and maintenance. Ablation cooling systems have proven to be substantially more efficient than other cooling methods due to the high values of heat of physicochemical transformations and to the injection heat effect [19]. While the other cooling approaches, namely, heat exchangers and solid-liquid and solid-solid hybrids, have been studied and developed for many years, hot gas cooling using a solid ablative material is a novel concept. Ablation is affected by the freestream conditions, the geometry of the ablative coolant apparatus, and the ablative surface material [20].

2.1.5 Ablative Coolant Configurations

The primary ablative coolant configurations for a SPWGG DAC application is a hollow cylinder configuration or a packed bed configuration. The following subsections describe the advantages and disadvantages of each configuration.

2.1.5.1 Hollow Cylinder Configuration

A hollow ablative cylinder configuration is the simplest of configurations. It consists of a hollow tube(s) that ablates and reduces the temperature of the high temperature combustion products. An example of a hollow ablative cylinder configuration is given in Figure 2.5. The primary advantage of this configuration is the simplicity and reliability. Illustrated in Figure 2.5, a boundary layer is formed and developed through the tube. There is no blockage and therefore the minimal momentum losses. The primary disadvantage of the configuration is the lack of mixing and surface area, which can cause a reduction in heat transfer efficiency.
A remedy for this disadvantage is to increase the tube length, which increases the surface area and allow for additional mixing potential. Additionally, baffles and other types of ablative fins can be used effectively for mixing and increasing the surface area [22].

2.1.5.2 Packed Bed Configuration

Another configuration is a packed bed. In this case, shaped pellets are placed inside of a tube. High temperature combustion products flow around the pellets cooling the gas. The gas cooling pellets can be either ablative or non-ablative depending on the configuration. Ablative pellets will ablate and mix with the high temperature combustion gas that is flowing around the pellets. Non-ablative pellets will not ablate, reducing the mixing and overall cooling effect. The advantage of ablative pellets when compared to non-ablative pellets is the increasing mixing effects which can increase the overall heat transfer effects. However, this can lead to clogging in the system due to agglomerates being produced at the pellet surface. Clogging is unlikely to happen in non-ablative pellet because no agglomerate will form. An example of a packed bed configuration is shown in Figure 2.6.
Where hot gas enters the packed bed from the top face, passes over a bed of spherical coolant materials, and exits through the bottom face with a reduced temperature. The primary advantage of this configuration when compared to a hollow cylinder configuration is the increased mixing ability and surface area [24]. This configuration can also act as a filter which can condition the combustion products to a desired composition. The primary disadvantage of this configuration when compared to a hollow cylinder configuration is the flow obstruction which can lead to degraded performance [25], [26].

2.1.6 Desired Gas Properties

The primary goal of the ablative coolant medium is to condition the high temperature combustion gas generated from a solid propellant to operable temperatures and gas
Typically, solid propellants generate high temperature combustion gas ranging from 1500-4000 K. Table 2.1 provides various solid propellants used in SPWGG DAC configurations and their respective flame temperatures. The flame temperatures will vary slightly with the specific composition of the propellant.

**Table 2.1**: Propellant types and respective flame temperatures [27]

<table>
<thead>
<tr>
<th>Propellant Type</th>
<th>Flame Temperature (K)</th>
<th>Isp (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>DB/AP/Al</td>
<td>3880</td>
<td>260-265</td>
</tr>
<tr>
<td>DB/AP-HMX/Al</td>
<td>4000</td>
<td>265-270</td>
</tr>
<tr>
<td>PVC/AP/Al</td>
<td>3380</td>
<td>260-265</td>
</tr>
<tr>
<td>CTPB/AP/Al</td>
<td>3440</td>
<td>260-265</td>
</tr>
<tr>
<td>HTPB/AP/Al</td>
<td>3440</td>
<td>260-265</td>
</tr>
<tr>
<td>HTPE/AP/Al</td>
<td>3538</td>
<td>248-269</td>
</tr>
<tr>
<td>PBAA/AP/Al</td>
<td>3440</td>
<td>260-265</td>
</tr>
<tr>
<td>AN/Polymer</td>
<td>1550</td>
<td>180-190</td>
</tr>
</tbody>
</table>

In this research, the propellant of choice is HTPB/AP, which is a fundamental propellant used in hobby rockets and other propulsion applications such as SPWGG DACS for micro-satellites. It is critical to effectively cool the high temperature combustion products before expulsion into the temperature-sensitive subsystems they drive.

The desired gas properties leaving the ablative coolant medium depends primary on the application. A list of SPWGG applications and their desired temperature for operation is provided in Table 2.2.
Table 2.2: Desired operational temperatures for SPWGG applications

<table>
<thead>
<tr>
<th>Application</th>
<th>Operation Temperature (°F)</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Micro-Satellite</td>
<td>62-98</td>
<td>[1]</td>
</tr>
<tr>
<td>Auxiliary Power Unit</td>
<td>1800-2250</td>
<td>[28]</td>
</tr>
<tr>
<td>Pneumatic Tools</td>
<td>130</td>
<td>[29]</td>
</tr>
<tr>
<td>Turbine Drive</td>
<td>1300</td>
<td>[30]</td>
</tr>
<tr>
<td>DACS</td>
<td>2400</td>
<td>[24]</td>
</tr>
</tbody>
</table>

Table 2.2 shows that the desired temperatures for operation are much lower than the flame temperature of the solid propellants listed in Table 2.1.

2.1.7 SPWGG Ablative Coolant Materials

Ablative coolant mediums for SPWGG DAC applications typically use Thermal Protection System (TPS) materials which have a high temperature of ablation used for mass rejection of heat. In general, ablative materials are used as part of thermal protection systems for atmospheric re-entry vehicle heat shields, ablative nozzle cooling, and ablative insulators because of their endothermic attributes [31]. Ablative materials have utility both internal and external to the propulsion system, but in general, ablation is used as passive TPS [32]. In the case of SPWGG’s, ablative materials could be utilized for gas cooling instead of thermal protection systems. The equivalence ratio is commonly used to indicate whether a fuel-oxidizer mixture is rich lean, or stoichiometric and is defined as [33],
\[ \phi = \frac{\frac{\dot{m}_o}{\dot{m}_f}}{\text{stoic}} = \frac{\frac{\dot{m}_f}{\dot{m}_o}}{\text{stoic}} \] (2.1)

where \( \dot{m}_o \) is the mass flow rate of the oxidizer, and \( \dot{m}_f \) is the mass flow rate of the fuel. When \( \phi > 1 \) the mixture is fuel-rich, when \( \phi < 1 \) the mixture is fuel lean, and when \( \phi = 1 \), the mixture is stoichiometric.

By increasing the fuel-to-oxidizer ratio above the stoichiometric condition, the adiabatic flame temperature can be decreased, as illustrated in Figure 2.7. Therefore, it is ideal to choose a material that will significantly increase the F/O ratio, i.e., increase the fuel mass flow.
rate. The following table illustrates materials that have been shown to increase oxygen levels when burned.

**Table 2.3: List of optimal ablative cooling additives [35]**

<table>
<thead>
<tr>
<th>Material</th>
<th>Empirical Formula</th>
<th>Density, g/cc</th>
<th>Oxidation Ratio (O/F)</th>
<th>$\Delta H_f^M / \text{Kcal/g}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oxalohydroxamic Acid</td>
<td>$C_2H_4O_4N_2$</td>
<td>1.85</td>
<td>1.0</td>
<td>-1.138</td>
</tr>
<tr>
<td>Hydroxylammonium Oxalate</td>
<td>$C_2H_8O_6N_2$</td>
<td>1.60</td>
<td>1.0</td>
<td>-1.85</td>
</tr>
<tr>
<td>Ammonium Dihydrogen Phosphate</td>
<td>$NH_4H_2PO_4$</td>
<td>1.803</td>
<td>1.33</td>
<td>-3.02</td>
</tr>
<tr>
<td>Ammonium Nitrate</td>
<td>$NH_4NO_3$</td>
<td>1.725</td>
<td>1.5</td>
<td>-1.09</td>
</tr>
<tr>
<td>Ammonium Perchlorate</td>
<td>$NH_4CLO_4$</td>
<td>1.95</td>
<td>2.0</td>
<td>-0.60</td>
</tr>
</tbody>
</table>

The oxidation ratio increases as the F/O ratio decreases; therefore, it is ideal to choose compounds with a large oxidation ratio. It is also ideal to choose the compound that requires the most energy for activation per gram. From the materials listed in Table 2.3, Ammonium Dihydrogen Phosphate would be the optimal choice for a coolant additive due to its oxidation ratio and activation energy attributes.

In general, a binder compound such as polymethyl methacrylate (PMMA) is used for structural integrity. The coolant material, in powder form, would be mixed with the binder before casting to provide uniform dispersion through the binder. However, for this research, PMMA with no additive is selected as the coolant material due to its low cost, manufacturability and desirable heat transfer attributes.
2.2 X-Ray Regression Rate Measurement Techniques

There are multiple methods to experimentally quantify the regression rate of an ablative material for a specific set of inputs such as temperature, pressure and geometry. The methods described in literature include x-rays, microwaves, ultrasound, resistance and plasma capacitance gauge [36]-[42]. Real-time radiography (RTR) is an ideal method to determine the surface regression rate of ablative material specimens in time and space [43]. Conventional methods described in literature are used to measure regression rate of an ablating material only at a single point. This prevents characterization of the entire surface thereby limiting the amount of information obtained from an experiment. Figure 2.8 represents a detailed experimental apparatus for characterizing a flat plate ablative material.

Figure 2.8: Detailed RTR setup for ablative material characterization [44]
In general, the apparatus consists of a high intensity source producing x-rays that are transmitted through x-ray translucent windows, and the ablative material sample. An image intensifier captures the x-rays and relays them to a camera. To ablate the material a solid propellant grain is ignited, and the high temperature combustion gases are passed over the ablative material heating it up before the combustion gases are dispersed through a nozzle. This method is typically used for characterizing regression rates and heat transfer attributes for flat-plates as ablating hollow cylinders is still a novel concept for SPWGG DACS.

A study conducted in 2000 used two RTR setups as a method to observe the difference between laminar and turbulent ablation of a flat plate for a hybrid rocket [43]. The flat plate material was a solid rocket fuel instead of a cooling material which is what our research focuses on. Gaseous oxygen (GOX) enters the chamber at a specified mass flux and exits through an interchangeable nozzle. The nozzle size is used to control chamber pressure which is related to the regression rate of the solid propellant material. Figure 2.9 illustrates the experimental setup.

![Figure 2.9: Laminar and turbulent ablative apparatus](image)

*Figure 2.9: Laminar and turbulent ablative apparatus [43]*
In the apparatus, x-ray viewing window #1 was used to observe the laminar flow regime and x-ray viewing window #2 was used to observe the turbulent flow regime. Thermocouples (TC) and pressure transducers (P) were also used to measure temperature and pressure as a function of time and location in the setup.

The author of the study was able to determine the regression rate of both regimes but was unable to predict uncertainties due the method of tracking the surface [43]. Figure 2.10 illustrates the regression rate of the solid propellant as a function of incoming mass flux.

![Graph](image)

**Figure 2.10:** Regression rate vs mass flux for flat plate hybrid rocket fuel [43]

The mass flux ranges from 20 kg/m²-s to 700 kg/m²-s with a corresponding regression rate ranging from 0.15 mm/s to 1.75 mm/s. The results produce a linear line with a positive slope. As the mass flux increases the regression rate increases. Unfortunately, the acquisition is relatively expensive, data interpretation is labor intensive, and the accuracy and error is difficult to predict with current methods [37], [43], [45].
2.2.1 RTR Transmission Techniques

RTR is a method to transmit x-rays through an axisymmetric sample, rather than a flat slab, to produce a video image of the internal movements. X-ray transmitted intensity profiles can more readily be used to identify features inside of solid and hybrid rockets such as the burning of the internal bore [37], [43], [46], propagating cracks [47], [48], and the accumulation of slag [49]. As the x-ray beam traverses through the object the photons are either absorbed, scattered, or complete the traversal without any absorption. The transmitted intensity levels are based on the attenuation of the x-ray beam. Figure 2.11 illustrates the RTR setup with a camera, object (motor in this case), screen or image intensifier and camera [50]. Figure 2.12 represents a theoretical transmitted intensity profile of a center perforated propellant grain having a crack aligned with the x-ray source with respect to Figure 2.11 [50].

![Diagram](image)

**Figure 2.11:** RTR motor analysis setup [50]
In Figure 2.12, the transmitted intensity at point A is representative of $d_1 + d_2$, the thickness of the propellant along that ray. Also, the point B represents the edge, or location of the inner bore of the cylinder. Note that it is theoretically a minimum value since this ray passes through the maximum length of material. These features are used to track the thickness and the degradation with time to determine a regression rate of the inner surface.

Theoretically, the intensity level of a narrow beam after the transmission is a function of the material-based attenuation constant and transmitted thickness [51],

$$I = I_0 e^{-\mu \text{m}_\text{trans}}$$

(2.2)

where $I$ is the instantaneous intensity after transmission (typically the intensity value that is received by an image intensifier and x-ray camera), $I_0$ is the instantaneous intensity before
transmission (typically the intensity value that is transmitted from the x-ray source), \( \mu_m \) is the linear attenuation coefficient, and \( h_{\text{trans}} \) is the length of transmission. The linear attenuation constant is a function of material density and can be described as [51],

\[
\mu_m = \left( \frac{\mu}{\rho} \right) \rho_m
\]

where \( \frac{\mu}{\rho} \) is the energy specific mass attenuation coefficient. A database for the mass attenuation coefficient is provided by NIST for various materials [52]. The \( \rho_m \) is the density of the material that the x-rays are being transmitted through. The linear attenuation coefficient defines the probability of interaction of an x-ray photon with the atoms of the material [48]. In general, lower energy x-ray photons have a higher probability to be intercepted by an electron in the material [48].

2.2.2 Edge Detection Techniques

There are several methods for detecting and tracking an edge or surface of an artifact, which include but are not limited to hand selection, moving average method, and weighted moving average method. Other methods such as the Canny-Gaussian methods and Sobel methods exist but are typically only used on images with abrupt changes in pixel intensity, i.e., non-radiographic images [53], [54]. Edges in a radiographic image do not have abrupt changes in pixel intensity and require the edge detection method described in this research.

2.2.2.1 Hand Selection Method

Perhaps the least complex but most time-consuming method for detecting an edge is through hand selection. This method has been used in the studies conducted in [37], [43],
The main advantage of this method is that no intricate software or algorithm is needed, only the eye of the examiner. This main disadvantage of this method is that it is extremely time consuming and it is almost impossible to quantify or repeat the results of the method. Therefore, it is deemed an insufficient method for detecting and edge in this research.

2.2.2.2 Simple Moving Average Method

A simple moving average filter (SMAF) is typically used in signal processing as a type of finite impulse response filter. A SMAF with the input signal $x(t)$ and the output signal $\bar{x}(t)$ can be described in continuous-time domain by, [55]

\[
\bar{x}(\tau) = \frac{1}{T_w} \int_{\tau-T_w/2}^{\tau+T_w/2} x(\tau) d\tau
\]

where $T_w$ is the moving average length. An example of the SMAF being applied to a raw signal is illustrated in Figure 2.13.

![Figure 2.13: SMAF applied [56]](image)
Using the SMAF and a gradient threshold to detect discontinuities is a common concept. A study conducted in 1987 attempted a similar variant of using a gradient threshold for selection of an edge or discontinuity without filtering the signal [57]. The study only applied the method to non-radiographic images which have a large mean to noise ratio of the signal. In radiographic images, a moving average filter is required because the mean to noise ratio is very small. If a filter of some sort is not implemented for radiographic images, the noise could be misinterpreted as a discontinuity.

### 2.2.2.3 Weighted Moving Average Method

Similar to the SMAF, a weighted moving average filter (WMAF) is an averaging filter that has multiplication factors to give different weights to data at different positions in the sample window, effectively weighting the filter. The \( t \)th weighted moving average \( z_t \) is defined as [58],

\[
    z_t = \lambda \bar{x}_t + (1 - \lambda)z_{t-1}
\]  

(2.5)

where \( \lambda \) is a smoothing constant, or weighting constant, \( 0 < \lambda < 1 \), \( \bar{x}_t \) is the average of the \( t \)th group data, \( z_t \) is the weighted average for the past data, the initial value of \( z_0 = \bar{x} \), or an estimate of the mean can be given. The main advantage of the WMAF when compared to the SMAF is that the increased sensitivity based on a weighted constant. This allows for more control over the algorithm so it can be finely tuned for a particular application, such as has been done in this research for radiographic edge detection. An example of a WMAF applied to raw signal data is provided in Figure 2.14.
Figure 2.14 displays raw data (without WMA) and the WMAF weighted based on the minimum amplitude. The minimum amplitude is captured in the WMAF instead of being an outlier. A similar method for weighting is applied in this research to produce a low-mean averaged weighted scheme. A threshold will be applied to the WMAF to detect outliers where possible discontinuities or edges would theoretically be located.

### 2.2.2.4 Probability and Statistics

A Normal or Gaussian distribution is an extremely important continuous probability distribution. The probability density function of the Gaussian distribution is [60]:

![Figure 2.14: WMAF applied [59]](image_url)
First, if the random variable \( x \) has the normal distribution then it can be any finite value. There are two parameters of the distribution: \( \mu \) is the mean and can take on any finite value, \( \sigma \) is the standard deviation and is a positive value. An example of a normal distribution is provided in Figure 2.15.

![Gaussian distribution and threshold](image)

**Figure 2.15:** Gaussian distribution and threshold [61]

The distribution is symmetric about \( \mu \), so it is not only the mean, but it is also the median of the Gaussian distribution. The thresholds on the edges of the distribution are a function of the standard deviation [60],

\[
f(x) = \frac{1}{\sqrt{2\pi \sigma}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}
\]
\[ p = \mu + k \times \sigma \]  \hspace{1cm} (2.7)

Where \( k \) is a positive constant dependent on the number of deviations from the mean. Table 2.4 provides a percentage of the Gaussian distribution that is capture for each standard deviation of the mean.

**Table 2.4:** Standard deviations related to percentage of captured distribution [60]

<table>
<thead>
<tr>
<th>Standard deviations from mean</th>
<th>Percentage of distribution captured</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>68%</td>
</tr>
<tr>
<td>2</td>
<td>95%</td>
</tr>
<tr>
<td>3</td>
<td>99.7%</td>
</tr>
</tbody>
</table>

There are an infinite number of different Gaussian distributions corresponding to the different values for \( \mu \) and \( \sigma \). The Gaussian distribution can be applied to statistical data to predict outliers as shown in Figure 2.16.

**Figure 2.16:** Gaussian threshold applied to statistical data [62]
In this case, the distribution acts as a threshold for WMA filtered data. Once the WMA is determined a Gaussian threshold is applied to detect outliers of the data. In Figure 2.16, the unfiltered data is represented by the individual points, the WMA is represented by the solid line, the Gaussian threshold is represented by the dashed line, and the WMA outliers are represented by the individual red points. This is still a novel concept to be applied as an edge detection method for radiographic images.

2.3 Ablation Heat Transfer Modeling

This section is a review of the literature relevant to the ablative heat transfer modeling related to solid ablative materials. It will cover topics such as overall processes, analytic models, and computation models as it pertains to the characterization of the ablative boundary in this research.

2.3.1 Overall Processes

Ablation is the physicochemical transformation of a solid substance due to momentum and thermal boundary layers. The fundamental mechanisms associated with ablation are:

- Convection heat transfer from the thermal boundary layer.
- Radiation heat transfer from the thermal boundary layer.
- Mechanical erosion of the virgin material.
- Conduction heat transfer in the virgin material.

Figure 2.17 illustrates the overall process and mechanisms involved with ablation heat transfer.
A high temperature thermal boundary layer heats the surface of an ablative material through convection and radiation heat transfer. As the surface heats up, endothermic chemical reactions take place at the wall generating gaseous products that are injected into the momentum boundary layer resulting in concentration gradients. The concentration gradients are associated with a net heat flux due to species diffusion. The heat flux toward the surface is partly conducted inside the material and partly re-radiated from the hot surface [64]. The injection of the ablation products is known as gas blowing and reduces the temperature of the boundary layer which results in a reduction in convection heat transfer to the wall. If there is enough gas blowing of the ablation products, then the high temperature gas will be cooled to a desired temperature.

The ablative process uses mechanical erosion as a sacrificial method for thermal protection. Mechanical erosion (spallation) takes place when the frictional forces at the
ablative surface overcome structural limits [63]. In this research, mechanical erosion is captured with real-time radiography and a newly developed edge detection algorithm.

As the high temperature boundary layer heats up the surface of the ablative material, the ablative material distributes the heat load throughout the virgin material due to conduction heat transfer. Since the ablating materials usually have very low thermal conductivity, it is reasonable to assume that the ablation occurs in a semi-infinite body [71]. This assumption is valid only if the ablative material is thick enough to keep the temperature on the face opposite of the combustion products at ambient. It is critical to size the ablative thickness and material to account for the ablation time. The lower the thermal conductivity the more cooling potential an ablative coolant system has.

The mechanisms described in this section are explored through analytic and computational models, in the next section, to gain further insight into the heat and mass transfer phenomena that takes place during the ablation process.

### 2.3.2 Analytic Models

Analytic models in literature provide deep insight into the mechanisms that drive ablation. An analytic model proposed by Mills and Coimbra in 2015 presents a foundation for predicting surface heat flux rates of an ablating surface [72]. The authors describe the surface heat flux to be a function of the mass flux leaving the surface, the latent heat of fusion of the ablative material and the sensible heat of the ablative material [72],

\[
q_s = \dot{m}_f \left[ l_f s + C_{p,f} (T_s - T_o) \right]
\]  

(2.8)
where $\dot{m}_f''$ is the fuel mass flux, $q_s$ is the heat transfer to the surface per unit area, $i_{fs}$ is the enthalpy required for sublimation, $C_{p,f}$ is the specific heat of the fuel, $T_s$ is the surface temperature, and $T_o$ is the initial temperature. A schematic of the surface is provided,

![Figure 2.18: Ablative melting analytic model [72]](image)

A study conducted in 2015 expanded upon this concept to characterize the surface heat flux as a function of fuel and oxidizer mass flux leaving the surface of a hybrid rocket fuel [73],

$$q_s = \dot{m}_f'' \left( \Delta h_{f,dec} + (h_{f,e} - h_{f,a}) + \Delta h_{f,melt} + (h_{f,m} - h_{f,o}) \right)$$

$$+ \dot{m}_{ox}'' \left( h_{ox,s} - h_{ox,o} \right)$$

(2.9)

where $\Delta h_{f,dec}$ is the enthalpy required to decompose the fuel from a liquid to a vapor (equivalent to the latent heat of vaporization if no decomposition is required) and $\Delta h_{f,melt}$ is the latent heat of fusion of the fuel. A schematic of the new surface model is provided,
Figure 2.19: Ablative model for oxidizer and fuel [73]

A model similar to both models discussed in this section will be used to describe surface heat flux for a polymer ablative surface.

2.3.2.1 Nusselt Number Review

The Nusselt number is the ratio of convective to conductive heat transfer across a boundary [74]. A Nusselt number for an ablative surface in literature has not been well defined because it is such a novel concept. This research aims to define a method for determining Nusselt number for internal flow through an ablative pipe using the ablative model described in previous sections. Table A.1 in Appendix A provides a list of proposed formulas for analytically determining a Nusselt number based on material and flow properties. In the early 1930s, the Dittus-Boelter equation was proposed for determining the Nusselt number as function of the Reynolds number and the Prandtl number [75]. The Reynolds number is defined as a ratio of the inertia forces to the viscous or friction forces [76]. The Prandtl number is defined as the ratio of momentum diffusivity to thermal diffusivity [76].
Later, in the 1970s, a new equation was proposed by Gnielinski to determine the Nusselt number as a function of Reynolds number, Prandtl number and an additional friction factor term which is used to describe the roughness of the pipe material [75]. Afterwards in the late 1970s, Jackson and Hall proposed calculating the Nusselt number with the Reynolds number, Prandtl number, and fluid and surface material density and specific heat [77].

Subsequently, researchers have been deriving new ways to describe the Nusselt number based on a set of parameters [78]-[82]. However, even in recent studies there have been no attempts to characterize the Nusselt number for internal flow of an ablating pipe.

### 2.3.3 Computational Models

A dependable numerical method for computation of surface regression rate, mass loss, surface ablation heat flux, and internal combustion product temperature time histories is crucial for the development and design of SPWGG coolant materials. The latest computational fluid dynamics (CFD) software has advanced the field of non-equilibrium, multiphase, and multidimensional Navier-Stokes abilities. However, the ability to accurately model surface boundary conditions is deficient and cannot be used to model realistic scenarios [63], [83]. This research focuses on using RTR to capture the ablative boundaries, thereby enhancing ablative computational models’ ability to model a realistic scenario.

CFD models are used to numerically solve the Navier-Stokes conservation equations. Some numerical methods solve the set of equations directly while others solve the equations using a Large-Eddy Simulation (LES). This section will expand upon both concepts; however, this research mainly focuses on solving the equations directly.
2.3.3.1 Direct Numerical Solution

A direct numerical solution is a simulation that solves the whole range of spatial and temporal scales for the set of Navier-Stokes equations without a turbulence model [84]. Typically, a Reynolds-averaged Navier-Stokes (RANS) decomposition is used to describe turbulence in a direct numerical solution [85]. In other research, turbulent flows, are described with the Favre-averaged Navier-Stokes equations, where time-averaged effects of the flow turbulence on the flow parameters are considered, whereas the other, i.e. large-scale, time-dependent phenomena are considered directly [86]. Through this procedure, extra terms known as the Reynolds stresses appear in the equations for which additional information must be provided [86]. To close the system of equations, this research employs transport equations for the turbulent kinetic energy and its dissipation rate, the so-called $k - \epsilon$ model [86].

2.3.3.2 Large-Eddy Solution

A Large-Eddy Simulation (LES) is another method for characterizing turbulence in a fluid flow. In this case, the Navier–Stokes equations are spatially filtered, the resolved scales of motion are directly computed, and the influence of the filtered scales on the resolved scales is modeled [87]. Although LES is a more computationally expensive technique than RANS, it offers two significant advantages. First, the large-scale motion of the turbulent fluid that contains most of the turbulent kinetic energy and controls the dynamics of the turbulence is resolved, and hence computed directly [87]. Second, knowledge of the large-scale dynamics and the assumption that an applied model should be valid independently of
the filter size leads to the formulation of the so-called dynamic models, where model coefficients are determined as part of the solution [88].

2.4 Summary and Assessment of Literature

In this section, a summary and assessment of the literature provided in this chapter is presented. In light of the constraints of this research problem, specifically SPWGG DACS, the summary is categorized into three sections: SPWGG, regression rate measurement, and ablation heat transfer summary and assessments. Each section described the significance, relation, and knowledge advancement that is benefited from this research.

2.4.1 SPWGG Summary and Assessment

In the SPWGG gas cooling configuration section, multiple configurations are illustrated and described. The configurations illustrated and described are shell and tube heat exchanger, regenerative cooling, solid-liquid or gas hybrid, solid-solid hybrid, and an ablative solid-solid hybrid. In addition, this research illustrated and described multiple ablative solid-solid hybrid configurations such as: hollow cylinder and packed bed configurations.

Furthermore, optimal coolant gas properties and materials are explored in this research. A polymer with no additive is used throughout this research. Ideally, the polymer would be casted with a coolant additive that would increase the F/O of the combustion products. An increased F/O ratio would significantly reduce the flame temperature of combustion.
Advantages and disadvantages are presented for each configuration in this research. The ablative solid-solid hybrid gas cooling configuration has the potential to have the most mass, volume and cost reduction of all of the configurations observed. Ablative hollow cylinders are more reliable and provide less obstruction to the flow which can cause unforeseen complexities than that of packed beds. Table 2.5 provides a summary of the concepts described and the corresponding advantages and disadvantages for each configuration.

**Table 2.5: SPWGG configurations summary**

<table>
<thead>
<tr>
<th>Concept</th>
<th>Advantage</th>
<th>Disadvantage</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shell and Tube Heat Exchanger</td>
<td>Structural integrity, high flow rates</td>
<td>Large volume, mass, cost</td>
<td>12, 13, 14</td>
</tr>
<tr>
<td>Solid-Liquid or Gas Hybrid</td>
<td>Mixing and cooling effects</td>
<td>Large volume, mass, cost</td>
<td>15, 16</td>
</tr>
<tr>
<td>Solid-Solid Hybrid</td>
<td>Reduced volume, mass, cost</td>
<td>Reduced cooling effects</td>
<td>17</td>
</tr>
<tr>
<td>Ablative Solid-Solid Hybrid Hollow Cylinder</td>
<td>Reduced volume, mass, cost</td>
<td>Reduced mixing effects</td>
<td>1, 6, 18, 19, 21</td>
</tr>
<tr>
<td>Ablative Solid-Solid Hybrid Packed Bed</td>
<td>Reduced volume, mass, cost, increase surface area and mixing</td>
<td>Reduced structural integrity, clogging</td>
<td>22</td>
</tr>
</tbody>
</table>

The ablative solid-solid hybrid hollow cylinder is selected for this research due to its significant advantages over the other configurations.

### 2.4.2 Regression Rate Measurement Summary and Assessment

In the regression rate measurement section, methods for acquiring regression rate of a regressing surface with RTR is illustrated and described. In previous literature, RTR has
been used for measuring burning rates of solid and hybrid rockets, propagating cracks and accumulation of slag. In this research, RTR is used for measuring the regression rate of an ablating polymer in a SPWGG ablative solid-solid hybrid hollow cylinder configuration. In previous literature, this has only been attempted for flat slabs, not hollow cylinder configurations.

In addition, a new surface tracking algorithm is developed. In previous literature, hand selection and commercial software have proven to be time consuming and costly and cannot quantify the error of the measurement. This research aims to decreases the expense, decrease the labor intensity of data interpretation, decrease the labor time, and quantify the accuracy and error of the regression rate measurement with a mathematical algorithm. Table 2.6 illustrates the advantages and disadvantages of each surface tracking method and provides the corresponding references.

**Table 2.6: RTR surface tracking techniques summary**

<table>
<thead>
<tr>
<th>Concept</th>
<th>Advantage</th>
<th>Disadvantage</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hand-Selection</td>
<td>Simple</td>
<td>Time consuming, unquantifiable error.</td>
<td>33, 39, 44, 45</td>
</tr>
<tr>
<td>Commercial Products</td>
<td>Works for regular images</td>
<td>Doesn’t work well for RTR images, unquantifiable error.</td>
<td>39</td>
</tr>
<tr>
<td>Moving Average with Gaussian Threshold</td>
<td>Works for RTR images, can be adjusted to specific experiment, quantifiable error.</td>
<td>Error is dependent on camera resolution</td>
<td>N/A</td>
</tr>
</tbody>
</table>
In this research, an advanced edge detection algorithm based on the maximum and minimum moving average slope values is developed to detect discontinuities in RTR videos. The discontinuity areas are related to the surface of the ablative material. Since RTR captures videos, the method enables the ablative surface to be captured over time. Therefore, a regression rate can be determined from the RTR videos by observing the surface recession. Surface recession rate will then be utilized to predict ablation heat transfer characteristics of the system.

2.4.3 Ablation Heat Transfer Modeling Summary and Assessment

The lack of both material data and the physics-based heat transfer formulations for computational modeling of hot gas ablative cooling necessitated the development of the RTR regression rate test and data reduction technique used in the current research. Therefore, the conceivable disadvantage of the SPWGG is the lack of information on ablative material regression rates and how effective selectable ablative materials perform at cooling the high temperature combustion products. This research aims to increase the amount of heat transfer information available for ablative polymer coolant mediums in a SPWGG DAC configuration.

**Table 2.7: Ablation heat transfer modeling summary**

<table>
<thead>
<tr>
<th>Concept</th>
<th>Type</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-porous</td>
<td>Analytical</td>
<td>68</td>
</tr>
<tr>
<td>Porous</td>
<td>Analytical</td>
<td>69</td>
</tr>
</tbody>
</table>

This research focuses primary on using the ablation heat transfer model for a non-porous compound. Fluid, surface and solid elements are expanded upon and will be used to predict
surface ablation heat flux rates and warm gas temperatures from RTR measured regression rates. This will provide a detailed database on ablation characteristics for SPWGG DACS applications.
CHAPTER 3

APPROACH

This chapter provides a detailed description of the SPWGG apparatus, the equipment used to conduct each test, and the edge detection algorithm used to track the ablating bore of the hollow coolant tube. In addition, this chapter provides a detailed description of the analytical method developed for solving for an ablation heat transfer coefficient through use of a surface energy balance. The description in this chapter lays out, step by step, the foundation for using RTR to characterize the ablation heat transfer attributes of a SPWGG polymer coolant tube.

3.1 SPWGG Experimental Design

This section provides a detailed description of the SPWGG apparatus and the equipment used to conduct each test. The SPWGG apparatus consists of three components: the solid propellant that drives the flow, a coolant tube, and a sample holder that holds the solid propellant and coolant tube in place. Solid rocket propellant is used to generate high temperature combustion products upon ignition. The high temperature combustion products travel through a polymer coolant tube that reduces the temperature of the combustion products due to ablation heat transfer. An illustration of the SPWGG used in this research is presented in Figure 3.1.
Figure 3.1: Diagram of mounted solid propellant and PMMA coolant tube in the aluminum sample holder

The bore diameter of the coolant tube and the combustion pressure are varied between each experiment in the test series to control the mass flux of the high temperature combustion gas produced from the solid propellant. The length of the coolant tube is constant for each experiment in the test series. An RTR system and a developed edge detection algorithm are utilized to observe and capture the surface characteristics of the polymer coolant tube.

3.1.1 Test Matrix

A test series of twelve experiments was devised to simulate different hot gas mass fluxes flowing through the cylindrical PMMA coolant bed. The hot gas mass flux is proportional to combustion chamber pressure and the bore diameter of the coolant bed. The combustion chamber pressure affects the propellant burn rate as seen in St. Roberts Law,

\[ \dot{r}_{prop} = a_o P_c^n \]  \hspace{1cm} (3.1)

where \( \dot{r}_{prop} \) is the average regression rate of the propellant, \( a_o \) is the temperature coefficient, \( P_c \) is the chamber pressure, and \( n \) is the pressure exponent or combustion index. The
coefficient \( a_o \) and the exponent \( n \) are propellant-based constants. Increasing the combustion pressure increases the burn rate, and therefore the mass flow rate of the solid propellant high temperature combustion gases. The mass flow rate of the solid propellant hot gas is,

\[
\dot{m}_{prop} = \rho_{prop}A_b\dot{r}_{prop}
\]

(3.2)

where \( \dot{m}_{prop} \) is the mass flow rate of the high temperature combustion products, \( \rho_{prop} \) is the density of the solid rocket propellant, \( A_b \) is the burn surface area of the solid rocket propellant, and \( \dot{r}_{prop} \) is the average regression rate of the propellant. The mass flux rate of the solid propellant high temperature combustion products is,

\[
\dot{m}_{prop}'' = \dot{m}_{prop}A_{bore}(3.3)
\]

where \( \dot{m}_{prop}'' \) is the mass flux rate of the high temperature combustion products, \( \dot{m}_{prop} \) is the mass flow rate of the high temperature combustion products, and \( A_{bore} \) is the cross-sectional area of the ablative coolant tube internal bore. The initial Reynolds number is calculated from,

\[
Re = \frac{\dot{m}_{prop}''D_{bore,initial}}{\mu_{prop}}
\]

(3.4)

where \( \dot{m}_{prop}'' \) is the mass flux rate of the high temperature combustion products, \( D_{bore,initial} \) is the initial bore diameter of the ablative coolant tube, and \( \mu_{prop} \) is the viscosity of the high temperature combustion products. The Reynolds number is useful in determining whether
the flow is laminar or turbulent. The predicted burn time, at constant pressure, of the solid driving propellant is determined from,

$$ t_b = \frac{L_{prop}}{\dot{r}_{prop}} $$  \hspace{1cm} (3.5)

where $t_b$ is the burn time of the solid driving propellant, $L_{prop}$ is the initial length of the solid driving propellant, and $\dot{r}_{prop}$ is the burn rate of the solid driving propellant.

Table 3.1 represents the test matrix of initial theoretical values used in this research. These values are compared to measured values to assess measurement accuracy. The goal is to control the conditions to cover a range of average flux and pressure levels. Since the pressure also influences the hot gas mass flux some care must be taken in the design. The test matrix allows control of the pressure level from 2,069 to 4,137 kPa and initial mass flux of the hot gas from 101 to 531 kg/m$^2$-s. All Reynolds numbers are greater than the turbulent threshold of 2,000 for pipe flow, therefore the flow through the tube is turbulence in all experiments of the test series. These ranges are representative of operating pressures for warm gas systems. The mass flux levels of the hot gas are representative of the flux levels through a packed bed of spheres with a packing ratio of 0.32-0.39. This research provides a foundation for expected results of ablative packed beds through use of a hollow cylinder coolant tube.
Table 3.1: Test matrix

<table>
<thead>
<tr>
<th>Test #</th>
<th>Desired Pressure, kPa</th>
<th>Propellant Mass Flow Rate, g/s</th>
<th>Initial Coolant Bore Diameter, mm</th>
<th>Initial Propellant Mass Flux, kg/m²-s</th>
<th>Re</th>
<th>Burn Time, sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3,103</td>
<td>3.75</td>
<td>4.775</td>
<td>209</td>
<td>11,315</td>
<td>5.07</td>
</tr>
<tr>
<td>2</td>
<td>4,137</td>
<td>4.21</td>
<td>3.175</td>
<td>531</td>
<td>19,115</td>
<td>4.54</td>
</tr>
<tr>
<td>3</td>
<td>2,069</td>
<td>3.19</td>
<td>6.350</td>
<td>101</td>
<td>7,272</td>
<td>5.92</td>
</tr>
<tr>
<td>4</td>
<td>3,103</td>
<td>3.75</td>
<td>4.775</td>
<td>209</td>
<td>11,315</td>
<td>5.07</td>
</tr>
<tr>
<td>5</td>
<td>2,069</td>
<td>3.19</td>
<td>3.175</td>
<td>403</td>
<td>14,507</td>
<td>5.92</td>
</tr>
<tr>
<td>6</td>
<td>4,137</td>
<td>4.21</td>
<td>6.350</td>
<td>133</td>
<td>9,575</td>
<td>4.54</td>
</tr>
<tr>
<td>7</td>
<td>2,069</td>
<td>3.19</td>
<td>3.175</td>
<td>403</td>
<td>14,507</td>
<td>5.92</td>
</tr>
<tr>
<td>8</td>
<td>4,137</td>
<td>4.21</td>
<td>6.350</td>
<td>133</td>
<td>9,575</td>
<td>4.54</td>
</tr>
<tr>
<td>9</td>
<td>3,103</td>
<td>3.75</td>
<td>4.775</td>
<td>209</td>
<td>11,315</td>
<td>5.07</td>
</tr>
<tr>
<td>10</td>
<td>3,103</td>
<td>3.75</td>
<td>4.775</td>
<td>209</td>
<td>11,315</td>
<td>5.07</td>
</tr>
<tr>
<td>11</td>
<td>4,137</td>
<td>4.21</td>
<td>3.175</td>
<td>531</td>
<td>19,115</td>
<td>4.54</td>
</tr>
<tr>
<td>12</td>
<td>2,069</td>
<td>3.19</td>
<td>6.350</td>
<td>101</td>
<td>7,272</td>
<td>5.92</td>
</tr>
</tbody>
</table>
3.1.2 Solid Rocket Propellant

In this research, the driving propellant is an Ammonium Perchlorate/Hydroxyl-Terminated Polybutadiene (AP/HTPB) non-metalized solid rocket propellant, which upon ignition, produces high temperature combustion products calculated to be 2700 K. AP/HTPB is a standard rocket propellant that is used in a variety of space and defense applications. The propellant is cast into a long cylinder and cut to the desired length. For all experiments in the test series the initial propellant mass and geometry is constant. An example of the epoxied propellant cut to proper length and mounted in a plexiglass holder is shown in Figure 3.2.

Figure 3.2: Plexiglass holder (left), epoxied propellant in Plexiglass holder (right)
Appendix B has detailed information on the propellant composition, predicted thermodynamic properties, manufacturing data, and the data used to determine the burning rate equation.

### 3.1.2 Ablative Coolant Tube

Once the high temperature combustion products have been generated, they travel through a Poly-Methyl Methacrylate (PMMA) coolant tube. As the high temperature combustion products travel through the PMMA coolant tube, the combustion products have a reduction in temperature due to the ablating surface of the PMMA coolant tube. An example of the PMMA coolant tube is provided in Figure 3.3.

![Figure 3.3: Milled PMMA coolant tube](image)

Appendix C has detailed information on the hollow cylinder PMMA coolant tube composition, predicted thermodynamic properties, and manufacturing data.
3.2 Laboratory Equipment

The following section describes the laboratory equipment that was used in this research. The laboratory equipment consists of a sample holder, combustion bomb, and radiography system.

3.2.1 High Pressure Laboratory

All experiments in the test series are conducted at the Johnson Research Center (JRC) High Pressure Laboratory (HPL). A floor plan of the laboratory is provided in Figure 3.4.

![Figure 3.4: Floor plan of JRC HPL](image)

In the HPL, there are two 12-inch-thick concrete reinforced test cells: Test Cell A and Test Cell B. Test Cell A is used for characterizing solid propellants with an ultrasonic measurement technique. Test Cell B is used for measuring regression rate using live
radiography. Both test cells are used in this research. Additionally, the HPL is equipped with a Digital Acquisition (DAQ) system, fume hood, assembly area, fabrication work bench, and pressurized Nitrogen (N₂) storage system.

3.2.2 Sample Holder

The sample holder serves two purposes. First, as a combustion chamber where the solid propellant can complete the reaction and contain the increase of pressure due to combustion. Second, as a mount for the solid propellant and PMMA coolant tube. An image of the aluminum combustion chamber is provided in Figure 3.5.

![Milled aluminum sample holder](image)

**Figure 3.5:** Milled aluminum sample holder

The solid propellant is mounted on one end, and on the other end the PMMA coolant tube is mounted, so that the aluminum engulfs the entire propellant and PMMA coolant tube bodies. Both are caulked into place with high pressure caulk. There is a gap between the solid
propellant and PMMA coolant tube that acts as a combustion chamber. In this configuration, the front face of the PMMA and the bore of the PMMA are the surfaces that are exposed to the high temperature combustion products. An illustration of the sample holder with mounted propellant and PMMA coolant tube is shown in Figure 3.1. Appendix D has detailed information on the aluminum sample holder geometric dimensions and manufacturing data.

### 3.2.3 Combustion Bomb

A copper combustion bomb completely encloses the aluminum sample holder with mounted propellant and PMMA coolant tube samples. An example of the configuration is displayed in Figure 3.6.

![Figure 3.6: Copper combustion bomb](image)
The combustion bomb has various ports on the outside that enables the ability to insert thermocouples and pressure transducers to monitor the temperature and pressure of the system as combustion takes place. Additionally, the combustion bomb is equipped with ports for inserting ignition leads from a battery that is used to ignite the solid propellant. The combustion bomb is pressurized with gaseous nitrogen as an inert gas. The operator sets the initial pressure for each experiment in the test series. The combustion bomb is connected to a surge tank so that the combustion bomb pressure only increases a few percent during the test. Appendix E provides a detailed schematic of the pressurization system.

The walls of the copper combustion bomb are thick enough to absorb any heat from the combustion process. The combustion bomb is equipped with two, 1-inch thick, circular, translucent plexiglass windows that enable the ability to use RTR to view the insides of the combustion bomb. This is due plexiglass having a lower attenuation compared to copper. The aluminum sample holder is placed inside of the combustion bomb level with the two plexiglass windows, allowing for x-rays to pass through the sample holder containing the mounted propellant and PMMA coolant tube.

3.2.4 Radiography system

In this research, the radiography system consists of three main components: the source, the image intensifier, and the camera. All the components of the radiography system and the combustion bomb are placed in Test Cell B in the JRC HPL capable of containing any radiation from the radiography system. This system acts as a non-destructive observation
mechanism for viewing the surface regression characteristics of the PMMA coolant tube. A schematic of the radiography system is illustrated in Figure 3.7.

![Real-time radiography system setup](image)

**Figure 3.7:** Real-time radiography system setup

The x-rays traverse through the first plexiglass window, then the sample holder with mounted propellant and PMMA coolant tube and leave through the second plexiglass window where they are transmitted to an image intensifier and camera. A computer outside of the bunker records using Basler’s video recording software and stores the RTR videos for later analysis on an encrypted drive.

### 3.2.4.1 X-Ray Source

The x-ray source used in all experiments in the test series is an ERESCO MF4 x-ray generator that is attached to a four-legged stand. It produces an x-ray beam in an elliptical shape with dimensions of 30° x 60°. A control unit is used to set the x-ray tube current and
voltage. The operating range for the ERESCO MF4 is 0.5-4.5 mA and 10-280kV. An example of the x-ray source and control unit is provided in Figure 3.8.

![Figure 3.8: X-Ray source](image)

The x-ray source is leveled and placed 17 inches from the center of the combustion bomb, allowing for maximum focus and resolution in the RTR image. After calibration, the optimal setting for the source was 3.0 mA at 75kV. Ideally, the x-ray beam will travel through the combustion bomb and PMMA coolant tube, then be received by a digital receiver.
3.2.4.2 Image Intensifier and Camera

An image intensifier and camera setup are used in this research to capture RTR videos of the PMMA surface regression. The video images represent the transmitted intensity of the x-rays and provide information on the changing internal bore of the coolant tube. The image intensifier is a 4-inch Toshiba E5877J-P1 distributed by North American Imaging. The camera is a Basler Ace acA2040-120um distributed by Edmonds Optics. A C-mount is utilized as a mounting bracket between the image intensifier and camera. The image intensifier with attached camera is shown in Figure 3.9.

Figure 3.9: Image intensifier and camera

Table 3.2 represents the corresponding resolution and framerate of both the image intensifier and the camera.
Table 3.2: Resolution of image intensifier and camera

<table>
<thead>
<tr>
<th>Component</th>
<th>Resolution</th>
<th>Framerate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Image Intensifier</td>
<td>77 Lp/cm</td>
<td>N/a</td>
</tr>
<tr>
<td>Camera</td>
<td>3.0 MP</td>
<td>120 fps</td>
</tr>
</tbody>
</table>

The image intensifier and camera are placed 17 inches from the center of the combustion bomb and 34 inches from the x-ray source.

3.3 Edge Detection Algorithm Overview

In this section, the edge detection algorithm developed from this research is described in detail. The edge detection algorithm requires three main steps: image rendering of each frame, determination of a weighted moving average of pixel intensity values, and identification of the maximum and minimum moving average slope values. With these three steps the inner surface of the PMMA coolant tube captured in the RTR videos is tracked as a function of time and axial location for each experiment in the test series. An accompanying probability and confidence level are determined with each edge that is detected allowing for measurement accuracy and error to be completely quantified.

3.3.1 Image Rendering

The first step in the edge detection algorithm is to render each frame in the x-ray video into operable images. This requires separating the x-ray video into individual frames, using a frame averaging technique to reduce image noise, and then adjusting the contrast and brightness of each image. ImageJ is an open source digital processing software created
and distributed by NIST, that is used to accomplish the image rendering step in this research. It can display, edit, analyze, process, save and print 8-bit, 16-bit and 32-bit images in formats including TIFF, GIF, JPEG, BMP, DICOM, FITS and "raw". For this research the RTR frames are saved after adjustment as TIFFs due to the lossless compression option. Saving the RTR frames as TIFF images provides the highest quality image for each frame which increases the accuracy of the edge detection algorithm. Figure 3.10 displays an RTR video frame of the experimental apparatus pre and post rendering.

![Figure 3.10: Pre (left) and post (right) image rendering](image)

The coolant sample and bore are shown in the middle of the frame. The bolts that hold the solid propellant, sample holder, and PMMA tube together are on the left and right. The entire view is framed by the circular plexiglass windows of the combustion chamber. The increase in image quality, in particular the bore surface of the PMMA coolant tube is more distinguishable in the post image rendering figure than it is in the pre-image rendering figure. A frame averaging scheme that averages three frames into one frame is used on the entire RTR video to reduce noise. After applying the frame averaging scheme, the camera
capture speed is reduced from 120fps to 40fps, which is still accepted for this research. The next process in the image rendering step is to adjust the contrast and brightness of each frame averaged image. Table 3.3 provides the image contrast and brightness setting used for every frame averaged image in each experiment of the test series. Meaning the values of contrast in Table 3.3 do not change.

Table 3.3: Image rendering settings

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Contrast</td>
<td>150%</td>
</tr>
<tr>
<td>Brightness</td>
<td>125%</td>
</tr>
</tbody>
</table>

This setting is used for all of the frame averaged images in each experiment of the test series to provide repeatability for the edge detection algorithm. It is ideal to have the highest color intensity difference in a plane where an edge will be tracked. The values of contrast and brightness in this research are set to a value that provides a large transmitted intensity difference between the plexiglass material and the bore of the plexiglass tube.

### 3.3.2 Moving Average Spatial Filter

Once the frames of the radiography video have been successfully rendered into operable images, pixel color intensities, based on a 32-bit color scheme, are mapped based on an x-y coordinate system for the entire image. Afterwards, a moving average filter and weighting scheme can be applied to the raw pixel intensity data. Detecting edges requires applying a moving average filter perpendicular to the edge surface. For this research, the moving average filter is applied in the x-direction because the tube bore surface is vertical.
(y-direction). A MATLAB code, developed from this research, is used to capture pixel intensities, and apply a moving average filter in the x-direction. Figure 3.11 is an illustration of an image rendered frame and the corresponding flow direction.

![Figure 3.11: Example of image rendered frame, flow direction, and plane of interest of the coolant tube bore](image)

In Figure 3.11, a red line represents a plane of interest. Notice the line travels in the x-direction and crosses both edges of the inner bore diameter. This is just a single plane for example purposes. In this research, the entire body is analyzed to characterize the entire surface of the inner bore diameter. The moving average filter is calculated based on,

\[
\bar{x}(\tau) = \frac{1}{T_w} \int_{\tau + T_w/2}^{\tau + T_w} x(\tau) d\tau
\]  

(3.6)
where $T_w$ is the moving average filter length. The moving average filter length is determined to be a value that does not significantly change the overall trajectory of the raw intensity data. For this research, a moving average filter length of twelve pixels is used to reduce noise but not alter the trends of the raw intensity data. The moving average in this research is determined by averaging six pixels to the left with six pixels to the right. Figure 3.12 displays the complete image rendering process and moving average spatial filter applied to the transmitted intensity profile of the plane of interest from Figure 3.11.

**Figure 3.12:** Unaltered transmitted intensity (top-left), 3-frame averaged transmitted intensity (top-right), contrast and brightness adjustment (bottom-left), moving average spatial filter (bottom-right) for the plane of interest
In Figure 3.12, the unaltered transmitted intensity profile for the plane of interest is displayed in the top-left graph, the three-frame averaged transmitted intensity profile for the plane of interest is displayed in the top-right graph, the contrast and brightness adjustment with three frame averaged transmitted intensity profile for the plane of interest is displayed in the bottom-left graph, and the moving average spatial filter applied to the image rendered transmitted intensity profile is displayed in the bottom-right graph. The noise in the unaltered transmitted intensity profile significantly reduces the possibility to detect an edge. The noise is significantly reduced in the three-frame averaged transmitted intensity profile when compared to the unaltered transmitted intensity profile.

The contrast and brightness adjustment provide a greater difference in intensity of the bore and coolant material. This leads to a large slope change in the moving average spatial filter, making it much easier to relate the transmitted intensity to an edge location. The moving average spatial filter significantly reduces the noise in the transmitted intensity data. The moving average has similar trajectory to the transmitted intensity data as it remains mean to the signal at all points, therefore it is an acceptable moving average length. The same parameter values for the three-frame averaging, contrast, brightness, and moving average spatial filter length are applied to every location along the axis of the coolant bore in each experiment of the test series. Additionally, the same parameter values are used for each time step in each experiment of the series. Therefore, repeatability is maintained throughout the test series.

Predicted intensity values for a hollow cylinder can be determined from Equation 2.2 and material from Appendix F. The predicted intensity values are determined by using geometric relations to find the total length of material a ray travels through, for each section
of the hollow cylinder. Local time dependent predicted (top) and transmitted (bottom) intensity values vs x-pixel location is shown in Figure 3.13.

Figure 3.13: Time dependent predicted (top) and transmitted (bottom) intensity values vs. x-pixel position
In Figure 3.13, the cylinder bore as a function of time and pixel position for the plane of interest is represented in bottom graphs by the arching lines. The arches expand depicting the ablation process expanding the coolant bore. The lines on both sides of the arches are the cylinder material. In the predicted intensity graph, the coolant material intensity profile has a positive slope. In the transmitted intensity graph, the coolant material intensity profile has a negative slope. This is because the predicted intensity model does not account for the flat plexiglass windows or the plastic blast shield that the x-ray pass through before passing through the coolant sample. It is important in that it reduces the level of the transmitted intensity and probably reduces the signal to noise ratio.

### 3.3.3 Moving Average Slope and Edge Location

In the final step of the edge detection algorithm developed from this research, the slope of the moving average spatial filter is used to detect and track the surface edge location of the coolant bore. This is accomplished by determining the maximum and minimum values of the slope of the moving average spatial filter at each location along the coolant tube axis. When the slope is maximum or minimum there is a large change in the moving average spatial filter. Since the contrast and brightness were adjusted during the image rendering step, the difference in transmitted intensity between the bore and the material will be enhanced. A large intensity change will correspond to a large moving average spatial filter change. This means the edge location can be determined by finding the maximum and minimum of the slope of the moving average spatial filter. Figure 3.14 illustrates the slope of the moving average spatial filter applied transmitted intensity profile from the plane of interest in Figure 3.11 as a function of pixel position. Edge location measurements are also
shown in Figure 3.14 as the blue dots, where the slope of the moving average spatial filter is maximum and minimum.

![Figure 3.14: Example of moving average slope and edge locations](image)

where the black line represents the moving average slope, and the blue dots represent the maximum and minimum moving average slope values which correspond to the surface edge location. The edge locations are accompanied with a set of error bars which is determined from the uncertainty analysis. A Monte Carlo simulation is developed to predict the uncertainty of the edge measurement and is discussed in detail more in Appendix H.

Figure 3.15 provides further illustration of the edge location in relation to the transmitted intensity profile and moving average spatial filter.
where the image rendered transmitted intensity profile is represented by the black line, the moving average spatial filter is represented by the red line, and the edge location is represented by the blue dots.

This set of parameters and procedure is used for each location along the axis of the coolant bore. There are more than one-hundred intensity profiles for each experiment and over 10,000 sample points from the entire test series. Therefore, a MATLAB program was developed to analyze the edge for each intensity profile to reduce the labor intensity and time required. These abundance of data points increases the amount of knowledge that can be obtained from a single test series.

Figure 3.15: Intensity profile and edge location
3.3.4 Edge-to-Diameter Conversion

In this section, the conversion from the two edge pixel locations to the coolant bore diameter measurement is presented. In the previous sections, a detailed discussion has been provided on the methodology used to track the coolant bore surface as a function of time and location.

The coolant bore diameter as a function of the edge location is represented as,

\[ D_{\text{bore}} = (E_{\text{right},i} - E_{\text{left},i})\varphi \]  \hspace{1cm} (3.7)

where \( D_{\text{bore}} \) is the diameter of the bore, \( E_{\text{right},i} \) is the right edge location of the intensity profile, \( E_{\text{left},i} \) is the left edge location of the intensity profile, and \( \varphi \) is a scale factor. The scale factor is based on the initial edge locations and the known initial diameter,

\[ \varphi = \frac{D_{\text{known}}}{E_{\text{right},0} - E_{\text{left},0}} \]  \hspace{1cm} (3.8)

where \( D_{\text{known}} \) is the mean initial coolant bore diameter, \( E_{\text{right},0} \) is the initial right edge location of the intensity profile, \( E_{\text{left},0} \) is the initial left edge location of the intensity profile. The initial coolant bore diameter is based on a known drill bit size that is used to drill the coolant bore.

In Table 3.4, the average initial pixel distance between the bore edges, the initial bore diameter, and the scale factor that were used for each experiment of the test series is provided.
### Table 3.4: Edge-to-Diameter Conversion Summary

<table>
<thead>
<tr>
<th>Test #</th>
<th>Average Initial ΔPixels Between Bore Edges</th>
<th>Initial Bore Diameter, mm</th>
<th>( \phi ), Pixels/mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>1)</td>
<td>78</td>
<td>4.775</td>
<td>16.33</td>
</tr>
<tr>
<td>2)</td>
<td>51</td>
<td>3.175</td>
<td>16.06</td>
</tr>
<tr>
<td>3)</td>
<td>97</td>
<td>6.350</td>
<td>15.27</td>
</tr>
<tr>
<td>4)</td>
<td>78</td>
<td>4.775</td>
<td>16.33</td>
</tr>
<tr>
<td>5)</td>
<td>53</td>
<td>3.175</td>
<td>16.69</td>
</tr>
<tr>
<td>6)</td>
<td>93</td>
<td>6.350</td>
<td>14.64</td>
</tr>
<tr>
<td>7)</td>
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<td>3.175</td>
<td>16.69</td>
</tr>
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<td>8)</td>
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</tr>
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<td>80</td>
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<tr>
<td>11)</td>
<td>52</td>
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<td>16.38</td>
</tr>
<tr>
<td>12)</td>
<td>100</td>
<td>6.350</td>
<td>15.75</td>
</tr>
</tbody>
</table>

The scale factor remains constant throughout the entire test. The scale factor ranges between 14.64 and 16.75 pixels/mm for the setup used in this research. Ideally, larger scale factors will lead to higher quality measurements.

### 3.4 Ablative Heat Transfer Model

In this section, the model for predicting local time dependent ablative heat flux and heat transfer coefficient from a measured local time dependent surface regression rate is presented.

#### 3.4.1 Governing Equations

The governing equations for the ablative heat transfer model from this research are derived from the mass, momentum, and energy conservation equations. The fundamental
conservation equations are the basis of all heat and mass transfer analytical models. The conservation of mass or continuity equation is defined as,

\[
\frac{\partial}{\partial t} \int_{CV} \rho \partial V + \int_{CS} \rho V_{rel} \partial A = 0 \tag{3.9}
\]

where \( \rho \) is the material density, \( \partial V \) is the change in volume in the control volume, \( V_{rel} \) is the relative velocity, and \( \partial A \) is the change in area over the control surface.

The conservation of momentum equation is defined as,

\[
\frac{\partial}{\partial t} \int_{CV} \rho V \partial V + \int_{CS} \rho V V_{rel} \partial A + \int_{CS} P \partial A + \int_{CS} \tau \partial A = 0 \tag{3.10}
\]

where \( \rho \) is the material density, \( V \) is the velocity of the mass in the control volume or at a control surface, \( \partial V \) is the change in volume in the control volume, \( V_{rel} \) is the relative velocity at a control surface, \( \partial A \) is the change in area at a control surface, \( P \) is the pressure at a control surface, and \( \tau \) is the shear stress at a control surface.

The conservation of energy equation is defined as,

\[
\frac{\partial}{\partial t} \int_{CV} \rho i \partial V + \int_{CS} \rho i V_{rel} \partial A + \int_{CS} \dot{q}'' \partial A + \int_{CS} \dot{w}'' \partial A = 0 \tag{3.11}
\]

where \( \rho \) is the material density, \( i \) is the enthalpy of the material in the control volume or at a control surface, \( V_{rel} \) is the relative velocity at a control surface, \( \partial A \) is the change in area at a control surface, \( \dot{q}'' \) is the heat flux at a control surface, and \( \dot{w}'' \) is the work flux at a control surface. The list of assumptions for the modified ablative model include,
• Steady-state,
• Uniform ablation between grid points,
• Outside temperature of the sample is ambient,
• Uniform coolant medium density
• Uniform heat transfer to ablative surface,
• Interface temperatures are equivalent to phase change temperatures,
• No work is done at any control surface.

Applying the assumptions, the mass conservation equation simplifies to,

\[ \int_{CS} \rho V_{rel} \partial A = 0 \quad (3.12) \]

the momentum conservation equation simplifies to,

\[ \int_{CS} \rho V V_{rel} \partial A + \int_{CS} P \partial A + \int_{CS} \tau \partial A = 0 \quad (3.13) \]

and the energy conservation equation simplifies to,

\[ \int_{CS} \rho i V_{rel} \partial A + \int_{CS} \dot{q}'' \partial A = 0 \quad (3.14) \]

The conservation equations will be applied to discrete locations along the coolant tube axis at each time step to determine ablative heat transfer attributes of the ablative solid-solid hybrid SPWGG system.
3.4.2 Ablative Heat Flux Model

Three control volumes are used in conjunction with the conservation equations are used in the ablative heat flux model. Figure 3.16 illustrates an example of the three control volumes and their relative location to the internal coolant bore diameter.

![Diagram showing three control volumes: Fully Developed Region, Fluid Element CV, Ablative Surface CV, Ablative Solid CV with r = 0 and r = R.]

**Figure 3.16:** Three control volumes

The amount of heat dissipating into the ablative solid due to convection and radiation will be equal to the available latent heat and conductive potential of the ablative material. It is ideal to choose an ablative material with a large latent heat or melting temperature to increase the amount of available cooling energy. Since the hot gas flow is removing the ablative material, the cooling enthalpy potential will always be replenished (until the material runs out). Heat will only penetrate the solid as deep as the thermal penetration length. The rest of the material beyond that will have a negligible temperature gradient. The temperature of the subsurface profile will remain stationary relative to the ablative surface during the entire ablation process assuming steady-state ablation and that it will be "attached" to the surface. It is important to understand the cooling potential to predict a
physics based hot gas temperature profile along the axis of the PMMA cylinder and exit temperature to assess design efficiency.

### 3.4.2.1 Ablative Mass Flux

In this section, the method for using RTR measured diameter to determine surface velocity of the ablative material, coolant mass flux and mass flow rate, and warm gas mass flux and flow rate is described. After the RTR measurements have been completed the local time dependent diameter is known for each $i$th location along the axis of the coolant bore. The change in diameter with respect to time can be used to determine an ablation rate of the bore surface,

\[
V_{\text{coolant},i} = \frac{\Delta D_{\text{bore},i}}{2\Delta t} \tag{3.15}
\]

where $V_{\text{coolant},i}$ is the surface velocity of the ablative material at the $i$th position from the inlet of the coolant tube, $\Delta D_{\text{bore},i}$ is the change in bore diameter of the coolant tube with respect to time at the $i$th position from the inlet of the coolant tube, and $\Delta t$ is the change in time between the time steps.

The surface ablation velocity can be used to determine the ablation mass flux of the bore surface,

\[
\dot{m}_{\text{coolant},i}'' = (\rho V_i)_{\text{coolant}} \tag{3.16}
\]

where $\dot{m}_{\text{coolant},i}''$ is the mass flux of the ablative surface at the $i$th position from the inlet of the coolant tube, $\rho_{\text{coolant}}$ is the coolant material density and is assumed to be constant at
each position along the coolant bore, and $V_{\text{coolant},i}$ is the surface velocity of the ablative material at the $i$th position from the inlet of the coolant tube. The mass flow rate of the coolant can be expressed as,

$$m_{\text{coolant},i} = m''_{\text{coolant},i} \pi D_{\text{bore},i} L_{\text{coolant},i}$$  \hspace{1cm} (3.17)$$

Where $m_{\text{coolant},i}$ is the mass flow rate of the coolant at the $i$th position from the inlet of the coolant tube, $m''_{\text{coolant},i}$ is the mass flux of the ablative surface at the $i$th position from the inlet of the coolant tube, $D_{\text{bore},i}$ is the diameter of the coolant bore at the $i$th position from the inlet of the coolant tube, $L_{\text{coolant},i}$ is the length of the section of coolant tube that is at the $i$th position from the inlet of the coolant tube. The mass flow rate at each section along the axis of the coolant bore can be added in a cumulative manner to the propellant mass flow rate as it passes through the bore of the coolant tube. This effectively defines a mass flow rate of the warm gas, which is a mixture of propellant hot gas and coolant.

The warm gas mass flow rate is defined as,

$$m_{\text{warmgas},i} = \sum_{i=1}^{n} m_{\text{coolant},i} + m_{\text{prop}}$$  \hspace{1cm} (3.18)$$

where $m_{\text{warmgas},i}$ is the mass flow rate of the warm gas at the $i$th position from the inlet of the coolant tube, $m_{\text{coolant},i}$ is the coolant mass flow rate at the $i$th position from the inlet of the coolant tube, and $m_{\text{prop}}$ is the hot gas mass rate and is defined in Equation 3.2. This equation enforces mass conservation of the flow throughout the coolant tube.

Therefore, the mass flux of the warm gas can be expressed as,
\[ m_{\text{warm gas},i}'' = \frac{4m_{\text{warm gas},i}}{\pi D_{\text{bore},i}^2} \] (3.19)

where \( m_{\text{warm gas},i}'' \) is the mass flux of the warm gas at the \( i \)th position from the inlet of the coolant tube, \( m_{\text{warm gas},i} \) is the mass flow rate of the warm gas at the \( i \)th position from the inlet of the coolant tube, and \( D_{\text{bore},i} \) is the diameter of the coolant bore at the \( i \)th position from the inlet of the coolant tube.

Now that the mass has been conserved, the next step is to balance the energy fluxes at each \( i \)th position from the inlet of the coolant tube.

### 3.4.2.2 Ablative Surface

An energy control volume analysis of the surface (s-surface) to just below the surface (u-surface) of the \( i \)th section of the ablative material is shown in Figure 3.17. It is also assumed that the liquid melt is removed as fast as it is formed by a fully developed flow.

![Ablative Surface Control Volume](image)

**Figure 3.17**: Ablative surface control volume
After balancing all the energies of the control volume with the conservation of energy, the following equation is developed,

$$\dot{m}_{coolant}''(i_{fs}) + (q''_u - q''_s) = 0$$  \hspace{1cm} (3.20)

where,

$$\dot{m}_s'' = \dot{m}_u'' = \dot{m}_{coolant}''$$  \hspace{1cm} (3.21)

and,

$$i_{fs} = i_s - i_u$$  \hspace{1cm} (3.22)

where $\dot{m}_{coolant}''$ is the mass flux of ablation of the ablative material and is measured with real-time radiography as the change in bore diameter with respect to time multiplied by the material density, $i$ is the enthalpy at the surface denoted in the subscript, $i_{fs}$ is the latent heat of fusion, and $q''$ is the heat flux at the surface denoted by the subscript.

Equation 3.20 will be used in conjunction with the ablative solid control volume to establish a relation for the overall ablative heat flux cooling potential of the material.

### 3.4.2.3 Ablative Solid

Furthermore, applying the same conservation of energy process from the subsurface material (u-surface) to a position deep enough into the solid where the temperature gradient is negligible (o-surface), the result is a control volume that is shown in the Figure 3.18.
Figure 3.18: Ablative solid control volume

After balancing all the energies in the control volume and combining the resultant equation with Equation 3.20, from the ablative surface control volume, the ablative heat flux becomes,

\[ \dot{q}_{s,l}'' = \dot{m}_{coolant,l}''[i_f + C_p(T_{melt} - T_o)] \]  

(3.23)

where,

\[ \dot{m}_o'' = \dot{m}_u'' = \dot{m}_{coolant}'' \]  

(3.24)

where \( \dot{q}_{s}' \) is the ablative heat flux potential at the surface of the ablative material, \( \dot{m}_{c}'' \) is the mass flux of the ablative coolant material leaving the control volume, \( i_f \) is the latent heat of fusion of the ablative coolant material, \( C_p \) is the specific heat at constant pressure for the ablative coolant material, \( T_{melt} \) is the melting temperature of the ablative coolant material, and \( T_o \) is the outer ambient temperature of the ablative coolant material. These equations show the ablative heat flux cooling potential is proportional to the ablative mass flux and can be measured locally with real-time radiography. The terms in the brackets in Equation 3.23
are a combination of latent and sensible heat, which are a function of ablative material properties and ambient temperature.

### 3.4.2.4 Steady-State Ablation Assumption

A steady-state ablation assumption is utilized in this research. Figure 3.19 is an illustration that represents the ablative surface.

![Figure 3.19: Steady-state ablative control volume](image)

Applying the conservation of energy, results in the following equation,

\[
\dot{m}[i_{\xi+\Delta\xi} - i_{\xi}] = [\dot{q}''A]_{\xi} - [\dot{q}''A]_{\xi+\Delta\xi}
\]  

(3.25)

where \(\dot{m}\) is the mass flow rate of the ablative coolant material, \(i\) is the enthalpy at the surface denoted in the subscript, \(\dot{q}''\) is the heat flux at the surface denoted by the subscript, and \(A\) is the surface area at the surface denoted by the subscript.

Dividing by \(A\Delta\xi\) and letting \(\Delta\xi \to 0\),
\[ \rho V \frac{\partial i}{\partial \zeta} = - \frac{\partial q}{\partial \zeta} \]  

(3.26)

where \( \rho \) is the density of the ablative coolant material, \( V \) is the velocity of the ablative coolant material leaving the control volume, \( \frac{\partial i}{\partial \zeta} \) is the change in enthalpy with respect to distance, and \( \frac{\partial q}{\partial \zeta} \) is the change in heat flux with respect to distance. This first order ordinary differential equation can be solved with boundary conditions,

\[
q = -k \frac{dT}{d\zeta} \quad \text{and} \quad \frac{\partial i}{\partial \zeta} = C_p \frac{dT}{d\zeta}
\]

(3.27)

where \( q \) is the heat flux, \( k \) is the thermal conductivity of the ablative coolant material, \( \frac{dT}{d\zeta} \) is the change in temperature with respect to distance, \( \frac{\partial i}{\partial \zeta} \) is the change in enthalpy, \( C_p \) is the specific heat at constant pressure for the ablative coolant material, and \( \frac{dT}{d\zeta} \) is the change in temperature. Substituting it in the governing equation,

\[
\frac{d}{d\zeta} \left( k \frac{dT}{d\zeta} \right) - \rho V C_p \frac{dT}{d\zeta} = 0
\]

(3.28)

Further simplification,

\[
\frac{d^2 T}{d\zeta^2} - \frac{V}{\alpha} \frac{dT}{d\zeta} = 0
\]

(3.29)

where \( \alpha = k/\rho c \) is the thermal diffusivity. This second order partial differential equation can be solved for the temperature profile using the following boundary conditions,
\[ \zeta = 0: \quad T = T_s \]
\[ \zeta \to \infty: \quad T \to T_o \]  \hfill (3.30)

The final form of the temperature profile through the solid is,

\[ \frac{T - T_o}{T_s - T_o} = e^{\left( \frac{V}{a} \right) \zeta} \]  \hfill (3.31)

which the temperature change shows the exponential dependency on surface velocity, thermal diffusivity, and thickness of the moving boundary.

### 3.4.3 Ablative Heat Transfer Coefficient

Figure 3.20 shows the surface energy balance between the hot propellant gas and ablative PMMA.

![Ablative Heat Transfer Coefficient](image)

**Figure 3.20**: Ablative surface energy balance between hot gas and ablative solid

The top section of Figure 3.20 represents the warm gas mixture and includes a convection and radiation term. Heat is transferred from the warm gas mixture to the ablative surface...
through convection and radiation heat transfer. The bottom section of Figure 3.20 represents the ablative solid and includes a latent and sensible heat term. The amount of cooling potential available from the ablative material is a combination of latent and sensible heat.

The first step in developing an ablative heat transfer coefficient is to determine a Reynolds number of the flow that passes through the $i$th position from the inlet of the coolant tube bore,

$$Re_i = \frac{\dot{m}_{\text{warmgas},i} D_{\text{bore},i}}{\mu_{\text{warmgas}}}$$

(3.32)

where $Re$ is the Reynolds number at the $i$th position from the inlet of the coolant tube bore, $\dot{m}_{\text{warmgas},i}$ is the mass flux of the warm gas at the $i$th position from the inlet of the coolant tube and is defined in Equation 3.19, $D_{\text{bore},i}$ is the diameter of the coolant bore at the $i$th position from the inlet of the coolant tube, and $\mu_{\text{warmgas}}$ is the viscosity of the warm gas which is assumed to be constant throughout the coolant tube bore. The Reynolds number is used to determine the regime of the flow; laminar or turbulent [9]. Each experiment in the test series of this research exhibits large Reynolds numbers, indicating a turbulent regime throughout the coolant tube. The following relationship for the Nusselt number for fully developed turbulent pipe flow is defined as,

$$Nu_i = \alpha_i Re_i^n Pr^m$$

(3.33)

for the Nusselt number is defined for turbulent flow through a hollow cylinder [9, 10]. Where $Nu_i$ is the Nusselt number at the $i$th position from the inlet of the coolant tube, $Re_i$ is the
Reynolds number at the $i$th position from the inlet of the coolant tube, and $Pr$ is the Prandtl number of the warm gas which is assumed to be constant throughout the coolant tube bore. The coefficient $\alpha_i$ and exponents $n$ and $m$ typically are based on the flow regime. The coefficient $\alpha_i$ is a constant value of 0.023, and exponents $n$ and $m$ are constant values of 0.8 and 0.3, respectively. However, since the flow in this research is regarding the undeveloped entrance region, the Nusselt number is solved using a surface energy balance.

Using Figure 3.20 and the conservation of energy at the surface of the ablative material, the following relation can be defined as a balance between surface, convective and radiative heat flux,

$$\dot{q}''_{s,i} - \dot{q}''_{\text{conv},i} - \dot{q}''_{\text{rad},i} = 0 \quad (3.34)$$

where $\dot{q}''_{s}$ is the ablative surface heat flux potential based on latent and sensible heat at the $i$th position from the inlet of the coolant tube, $\dot{q}''_{\text{conv},i}$ is the convective heat flux at the $i$th position from the inlet of the coolant tube, and $\dot{q}''_{\text{rad},i}$ is the radiative heat flux at the $i$th position from the inlet of the coolant tube.

The convection heat flux is defined as,

$$\dot{q}''_{\text{conv},i} = Nu_i \frac{k_{\text{warm gas}}}{D_i}(T_{\text{warm gas},i} - T_{\text{melt}}) \quad (3.35)$$

where $\dot{q}''_{\text{conv},i}$ is the convection heat flux at the $i$th position from the inlet of the coolant tube, $Nu$ is the Nusselt number at the $i$th position from the inlet of the coolant tube, $k_{\text{warm gas}}$ is the conduction coefficient of the warm gas which is assumed to be constant throughout the coolant bore, $D_i$ is the diameter of the coolant tube bore at the $i$th position from the inlet of
the coolant tube, \( T_{\text{warm gas},i} \) is the temperature of the warm gas at the \( i \)th position from the inlet of the coolant tube, and \( T_{\text{melt}} \) is the melting temperature of the ablative material which is assumed to be constant throughout the coolant tube bore.

The radiation heat flux is defined as,

\[
q_{\text{rad},i}'' = \varepsilon \sigma (T_{\text{warm gas},i}^4 - T_{\text{melt}}^4) = \varepsilon \sigma (T_{\text{warm gas},i}^3 + T_{\text{melt}}^3)(T_{\text{warm gas},i} - T_{\text{melt}}) \tag{3.36}
\]

where \( q_{\text{rad},i}'' \) is the radiative heat flux at the \( i \)th position from the inlet of the coolant tube, \( \varepsilon \) is the emissivity of the ablative material surface which is assumed to be constant throughout the coolant tube, \( \sigma \) is the Stefan-Boltzmann constant, \( T_{\text{warm gas},i} \) is the temperature of the warm gas at the \( i \)th position from the inlet of the coolant tube, and \( T_{\text{melt}} \) is the melting temperature of the ablative material. However, for this research the radiation term is neglected due to three main reasons: 1) the mass flux of the warm gas is very large, 2) the warm gas is diffuse, and 3) the surface of the coolant tube is opaque.

Therefore, combining Equations 3.23 and 3.35 and neglecting radiation the surface energy balance at the ablating surface becomes,

\[
\dot{m}_{\text{coolant},i}'[i_s + C_p(T_{\text{melt}} - T_o)] - Nu_i \frac{k_{\text{warm gas}}}{D_{\text{bore},i}}(T_{\text{warm gas},i} - T_{\text{melt}}) = 0 \tag{3.37}
\]

This equation can be used to solve for the Nusselt number as seen in the following relation,

\[
Nu_i = D_{\text{bore},i} \left\{ \frac{\dot{m}_{\text{coolant},i}'[i_s + C_p(T_{\text{melt}} - T_o)]}{k_{\text{warm gas}}(T_{\text{warm gas},i} - T_{\text{melt}})} \right\} \tag{3.38}
\]
The Nusselt number can be related to the heat transfer coefficient from the following definition,

\[ Nu_i = \frac{h_{\text{ablation},i} D_{\text{bore},i}}{k_{\text{warm\_gas}}} \] (3.39)

Since this research focuses on the undeveloped entrance region of the flow configuration, the Graetz number is investigated and is defined as,

\[ G_z = \left( \frac{D_{\text{bore}}}{L} \right) Re Pr \] (3.40)

The Graetz number is used to assess the thermal entry length in pipe flow. A Graetz number of approximately 1000 or less is the point at which the flow is thermally fully developed.

Finally, combining Equation 3.38 and Equation 3.39, the following relation for the ablative heat transfer coefficient is obtained.

\[ h_{\text{ablation},i} = \left\{ \frac{m''_{\text{coolant},i} \left[ i_s + C_p (T_{\text{melt}} - T_o) \right]}{(T_{\text{warm\_gas},i} - T_{\text{melt}})} \right\} \] (3.41)

This can be used at each \( i \)th position along the axis of the ablative coolant tube to determine a local ablative heat transfer coefficient. The uncertainty of each variable in the ablative heat transfer model is evaluated using a Monte Carlo analysis and is discussed in detail in Appendix H.
3.6 Summary of Approach

In this section, the procedure for characterizing the local time dependent ablation heat transfer coefficient of an ablating hollow cylinder coolant tube from RTR images is summarized. The procedure can be divided into three sub procedures, which are: the experimental procedure, the edge detection procedure, and the analytical heat transfer procedure.

In the experimental procedure, the driving solid propellant and the ablative coolant cylinder are mounted in the sample holder. The sample holder with mounted propellant and coolant tube is secured inside the copper combustion bomb. Ignition leads, thermocouples, and pressure transducers are connected to the combustion bomb. Next, the radiography system is powered on and made sure the coolant sample is in view of the x-ray camera. Then the solid propellant is ignited and the high temperature combustion products traverse through the coolant tube, ablating the tube. Real-time radiography is used to capture the bore regression as a function of time and location. Table 3.5 provides a summary of the experimental procedure.

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1)</td>
<td>Mount propellant and coolant tube in sample holder.</td>
</tr>
<tr>
<td>2)</td>
<td>Build combustion bomb around sample holder.</td>
</tr>
<tr>
<td>3)</td>
<td>Attach ignition leads, thermocouples, and pressure transducers.</td>
</tr>
<tr>
<td>4)</td>
<td>Power on radiography system and confirm coolant tube is in view.</td>
</tr>
<tr>
<td>5)</td>
<td>Ignite solid propellant.</td>
</tr>
<tr>
<td>6)</td>
<td>Capture coolant tube ablation process.</td>
</tr>
</tbody>
</table>
Once the each of the twelve experiments in the test series have been conducted, the RTR videos are analyzed in the edge detection procedure.

In the edge detection procedure, the first step is to acquire the RTR video and separate it into individual frames. The second step is to render the image into an optimal format using a three-frame average, and contrast and brightness adjustment. The third step is to extract the transmitted pixel intensity profiles for each location along axis of the coolant bore. The fourth step is to implement a moving average spatial filter to each image rendered intensity profile. The fifth step is to determine the slope of each moving average spatial filter and find the maximum and minimum of each slope. Table 3.6 provides a summary of the edge detection procedure developed in this research.

Table 3.6: Edge detection step-by-step procedure

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1)</td>
<td>Separate videos into individual frames.</td>
</tr>
<tr>
<td>2)</td>
<td>Image render each frame.</td>
</tr>
<tr>
<td>3)</td>
<td>Pixel Intensity Extraction along each plane across the coolant bore axis of each frame.</td>
</tr>
<tr>
<td>4)</td>
<td>Apply moving average spatial filter.</td>
</tr>
<tr>
<td>5)</td>
<td>Determine max. and min. of moving average slope.</td>
</tr>
</tbody>
</table>

Once these steps have been completed, the entire surface of the coolant bore has been characterized, as the location of the edges along the axis are known. The next steps are used to predict the heat transfer coefficient and exit gas temperature using a surface energy balance specific to the ablation process.

In the analytical heat transfer procedure, the first step is to use the two edge locations to determine a diameter of the coolant bore as a function of time and $i$th position from the
inlet of the coolant tube. Once that is complete, the change in diameter per change in time can be related to the surface velocity of the coolant bore. The surface velocity, density and surface area at the \( i \)th position from the inlet of the coolant tube are used to determine the mass flow rate of the coolant leaving the surface at the \( i \)th position from the inlet of the coolant tube due to ablation. The mass flux is determined from the mass flow rate and surface area. The mass flux leaving the surface in conjunction with material properties are used to determine the heat flux potential of the surface. The Reynolds number at the \( i \)th position from the inlet of the coolant tube can be determined for the warm gas, which can be used to determine the Nusselt number. The unknown coefficient and exponents applied to the Reynolds, Prandtl and initial diameter to length ratio are derived by minimizing the residual energy at the ablative boundary. Once the coefficient and exponents are known and the surface energy is balanced, the Nusselt number for the hot combustion gas can be characterized. After the Nusselt number based on the surface energy balance is evaluated, a Graetz number is used to determine if the flow is thermally developed. Additionally, the coolant-to-propellant ratio is used in conjunction with a thermochemical analysis from CEQUEL to predict warm gas temperature as it flows through the coolant tube. The data reduction loop is then repeated for each axial location of the coolant tube and at each timestep during the ablation process.

Table 3.7 provides a summary of the analytical heat transfer procedure for determining the ablative heat transfer characteristics of the polymer coolant tube.
Table 3.7: Analytical heat transfer step-by-step procedure for $i$th section of coolant tube

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1)</td>
<td>Determine diameter from edge locations, $D_{bore,i}$.</td>
<td></td>
</tr>
<tr>
<td>2)</td>
<td>Determine surface velocity, $V_{coolant,i}$.</td>
<td></td>
</tr>
<tr>
<td>3)</td>
<td>Determine coolant mass flow rate, $\dot{m}_{coolant,i}$.</td>
<td></td>
</tr>
<tr>
<td>4)</td>
<td>Determine from warm gas mass flow rate from mass balance, $\dot{m}_{warmgas,i}$.</td>
<td>(3.17)</td>
</tr>
<tr>
<td>5)</td>
<td>Determine ablative heat flux potential, $q_{s,i}$.</td>
<td>(3.23)</td>
</tr>
<tr>
<td>6)</td>
<td>Determine Reynolds number, $Re_i$.</td>
<td>(3.32)</td>
</tr>
<tr>
<td>7)</td>
<td>Use Surface Balance to Determine the Nusselt number, $Nu_i$.</td>
<td>(3.38)</td>
</tr>
<tr>
<td>8)</td>
<td>Determine Graetz number, $Gz_i$, to assess if the flow is thermally developed.</td>
<td></td>
</tr>
<tr>
<td>9)</td>
<td>Use $Nu_i$ to determine ablation heat transfer coefficient, $h_{ablation,i}$.</td>
<td>(3.41)</td>
</tr>
<tr>
<td>10)</td>
<td>Determine $(\frac{C}{P})_i$ ratio and use material from Appendix C to determine warm gas temperature.</td>
<td></td>
</tr>
<tr>
<td>11)</td>
<td>Repeat from step 1) for each $i$th position from the inlet of the coolant tube.</td>
<td></td>
</tr>
<tr>
<td>12)</td>
<td>Repeat all steps for each time step.</td>
<td></td>
</tr>
</tbody>
</table>

Once these steps have been completed for each location along the axis of the bore, the surface ablation characteristics and warm gas temperature of a hollow cylinder coolant tube are completely described. Results for the ablation characteristic using these three procedures are presented in Chapter 4.
CHAPTER 4

RESULTS AND DISCUSSION

This chapter presents and discusses the results from each experiment in the test series. Included in this section are RTR local time dependent diameter measurements and time evolutions, local ablative coolant surface velocity of the coolant bore, local ablative coolant mass flux, local ablative surface heat flux cooling potential of the coolant bore, local time dependent warm gas mass fluxes, local time dependent Reynolds number, Nusselt number, Graetz number, local time dependent ablation heat transfer coefficient, an entrance effect study, and local warm gas temperature analysis. Additionally, this research correlates all of the data in each experiment of the test series. As an example, the detailed results for Experiment #3 are first presented to show details of the analysis, then overall results for all tests are shown and discussed. Additional information regarding each test is detailed in Appendix G and Appendix H. These results are paramount to completely characterizing the ablation heat transfer attributes of a SPWGG ablative polymer coolant tube.

4.1 RTR Coolant Bore Radius Measurement

This section presents results for the time evolution of the bore of the polymer coolant tube using RTR and the edge detection algorithm developed from this research. Figure 4.1 displays the time evolution for Experiment #3 of the test series as an example. The time evolution for the rest of the test series can be found in Appendix G.1.
Figure 4.1: RTR time evolution

The time evolution incorporates images for 0.0, 2.5, 5.0, and 8.5 seconds from the solid propellant ignition. The edges of the bore are tracked and marked by the yellow line shown in Figure 4.1. At 0.0s the bore is uniform without any flaws. At 2.5s the bore shows the effects of the ablation process. The first area to ablate are the inlet corners of the coolant tube. This is due to the large velocity and pressure gradients that arise from entrance effects. At 5.0s the bore completely changes shape. The inlet of the coolant bore has increased drastically due to entrance effects. A throat area appears just downstream from the inlet where the bore radius is minimum. This could be due to the developing turbulent boundary layer that is incorporated with the entrance effects. The flow through the bore is incompressible with
a Mach number of 0.15 at the inlet. Downstream of the throat the ablation is uniform. This could be due to the boundary layer inside of the coolant tube been fully developed after the throat. At 8.5s the shape has more radius fluctuations than the shape at 5.0s.

The illustration in Figure 4.2 displays how the edge detection algorithm, developed from this research, tracks an edge during the ablation process. The plane of interest shown in Figure 4.2 is downstream of the inlet at the same location as shown in Figure 3.11, for example purposes. The times sampled are for 0.0, 2.5, 5.0, and 8.5 seconds after ignition of the solid propellant.

![Figure 4.2: Example edge location over time (t = 0.0, 2.5, 5.0, 8.5s)]](image)

In Figure 4.2, the black lines represent the image rendered pixel intensity values at each time step, the red lines represent the corresponding moving average spatial filter for each time step, and the blue hollow circles are the edge location for each time step.
A detailed illustration of the local time dependent bore radius is shown in Figure 4.3. The results shown are determined by taking the bore diameter based on two edge measurements and dividing by two.

**Figure 4.3:** Local time dependent coolant bore radius (Experiment #3), $\Delta t = 0.5s$

Where the blue dotted line represents the initial bore radius as a function of axial distance from the inlet, the red dotted line represents the final bore radius as a function of axial distance from the inlet, and the black lines with no markers represents the intermediate bore radius as a function of axial distance from the inlet as the bore ablates. The time step for the intermediate profiles is 0.5 seconds. The local time dependent bore radius for the rest of the experiments in test series can be found in Appendix G.2. The initial radius is uniform with a radius of 3.175 mm. Due to ablation, entrance effects and the developing boundary layer the inlet radius expands drastically to 10mm, the throat or minimum radius just beyond the entrance only expands to 5mm, and the rest of the bore beyond the throat area expand
relatively uniform to 5.75mm. Interestingly, the throat distance from the inlet is initially at 2.5mm. Due to the expanding inlet, the throat distance from the inlet finishes at 4.75mm, which is a 2.25mm shift from the initial position.

In Figure 4.4, an illustration of the developing momentum boundary layer at the entrance region to the coolant tube is shown.

![Diagram of coolant tube flow](image)

**Figure 4.4:** Upstream and downstream separations in the vicinity of the developed throat.

As the flow enters the coolant tube, a recirculation zone outside of the coolant tube is formed. At the leading edge of the coolant tube, the momentum boundary layer separates from the wall and the hot combustion gas is compressed as it enters the coolant tube. The compression of the hot combustion gas is due to the contraction of the momentum boundary layer inside the tube. Ablation causes the formation of a throat where the momentum boundary layer thickness is largest. Downstream from the throat, the momentum boundary layer reattaches to the wall and becomes fully developed flow. The flow separation in the
vicinity of the throat plane causes an increase in pressure loss, which affects erosion rates and heat and mass transfer rates at the separation and reattachment regions.

4.2 Ablative Coolant Surface Velocity

In Figure 4.5, a graph of the time dependent bore radius for five locations along the axis of the coolant tube are presented. The five locations are the inlet, 5mm from the inlet, 10mm from the inlet, 15mm from the inlet, and 20mm from the inlet.

![Figure 4.5: Coolant bore radius vs. time for five locations relative to the inlet](image)

The inlet is represented by the cross symbolled line, 5mm from the inlet is represented by the blue squared line, 10mm from the inlet is represented by the red triangle line, 15mm from the inlet is represented by the green diamonded line, and 20mm from the inlet is represented by the cyan circled line. The inlet of the coolant tube increases in radius the
fastest, while the other four points along the axis of the coolant tube increases at a similar rate.

The local time averaged ablative coolant surface velocity is determined from Equation 3.15. In Figure 4.6, the local time averaged ablative coolant surface velocity along the axis of the coolant tube is presented. The results shown are time averaged over the entire duration of the experiment.

The ablation rate at the inlet is maximum with a value of 0.77 mm/s. The ablation rate at the throat (5mm from the inlet) is minimum with a value of 0.21mm/s. The ablation rate beyond the throat to the outlet is relatively the same with a value that ranges between 0.25 and 0.30 mm/s.

**Figure 4.6:** Local ablative coolant surface velocity time-averaged over entire test
4.3 Ablative Coolant Mass Flux

The local time averaged ablative coolant mass flux is determined by Equation 3.16. In Figure 4.7, the local time averaged ablative coolant mass flux for Experiments #3 of the test series, is presented. The results shown are time averaged over the entire duration of the experiment.

![Ablative Coolant Mass Flux](image)

**Figure 4.7:** Local ablative coolant mass flux time-averaged over entire test

The local time averaged ablative coolant mass flux for the rest of the experiments in test series can be found in Appendix G.3. The mass flux is maximum (0.9 kg/m²-s) and minimum (0.25 kg/m²-s) at the inlet and the throat, respectively. Just beyond the throat the coolant material experiences a relatively uniform ablation and mass flux rate that ranges between 0.3 and 0.35 kg/m²-s.
4.4 Ablative Heat Flux Cooling Potential

As shown in Equation 3.23, the ablative coolant mass flux of the coolant can be directly related to the ablative coolant heat flux potential. Figure 4.8 presents the local ablative coolant heat flux potential, time average over the entire test, for Experiment #3.

![Graph](image)

**Figure 4.8:** Local ablative coolant heat flux potential time-averaged over entire test

Since the ablative coolant heat flux potential is a function of the ablative coolant mass flux and material properties, the coolant heat flux potential has a similar trend as the ablative coolant mass flux. The local time averaged ablative coolant heat flux potential for the rest of the experiments in test series can be found in Appendix G.4. The ablative coolant heat flux potential is maximum (3750 kW/m²) and minimum (1000 kW/m²) at the inlet and the throat, respectively. The section just beyond the throat to the outlet is relatively uniform with an ablative coolant heat flux potential between 1000-1500 kW/m².
4.5 Warm Gas Mass Flux

The local time dependent warm mass flux is determined by Equation 3.18. Figure 4.9 presents the local time dependent mass flux of the warm gas as it traverses through the coolant bore for Experiment #3 of the test series, for $\Delta t = 0.5s$.

![Graph showing local time dependent warm gas mass flux, $\Delta t = 0.5s$](image)

**Figure 4.9:** Local time dependent warm gas mass flux, $\Delta t = 0.5s$

Where the initial local mass flux of the warm gas is represented by the blue dotted line, the final local mass flux of the warm gas is represented by the red dotted line and the black unmarked lines are representative of the intermediate mass fluxes of the warm gas as it traverses through the coolant tube bore. The rest of the experimental data for the test series is found in Appendix G.5. Initially, the local mass flux of the warm gas is relatively uniform between 110-90 kg/m$^2$-s. As the bore ablates due to the hot combustion gas, the mass flux of the warm gas mixture decreases throughout the burn time. The final local mass flux of the
warm gas at the inlet is the lowest at 10 kg/m²-s, The throat area is the largest final mass flux of the warm gas at 38 kg/m²-s, and the region just beyond the throat to the outlet is relatively uniform between 38-28 kg/m²-s.

4.6 Reynolds Number

In this section, the local time dependent mass flux of the warm gas is used to determine the local time-dependent Reynolds number of the warm gas as it traverses through the coolant tube bore for Experiment #3, as shown in Equation 3.32. All experiments in the test series experience turbulent regime flows from the initial to the final burn time. Figure 4.10 provides a representation of the local time depend Reynolds number of the warm gas for Experiment #3, for Δt = 0.5s. The rest of the local time dependent Reynolds number results for each experiment of the test series is shown in Appendix G.6.

![Graph](image)

**Figure 4.10:** Local time dependent warm gas Reynolds number, Δt = 0.5s
In Figure 4.10, the initial local Reynolds number of the warm gas is represented by the blue dotted line, the final local Reynolds number of the warm gas is represented by the red dotted line, and the intermediate local time dependent Reynolds number of the warm gas are represented by the black unmarked lines. The initial Reynolds number is relatively uniform along the axis between 8,800-9,200. The final Reynolds number at the inlet is the lowest Reynolds number at 3,000. The final Reynolds number at the throat is the largest final Reynolds number at 5,500. The rest of the bore beyond the throat to the outlet is a uniform Reynolds number at 5,000. As ablation occurs, the bore expands and the Reynolds number of the warm gas decreases over time. This is trend is similar in all experiments of the test series. The flow is turbulent at all times in each experiment of the test series.

4.7 Nusselt Number Correlation

Once the local time dependent Reynolds number has been determined, the surface energy balance between the heat flux potential of the ablative material and the convective heat flux of the warm gas can be used to determine the Nusselt number of the ablative boundary. This is accomplished using the surface energy balance presented in Equation 3.37,

\[
\dot{m}_{\text{coolant},i}'' i_{fs} + C_{p}(T_{\text{melt}} - T_o) - N_{\text{sh}}\frac{k_{\text{warm gas}}}{D_{\text{bore},i}} (T_{\text{warm gas},i} - T_{\text{melt}}) = 0
\]

where all of the variables in the equation are known, except the Nusselt number. The \(\dot{m}_{\text{coolant},i}''\) value is a measurement made with the RTR system and data reduction analysis.
The Equation in 3.37 can be used to solve for the Nusselt number as described in Equation 3.38,

\[ Nu_i = D_{bore,i} \left( \frac{\dot{m}'_{\text{coolant},i} [i_{fs} + C_p(T_{\text{melt}} - T_o)]}{k_{\text{warmgas}}(T_{\text{warmgas},i} - T_{\text{melt}})} \right) \]

Figure 4.11 presents the local Nusselt number, time-averaged over the entire test, based on the surface energy balance and Equations 3.38 and 3.39 for Experiment #3 of the test series. The rest of the local Nusselt number results for each experiment of the test series is presented in Appendix G.7.

![Graph](image)

**Figure 4.11:** Local Nusselt number time-averaged over entire test

The Nusselt number in Experiment #3 is maximum at the inlet of the coolant tube with a value of 600, the minimum is at the throat with a value of 90, and the region just beyond the
throat to the outlet is relatively uniform between a value of 100-130. A similar trend is observed in each experiment of the test series.

The Nusselt number for fully developed turbulent flow can be expressed in terms of the Reynolds and Prandtl number, according to the Dittus-Boelter equation for pipe flow as described in Equation 3.33,

\[ Nu = 0.023 Re^{0.8} Pr^{0.3} \]

To assess if the flow is fully developed, the Graetz number from Equation 3.40 is employed,

\[ Gz = \left( \frac{D_{\text{bore}}}{L} \right) Re Pr \]

Figure 4.12 shows the local Graetz number time-averaged over the entire test.

**Figure 4.12**: Local Graetz number time-averaged over entire test
The Graetz number is always larger than 1000, therefore the flow is never fully developed. This means in these experiments the entrance effects dominate as the momentum boundary layer develops.

The average Nusselt number for the average Reynolds number of the test series is plotted in Figure 4.13.

![Graetz vs Nusselt for location dependent time averaged results](image)

**Figure 4.13:** Graetz vs Nusselt for location dependent time averaged results

The $\frac{X}{D}$ is a dimensionless axial distance from the coolant tube inlet. Additionally, the Nusselt number is normalized with the fully developed form of the Nusselt number. This means the values shown indicate the degree of deviation from the fully developed values. A previous study conducted by Al-Arabi for pipe flows with Reynolds numbers of 9,000 is shown in comparison to the experimental data obtained from this research [92]. The comparison with Al-Arabi's correlation shows the data obtain from this research is similar.
In addition, to the comparison with Al-Arabi’s correlation, the experimental data obtained from this research was also compared to Patel’s Nusselt number correlation [93]. This is accomplished by observing the Nusselt number as a function of Graetz number, as shown in Figure 4.14.

![Figure 4.14: Nusselt number comparisons](image)

The figure shows a region for the undeveloped flow and developed flow. The experimental data obtained in the research ranged between 19,000 and 7,000, similar Reynolds numbers were tested in Patel’s research. The figure also shows the Dittus-Boelter Nusselt number correlation for comparison purposes. The Dittus-Boelter and Patel’s correlations are specifically used for the developed flow region and is shown as the solid black and red lines, respectively. Those same correlations are extrapolated into the undeveloped region for a “comparison aid” or “guide of eye” comparison to the experimental data obtained from this
research and are shown as the black and red dashed lines, respectively. The comparison shows the experimental data obtained from the research has similarities to both the Dittus Boelter and Patel’s Nusselt number formulation.

4.8 Ablative Heat Transfer Coefficient

After the Nusselt number is evaluated with the surface energy balance and the Reynolds number has been calculated, the local time dependent ablative heat transfer coefficient can be solved for using Equation 3.41. In Figure 4.15, the local time dependent ablative heat transfer coefficient for Experiment #3 of the test series is presented, for $\Delta t = 0.5s$.

![Figure 4.15: Local time dependent ablative heat transfer coefficient, $\Delta t = 0.5s$](image-url)
where the initial local ablative heat transfer coefficient is represented by the blue dotted line, the final local ablative heat transfer coefficient is represented by the red dotted line, and the intermediate local ablative heat transfer coefficients are represented by the black unmarked lines. Initially, the local ablative heat transfer coefficient at the inlet is 0.65 kW/m$^2$-K, at the throat it is 0.10 kW/m$^2$-K, and just beyond the throat to the outlet ranges between 0.10 and 0.16 kW/m$^2$-K. Due to the expanding coolant tube bore diameter and the decrease in Reynolds number, over time, the final ablative heat transfer coefficient decreases. The final ablative heat transfer coefficient at the inlet is 0.20 kW/m$^2$-K, at the throat it is 0.05 kW/m$^2$-K, and just beyond the throat to the outlet is ranges between 0.07 and 0.08 kW/m$^2$-K. A similar trend is observed in all of the experiments of the test series. The ablative heat transfer is calculated as three to five times larger in at the inlet due to the one-dimensional integration scheme and the compounding effects of radiation directed at the surface of the specimen face.

4.9 Entrance Effect Study

The inlet of each experiment in the test series experiences significant entrance effects. This is due to the high mass flux and temperature that is associated with solid propellant combustion. Interestingly, the presence of a thermocouple just before the inlet can alter the entrance effects by providing additional turbulence to the flow. Figure 4.16 illustrates the difference in entrance effect between the presence and absence of a thermocouple just before the inlet of the coolant tube.
Figure 4.16: Initial time step with no thermocouple (top-left), final time step with no thermocouple (top-right), initial time step with thermocouple (bottom-left), final time step with thermocouple (bottom-right)

where the initial time step with the absence of a thermocouple is presented in the top-left image, the final time step with the absence of a thermocouple is presented in the top-right image, the initial time step with the presence of a thermocouple is presented in the bottom-left image, and the final time step with the presence of a thermocouple is presented in the bottom-right image. Visually, it is difficult to observe the entrance effects on the ablation process. However, it is drastically apparent when using the developed edge detection algorithm to measure the diameter along the axis of the coolant tube bore. Figure 4.17
illustrates the entrance effect difference between the absence and presence of a thermocouple obstructing the flow just before the inlet of the coolant tube bore.

**Figure 4.17:** Local time dependent coolant tube bore radius with absence of thermocouple (left), local time dependent coolant tube bore radius with presence of thermocouple (right), $\Delta t = 0.5s$

In the left and right image, the local time dependent coolant tube bore radius with the absence and presence of an inlet thermocouple obstructing the flow is presented, respectively. In the left graph, the inlet entrance effects are much stronger as the inlet of the bore is greater than 20 mm and the throat of the coolant tube bore is positioned 5mm from the inlet of the coolant tube bore. In the right graph, the inlet of the bore is less than 18mm and the throat of the bore is positioned at only 2.5mm. This illustrates the difference between the presence and absence of a thermocouple obstructing the inlet of the coolant tube bore. The thermocouple obstructing the inlet of the coolant tube bore could be providing radiation shielding and extra turbulence in the form of small vortices when compared to the absence of a thermocouple. This also reduces the amount of ablative heat
In Figure 4.18, the ablation heat flux potential difference between the absence and presence of a thermocouple at the inlet of the coolant tube bore is presented.

**Figure 4.18:** Local ablative heat flux potential of the coolant tube bore with absence of thermocouple (left), local ablative heat flux potential of the coolant tube bore with presence of thermocouple (right), time-averaged over entire test.

In the left and right graphs, the local ablative heat flux of the coolant tube bore with the absence and the presence of an inlet thermocouple obstructing the flow into the coolant tube bore is used to illustrate the difference in entrance effects, respectively. In the left graph, the inlet ablative heat flux potential is 550 kW/m² which is much greater than the right graph which is 415 kW/m² at the inlet of the coolant tube bore. This is due to the change in diameter being reduced in the case with a thermocouple at the inlet when compare to the case with the absence of a thermocouple. This means less flow cooling will occur in the case
without the presence of an inlet thermocouple obstructing the flow into the coolant bore. When observing the throat area of the coolant tube bore, the heat fluxes only differ by 20 kW/m². Which shows the entrance effects are only significant from the inlet to the throat section of the coolant tube bore.

4.10 Warm Gas Temperature

The temperature of the warm gas can be described as a function of coolant-to-propellant (C/P) gas ratio using material from Appendix C. Therefore, local time dependent C/P along the axis of the coolant tube bore is presented in Figure 4.19, for \( \Delta t = 0.5s \).

![Figure 4.19: Local time dependent C/P ratio, \( \Delta t = 0.5s \)](image)

The initial local C/P ratio is represented by the blue circled line, the final C/P ratio is represented by the red squared line, and the C/P ratio of the intermediate times are
represented by the solid black unmarked lines. These values are time-averaged to estimate the C/P ratio independent of time. In Figure 4.20, the local C/P ratio, time-averaged over the entire test, is presented.

![Graph showing C/P ratio against axial distance](image)

**Figure 4.20:** Local C/P ratio time-averaged over entire test

Figure 4.20 shows that C/P ratio increases as the flow progresses along the axis of the coolant tube bore. This is because the coolant is tracked in a cumulative manner, meaning the mass is conserved throughout the tube. As mass of the coolant enters the flow the C/P ratio increases from an inlet value of 0.0 to an exit value of 0.09. In Figure 4.21, the local warm gas temperature, time-averaged over the entire test, is presented based on the C/P ratio and material from Appendix C.
Figure 4.21 shows the warm gas temperature flowing through the coolant bore decreases from the inlet to the outlet in a linear fashion. This trend is observed in each experiment of the test series. For Experiment #3, the exit temperature is predicted to be 2240 K.

### 4.11 Monte Carlo Uncertainty Results

A Monte Carlo uncertainty analysis is utilized to predict uncertainties based on a 95% confidence interval ($2\sigma$) and 1,000,000 iterations for each parameter evaluated. More detail regarding the Monte Carlo simulation is provided in Appendix H. In Appendix H, the variable mean, and error values are documented, and a step-by-step procedure is presented. Example probability distributions of the calculated parameters for a single representative test are presented in Appendix H. Also, the percent contribution of each variable is shown in Appendix H for each calculated parameter.
In Table 4.1, the percent uncertainty for each calculated parameter for Experiment #3 is summarized. These results are based on using a 2.0 second time step in the data reduction. These results are used to plot the error bars on each of the figures throughout this chapter.

**Table 4.1: Data reduction uncertainty summary**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Uncertainty</th>
<th>Equation Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Edge Location</td>
<td>1.88%</td>
<td>Section 3.3</td>
</tr>
<tr>
<td>(D_{bore})</td>
<td>3.75%</td>
<td>(3.7)</td>
</tr>
<tr>
<td>(V_{coolant})</td>
<td>20.7%</td>
<td>(3.15)</td>
</tr>
<tr>
<td>(\dot{m}''_{coolant})</td>
<td>20.8%</td>
<td>(3.16)</td>
</tr>
<tr>
<td>(q''_{s})</td>
<td>20.8%</td>
<td>(3.23)</td>
</tr>
<tr>
<td>(\dot{m}''_{warm, gas})</td>
<td>9.01%</td>
<td>(3.19)</td>
</tr>
<tr>
<td>(Re)</td>
<td>9.18%</td>
<td>(3.32)</td>
</tr>
<tr>
<td>(Nu)</td>
<td>21.4%</td>
<td>(3.38)</td>
</tr>
<tr>
<td>(Gz)</td>
<td>10.8%</td>
<td>(3.40)</td>
</tr>
<tr>
<td>(h_{ablation})</td>
<td>22.3%</td>
<td>(3.41)</td>
</tr>
<tr>
<td>(T_{out})</td>
<td>19.3%</td>
<td>Appendix C</td>
</tr>
</tbody>
</table>

Uncertainty for each parameter was calculated for Test 3 using time steps ranging from 0.5 sec to 6.5 seconds to assess the sensitivity of the resulting uncertainty to the observation window size. This analysis can also be used to determine an ideal timestep (with the least amount of uncertainty) for observing and calculating each parameter. Figure 4.22 shows the uncertainty of each parameter as a function of timestep.
Figure 4.22: Parameter uncertainty vs. timestep

As shown, the uncertainty of each variable is maximum at a small timestep and minimum at a larger timestep. The uncertainty results that are most sensitive to the time step (ablation surface velocity, coolant mass flux, ablative cooling heat flux potential, coolant-to-propellant ratio, Nusselt number, ablative heat transfer coefficient, and outlet warm gas temperature) increase in uncertainty as the time step decreases. This result occurs because as the time step is lowered, the movement of the surface between time steps starts to match the uncertainty of the distance measurement. The other parameters such as coolant bore diameter, Reynolds number and Graetz number have no time dependence and therefore have a constant uncertainty regardless of the timestep. These results suggest that a time step of 2 seconds results in an uncertainty that is optimal. Timesteps below 2 seconds will
have large associated uncertainties. Timesteps above 2 seconds will reduce the amount of data captured with larger timesteps.

### 4.12 Summary of Results

In this section, a summary of the results presented from this research is provided.

**Table 4.2:** Experimental test series results I

<table>
<thead>
<tr>
<th>Test #</th>
<th>Ablation Time (sec)</th>
<th>Recorded Pressure (kPa)</th>
<th>Initial-Final $D_{bore}$ (mm)</th>
<th>$V_{coolant}$ (mm/s)</th>
<th>$\dot{m}_{coolant}''$ (kg/m²·s)</th>
<th>$q_s$ (kW/m²)</th>
</tr>
</thead>
<tbody>
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<td>1</td>
<td>5.00</td>
<td>3157</td>
<td>4.78 - 9.75</td>
<td>0.280</td>
<td>0.333</td>
<td>1368</td>
</tr>
<tr>
<td>2</td>
<td>4.50</td>
<td>4254</td>
<td>3.18 - 8.00</td>
<td>0.252</td>
<td>0.300</td>
<td>1231</td>
</tr>
<tr>
<td>3</td>
<td>6.50</td>
<td>2123</td>
<td>6.35 - 12.24</td>
<td>0.331</td>
<td>0.300</td>
<td>1614</td>
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<td>6.50</td>
<td>3164</td>
<td>4.78 - 10.59</td>
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<td>5</td>
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<td>6.35 - 11.94</td>
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<td>0.338</td>
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<td>6.35 - 11.41</td>
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<td>6.35 - 11.15</td>
<td>0.239</td>
<td>0.285</td>
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The values are determined using the equations in Section 3.4 and the average initial and final RTR diameter measurements.

Table 4.3: Experimental test series results II

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<th>Test #</th>
<th>( m''_{\text{w}armsgas} ) kg/m(^2)-s</th>
<th>Re</th>
<th>Nu</th>
<th>( h_c ), kW/m(^2)-K</th>
<th>Outlet C/P</th>
<th>( T_{\text{out}} ) K</th>
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<td>0.025</td>
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<td>88</td>
<td>0.582</td>
<td>0.032</td>
<td>2498</td>
</tr>
<tr>
<td>12</td>
<td>68</td>
<td>7552</td>
<td>103</td>
<td>0.515</td>
<td>0.052</td>
<td>2395</td>
</tr>
</tbody>
</table>

The new experimental data established in this chapter are paramount to the development of new SPWGG DAC systems that operate within the tested bounds of this research.
CHAPTER 5

CONCLUSION

5.1 Conclusions

The RTR measurements and new data reduction algorithm developed from this research provides time-dependent, two-dimensional ablation surface profiles to an accuracy of 0.10 mm, and error of 1.88%. The detail surface profile measurements consequently provide the data essential to a surface energy balance between the warm gas and the ablative material that is used to predict temporally and spatially dependent heat transfer coefficients, warm gas temperatures, and dimensionless parameters. Additionally, dimensionless parameters are used to determine if the flow through the coolant tube is ever fully developed or if the momentum boundary layer is developing through the tube. The results show that the flow through the coolant tube is always developing and never fully develops. Comparing the experimental data obtained from this research with Nusselt number formulations from literature shows similar trends and therefore confirms the validity of the data.

The objective of cooling the high temperature combustion gas generated from the solid propellant was accomplished. The high temperature combustion gas was cooled on average of 300 K for each experiment in the test series.

Interestingly, the ablative coolant tube regressed in a non-uniform fashion. The inlet of the tube had the highest mass loss while exhibiting a noticeable sensitivity to entrance flow disturbances. The data shows that a throat forms in all experiments 2.5-5mm
downstream of the inlet which could be attributed to entrance effects and the developing boundary layer. The region beyond the throat to the exit of the coolant tube had relatively uniform mass loss, which could be attributed to the boundary layer being fully developed downstream of the throat. This detail of analysis concerning the non-uniform regression and ablation rates would not be possible through a simple posttest analysis of the net coolant loss. The novel edge detection algorithm and analytical heat transfer relations developed in this research can be used to describe and map in detail, the local time-dependent ablation heat transfer characteristics of an ablative coolant tube in a SPCGG DAC application.

Also, a detailed Monte Carlo simulation was used to assess the uncertainty of each parameter in the data reduction approach. The results showed that the uncertainty of each variable does not exceed 20% for a timestep of 2 seconds.

5.2 Future Work

Future experiments can be conducted with longer burn times and longer tubes. This will allow the flow to complete the transition from undeveloped to fully developed flow. Additionally, increasing the duration of the experiment will decrease the uncertainty of the measurement methodology. This would allow for larger timesteps to be used in the data reduction approach.

Improvement for future work is to implement a ramp design at the inlet to the coolant tube. This will also allow for the thermal and momentum boundary layer to be developed further upstream. Additionally, this would protect the front face from experiencing any flow or temperature gradients and would reduce the uncertainty of the inlet data.
The uncertainty of each parameter such as coolant mass flux, ablative heat flux cooling potential, Nusselt number, and heat transfer coefficient have a primary dependence on the diameter measurement. Therefore, to further reduce the uncertainty of the data reduction approach, the diameter measurement uncertainty would have to decrease. The diameter measurement is primarily dependent on the camera resolution, projection, and standard deviation of the measurements. Each of those dependencies could be improved to further reduce the uncertainty of the measurement.

With the diameter and regression experimental data from this research, new computational fluid dynamics models can be generated to enhanced designs for optimal solid propellant gas generator systems.
## APPENDIX A
### NUSSELT NUMBER FORMULATIONS

**Table A.1:** Nusselt number formulations from literature

<table>
<thead>
<tr>
<th>Formula</th>
<th>Ref.</th>
<th>Eq.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Nu = 0.023Re^{0.8}Pr^n )</td>
<td>[78]</td>
<td>(A.1)</td>
</tr>
<tr>
<td>( Nu = Nu_{0,b} \left( \frac{c_p}{c_{p,b}} \right)^{0.35} \left( \frac{k_b}{k_w} \right)^{-0.33} \left( \frac{\mu_b}{\mu_w} \right)^{0.11} )</td>
<td>[82]</td>
<td>(A.2)</td>
</tr>
<tr>
<td>( Nu_{0,b} = \left( \frac{f_b}{8} Re_b Pr \right) \left( 12.7 \left( \frac{f_b}{8} \right)^{0.5} \left( Pr^{-\frac{3}{5}} - 1 \right) + 1.07 \right) )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( f = (1.82 \log_{10}(Re_b) - 1.64)^{-2} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( Nu = \left( \frac{f}{8} \right) (Re - 1000)Pr \left( 1 + 12.7 \left( \frac{f}{8} \right)^{0.5} \left( Pr^{-\frac{3}{5}} - 1 \right) \right) )</td>
<td>[82]</td>
<td>(A.3)</td>
</tr>
<tr>
<td>( f = (0.790 \ln(Re) - 1.64)^{-2} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( 10^4 &lt; Re &lt; 10^6 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( Nu = 0.0183Re^{0.82}Pr^{0.5} \left( \frac{\rho_w}{\rho_b} \right)^{0.3} \left( \frac{\bar{c}<em>p}{c</em>{p,b}} \right)^n )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \bar{c}<em>p = \left( \int</em>{T_w}^{T_b} c_p dT \right) \left( T_b - T_w \right) )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( n = 0.4 ) for ( T_b &lt; T_w &lt; T_{pc} ) and ( 1.2T_{pc} &lt; T_b &lt; T_W )</td>
<td></td>
<td>(A.4)</td>
</tr>
<tr>
<td>( n = 0.4 + 0.2 \left( \frac{T_w}{T_{pc}} - 1 \right) ) for ( T_b &lt; T_{pc} &lt; T_w )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( n = 0.4 + 0.2 \left( \frac{T_w}{T_{pc}} - 1 \right) \left[ 1 - 5 \left( \frac{T_b}{T_{pc}} - 1 \right) \right] ) for</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
\( T_{pc} < T_b < 1.2T_{pc} \) and \( T_b < T_W \)

<table>
<thead>
<tr>
<th>Equation</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ Nu = \frac{\left(\frac{f}{8}\right)RePr}{1 + \frac{900}{Re} + 12.7 \sqrt{\frac{f}{8}} (\overline{Pr}^2 - 1)} ]</td>
<td>[82] (A.5)</td>
</tr>
<tr>
<td>[ f = f_0 \left(\frac{\rho_w}{\rho_b}\right)^{0.4} \left(\frac{\mu_w}{\mu_b}\right)^{0.2} ]</td>
<td></td>
</tr>
<tr>
<td>[ f_0 = (1.82 \log_{10}(Re) - 1.64)^{-2} ]</td>
<td></td>
</tr>
<tr>
<td>[ Nu = \frac{(fr)RePr}{1.07 + 12.7 \sqrt{\frac{f_r}{8}} (\overline{Pr}^2 - 1)} \left(\frac{c_p}{c_{p,b}}\right)^{0.65} ]</td>
<td>[82] (A.6)</td>
</tr>
<tr>
<td>[ f_r = f_0 \left(\frac{\rho_w}{\rho_b}\right)^{0.18} \left(\frac{\mu_w}{\mu_b}\right)^{0.18} ]</td>
<td></td>
</tr>
<tr>
<td>[ Nu = 0.0244Re^{0.762}Pr^{-0.552} \left(\frac{\rho_w}{\rho_b}\right)^{0.0293} ]</td>
<td>[79] (A.7)</td>
</tr>
<tr>
<td>[ \overline{Pr} = \frac{c_p}{k} ]</td>
<td></td>
</tr>
<tr>
<td>[ Nu = 0.0061Re^{0.904}Pr^{0.833} \left(\frac{\rho_w}{\rho_b}\right)^{0.564} ]</td>
<td>[82] (A.8)</td>
</tr>
<tr>
<td>[ Nu = 0.039Re^{0.79}Pr^{0.3} ]</td>
<td>[93] (A.9)</td>
</tr>
</tbody>
</table>
APPENDIX B

SOLID PROPELLANT INFORMATION

In Appendix B, the relevant solid propellant information used in this research is provided. This includes the solid propellant geometry and gas properties, ultrasonic burn rate measurements and the correspond values for the burning rate law, solid propellant burn rate as a function of pressure, and a CEQUEL generated flame temperature of the solid propellant. The material presented in this Appendix completely characterizes the driving solid propellant used in this research.

Table B.1: Solid propellant geometry and gas properties

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>25</td>
<td>mm</td>
</tr>
<tr>
<td>Outer Diameter</td>
<td>25</td>
<td>mm</td>
</tr>
<tr>
<td>Flame Temperature</td>
<td>2700</td>
<td>°K</td>
</tr>
<tr>
<td>Combustion Pressure</td>
<td>2069, 3103, 4137</td>
<td>kPa</td>
</tr>
<tr>
<td>Gas Density</td>
<td>3.36</td>
<td>kg/m^3</td>
</tr>
<tr>
<td>Gas Viscosity</td>
<td>7.23e-5</td>
<td>N-s/m^2</td>
</tr>
<tr>
<td>Gas Thermal Conductivity</td>
<td>0.047</td>
<td>W/m-K</td>
</tr>
<tr>
<td>Gas Prandtl Number</td>
<td>0.565</td>
<td></td>
</tr>
</tbody>
</table>
Figure B.1: Sample 1H instantaneous length and pressure vs. time

Figure B.2: Sample 1I instantaneous length and pressure vs. time
Figure B.3: Burning rate vs. chamber pressure

Table B.2: Propellant sample burn time and max pressure

<table>
<thead>
<tr>
<th>Sample</th>
<th>Burn Time, s</th>
<th>Max Pressure (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1H</td>
<td>4.282</td>
<td>14873</td>
</tr>
<tr>
<td>1I</td>
<td>4.845</td>
<td>15266</td>
</tr>
</tbody>
</table>

Table B.3: Propellant sample burn rate law characteristics

<table>
<thead>
<tr>
<th>Sample</th>
<th>a</th>
<th>n</th>
<th>a (bulk fit)</th>
<th>n (bulk fit)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1H</td>
<td>0.019</td>
<td>0.380</td>
<td>0.019</td>
<td>0.383</td>
</tr>
<tr>
<td>1I</td>
<td>0.019</td>
<td>0.387</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8441C</td>
<td>0.030</td>
<td>0.323</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table B.4: Propellant sample burn rate for various pressures

<table>
<thead>
<tr>
<th>Sample</th>
<th>690 kPa</th>
<th>2069 kPa</th>
<th>4137 kPa</th>
<th>6895 kPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>1H</td>
<td>2.80</td>
<td>4.23</td>
<td>5.50</td>
<td>6.70</td>
</tr>
<tr>
<td>1I</td>
<td>2.85</td>
<td>4.35</td>
<td>5.68</td>
<td>6.93</td>
</tr>
<tr>
<td>8441C</td>
<td>3.38</td>
<td>4.80</td>
<td>6.00</td>
<td>7.08</td>
</tr>
</tbody>
</table>

Figure B.4: CEQUEL flame temperature vs. chamber pressure
APPENDIX C

PMMA COOLANT TUBE INFORMATION

In Appendix C, the relevant PMMA coolant tube information used in this research is provided. This includes the coolant tube geometry and material properties, and a CEQUEL generated warm gas temperature as a function of coolant-to-propellant ratio. The material presented in this Appendix completely characterizes the driving solid propellant used in this research.

Table C.1: PMMA coolant tube geometry and material properties

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>25</td>
<td>mm</td>
</tr>
<tr>
<td>Outer Diameter</td>
<td>25</td>
<td>mm</td>
</tr>
<tr>
<td>Inner Diameter</td>
<td>3.175, 4.775, 6.35</td>
<td>mm</td>
</tr>
<tr>
<td>Melting Temperature</td>
<td>433</td>
<td>&quot;K</td>
</tr>
<tr>
<td>Ambient Temperature</td>
<td>297</td>
<td>&quot;K</td>
</tr>
<tr>
<td>Density</td>
<td>1190</td>
<td>kg/m³</td>
</tr>
<tr>
<td>Specific Heat</td>
<td>1810</td>
<td>J/kg-K</td>
</tr>
<tr>
<td>Latent Heat of Fusion</td>
<td>3860000</td>
<td>J/kg</td>
</tr>
</tbody>
</table>
Figure C.1: Warm gas temperature vs. C/P ratio (2000 kPa)

Figure C.2: Warm gas temperature vs. C/P ratio (4100 kPa)
APPENDIX D

SAMPLE HOLDER INFORMATION

Table D.1: Sample holder geometric dimensions

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total length</td>
<td>75</td>
<td>mm</td>
</tr>
<tr>
<td>Overlapping length, propellant side</td>
<td>25</td>
<td>mm</td>
</tr>
<tr>
<td>Overlapping length, PMMA side</td>
<td>25</td>
<td>mm</td>
</tr>
<tr>
<td>Chamber length</td>
<td>25</td>
<td>mm</td>
</tr>
<tr>
<td>Outer Diameter</td>
<td>31</td>
<td>mm</td>
</tr>
<tr>
<td>Inner Diameter</td>
<td>25</td>
<td>mm</td>
</tr>
</tbody>
</table>

Figure D.1: Schematic of dimension of sample holder
APPENDIX E

PRESSURIZATION SYSTEM

In Appendix E, the gaseous nitrogen pressurization system is presented in detail. This includes the pressurization system in the test cell, the high-pressure line clamps, and the nitrogen supply tanks. Inside of the test cell nitrogen is supplied to the system from the N2 supply line.

Figure E.1: Test cell pressurization system with experimental apparatus
Figure E.2: High-pressure line clamps

Figure E.3: Nitrogen supply tanks
## APPENDIX F

### PREDICTED INTENSITY VALUE MODEL

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_i$</td>
<td>radius of $i$th cylinder</td>
<td>in.</td>
</tr>
<tr>
<td>$x$</td>
<td>x-coordinate, origin at source</td>
<td>in.</td>
</tr>
<tr>
<td>$y$</td>
<td>y-coordinate, origin at source</td>
<td>in.</td>
</tr>
<tr>
<td>$z$</td>
<td>z-coordinate, origin at source</td>
<td>in.</td>
</tr>
<tr>
<td>$h_i$</td>
<td>x-offset of the $i$th cylinder</td>
<td>in.</td>
</tr>
<tr>
<td>$k_i$</td>
<td>y-offset of the $i$th cylinder</td>
<td>in.</td>
</tr>
<tr>
<td>$m_k$</td>
<td>slope of the $k$th x-ray</td>
<td>in.</td>
</tr>
<tr>
<td>$b$</td>
<td>Offset in y-direction of x-ray source</td>
<td>in.</td>
</tr>
<tr>
<td>$x_{\text{DET}}$</td>
<td>x-position of the x-ray detector</td>
<td>in.</td>
</tr>
<tr>
<td>$y_{\text{DET,k}}$</td>
<td>y-position of the $k$th x-ray line</td>
<td>in.</td>
</tr>
<tr>
<td>$A$</td>
<td>Quadratic factor</td>
<td>$\frac{1}{\text{in}^2}$</td>
</tr>
<tr>
<td>$B$</td>
<td>Quadratic factor</td>
<td>$\frac{1}{\text{in}}$</td>
</tr>
<tr>
<td>$C$</td>
<td>Quadratic factor</td>
<td>-</td>
</tr>
<tr>
<td>$x_1$</td>
<td>First root of quadratic solution</td>
<td>in.</td>
</tr>
<tr>
<td>$x_2$</td>
<td>Second root of quadratic solution</td>
<td>in.</td>
</tr>
<tr>
<td>$y_1$</td>
<td>First root of quadratic solution</td>
<td>in.</td>
</tr>
<tr>
<td>$y_2$</td>
<td>Second root of quadratic solution</td>
<td>in.</td>
</tr>
<tr>
<td>$l_{i,k}$</td>
<td>Length of $k$th ray through $i$th material</td>
<td>in.</td>
</tr>
</tbody>
</table>
Figure F.1 shows a diagram of the two-dimensional ray tracing system. Start by assuming an x-ray source is located at the origin of an (x, y, z) coordinate system at (0, b, 0) and is pointing in the x-direction. A detector is located at x = x_{DET}. The plane of the detector is parallel to the y-z plane. One ray emits from the source, the $k$th ray, and it intercepts the detector at $(x_{DET}, y_{DET})$.

![Figure F.1: X-ray coordinate system](image)

The equation for the $k$th ray is then,

$$y_k = m_k x + b$$  \hspace{1cm} (F.1)
where,

\[ m = \frac{y_{DET} - b}{x_{DET}} \quad (F.2) \]

For this analysis, the ray path through a semi-infinite solid is determined. Figure F.2 shows the configurations.

**Figure F.2: Ray path through cylinder**

Figure F.2 shows the \( k \)th ray going through a cylinder centered at \((h_i, k_i)\) and having a radius, \( R_i \). It is represented mathematically as,

\[ R_i = (x - h_i)^2 + (y - y_i)^2 \quad (F.3) \]

Combining Equations F.1, F.2, and F.3 yields,
\[ (1 + m^2)x^2 + [-2h_i + 2m_kb - 2m_kk_i]x + [h_i^2 + b^2 - R_i^2] - 2k_ib_k = 0 \]  \hspace{1cm} (F.4)

Which is a quadratic in \( x \). Defining,

\[ A = 1 + m_k^2 \]  \hspace{1cm} (F.5)

\[ B = 2[-h_i + m_kb - m_kk_i] \]  \hspace{1cm} (F.6)

\[ C = h_i^2 + b^2 + k_i^2 - R_i^2 - 2k_ib \]  \hspace{1cm} (F.7)

and using Equation F.2 for \( m \), the quadratic formula is used to determine the possible intersection points of the ray with the circle.

\[ [x_{1,2}]_k = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \]  \hspace{1cm} (F.8)

Equation F.8 has three types of solutions depending on where the ray intersects with the cylinder.

For case I,

\[ \sqrt{B^2 - 4AC} = 0 \]  \hspace{1cm} (F.9)

Is the case where the ray intersects the cylinder at one point (tangent to the edge).

For case II,

\[ \sqrt{B^2 - 4AC} < 0 \]  \hspace{1cm} (F.10)
The square root term is imaginary. The represents the case where the ray misses the cylinder altogether.

For case III,

\[ \sqrt{B^2 - 4AC} > 0 \]  \hspace{1cm} (F.11)

Which gives two solutions, and represents the configuration shown in Figure F.2, and \( x_1 \) and \( x_2 \) are solved using Equation F.8. The \( y \) solution(s) to the intersection points are calculated using Equation F.1 for each \( x \) intersection point,

\[ y_{1,k} = m_k x_{1,k} + b \]  \hspace{1cm} (F.12)

\[ y_{2,k} = m_k x_{2,k} + b \]  \hspace{1cm} (F.13)

This give the entry and exit coordinates of the ray,

Case III \( (x_1, y_1)_k, (x_2, y_2)_k \)  \hspace{1cm} (F.14)

Case II \( (x_1, y_1)_k \)  \hspace{1cm} (F.15)

Case I \( (-, -)_k \)  \hspace{1cm} (F.16)

The length through the cylinder is then,

\[ l_k = \sqrt{(x_2 - x_1)_k^2 + (y_2 - y_1)_k^2} \]  \hspace{1cm} (F.17)
The transmitted radiation that reaches the detector is reduced by the path length of each ray through the cylinder using Beer’s Law,

\[ I_k = \frac{I_o}{R_k^2} e^{-\mu_k l_k} \quad \text{(F.18)} \]

By examining a number of rays by changing the \( y_{DET} \) position on the detector, the transmitted intensity profile at the detector can be calculated. Figure F.3 shows an example transmitted intensity profile for a solid rod.

![Diagram](image)

**Figure F.3:** Transmitted intensity profile for a solid rod
When the beam touches the cylinder at "A", the intensity starts to drop. It reaches a minimum at the center, which is the longest path through the cylinder, the profile then returns to the unobstructed value at point "B".

The same setup can be used to analyze, \( i \), concentric cylinders, each having different attenuations. Figure F.4 shows the layout,

\[
l_1 = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
\]  

(F.19)
\[ l_2 = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \text{(F.20)} \]

\[ l_3 = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \text{(F.21)} \]

But the path through each material is,

\[ ll_1 = l_1 \quad \text{(F.22)} \]

\[ ll_2 = l_2 - l_1 \quad \text{(F.23)} \]

\[ ll_3 = l_3 - l_2 \quad \text{(F.24)} \]

So now the transmitted x-ray intensity for the \( k \)the ray is calculated as,

\[ I_k = \frac{I_o}{R_k^2} e^{[\Sigma - \mu_i ll_i]_k} \quad \text{(F.25)} \]

Figure F.5 shows an example transmitted intensity profile for a hollow cylinder surrounded by a single material,
The profile is similar to Figure F.3 (dashed line), but in this case the hollow curve causes the transmitted intensity to rise in the middle and then fall. Points “C” and “D” correspond to the ray that is tangent to the inner bore of the cylinder. This “edge” can be used as a feature to measure the inner diameter of the cylinder when the x-ray system is spatially calibrated. Likewise, the \( y_{DET,A} \) and the \( y_{DET,B} \) locations can be used to measure the outer diameter of the cylinder.

In the case where the inner cylinder is changing diameters with time, such as a burning propellant or an eroding nozzle, the inner edge can be used to track the bore
Figure F.6 illustrates how the profile would change for a bore that is burning away with time.
Figure F.6: Transmitted intensity for a hollow cylinder burning steadily and evenly inside

This means that the surface position and hence the rate of regression can be calculated with time.
In Figure F.7, the slope of these lines represents the regression rate of each surface and can be calibrated to actual values.
In Appendix G, the results for the real-time radiography measurements are presented for each experiment in the test series. This includes RTR time evolutions, coolant mass flux and heat flux potential, hot gas mass flux, Reynolds number and Nusselt number, and ablative heat transfer coefficient.

G.1 RTR Time Evolution

Figure G.1: RTR time evolution (Experiment #1)
**Figure G.2:** RTR time evolution (Experiment #2)

**Figure G.3:** RTR time evolution (Experiment #3)
Figure G.4: RTR time evolution (Experiment #4)

Figure G.5: RTR time evolution (Experiment #5)
Figure G.6: RTR time evolution (Experiment #6)

Figure G.7: RTR time evolution (Experiment #7)
Figure G.8: RTR time evolution (Experiment #8)

Figure G.9: RTR time evolution (Experiment #9)
Figure G.10: RTR time evolution (Experiment #10)

Figure G.11: RTR time evolution (Experiment #11)
Figure G.12: RTR time evolution (Experiment #12)

G.2 Coolant Bore Radius Measurement

Figure G.13: Local time dependent coolant bore radius (Experiment #1)
Figure G.14: Local time dependent coolant bore radius (Experiment #2)

Figure G.15: Local time dependent coolant bore radius (Experiment #3)
Figure G.16: Local time dependent coolant bore radius (Experiment #4)

Figure G.17: Local time dependent coolant bore radius (Experiment #5)
Figure G.18: Local time dependent coolant bore radius (Experiment #6)

Figure G.19: Local time dependent coolant bore radius (Experiment #7)
Figure G.20: Local time dependent coolant bore radius (Experiment #8)

Figure G.21: Local time dependent coolant bore radius (Experiment #9)
**Figure G.22:** Local time dependent coolant bore radius (Experiment #10)

**Figure G.23:** Local time dependent coolant bore radius (Experiment #11)
Figure G.24: Local time dependent coolant bore radius ( Experiment #12)

G.3 Ablative Coolant Mass Flux

Figure G.25: Ablative coolant mass flux (Experiment #1)
Figure G.26: Ablative coolant mass flux (Experiment #2)

Figure G.27: Ablative coolant mass flux (Experiment #3)
Figure G.28: Ablative coolant mass flux (Experiment #4)

Figure G.29: Ablative coolant mass flux (Experiment #5)
Figure G.30: Ablative coolant mass flux (Experiment #6)

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Figure G.32: Ablative coolant mass flux (Experiment #8)

Figure G.33: Ablative coolant mass flux (Experiment #9)
Figure G.34: Ablative coolant mass flux (Experiment #10)

Figure G.35: Ablative coolant mass flux (Experiment #11)
Figure G.36: Ablative coolant mass flux (Experiment #12)

G.4 Ablative Coolant Heat Flux Potential

Figure G.37: Ablative coolant heat flux potential (Experiment #1)
**Figure G.38**: Ablative coolant heat flux potential (Experiment #2)

**Figure G.39**: Ablative coolant heat flux potential (Experiment #3)
Figure G.40: Ablative coolant heat flux potential (Experiment #4)

Figure G.41: Ablative coolant heat flux potential (Experiment #5)
Figure G.42: Ablative coolant heat flux potential (Experiment #6)

Figure G.43: Ablative coolant heat flux potential (Experiment #7)
Figure G.44: Ablative coolant heat flux potential (Experiment #8)

Figure G.45: Ablative coolant heat flux potential (Experiment #9)
Figure G.46: Ablative coolant heat flux potential (Experiment #10)

Figure G.47: Ablative coolant heat flux potential (Experiment #11)
Figure G.48: Ablative coolant heat flux potential (Experiment #12)

G.5 Warm Gas Mass Flux

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Figure G.50: Warm gas mass flux (Experiment #2)

Figure G.51: Warm gas mass flux (Experiment #3)
Figure G.52: Warm gas mass flux (Experiment #4)

Figure G.53: Warm gas mass flux (Experiment #5)
**Figure G.54:** Warm gas mass flux (Experiment #6)

**Figure G.55:** Warm gas mass flux (Experiment #7)
Figure G.56: Warm gas mass flux (Experiment #8)

Figure G.57: Warm gas mass flux (Experiment #9)
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Figure G.59: Warm gas mass flux (Experiment #11)
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G.6 Reynolds Number

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Figure G.63: Reynolds number (Experiment #3)
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G.7 Nusselt Number

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Figure G.83: Nusselt number (Experiment #11)
Figure G.84: Nusselt number (Experiment #12)

G.8 Ablative Heat Transfer Coefficient

Figure G.85: Local time dependent ablation heat transfer coefficient (Experiment #1)
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Figure G.89: Local time dependent ablation heat transfer coefficient (Experiment #5)
Figure G.90: Local time dependent ablation heat transfer coefficient (Experiment #6)

Figure G.91: Local time dependent ablation heat transfer coefficient (Experiment #7)
Figure G.92: Local time dependent ablation heat transfer coefficient (Experiment #8)

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Figure G.94: Local time dependent ablation heat transfer coefficient (Experiment #10)

Figure G.95: Ablative heat transfer coefficient (Experiment #11)
Figure G.96: Ablative heat transfer coefficient (Experiment #12)

G.10 Warm Gas Temperature

Figure G.97: Warm gas temperature (Experiment #1)
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Figure G.99: Warm gas temperature (Experiment #3)
Figure G.100: Warm gas temperature (Experiment #4)

Figure G.101: Warm gas temperature (Experiment #5)
Figure G.102: Warm gas temperature (Experiment #6)

Figure G.103: Warm gas temperature (Experiment #7)
Figure G.104: Warm gas temperature (Experiment #8)

Figure G.105: Warm gas temperature (Experiment #9)
Figure G.106: Warm gas temperature (Experiment #10)

Figure G.107: Warm gas temperature (Experiment #11)
Figure G.108: Warm gas temperature (Experiment #12)
APPENDIX H

UNCERTAINTY ANALYSIS

The total measurement error is the sum of the systematic errors and the random error associated with the measurement. Systematic errors may result from multiple sources such as the uncertainty in a calibration device, the resolution of a system, the response time of a sensor, etc. These errors are reproducible and affect all measurements made with a sensor in the same way. Random errors are the unrepeatable errors which may be caused by sources like random noise or inherent “unsteadiness” in the presumed steady-state process. The total measurement error is thus calculated as the sum of these two categories of errors.

\[ \delta_{total} = \sum_{i=1}^{N} \delta_{systematic} + \delta_{random} \]  

(H.1)

For a steady-state measurement, the random uncertainty is estimated as the standard deviation, \( s_x \), of the set of measurements according to.

\[ \sigma_x = \left[ \frac{1}{N-1} \sum_{i=1}^{N} (X_i - \bar{X})^2 \right]^{\frac{1}{2}} \]  

(H.2)

where \( X_i \) is the \( i^{th} \) measurement from the data set of \( N \) total measurements and \( \bar{X} \), the mean value of the dataset, is calculated from,
The elemental standard uncertainty for a measurement is calculated by combining the estimated systematic and random uncertainties,

\[
\bar{X} = \frac{1}{N} \sum_{i=1}^{N} X_i
\]

\[
u_X^2 = \sum_{i=1}^{n} b_i^2 + s_i^2
\]

where \(b_i\) is the systematic uncertainty of the measurement, and \(s_i\) is the random uncertainty of the measurement. The elemental standard uncertainty is then multiplied by a confidence factor to produce the expanded uncertainty estimate for the measurement. For a large number of samples, a confidence factor 2 can be used for a 95% confidence interval.

Typically, multiple measurements are combined through a data reduction equation to calculate a parameter of interest. The general form for a Data Reduction Equations (DRE) is illustrated by the following relation,

\[
r(i) = f[X(i), Y(i)]
\]

Where \(r(i)\) is the parameter of interest, and \(X(i)\) and \(Y(i)\) are input variables to the DRE. The uncertainty associated with the measured quantities \(X(i)\) and \(Y(i)\) will result in uncertainty of the calculated quantity \(r(i)\).

A Monte Carlo method (MCM) can be used to estimate the uncertainty of a result determined from multiple measurements. The MCM is a statistical approach to uncertainty analysis that is used to determine probable outcomes based on random variation on input
parameters. When applied to a DRE, with random variation of input parameters based on their uncertainties, the MCM produces a distribution of results that can be used to estimate the uncertainty of that result.

Figure H.1 illustrates a detailed approach for the MCM simulation. The simulation is implemented through an iterative process. For each iteration, random values for the elemental uncertainties are pulled from an assumed probability distribution for that error source. These elemental errors are then added to the mean values of the independent parameters and used to calculate a result. The simulation is repeated $M$ number of times to produce $M$ results. The distribution of the $M$ results is then taken as the probability distribution for the result and used to determine the result expanded uncertainty.
Figure H.1: Schematic of MCM for uncertainty propagation when random standard uncertainties for individual variables are used. [91]
In this research, the MCM method was applied to all of the calculated parameters. 1,000,000 iterations were used to determine the probability distributions of each result. Because the result distributions were normal distributions, the result uncertainties are based on a standard deviation and a 95% confidence interval (t=2). In Table H.1, the mean values utilized in the Monte Carlo simulation are documented. Mean values for the initial bore diameter, $D_{bore}$, timestep, $\Delta t$, and initial length, $L$, were based on laboratory measurements. The mean value for the propellant mass flow rate was calculated from a solid propellant burning rate equation determined in a parallel study of the propellant used. Finally, the material properties for PMMA and the thermodynamic constants listed in the table were obtained from MATWEB [90] and the NIST Chemical WebBook [94] and were assumed to be constant throughout the simulation for each experiment in the test series. In Table H.2 the uncertainty values for the independent variables for the Monte Carlo simulation are documented. For measured parameters, the uncertainty was estimated based on the suspected error sources in the measurements. PMMA property and thermodynamic constant uncertainties were based on values are MATWEB [90] and the NIST Chemical WebBook [94] when possible and were estimated if no uncertainty could be found.

Uncertainty Percent Contributions (UPC) are used to determine the dominate variable of the uncertainty of the DRE. UPC’s are defined as,

$$\text{UPC}_i = \frac{U_{X_i}^2}{U_{total}^2}$$  \hspace{1cm} (H.6)

Where $U_{X_i}$ represents the uncertainty of variable $X_i$, and $U_{total}$ represents the total uncertainty of the DRE.
Table H.1: Mean values for each variable

<table>
<thead>
<tr>
<th>Description</th>
<th>Variable</th>
<th>Mean Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drill bit known bore diameter</td>
<td>$D_{bore}$</td>
<td>3.175, 4.775, 6.35</td>
<td>mm</td>
</tr>
<tr>
<td>Timestep</td>
<td>$\Delta t$</td>
<td>2.0</td>
<td>sec</td>
</tr>
<tr>
<td>PMMA density</td>
<td>$\rho_{coolant}$</td>
<td>1190</td>
<td>kg/m³</td>
</tr>
<tr>
<td>PMMA latent heat</td>
<td>$i_{fs}$</td>
<td>3860000</td>
<td>J/kg</td>
</tr>
<tr>
<td>PMMA specific heat</td>
<td>$C_p$</td>
<td>1810</td>
<td>J/kg-K</td>
</tr>
<tr>
<td>PMMA initial outer temperature</td>
<td>$T_o$</td>
<td>297</td>
<td>K</td>
</tr>
<tr>
<td>PMMA melt temperature</td>
<td>$T_{melt}$</td>
<td>433</td>
<td>K</td>
</tr>
<tr>
<td>PMMA tube length</td>
<td>$L$</td>
<td>0.023</td>
<td>m</td>
</tr>
<tr>
<td>Propellant mass flow rate</td>
<td>$\dot{m}_{prop}$</td>
<td>0.00311</td>
<td>kg/s</td>
</tr>
<tr>
<td>Warm gas thermal conductivity</td>
<td>$k_{warmgas}$</td>
<td>0.047</td>
<td>W/m-K</td>
</tr>
<tr>
<td>Warm gas temperature</td>
<td>$T_{warmgas}$</td>
<td>2700</td>
<td>K</td>
</tr>
<tr>
<td>Warm gas Prandtl number</td>
<td>$Pr_{warmgas}$</td>
<td>0.565</td>
<td>-</td>
</tr>
</tbody>
</table>
Table H.2: Systematic and random uncertainties for each variable

<table>
<thead>
<tr>
<th>Description</th>
<th>Variable</th>
<th>Uncertainty</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Edge resolution uncertainty</td>
<td>$b_{1,E}$</td>
<td>0.5</td>
<td>pixels</td>
</tr>
<tr>
<td>Edge projection uncertainty</td>
<td>$b_{2,E}$</td>
<td>1</td>
<td>pixel</td>
</tr>
<tr>
<td>Edge random uncertainty</td>
<td>$s_E$</td>
<td>1.225</td>
<td>pixels</td>
</tr>
<tr>
<td>Drill bit known diameter uncertainty</td>
<td>$b_{1,dk}$</td>
<td>0.1</td>
<td>mm</td>
</tr>
<tr>
<td>PMMA length uncertainty</td>
<td>$b_{1,\text{coolant}}$</td>
<td>0.1</td>
<td>mm</td>
</tr>
<tr>
<td>Timestep uncertainty</td>
<td>$b_{1,t}$</td>
<td>0.00067</td>
<td>sec</td>
</tr>
<tr>
<td>PMMA density uncertainty</td>
<td>$b_{1,\rho}$</td>
<td>0.8% of mean</td>
<td>kg/m³</td>
</tr>
<tr>
<td>PMMA latent heat uncertainty</td>
<td>$b_{1,\text{fs}}$</td>
<td>10% of mean</td>
<td>J/kg</td>
</tr>
<tr>
<td>PMMA specific heat uncertainty</td>
<td>$b_{1,\text{c}_p}$</td>
<td>19% of mean</td>
<td>J/kg·K</td>
</tr>
<tr>
<td>PMMA initial outer temperature uncertainty</td>
<td>$b_{1,T_0}$</td>
<td>0.75% of mean</td>
<td>K</td>
</tr>
<tr>
<td>PMMA melt temperature uncertainty</td>
<td>$b_{1,T_{\text{melt}}}$</td>
<td>1% of mean</td>
<td>K</td>
</tr>
<tr>
<td>Solid propellant mass flow rate</td>
<td>$\dot{m}_{\text{prop}}$</td>
<td>5% of mean</td>
<td>kg/s</td>
</tr>
<tr>
<td>Combustion temperature uncertainty</td>
<td>$b_{1,T_{\text{prop}}}$</td>
<td>6.7% of mean</td>
<td>K</td>
</tr>
<tr>
<td>Warm gas thermal conductivity uncertainty</td>
<td>$b_{1,k_{\text{warmgas}}}$</td>
<td>6.7% of mean</td>
<td>W/m·K</td>
</tr>
<tr>
<td>Warm gas viscosity uncertainty</td>
<td>$b_{1,\mu_{\text{warmgas}}}$</td>
<td>6.7% of mean</td>
<td>N·s/m²</td>
</tr>
<tr>
<td>Warm gas Prandtl number uncertainty</td>
<td>$b_{1,\text{Pr}}$</td>
<td>6.7% of mean</td>
<td>-</td>
</tr>
</tbody>
</table>

The following figures are probability densities for each parameter of the data reduction approach of the research.
**Figure H.2**: Edge location probability density

**Figure H.3**: Edge location uncertainty percent contributions
Figure H.4: Coolant bore diameter probability density

Figure H.5: Coolant bore diameter uncertainty percent contributions
**Figure H.6:** Bore surface velocity probability density

**Figure H.7:** Bore surface uncertainty percent contributions
Figure H.8: Coolant mass flux probability density

Figure H.9: Coolant mass flux uncertainty percent contributions
Figure H.10: Ablative heat flux cooling potential probability density

Figure H.11: Ablative heat flux cooling potential uncertainty percent contributions
Figure H.12: Warm gas mass flux probability density

Figure H.13: Warm gas mass flux uncertainty percent contributions
Figure H.14: Reynolds number probability density

Figure H.15: Reynolds number uncertainty percent contributions
Figure H.16: Nusselt number probability density

Figure H.17: Nusselt number uncertainty percent contributions
**Figure H.18**: Graetz number probability density

**Figure H.19**: Graetz number uncertainty percent contributions
Figure H.20: Ablative heat transfer coefficient probability density

Figure H.21: Ablative heat transfer coefficient uncertainty percent contributions
Figure H.22: Coolant-to-propellant probability density

Figure H.23: Coolant-to-propellant uncertainty percent contributions
Figure H.24: Outlet temperature probability density

Figure H.25: Outlet temperature uncertainty percent contributions
REFERENCES


http://www.nist.gov/pml/data/xraycoef/, retrieved May 2020


