Examination of dichotomous and centroid tracking of a monopulse angle tracking unit

Jonathan Thomas Andrews

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EXAMINATION OF DICHOTOMOUS AND CENTROID TRACKING
OF A MONOPULSE ANGLE TRACKING UNIT

by

JONATHAN THOMAS ANDREWS

A THESIS

Submitted in partial fulfillment of the requirements
for the degree of Master of Science in Engineering
in
The Department of Electrical and Computer Engineering
to
The School of Graduate Studies
of
The University of Alabama in Huntsville

HUNTSVILLE, ALABAMA

2017
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Jonathan Thomas Andrews

Date 3/21/2017
THESIS APPROVAL FORM

Submitted by Jonathan Thomas Andrews in partial fulfillment of the requirements for the degree of Master of Science in Engineering and accepted on behalf of the Faculty of the School of Graduate Studies by the thesis committee.

We, the undersigned members of the Graduate Faculty of The University of Alabama in Huntsville, certify that we have advised and/or supervised the candidate on the work described in this thesis. We further certify that we have reviewed the thesis manuscript and approve it in partial fulfillment of the requirements for the degree of Master of Science in Engineering.

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ABSTRACT

The School of Graduate Studies

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Degree Master of Science in Engineering  College/Dept. Engineering / Electrical and Computer Engineering

Name of Candidate Jonathan Thomas Andrews

Title Examination of Dichotomous and Centroid Tracking of a Monopulse Angle Tracking Unit

This thesis mathematically describes the overall behavior of a monopulse angle tracker as it relates to transitioning to an interfering target signal. The engineering community is confident in its assessment on how an angle tracker will perform in a multi-target environment as the target-to-tracker range decreases, causing targets to leave the tracker antenna beam. When one target signal dominates due to the angular separation of the targets, the tracker will center on the dominating signal. It is not well understood how the tracker will behave when angle separation is not the limiting factor in signal strength.

Analytical studies presented in this thesis will evaluate how the open-loop error signal normalization impacts the track loop dynamics. Specifically, if the error signal is normalized by an Automatic Gain Control (AGC), the behavior will depend upon the target Doppler frequency separation of the two targets relative to the AGC bandwidth. Along with verification of the theoretical derivations, simulation efforts will consider multiple target signals within the main beam of the antenna, allowing for a more thorough investigation of closed-loop tracker performance.
Abstract Approval:

Committee Chair

Department Chair

Graduate Dean
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<td>automatic gain control</td>
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<tr>
<td>BPL</td>
<td>bandpass limiter</td>
</tr>
<tr>
<td>CW</td>
<td>continuous wave</td>
</tr>
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<td>DC</td>
<td>direct current</td>
</tr>
<tr>
<td>Hz</td>
<td>Hertz</td>
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<td>IF</td>
<td>intermediate frequency</td>
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CHAPTER 1

INTRODUCTION

A standard textbook approach to multi-target signal interaction indicates that the combination of the reflected waves for two unresolvable targets separated in angle tends to result in a perceived angle of arrival near the power centroid of the two targets [1]. However, the main assumption upon which this observation is based is that the Doppler frequencies of the two targets are the same. As an extension of that foundation, Isaac Kliger and Irving Kanter investigated the case where the Doppler frequencies of the two targets were not equal and demonstrated that the radar angle track did not follow a power centroid behavior but, instead, quickly switched to the larger target [2, 3, 4]. This behavior is termed dichotomous track.

Further publications outline a framework for dichotomous tracking, however, there has not been much published on the subject in the past decades [5, 6]. The lack of scholarly articles concerning this phenomenon is striking given its importance in modern situations where an aircraft employs a towed-decoy [7]. In particular, if the seeker angle tracker is dichotomous, it could impact the effectiveness of the decoy. This thesis revisits the theory of dichotomous tracking and uses simulations to determine how well practical systems followed the theory. The cause of a track transition between multiple targets has a direct impact on current academic studies and real-world applications [8].
Extending unpublished work by Dr. Larry Bennet, Mark Tillman provided a more rigorous investigation of Kanter’s observations [9, 10]. Tillman delineated distinct angle tracking regions defined by the Doppler separation between the two targets relative to the Automatic Gain Control (AGC) bandwidth; however, it was constrained to the limiting cases of either an infinite or zero-bandwidth AGC. Further, hardware verification tests included in Tillman’s research were intended to characterize the effects of a specific system.

This research addresses AGC bandwidths other than zero or infinite, along with a more general representation of the monopulse receiver and associated angle tracking characteristics. This thesis revisits the theoretical development of Kanter, Bennet, and Tillman, and provides verification simulations extended to a full, closed-loop angle tracker to investigate how the angle track loop impacts the dichotomous track characteristics. The overall observations of this research are consistent with the results derived by Kanter and observed by Bennet and Tillman. Specifically, the angle track behavior depends on the difference in Doppler frequency of the two targets relative to receiver component bandwidths. This is illustrated in Figure 1.1.
Region 1 corresponds to the case where the two targets have identical Doppler frequencies. In this case, the angle track is a power centroid between the two targets. Although this is an unrealistic tracking situation, as it is unlikely the signal returns from two targets would produce identical Doppler frequencies, it is still considered during this research to verify theoretical predictions.

Region 2 corresponds to a non-zero Doppler frequency separation between the two targets that is less than both the angle track filter and AGC bandwidths. For this case, a dichotomous track emerges where the angle track immediately follows the target with the largest signal amplitude. This region was established by Kanter and verified through simulation by Tillman. Since the Doppler frequency separation of the two targets fall within the AGC bandwidth, the AGC can follow the beat signal created by the two target returns and thus, ideally, instantaneously follow that beat signal. This case is referred to as an instantaneous AGC model and is typically analyzed by assuming an infinite bandwidth AGC.
Region 3 corresponds to the case where the Doppler frequency separation between the two targets is greater than the AGC bandwidth but less than the Doppler track loop bandwidth. This case is similar to the power centroid track with the transition between the two targets characterized by what is termed an elliptic centroid. Since the Doppler separation is greater than the AGC bandwidth, AGC normalization is based upon the root-mean-square (RMS) value of the received target signal. This case is referenced as a slow AGC model due to the power averaging nature of the signal normalization. It is usually analyzed by assuming an AGC bandwidth of zero (a perfect integrator).

During this study, it was observed that practical angle trackers do not precisely follow the dichotomous or elliptic centroid behavior predicted by the theory as the target Doppler frequency separation increased. Because of this discrepancy, the theoretical analysis was expanded to consider a finite, non-zero AGC bandwidth. As described in Chapter 3, this extension was able to explain the observed behavior by relating the transition between regions 2 and 3 to the AGC bandwidth. This region exhibits a gradual transition between the dichotomous and elliptic centroid tracking regions as opposed to an immediate shift in behavior.
CHAPTER 2

SYSTEM DEFINITION

The system considered in this investigation is a semi-active missile seeker which tracks a continuous wave (CW) signal reflected from a target illuminated by an associated ground radar. This analysis focused on the behavior of the seeker receiver, and as such, the specifics of the ground radar transmitter were not considered.

A typical semi-active missile seeker utilizes a three-channel design consisting of one sum and two difference channels to provide angle tracking in two orthogonal dimensions [11]. The two difference channels have identical receiver chains; therefore, this analysis considered only a single difference channel, since that was sufficient to capture the phenomena of interest.

Figure 2.1 contains a block diagram of the seeker receiver considered in this study. The seeker antenna receives a portion of the illumination signal after being reflected from the target(s). This signal is processed by the monopulse arithmetic and amplified by the RF amplifiers to produce sum ($\Sigma$) and difference ($\Delta$) signals that are processed by the rest of the receiver. In general, both the sum and difference signals are a function of the difference between the target angle and the antenna pointing angle.
The RF signal is mixed to a lower intermediate frequency (IF) to facilitate further processing. Figure 2.1 shows only a single pair of mixers. Because of implementation issues, an actual seeker could contain multiple mixers and IF amplifier stages [12]. However, the net effect of the multiple stages can be represented by a single stage for purposes of this analysis.

The mixer heterodynes the RF signals to IF using a signal derived from the Doppler tracker, \( h(t) \). The Doppler tracker produces a sinusoid with a frequency of 

\[
 f_c + f_{IF} + f_{drk}
\]

where \( f_c \) is the RF, \( f_{IF} \) is the IF and \( f_{drk} \) is the Doppler frequency produced by the Doppler tracker. In a single target scenario, the Doppler tracker keeps \( f_{drk} \) very close to the target Doppler frequency so that the signals out of the mixers are very close to \( f_{IF} \). For two targets, the Doppler tracker would yield a Doppler frequency that is a form of centroid between the Doppler frequencies of the two targets. In this manner, the relative Doppler frequency between the two targets is preserved. This property is critical to the analyses discussed herein. A Doppler tracker was not modeled.
as part of this analysis. Rather, $f_{drk}$ was artificially set to the average of the Doppler frequencies of the two targets.

The output of the sum channel mixer (the top mixer in Figure 2.1) goes to both an IF amplifier and a bandpass limiter (BPL). The BPL consists of a voltage limiter followed by a bandpass filter that isolates the spectral line nominally centered on the IF. The purpose of the BPL is to provide a constant amplitude reference signal for the synchronous detector.

The two IF amplifiers are identical with gains controlled by an Automatic Gain Control (AGC). The AGC regulates the amplitude of the sum channel IF amplifier output to a fixed steady-state value by essentially dividing by a signal proportional to the amplitude of the signal into the sum channel IF amplifier. The action of the AGC on the difference channel amplifier output provides the difference-to-sum ratio needed by the angle track loop. The sign of this ratio is provided by the synchronous detector.

The output of the synchronous detector, $e_s(t)$, is multiplied by a scale factor, $\theta_d$, to produce an error signal, $e(t)$, that is proportional to the angle between the target (for the single target case) and the pointing angle of the antenna. The combination of seeker receiver components of Figure 2.1 is termed the angle discriminator of the seeker.

The mixer and synchronous detector symbol (⊗) of Figure 2.1 consist of a signal multiplier followed by a low-pass filter (LPF), as shown in Figure 2.2. The multiplier output contains a signal with frequency components at the sum and difference of the frequencies of the two signals into the multiplier. The LPF eliminates the component at
the sum of the two input frequencies and passes the component at the difference of the two input frequencies (see Figure 2.2).

![Diagram of Signal Mixer Operations]

**Figure 2.2 Signal Mixer Operations**

The next two sections contain the mathematics that describe how target signals are processed through the receiver and discriminator of Figure 2.1 to produce the error signal, \( e(t) \). Development begins with consideration of a single target and then progresses to consideration of two targets.

### 2.1 Single Target

The sum and difference signals into the mixer for the return from a single target can be written as

\[
\Sigma(t) = S_i G_{\Sigma} (\Delta \theta) \cos(2\pi f_s t + 2\pi f_d t) \quad (2.1)
\]

and

\[
\Delta(t) = S_i G_{\Delta} (\Delta \theta) \cos(2\pi f_s t + 2\pi f_d t) \quad (2.2)
\]

where
$S_\gamma$ is the target amplitude, in v.

$G_\Sigma(\Delta \theta)$ is the antenna sum pattern gain, in V / V.

$G_\Delta(\Delta \theta)$ is the antenna difference pattern gain, in V / V.

$f_c$ is the carrier frequency, in Hz.

$f_d$ is the target Doppler frequency, in Hz.

$\Delta \theta = \theta_{tgt} - \theta_{trk}$ is the angle error, with $\theta_{tgt}$ as the angle of the target and $\theta_{trk}$ as the angle to which the antenna is steered. $\theta_{trk}$ is determined by the angle tracker. For notational convenience, the antenna gains are represented by $G_\Sigma = G_\Sigma(\Delta \theta)$ and $G_\Delta = G_\Delta(\Delta \theta)$ henceforth.

The heterodyne signal from the Doppler tracker, $h(t)$, (see Figure 2.1) is given by

$$h(t) = S_h \cos \left( 2\pi f_c t + 2\pi f_{dtrk} t + 2\pi f_{IF} t \right)$$

(2.3)

where

$S_h$ is the heterodyne signal amplitude, in V.

$f_{IF}$ is the intermediate frequency, in Hz.

$f_{dtrk}$ is the Doppler frequency out of the Doppler tracker, in Hz.

As indicated earlier, $f_{dtrk}$ is approximately equal to $f_d$ for a single target, and to the average of the target Doppler frequencies for two targets.

The sum and difference signals out of the mixers are

$$\Sigma_{IF}(t) = \Sigma(t) \otimes h(t)$$

$$= \text{LPF}\left\{ S_\gamma G_\Sigma \cos \left( 2\pi f_c t + 2\pi f_{d} t \right) \times S_h \cos \left( 2\pi f_c t + 2\pi f_{dtrk} t + 2\pi f_{IF} t \right) \right\}$$
= LPF \left\{ \frac{S_s S_h G_s}{2} \cos \left( 4\pi f_s t + 2\pi f_d t + 2\pi f_{dtrk} t + 2\pi f_{IF} t \right) \\
+ \frac{S_s S_h G_s}{2} \cos \left( 2\pi f_d t - 2\pi f_{dtrk} t - 2\pi f_{IF} t \right) \right\} \\
= \frac{S_s S_h G_s}{2} \cos \left( 2\pi f_d t - 2\pi f_{dtrk} t - 2\pi f_{IF} t \right) \\
(2.4)

\text{and}

\Delta_{IF} (t) = \Delta (t) \otimes h(t)

= LPF \left\{ S_s G_s \cos \left( 2\pi f_c t + 2\pi f_d t \right) \times S_h \cos \left( 2\pi f_c t + 2\pi f_{dtrk} t + 2\pi f_{IF} t \right) \right\} \\
= LPF \left\{ \frac{S_s S_h G_s}{2} \cos \left( 4\pi f_c t + 2\pi f_d t + 2\pi f_{dtrk} t + 2\pi f_{IF} t \right) \\
+ \frac{S_s S_h G_s}{2} \cos \left( 2\pi f_d t - 2\pi f_{dtrk} t - 2\pi f_{IF} t \right) \right\} \\
= \frac{S_s S_h G_s}{2} \cos \left( 2\pi f_d t - 2\pi f_{dtrk} t - 2\pi f_{IF} t \right) \\
(2.5)

where the nomenclature LPF\{\ldots\} represents the signal out of the low-pass filter of the mixer of Figure 2.1.

The signal at the bandpass limiter output is

\Sigma_{lim} (t) = \Sigma_{IF} (t) \big|_{\text{limited}}

= \frac{S_s S_h G_s}{2} \cos \left( 2\pi f_d t - 2\pi f_{dtrk} t - 2\pi f_{IF} t \right) \big|_{\text{limited}}
\[ V_{\text{lim}} \cos \left( 2\pi f_d t - 2\pi f_{\text{drk}} t - 2\pi f_{\text{IF}} t \right), \tag{2.6} \]

where \( V_{\text{lim}} \) is the limited (constant) voltage level.

In steady-state, the AGC regulates the gains of the two IF amplifiers in order to maintain the amplitude of the sum channel IF amplifier output at some constant value of \( V \). It does this by effectively dividing the input of both amplifiers by the amplitude of the sum signal and multiplying them by \( V \). Mathematically, this is written as

\[ G_{\text{AGC}}(t) = \frac{V}{|\Sigma_{\text{IF}}(t)|} = \frac{2V}{S_1 S_h G_x}, \tag{2.7} \]

where \( V \) is the voltage output of the main IF amplifier.

With this, the output of the IF sum and difference channel amplifiers of Figure 2.1 can be written as

\[ \Sigma_{\text{AGC}}(t) = G_{\text{AGC}}(t) \Sigma_{\text{IF}}(t) \]

\[ = \frac{2V}{S_1 S_h G_x} \left[ \frac{S_1 S_h G_x}{2} \cos \left( 2\pi f_d t - 2\pi f_{\text{drk}} t - 2\pi f_{\text{IF}} t \right) \right] \]

\[ = V \cos \left( 2\pi f_d t - 2\pi f_{\text{drk}} t - 2\pi f_{\text{IF}} t \right) \tag{2.8} \]

and

\[ \Delta_{\text{AGC}}(t) = G_{\text{AGC}}(t) \Delta_{\text{IF}}(t) \]

\[ = \frac{2V}{S_1 S_h G_x} \left[ \frac{S_1 S_h G_x}{2} \cos \left( 2\pi f_d t - 2\pi f_{\text{drk}} t - 2\pi f_{\text{IF}} t \right) \right] \]
\[ V \frac{G_{\Delta}}{G_{\Sigma}} \cos (2\pi f_d t - 2\pi f_{d\text{dr}} t - 2\pi f_{f_{\text{IF}}} t). \]  

(2.9)

The output of the synchronous detector, \( e_s(t) \), is formed by mixing the amplitude-limited sum signal, \( \Sigma_{\text{lim}}(t) \), with the AGC-normalized difference signal, \( \Delta_{\text{AGC}}(t) \). Mathematically,

\[ e_s(t) = \Sigma_{\text{lim}}(t) \otimes \Delta_{\text{AGC}}(t) \]

\[ = \text{LPF}\left\{ V_{\text{lim}} \cos (2\pi f_d t - 2\pi f_{d\text{dr}} t - 2\pi f_{f_{\text{IF}}} t) \right\} \]

\[ \quad \times V \frac{G_{\Delta}}{G_{\Sigma}} \cos (2\pi f_d t - 2\pi f_{d\text{dr}} t - 2\pi f_{f_{\text{IF}}} t) \}

\[ = \text{LPF}\left\{ V_{\text{lim}} \frac{V}{2} \frac{G_{\Delta}}{G_{\Sigma}} \cos (4\pi f_d t - 4\pi f_{d\text{dr}} t - 4\pi f_{f_{\text{IF}}} t) + \frac{V_{\text{lim}} V}{2} \frac{G_{\Delta}}{G_{\Sigma}} \right\} \]

\[ = \frac{V_{\text{lim}} V}{2} \frac{G_{\Delta}}{G_{\Sigma}}. \]  

(2.10)

Finally, the output of the synchronous detector is scaled by \( \theta_d \) to produce the error signal, \( e(t) \), that is sent to the angle track filter. Specifically,

\[ e(t) = e_s(t) \theta_d \]

\[ = \frac{V_{\text{lim}} V}{2} \frac{G_{\Delta}}{G_{\Sigma}} \theta_d. \]  

(2.11)

Assuming \( \theta_d = \frac{2}{V_{\text{lim}}} \), Equation (2.11) results in
\[ e(t) = \frac{G_z(\Delta \theta)}{G_x(\Delta \theta)} \]  \hspace{1cm} (2.12)

where the argument \( \Delta \theta \) has been reintroduced to explicitly denote the fact that \( G_x \) and \( G_\lambda \) are a function of the angle separation, \( \Delta \theta \), between the target location and the antenna pointing angle.

### 2.2 Dual Targets

The sum and difference signals into the mixer for the two target case can be written as

\[
\Sigma(t) = S_1G_x(\Delta \theta_1) \cos(2\pi f_c t + 2\pi f_{d1} t) + S_2G_x(\Delta \theta_2) \cos(2\pi f_c t + 2\pi f_{d2} t) \]  \hspace{1cm} (2.13)

and

\[
\Delta(t) = S_1G_x(\Delta \theta_1) \cos(2\pi f_c t + 2\pi f_{d1} t) + S_2G_x(\Delta \theta_2) \cos(2\pi f_c t + 2\pi f_{d2} t) \]  \hspace{1cm} (2.14)

where

- \( S_n \) is the target \( n \) \( (n = 1, 2) \) amplitude, in V.
- \( G_x(\Delta \theta_n) \) is the antenna sum pattern gain for target \( n \), in V / V.
- \( G_\lambda(\Delta \theta_n) \) is the antenna difference pattern gain for target \( n \), in V / V.
- \( f_c \) is the carrier frequency, in Hz.
- \( f_{dn} \) is target \( n \) Doppler frequency, in Hz.

As before, \( \Delta \theta_n = \theta_{tn} - \theta_{nk} \) is the angle error between target \( n \) and the antenna beam pointing angle. The convention \( G_{\Sigma n} = G_x(\Delta \theta_n) \) and \( G_{\Delta n} = G_\lambda(\Delta \theta_n) \) is used to simplify notation.

The heterodyne signal, \( h(t) \), is given by
\[ h(t) = S_h \cos \left( 2\pi f_{\text{IF}} t + 2\pi f_{\text{dtrk}} t + 2\pi f_{\text{IP}} t \right) \]  

(2.15)

where

\( S_h \) is the heterodyne signal amplitude, in V.
\( f_{\text{IF}} \) is the intermediate frequency, in Hz.
\( f_{\text{dtrk}} \) is the Doppler frequency out of the Doppler tracker, in Hz.

The IF signal out of the sum channel mixer is

\[ \Sigma_{\text{IF}}(t) = \Sigma(t) \otimes h(t) \]

\[ = \text{LPF}\left\{ \frac{S_h G_{S1}}{2} \cos \left( 4\pi f_{\text{IF}} t + 2\pi f_{d1} t + 2\pi f_{\text{dtrk}} t + 2\pi f_{\text{IP}} t \right) \right\} \]

\[ + \frac{S_h G_{S1}}{2} \cos \left( 2\pi f_{d1} t - 2\pi f_{\text{dtrk}} t - 2\pi f_{\text{IP}} t \right) \]

\[ + \frac{S_h G_{S2}}{2} \cos \left( 4\pi f_{\text{IF}} t + 2\pi f_{d2} t + 2\pi f_{\text{dtrk}} t + 2\pi f_{\text{IP}} t \right) \]

\[ + \frac{S_h G_{S2}}{2} \cos \left( 2\pi f_{d2} t - 2\pi f_{\text{dtrk}} t - 2\pi f_{\text{IP}} t \right) \}

\[ = \frac{S_h G_{S1}}{2} \cos \left( 2\pi f_{d1} t - 2\pi f_{\text{dtrk}} t - 2\pi f_{\text{IP}} t \right) \]  

(2.16)

Similarly, the IF signal out of the difference channel mixer is

\[ \Delta_{\text{IF}}(t) = \Delta(t) \otimes h(t) \]
As with the single target case, the AGC regulates the gains of the two IF amplifiers to maintain the amplitude of the sum channel IF amplifier output at some constant value of $V$. It does this by effectively dividing the input of both amplifiers by the overall amplitude of the sum signal, and multiplying them by $V$. To determine the overall amplitude of the sum signal, the two terms in $\Sigma_{IF}(t)$ must be combined, which is accomplished by manipulating the terms of the second cosine of $\Sigma_{IF}(t)$. Specifically, by adding and subtracting $2\pi f_{d1}t$ to the argument of the cosine, $\Sigma_{IF}(t)$ can be written as

\[
\begin{align*}
\Sigma_{IF}(t) &= \frac{S_1 S_h G_{A1}}{2} \cos(2\pi f_{d1}t - 2\pi f_{d1}t - 2\pi f_{d1}t - 2\pi f_{d1}t) \\
&\quad + \frac{S_2 S_h G_{A2}}{2} \cos(2\pi f_{d2}t - 2\pi f_{d2}t - 2\pi f_{d2}t - 2\pi f_{d2}t) \\
&\quad + \frac{S_2 S_h G_{A1}}{2} \cos(2\pi f_{d1}t - 2\pi f_{d1}t - 2\pi f_{d1}t - 2\pi f_{d1}t) \\
&\quad + \frac{S_2 S_h G_{A2}}{2} \cos(2\pi f_{d2}t - 2\pi f_{d2}t - 2\pi f_{d2}t - 2\pi f_{d2}t).
\end{align*}
\]

The second cosine term can be expanded to yield
\[ \Sigma_{IF}(t) = \frac{S_1 S_h G_{z1}}{2} \cos(2\pi f_{dt} t - 2\pi f_{drtk} t - 2\pi f_{IF} t) \\
+ \frac{S_2 S_h G_{z2}}{2} \cos(2\pi f_{dt} t - 2\pi f_{drtk} t - 2\pi f_{IF} t) \cos(2\pi f_{d2} t - 2\pi f_{dt} t) \\
- \frac{S_2 S_h G_{z2}}{2} \sin(2\pi f_{dt} t - 2\pi f_{drtk} t - 2\pi f_{IF} t) \sin(2\pi f_{d2} t - 2\pi f_{dt} t). \]

Combining the first two terms results in

\[ \Sigma_{IF}(t) = \left[ \frac{S_1 S_h G_{z1}}{2} + \frac{S_2 S_h G_{z2}}{2} \cos(2\pi f_{dt} t - 2\pi f_{dt} t) \right] \cos(2\pi f_{dt} t - 2\pi f_{drtk} t - 2\pi f_{IF} t) \\
- \left[ \frac{S_2 S_h G_{z2}}{2} \sin(2\pi f_{dt} t - 2\pi f_{dt} t) \right] \sin(2\pi f_{dt} t - 2\pi f_{drtk} t - 2\pi f_{IF} t). \]

Equation (2.18) is of the form

\[ \Sigma_{IF}(t) = \left[ A(t) \cos(\theta(t)) \right] \cos(2\pi f_{dt} t - 2\pi f_{drtk} t - 2\pi f_{IF} t) \\
- \left[ A(t) \sin(\theta(t)) \right] \sin(2\pi f_{dt} t - 2\pi f_{drtk} t - 2\pi f_{IF} t). \]

(2.19)

where

\[ A(t) \cos(\theta(t)) = \frac{S_1 S_h G_{z1}}{2} + \frac{S_2 S_h G_{z2}}{2} \cos(2\pi f_{d2} t - 2\pi f_{dt} t) \]

(2.20)

and

\[ A(t) \sin(\theta(t)) = \frac{S_2 S_h G_{z2}}{2} \sin(2\pi f_{d2} t - 2\pi f_{dt} t). \]

(2.21)

This leads to the observation that

\[ A(t) = \sqrt{\left[ A(t) \cos(\theta(t)) \right]^2 + \left[ A(t) \sin(\theta(t)) \right]^2}. \]
\[
\sqrt{\left[ \frac{S_1 S_h G_{z1}}{2} + \frac{S_2 S_h G_{z2}}{2} \cos \left( 2\pi f_{d2} t - 2\pi f_{d1} t \right) \right]^2 + \left[ \frac{S_2 S_h G_{z2}}{2} \sin \left( 2\pi f_{d2} t - 2\pi f_{d1} t \right) \right]^2} \\
= \sqrt{\left( \frac{S_1 S_h G_{z1}}{2} \right)^2 + \left( \frac{S_2 S_h G_{z2}}{2} \right)^2 + 2 \left( \frac{S_1 S_h G_{z1}}{2} \right) \left( \frac{S_2 S_h G_{z2}}{2} \right) \cos \left( 2\pi f_{d2} t - 2\pi f_{d1} t \right)}
\]

(2.22)

and

\[
\theta(t) = \tan^{-1} \left( \frac{A(t) \sin \left( \theta(t) \right)}{A(t) \cos \left( \theta(t) \right)} \right).
\]

Introducing the following terms will simplify notation during further analysis:

\[
A_1 = \frac{S_1 S_h G_{z1}}{2},
\]

(2.23)

\[
A_2 = \frac{S_2 S_h G_{z2}}{2},
\]

(2.24)

\[
\Delta \varphi(t) = 2\pi f_{d2} t - 2\pi f_{d1} t,
\]

(2.25)

and

\[
R = \frac{A_2}{A_1} = \frac{S_2 G_{z2}}{S_1 G_{z1}}.
\]

(2.26)

This allows \( A(t) \) to be written as
\[ A(t) = A_0 \sqrt{1 + R^2 + 2R \cos(\Delta \varphi(t))}. \] \hspace{1cm} (2.27)

Using the above definitions of \( A(t) \) and \( \theta(t) \) allows \( \Sigma_{IF}(t) \) to be written as

\[ \Sigma_{IF}(t) = A(t) \cos\left(2\pi f_{d,t}t - 2\pi f_{drk,t}t - 2\pi f_{IF}t + \theta(t)\right). \] \hspace{1cm} (2.28)

The signal at the band-pass limiter output is

\[
\Sigma_{\text{lim}}(t) = \left. \Sigma_{IF}(t) \right|_{\text{limited}} \\
= A(t) \cos\left(2\pi f_{d,t}t - 2\pi f_{drk,t}t - 2\pi f_{IF}t + \theta(t)\right)_{\text{limited}} \\
= V_{\text{lim}} \cos\left(2\pi f_{d,t}t - 2\pi f_{drk,t}t - 2\pi f_{IF}t + \theta(t)\right), \hspace{1cm} (2.29)
\]

where \( V_{\text{lim}} \) is the limit level. In the remaining analysis, two distinct cases are considered based upon an assumption about the Doppler frequency separation between the two targets.

### 2.2.1 Doppler Separation Less Than the AGC Bandwidth

For case 1 it is assumed the Doppler frequency separation between the two targets is less than the AGC bandwidth (defined as region 2 in Figure 1.1). In this case, the AGC circuitry will be able to follow the variations in \( A(t) \) and produce a gain that is close to the inverse of \( A(t) \). In the limit, the AGC gain would be exactly the inverse of \( A(t) \). That is,

\[ G_{AGC}(t) = \frac{V}{A(t)}, \] \hspace{1cm} (2.30)
where $V$ is the regulated voltage output of the IF amplifier. This limiting case corresponds to an infinite bandwidth AGC and is often termed the instantaneous AGC case. With Equation (2.30), the output of the IF amplifier in the sum channel is

$$\Sigma_{AGC} (t) = G_{AGC}(t) \Sigma_{IF} (t)$$

$$= \frac{V}{A(t)} \left[ A(t) \cos(2\pi f_{d1} t - 2\pi f_{dfrk} t - 2\pi f_{IF} t + \theta(t)) \right]$$

$$= V \cos\left(2\pi f_{d1} t - 2\pi f_{dfrk} t - 2\pi f_{IF} t + \theta(t)\right). \quad (2.31)$$

The output of the IF amplifier in the difference channel is

$$\Delta_{AGC} (t) = G_{AGC}(t) \Delta_{IF} (t)$$

$$= \frac{V}{A(t)} \left[ \frac{S_h S_h G_{M1}}{2} \cos(2\pi f_{d1} t - 2\pi f_{dfrk} t - 2\pi f_{IF} t) 
+ \frac{S_2 S_h G_{M2}}{2} \cos(2\pi f_{d2} t - 2\pi f_{dfrk} t - 2\pi f_{IF} t) \right]$$

$$= \frac{V S_h S_h G_{M1}}{2A(t)} \cos(2\pi f_{d1} t - 2\pi f_{dfrk} t - 2\pi f_{IF} t) 
+ \frac{V S_2 S_h G_{M2}}{2A(t)} \cos(2\pi f_{d2} t - 2\pi f_{dfrk} t - 2\pi f_{IF} t). \quad (2.32)$$

The difference and amplitude-limited sum signals (see Equation (2.29)) are processed through the synchronous detector to yield

$$e_s (t) = \Sigma_{lim} (t) \otimes \Delta_{AGC} (t)$$
\[
= \text{LPF} \left\{ V_{\text{lim}} \cos \left( 2\pi f_{d1} t - 2\pi f_{d2} t - 2\pi f_{\text{IF}} t + \theta(t) \right) \right.
\times \left[ \frac{V_{S1} S_h G_{\Delta 1}}{2A(t)} \cos \left( 2\pi f_{d1} t - 2\pi f_{d2} t - 2\pi f_{\text{IF}} t \right) \\
+ \frac{V_{S2} S_h G_{\Delta 2}}{2A(t)} \cos \left( 2\pi f_{d2} t - 2\pi f_{d1} t - 2\pi f_{\text{IF}} t \right) \right] \right\}
\]

or

\[
e(t) = \frac{V_{\text{lim}} V_{S1} S_h G_{\Delta 1}}{4A(t)} \cos \theta(t) + \frac{V_{\text{lim}} V_{S2} S_h G_{\Delta 2}}{4A(t)} \cos \left( \theta(t) - (2\pi f_{d2} t - 2\pi f_{d1} t) \right). \tag{2.33}
\]

Using Equations (2.23) and (2.24), Equation (2.33) can be rewritten as

\[
e(t) = \frac{V_{\text{lim}} V_{A1} G_{\Delta 1}}{2A(t)} \cos \theta(t) + \frac{V_{\text{lim}} V_{A2} G_{\Delta 2}}{2A(t)} \cos \left( \theta(t) - \Delta \phi(t) \right) \tag{2.34}
\]

where

\[
\Delta \phi(t) = 2\pi f_{d2} t - 2\pi f_{d1} t = 2\pi(f_{d2} - f_{d1}) t.
\]

Expanding the second cosine term in Equation (2.34) results in
\[ e_s(t) = \frac{V_{\text{lim}}}{2A(t)} G_{\Delta_1} \cos(\theta(t)) + \frac{V_{\text{lim}}}{2A(t)} G_{\Delta_2} \cos(\theta(t)) \cos(\Delta \phi(t)) \]
\[ + \frac{V_{\text{lim}}}{2A(t)} G_{\Delta_2} \sin(\theta(t)) \sin(\Delta \phi(t)) \]  
(2.35)

or

\[ e_s(t) = \frac{V_{\text{lim}}}{2A(t)} \left[ A_1 \frac{G_{\Delta_1}}{G_{\Sigma_1}} + A_2 \frac{G_{\Delta_2}}{G_{\Sigma_2}} \cos(\Delta \phi(t)) \right] \cos(\theta(t)) \]
\[ + \frac{V_{\text{lim}}}{2A(t)} \left[ A_2 \frac{G_{\Delta_2}}{G_{\Sigma_2}} \sin(\Delta \phi(t)) \right] \sin(\theta(t)). \]  
(2.36)

Referring to the expansion of \( \Sigma_{HF}(t) \) (see Equation (2.19)), it was observed that

\[ A(t) \sin(\theta(t)) = A_2 \sin(\Delta \phi(t)) \]

and

\[ A(t) \cos(\theta(t)) = A_1 + A_2 \cos(\Delta \phi(t)), \]

which can be rearranged as

\[ \sin(\theta(t)) = \frac{A_2 \sin(\Delta \phi(t))}{A(t)} \]  
(2.37)

and

\[ \cos(\theta(t)) = \frac{A_1 + A_2 \cos(\Delta \phi(t))}{A(t)}. \]  
(2.38)

Substituting Equations (2.37) and (2.38) into \( e_s(t) \) and manipulating produces
\[
e_s(t) = \frac{V_{\text{lim}} V}{2} \left[ \frac{G_{\Delta_1}}{G_{\Sigma_1}} \left( \frac{A_1^2 + A_1 A_2 \cos(\Delta \varphi(t))}{A^2(t)} \right) + \frac{G_{\Delta_2}}{G_{\Sigma_2}} \left( \frac{A_2^2 + A_1 A_2 \cos(\Delta \varphi(t))}{A^2(t)} \right) \right].
\]

Finally, substituting for \( A^2(t) \) (see Equation (2.27)) and manipulating produces the final value of \( e_s(t) \) as

\[
e_s(t) = \frac{V_{\text{lim}} V}{2} \left[ \frac{G_{\Delta_1}}{G_{\Sigma_1}} \left( \frac{1 + R \cos(\Delta \varphi(t))}{1 + R^2 + 2R \cos(\Delta \varphi(t))} \right) \right.
+ \left. \frac{G_{\Delta_2}}{G_{\Sigma_2}} R \left( \frac{R + \cos(\Delta \varphi(t))}{1 + R^2 + 2R \cos(\Delta \varphi(t))} \right) \right].
\] (2.39)

As done in the single target case, the error signal \( e_s(t) \) is scaled by \( \theta_d \) to give

\[
e(t) = e_s(t) \theta_d
= \frac{V_{\text{lim}} V}{2} \theta_d \left[ \frac{G_{\Delta_1}}{G_{\Sigma_1}} \left( \frac{1 + R \cos(\Delta \varphi(t))}{1 + R^2 + 2R \cos(\Delta \varphi(t))} \right) \right.
+ \left. \frac{G_{\Delta_2}}{G_{\Sigma_2}} R \left( \frac{R + \cos(\Delta \varphi(t))}{1 + R^2 + 2R \cos(\Delta \varphi(t))} \right) \right].
\] (2.40)

Recall that \( \Delta \varphi(t) = 2\pi (f_{d_2} - f_{d_1}) t = 2\pi (\Delta f) t \) (See Equation (2.25)). This means \( e(t) \) is a non-linear function of \( \Delta f \) and will have a DC term and harmonics at multiples of \( \Delta f \). The signal of interest is the steady-state error signal, \( e_{\text{dc}}(t) \), which is the DC component of \( e(t) \). From Fourier series analysis, the DC component of \( e(t) \) is given by

\[
e_{\text{dc}} = \Delta f \int_{0}^{1/\Delta f} e(t) dt.
\]
Using the change of variable of $\Delta \varphi(t) = 2\pi \Delta f t$ results in

$$e_{DC} = \frac{1}{2\pi} \int_{0}^{2\pi} e(\Delta \varphi(t)) d\Delta \varphi(t)$$

or, using Equation (2.40),

$$e_{DC} = \frac{V_{lim} V}{4\pi} \theta_d \int_{0}^{2\pi} \left[ \frac{G_{A1}}{G_{S1}} \left( \frac{1 + R \cos(\Delta \varphi(t))}{1 + R^2 + 2R \cos(\Delta \varphi(t))} \right) \right. + \left. \frac{G_{A2}}{G_{S2}} R \left( \frac{R \cos(\Delta \varphi(t))}{1 + R^2 + 2R \cos(\Delta \varphi(t))} \right) \right] d\Delta \varphi(t).$$

Evaluation of the integral of Equation (2.41) is provided in Appendix A. With the results of the appendix, $e_{DC}$ can be written as

$$e_{DC} = \begin{cases} 
\frac{V_{lim} V}{2} \frac{G_{A1}}{G_{S1}} \theta_d, & \text{when } A_i > A_2 \\
\frac{V_{lim} V}{2} \frac{G_{A2}}{G_{S2}} \theta_d, & \text{when } A_2 > A_i \\
\frac{V_{lim} V}{4} \left( \frac{G_{A1} + G_{A2}}{G_{S1} + G_{S2}} \right) \theta_d, & \text{when } A_i = A_2 
\end{cases}$$

With $\theta_d = \frac{2}{V_{lim} V}$, Equation (2.42) becomes

$$e_{DC} = \begin{cases} 
\frac{G_{A1}}{G_{S1}}, & \text{when } A_i > A_2 \\
\frac{G_{A2}}{G_{S2}}, & \text{when } A_2 > A_i \\
\frac{1}{2} \left( \frac{G_{A1} + G_{A2}}{G_{S1} + G_{S2}} \right), & \text{when } A_i = A_2 
\end{cases}$$
Using the definitions $A_1 = \frac{S_1}{2} S_G S_1$ and $A_2 = \frac{S_2}{2} S_G S_2$ (see Equations (2.23) and (2.24)),

$e_{DC}$ can be written as

$$e_{DC} = \begin{cases} 
\frac{G_{\Delta 1}}{G_{\Delta 1}}, & \text{when } S_1 G_{\Sigma 1} > S_2 G_{\Sigma 2} \\
\frac{G_{\Delta 2}}{G_{\Delta 2}}, & \text{when } S_2 G_{\Sigma 2} > S_1 G_{\Sigma 1} \\
\frac{1}{2} \left( \frac{G_{\Delta 1}}{G_{\Delta 1}} + \frac{G_{\Delta 2}}{G_{\Delta 2}} \right), & \text{when } S_1 G_{\Sigma 1} = S_2 G_{\Sigma 2}
\end{cases}$$

(2.44)

It is interesting to note when the target 1 signal in the sum channel ($S_1 G_{\Sigma 1}$) is greater than the target 2 signal in the sum channel ($S_2 G_{\Sigma 2}$), the steady-state angle error signal depends only on the target 1 signals in the difference and sum channels. Similarly, if the target 2 signal in the sum channel is larger than the target 1 sum channel signal, the steady-state angle error signal depends only on the target 2 signals in the difference and sum channels. If the two target signals in the sum channel are the same, the steady-state error is a weighted sum of the ratio of the two difference-to-sum channel signals. The impact of this is that if $S_1 G_{\Sigma 1} > S_2 G_{\Sigma 2}$, the angle tracker will track target 1 and if $S_2 G_{\Sigma 2} > S_1 G_{\Sigma 1}$, the angle tracker will track target 2. In other words, the angle tracker dichotomously tracks the two targets.

The above phenomenon becomes clearer if one considers ideal antenna patterns. That is, if $G_{\Delta n} (\Delta \theta_n) = 1$ and $G_{\Delta n} (\Delta \theta_n) = \Delta \theta_n$. With

$$G_{\Sigma 1} (\Delta \theta_1) = G_{\Sigma 2} (\Delta \theta_2) = 1,$$

(2.45)
\[ G_{\Delta 1}(\Delta \theta_1) = \Delta \theta_1, \]  

(2.46)

and

\[ G_{\Delta 2}(\Delta \theta_2) = \Delta \theta_2, \]  

(2.47)

Equation (2.44) becomes

\[
e_{DC} = \begin{cases} 
\Delta \theta_1, & \text{when } S_1 G_{\Sigma 1} > S_2 G_{\Sigma 2} \\
\Delta \theta_2, & \text{when } S_2 G_{\Sigma 2} > S_1 G_{\Sigma 1} \\
\frac{\Delta \theta_1 + \Delta \theta_2}{2}, & \text{when } S_1 G_{\Sigma 1} = S_2 G_{\Sigma 2}
\end{cases}
\]  

(2.48)

Equation (2.48) clearly illustrates the aforementioned observation that when the Doppler frequency separation between the two target signals is less than the AGC bandwidth, the error angle track signal, \( e_{DC} \), will depend on the angle location of the target with the largest sum channel amplitude. Thus, the angle tracker will tend to track the larger target. When the sum channel signal amplitudes are equal, the angle tracker will point to the average of the two angle offsets.

### 2.2.2 Doppler Separation Greater Than the AGC Bandwidth

For case 2, it is assumed the Doppler frequency separation between the two targets is greater than the AGC bandwidth (defined as region 3 in Figure 1.1). In that case, the AGC circuitry will not be able to follow variations in the received target signals caused by Doppler frequency interaction. Rather, the AGC circuitry will produce a gain inversely proportional to the root-mean-square (RMS) of the IF sum channel input, \( \Sigma_{RMS} \).
This can be modeled as a limiting case by assuming a zero-bandwidth AGC. This results in what is termed a slow AGC. Under the AGC limitations of this case, the AGC gain is mathematically defined as

\[
G_{\text{AGC}} (t) = \frac{V}{\Sigma_{\text{RMS}}},
\]

where \( V \) is the regulated voltage output of the IF amplifier, and the RMS value of \( \Sigma_{\text{IF}} (t) \) is

\[
\Sigma_{\text{RMS}} = \sqrt{\lim_{T \to \infty} \frac{1}{T} \int_0^T \Sigma_{\text{IF}}^2 (t) \, dt},
\]

where the limit as \( T \) approaches infinity is dictated by the fact that the AGC bandwidth is zero. Solving for \( \Sigma_{\text{IF}}^2 (t) \) using Equation (2.28) results in

\[
\Sigma_{\text{IF}}^2 (t) = A_1 \sqrt{1 + R^2 + 2R \cos (\Delta \phi(t)) \cos \left( 2\pi \left( f_{d1} - f_{\text{drk}} - f_{\text{IF}} \right) t + \theta(t) \right)} \bigg]^2
\]

\[
= A_1^2 \left[ 1 + R^2 + 2R \cos (\Delta \phi(t)) \right] \cos^2 \left( 2\pi \left( f_{d1} - f_{\text{drk}} - f_{\text{IF}} \right) t + \theta(t) \right).
\]

Expanding \( \cos^2 \left( 2\pi \left( f_{d1} - f_{\text{drk}} - f_{\text{IF}} \right) t + \theta(t) \right) \) and manipulating

\[
\Sigma_{\text{IF}}^2 (t) = A_1^2 \left[ 1 + R^2 \right] - \frac{A_1^2 \left( 1 + R^2 \right)}{2} \cos \left( 4\pi \left( f_{d1} - f_{\text{drk}} - f_{\text{IF}} \right) t + 2\theta(t) \right) + A_1^2 R \cos (\Delta \phi(t)) - A_1^2 R \cos (\Delta \phi(t)) \cos \left( 4\pi \left( f_{d1} - f_{\text{drk}} - f_{\text{IF}} \right) t + 2\theta(t) \right).
\]

Expanding using the product of cosines yields
\[
\Sigma_{IF}^2(t) = \frac{A_i^2(1+R^2)}{2} - \frac{A_i^2(1+R^2)}{2} \cos(4\pi(f_{d1} - f_{dtrk} - f_{IF})t + 2\theta(t)) \\
+ \frac{A_i^2R}{2} \cos(\Delta\varphi(t)) - \frac{A_i^2R}{2} \cos(4\pi(f_{d1} - f_{dtrk} - f_{IF})t + 2\theta(t) + \Delta\varphi(t)) \\
- \frac{A_i^2R}{2} \cos(4\pi(f_{d1} - f_{dtrk} - f_{IF})t + 2\theta(t) - \Delta\varphi(t)).
\]

(2.51)

Using Equation (2.51) in Equation (2.50) gives

\[
\Sigma_{RMS} = \left\{ \lim_{T \to \infty} \frac{1}{T} \int_0^T \left[ \frac{A_i^2(1+R^2)}{2} - \frac{A_i^2(1+R^2)}{2} \cos(4\pi(f_{d1} - f_{dtrk} - f_{IF})t + 2\theta(t)) \\
+ \frac{A_i^2R}{2} \cos(\Delta\varphi(t)) \\
- \frac{A_i^2R}{2} \cos(4\pi(f_{d1} - f_{dtrk} - f_{IF})t + 2\theta(t) + \Delta\varphi(t)) \\
- \frac{A_i^2R}{2} \cos(4\pi(f_{d1} - f_{dtrk} - f_{IF})t + 2\theta(t) - \Delta\varphi(t)) \right] dt \right\}^{\frac{1}{2}}.
\]

As \( T \) approaches infinity, the terms containing the cosines go to zero which leaves only the first term of the argument of the integral. Thus,

\[
\Sigma_{RMS} = \sqrt{\frac{A_i^2(1+R^2)}{2}} \\
= A_i \sqrt{\frac{1+R^2}{2}}. 
\]

(2.52)

With this, the output of the IF amplifier in the sum channel is

\[
\Sigma_{AGC}(t) = G_{AGC}(t) \Sigma_{IF}(t)
\]
Similarly, the output of the IF amplifier in the difference channel is

$$
\Delta_{AGC}(t) = G_{AGC}(t) \Delta_{IF}(t)
$$

$$
= \frac{V}{\Sigma_{RMS}} \left[ \frac{S_h S_{\Delta 1}}{2} \cos \left( 2\pi f_{d1}t - 2\pi f_{dtrk}t - 2\pi f_{IF}t \right) 
+ \frac{S_h^2 G_{\Delta 2}}{2} \cos \left( 2\pi f_{d2}t - 2\pi f_{dtrk}t - 2\pi f_{IF}t \right) \right].
$$

(2.54)

The difference and the amplitude-limited sum signal (see Equation (2.29)) are then processed through the synchronous detector to yield

$$
e_s(t) = \Sigma_{lim}(t) \otimes \Delta_{AGC}(t)
$$

$$
= \text{LPF} \left\{ \frac{V_{lim}}{4 \Sigma_{RMS}} \left[ \cos \left( 4\pi f_{d1}t - 4\pi f_{dtrk}t - 4\pi f_{IF}t + \theta(t) \right) + \cos \left( \theta(t) \right) \right] 
+ \frac{V_{lim}}{4 \Sigma_{RMS}} \left[ \cos \left( 2\pi f_{d1}t + 2\pi f_{d2}t - 4\pi f_{dtrk}t - 4\pi f_{IF}t + \theta(t) \right) 
+ \frac{V_{lim}}{4 \Sigma_{RMS}} \left[ \cos \left( 2\pi f_{d1}t - 2\pi f_{d2}t + \theta(t) \right) \right]
\right\}

= \frac{V_{lim} V S_h S_{\Delta 1}}{4 \Sigma_{RMS}} \cos \left( \theta(t) \right) + \frac{V_{lim} V S_h^2 G_{\Delta 2}}{4 \Sigma_{RMS}} \cos \left( 2\pi f_{d1}t - 2\pi f_{d2}t + \theta(t) \right)
$$

28
or

\[ e_s(t) = \frac{V_{\text{lim}}}{4\Sigma_{RMS}} V S I \bar{S}_A G_{\Delta A} \cos(\theta(t)) + \frac{V_{\text{lim}}}{4\Sigma_{RMS}} V S I \bar{S}_A G_{\Delta A} \cos(\theta(t) - (2\pi f_{dA} t - 2\pi f_{dI} t)) \]

(2.55)

Using Equations (2.23) and (2.24), Equation (2.55) can be rewritten as

\[ e_s(t) = \frac{V_{\text{lim}}}{2\Sigma_{RMS}} V A I \bar{S}_A G_{\Delta A} \cos(\theta(t)) + \frac{V_{\text{lim}}}{2\Sigma_{RMS}} V A I \bar{S}_A G_{\Delta A} \cos(\theta(t) - \Delta\varphi(t)) \]

(2.56)

where, again

\[ \Delta\varphi(t) = 2\pi f_{dA} t - 2\pi f_{dI} t = 2\pi(f_{dA} - f_{dI}) t. \]

Following the procedure of Section 2.2.1, Equation (2.56) can be manipulated to give

\[ e_s(t) = \frac{V_{\text{lim}}}{2\Sigma_{RMS}} V A I \bar{S}_A G_{\Delta A} \cos(\theta(t)) + \frac{V_{\text{lim}}}{2\Sigma_{RMS}} V A I \bar{S}_A G_{\Delta A} \cos(\theta(t)) \cos(\Delta\varphi(t)) \]

(2.57)

or

\[ e_s(t) = \frac{V_{\text{lim}}}{2\Sigma_{RMS}} V A I \bar{S}_A G_{\Delta A} \left[ A_1 G_{\Delta A} + A_2 G_{\Delta A} \cos(\Delta\varphi(t)) \right] \cos(\theta(t)) \]

(2.58)

or

\[ e_s(t) = \frac{V_{\text{lim}}}{2\Sigma_{RMS}} V A I \bar{S}_A G_{\Delta A} \left[ A_2 G_{\Delta A} \sin(\Delta\varphi(t)) \right] \sin(\theta(t)). \]

As done in the previous case (referring to the expansion of \( \Sigma_{IF}(t) \) – see Equation (2.19))

it is observed that

29
\[ A(t) \sin(\theta(t)) = A_2 \sin(\Delta \phi(t)) \]

and
\[ A(t) \cos(\theta(t)) = A_1 + A_2 \cos(\Delta \phi(t)) \]

which can be rearranged as
\[ \sin(\theta(t)) = \frac{A_2 \sin(\Delta \phi(t))}{A(t)} \]  \hspace{1cm} (2.59)

and
\[ \cos(\theta(t)) = \frac{A_1 + A_2 \cos(\Delta \phi(t))}{A(t)} . \]  \hspace{1cm} (2.60)

Substituting Equations (2.59) and (2.60) into \( e_s(t) \) and manipulating produces
\[
e_s(t) = \frac{V_{\text{lim}} V A_1}{2 \Sigma_{\text{RMS}}} \left[ G_{A1} \left( \frac{A_1^2 + A_1 A_2 \cos(\Delta \phi(t))}{A(t)} \right) + G_{A2} \left( \frac{A_2^2 + A_1 A_2 \cos(\Delta \phi(t))}{A(t)} \right) \right].
\]

Finally, substituting \( A(t) \) from Equation (2.27) and manipulating produces
\[
e_s(t) = \frac{V_{\text{lim}} V A_1}{2 \Sigma_{\text{RMS}}} \left[ G_{A1} \left( \frac{1 + R \cos(\Delta \phi(t))}{\sqrt{1 + R^2 + 2R \cos(\Delta \phi(t))}} \right) + G_{A2} R \left( \frac{R \cos(\Delta \phi(t))}{\sqrt{1 + R^2 + 2R \cos(\Delta \phi(t))}} \right) \right]. \hspace{1cm} (2.61)
\]

The error signal \( e_s(t) \) is scaled by \( \theta_d \) to give
\[ e(t) = e_s(t) \theta_d \]

\[ \frac{V_{\text{lim}} V}{2 \Sigma_{RMS}} \theta_d \left[ \frac{G_{\Delta 1}}{G_{\Delta 0}} \left( \frac{1 + R \cos(\Delta \phi(t))}{\sqrt{1 + R^2 + 2R \cos(\Delta \phi(t))}} \right) \right] + \frac{G_{\Delta 2}}{G_{\Delta 0}} R \left( \frac{R + \cos(\Delta \phi(t))}{\sqrt{1 + R^2 + 2R \cos(\Delta \phi(t))}} \right). \]  

(2.62)

As with the previous case, recall that \( \Delta \phi(t) = 2\pi(f_d - f_n) t = 2\pi(\Delta f) t \). This means \( e(t) \) is a non-linear function of \( \Delta f \), and will thus have a DC term and harmonics at multiples of \( \Delta f \). The signal of interest is the DC component of \( e(t) \), which is given by

\[ e_{DC} = \Delta f \int_0^{\Delta f} e(t) \, dt. \]

Using the change of variable of \( \Delta \phi(t) = 2\pi(\Delta f) t \) results in

\[ e_{DC} = \frac{1}{2\pi} \int_0^{2\pi} e(\Delta \phi(t)) \, d\Delta \phi(t) \]

(2.63)

or, using Equation (2.62),

\[ e_{DC} = \frac{V_{\text{lim}} V}{4 \pi \Sigma_{RMS}} \theta_d \left[ \frac{G_{\Delta 1}}{G_{\Delta 0}} \left( \frac{1 + R \cos(\Delta \phi(t))}{\sqrt{1 + R^2 + 2R \cos(\Delta \phi(t))}} \right) \right] + \frac{G_{\Delta 2}}{G_{\Delta 0}} R \left( \frac{R + \cos(\Delta \phi(t))}{\sqrt{1 + R^2 + 2R \cos(\Delta \phi(t))}} \right) \, d\Delta \phi(t). \]  

(2.64)
Evaluation of the integral of Equation (2.64) is provided in Appendix B. With the results of the appendix, Equation (2.64) can be written as

\[
e_{DC} = \frac{V_{lim} V A}{2\pi \sigma_{RMS}} \theta_d \left[ \left( \frac{G_{\Delta_1}}{G_{\Sigma_1}} - \frac{G_{\Delta_2}}{G_{\Sigma_2}} \right) (1 - R) K(k) + \left( \frac{G_{\Delta_1}}{G_{\Sigma_1}} + \frac{G_{\Delta_2}}{G_{\Sigma_2}} \right) (1 + R) E(k) \right]
\]  

(2.65)

where

- \( K(k) \) is the elliptic integral of the first kind.
- \( E(k) \) is the elliptic integral of the second kind.
- \( k \) is the elliptic integral parameter \( = \sqrt{4R/(1+R)} \).

Substituting \( \Sigma_{RMS} \) from Equation (2.52) reduces \( e_{DC} \) to

\[
e_{DC} = \frac{V_{lim} V \sqrt{2}}{2\pi \sqrt{1+R^2}} \theta_d \left[ \left( \frac{G_{\Delta_1}}{G_{\Sigma_1}} - \frac{G_{\Delta_2}}{G_{\Sigma_2}} \right) (1 - R) K(k) + \left( \frac{G_{\Delta_1}}{G_{\Sigma_1}} + \frac{G_{\Delta_2}}{G_{\Sigma_2}} \right) (1 + R) E(k) \right].
\]

(2.66)

With \( \theta_d = \frac{\sqrt{2}}{V_{lim} V} \), Equation (2.66) becomes

\[
e_{DC} = \frac{1}{\pi \sqrt{1+R^2}} \left[ \left( \frac{G_{\Delta_1}}{G_{\Sigma_1}} - \frac{G_{\Delta_2}}{G_{\Sigma_2}} \right) (1 - R) K(k) + \left( \frac{G_{\Delta_1}}{G_{\Sigma_1}} + \frac{G_{\Delta_2}}{G_{\Sigma_2}} \right) (1 + R) E(k) \right].
\]  

(2.67)

The ideal antenna assumption of Section 2.2.1 can again be used to update Equation (2.67) to a form that specifically includes the angles of the two targets. Specifically with

\[
G_{\Sigma_1} (\Delta \theta_1) = G_{\Sigma_2} (\Delta \theta_2) = 1,
\]

\[
G_{\Delta_1} (\Delta \theta_1) = \Delta \theta_1,
\]
and

\[ G_{\Delta_2}(\Delta_2) = \Delta_2 \]

Equation (2.67) becomes

\[
e_{dc} = \frac{1}{\pi \sqrt{1+R^2}} \left[ (\Delta \theta_1 - \Delta \theta_2)(1-R)K(k) + (\Delta \theta_1 + \Delta \theta_2)(1+R)E(k) \right]. \tag{2.68}
\]

Figure 2.3 contains a plot of \( e_{dc} \) as a function of \( R \) (recall \( R = A_2/A_1 \), see Equation (2.26)) for the case where \( \Delta \theta_1 = 1 \) milliradian and \( \Delta \theta_2 = -1 \) milliradian. This illustrates how the overall angle track varies as the relative size of the two targets in the sum channel varies. Note that when \( R < 1 \) (\( R < 0 \) in dB), meaning the sum channel signal for target 1 is greater than the sum channel signal for target 2, the error signal goes toward \( \Delta \theta_1 \). Conversely, when \( R > 1 \) (\( R > 0 \) in dB), meaning the target 2 sum channel signal is greater than the target 1 sum channel signal, the error signal goes toward \( \Delta \theta_2 \).

This is different than the instantaneous AGC case of Section 2.2.1 as the error, and thus the angle track, does not instantly switch between the two targets. Instead, the track follows a centroid between the targets defined by the elliptic functions of Equation (2.68), thus an elliptic centroid.
Figure 2.3 Elliptic Centroid Tracking Behavior
CHAPTER 3

SIMULATION

A series of MATLAB simulations were configured to verify the theoretical predictions of Section 2.2. The target signals were then independently processed through an antenna model generating target signal voltages, which ultimately generated a sum and difference channel signal. Signal normalization was performed using an AGC configured with a 20 Hz bandwidth. The first series of simulations validated the theoretical prediction of the discriminator output. The simulation was then extended to include an analog angle tracker to examine closed-loop angle tracking characteristics. A final series of simulations were created to include a realistic antenna model to capture more realistic system behavior.

Each series of simulations contain three test cases corresponding to the three primary angle tracking regions defined by the Doppler separation between the two targets (see Figure 1.1). The three test cases used (1) equal target Doppler frequencies, (2) a target Doppler frequency separation less than the AGC bandwidth, and (3) a target Doppler frequency separation greater than the AGC bandwidth.

Two stationary targets were centered about the seeker antenna pointing angle and separated by 8.7 milliradians (±1/4° from the seeker antenna pointing angle), as shown in
Figure 3.1. The amplitude of target 1† was held at a constant 0 dB, while the target 2 amplitude was configured to ramp from -10 dB to +10 dB in 1 dB steps. The target 2 amplitude was held at each value for five seconds to allow time for the AGC transients to settle.

![Figure 3.1 Dual Target Configuration](image)

3.1 Discriminator Verification

Figure 3.2 shows a block diagram of the simulation receiver configuration. Through the use of an ideal antenna model (see Equations (2.45)-(2.47)), the received target signals combine to generate a sum and difference channel voltage. The

† In the ensuing, the phrase “the amplitude of target n” is a short-hand way of saying “the amplitude of the target n signal in the sum channel.”
discriminator produces an AGC-normalized error signal, \( e(t) \). The use of an open-loop configuration allows for a comparison to the theoretical results of Section 2.2. Specifically, the antenna was fixed at a constant angle of 0° and the angle track loop was not closed for this set of experiments.

![Simulation Block Diagram for Discriminator Verification](image)

Figure 3.2 Simulation Block Diagram for Discriminator Verification

The first simulation was configured with equal target Doppler frequencies. When the target Doppler frequency separation, \( \Delta f \), is equal to zero, the expected angle error (see Equation (2.40)) has a DC value of

\[
e_{\text{DC}} = \frac{V_{\text{lim}} V}{2(1+R)} \theta_d \left( \frac{G_{\Delta 1}}{G_{\Sigma 1}} + \frac{G_{\Delta 2}}{G_{\Sigma 2}} R \right). \tag{3.1}
\]

With \( \theta_d = \frac{2}{V_{\text{lim}} V} \) and the use of an ideal antenna (see Equations (2.45) – (2.47)), Equation (3.1) becomes

\[
e_{\text{DC}} = \frac{\Delta \theta_1 + \Delta \theta_2 R}{(1+R)}. \tag{3.2}
\]

\(^{\dagger}\) The details of the AGC signal normalization process are discussed in Appendix C.
The simulation results for this case are shown in Figure 3.3 with the true target azimuth angles shown as solid lines. The theoretical steady-state angle error from Equation (3.2) is represented by a dashed magenta line with the experimental steady-state angle error represented by green dots. Since the amplitude separation between the two targets was configured to change by 1 dB every five seconds, the experimental steady-state error was calculated by computing the time-average of the discriminator output over the last three seconds of each five second interval. This calculation reduces the possible impact of filter transients. This allows a more valid comparison of the experimental and theoretical results. As can be seen, the experimental results match the theory.
As the target Doppler frequency separation increases to a non-zero value with a magnitude less than the AGC bandwidth, the steady-state discriminator output is theoretically characterized by a dichotomous track (see Equation (2.48)). Simulation results for a target Doppler frequency separation of 1 Hz are shown in Figure 3.4. The experimental tracking error closely follows the theoretical prediction whereby the discriminator output immediately transitions to the larger target in a dichotomous fashion. When the two target amplitudes are equal, the angle track error exhibits a voltage centroid between the two targets, as predicted by theory.
Figure 3.5 shows the simulation results for a target Doppler frequency separation of 30 Hz, which is greater than the 20 Hz bandwidth of the AGC filter. Since the Doppler frequency separation is greater than the AGC bandwidth, the ACG filter is theoretically expected to respond to the RMS value of the received target signals and cause the steady-state angle error to follow an elliptic centroid track between the two targets (see Equation (2.68)). Similar to the previous cases, when the two target amplitudes are equal ($R = 0$ in dB), the angle error is the centroid of the two target angles.
While the experimental angle track characteristics of Figure 3.5 followed the overall trend of the theoretical prediction, the results were not as consistent with theory as were the simulation results for regions 1 and 2. This difference can be attributed to the AGC assumptions made during theoretical analysis. The theoretical analysis considered the limiting case of a zero-bandwidth AGC (see Section 2.2.2), while the simulated AGC had a finite, non-zero, bandwidth.

In an attempt to explain how the use of a finite AGC bandwidth might cause the behavior illustrated in Figure 3.5, the theoretical results of Section 2.2 were revisited. Recall that the definition of $e(t)$ for an instantaneous AGC is

Figure 3.5 Angle Error – 30 Hz Doppler Separation
\[ e(t) = \frac{V_{\text{im}} V}{2} \theta_d \left[ \frac{G_{\alpha_1}}{G_{\Sigma_1}} \left( \frac{1 + R \cos(\Delta \phi(t))}{1 + R^2 + 2R \cos(\Delta \phi(t))} \right) \right. \\
\left. \quad + \frac{G_{\alpha_2}}{G_{\Sigma_2}} R \left( \frac{R + \cos(\Delta \phi(t)\right)}{1 + R^2 + 2R \cos(\Delta \phi(t))} \right) \right] \]  

(3.3)

while for a zero-bandwidth AGC, \( e(t) \) is defined as

\[ e(t) = \frac{V_{\text{im}} V}{2} \theta_d \left[ \frac{G_{\alpha_1}}{G_{\Sigma_1}} \left( \frac{1 + R \cos(\Delta \phi(t))}{\sqrt{1 + R^2 + 2R \cos(\Delta \phi(t))} \left(1 + R^2\right)} \right) \right. \\
\left. \quad + \frac{G_{\alpha_2}}{G_{\Sigma_2}} R \left( \frac{R + \cos(\Delta \phi(t))}{\sqrt{1 + R^2 + 2R \cos(\Delta \phi(t))} \left(1 + R^2\right)} \right) \right] \]

(3.4)

By examining Equations (3.3) and (3.4), it can be observed that the only difference between the two equations is in the denominator. A comparison of the denominator terms led to the conjecture that a unified equation for both cases could be formulated by introducing a term that represents the AGC filter bandwidth. Specifically, Equation (3.4) was modified to include a scaling factor, \( G \), that could be related to the AGC bandwidth and the structure of the AGC. The result is

\[ e(t) = \frac{V_{\text{im}} V}{2} \theta_d \left[ \frac{G_{\alpha_1}}{G_{\Sigma_1}} \left( \frac{1 + R \cos(\Delta \phi(t))}{\sqrt{1 + R^2 + 2R \cos(\Delta \phi(t))} \left[1 + R^2 + 2RG \cos(\Delta \phi(t))\right]} \right) \right. \\
\left. \quad + \frac{G_{\alpha_2}}{G_{\Sigma_2}} R \left( \frac{R + \cos(\Delta \phi(t))}{\sqrt{1 + R^2 + 2R \cos(\Delta \phi(t))} \left[1 + R^2 + 2RG \cos(\Delta \phi(t))\right]} \right) \right] \]

(3.5)
If $G = 1$, Equation (3.5) reduces to Equation (3.3), which corresponds to an infinite bandwidth AGC. If $G = 0$, Equation (3.5) reduces to Equation (3.4), which corresponds to a zero-bandwidth AGC. This led to the conjecture that $G$ could be used to capture the effects of AGC bandwidth on the transition between dichotomous and elliptic centroid track behavior. As discussed earlier, the steady-state tracking error, $e_{DC}$, is the integral of $e(t)$ (see Equation (2.63)). Unfortunately, a closed-form expression for the integral could not be determined. In lieu of this, the integral of Equation (3.5) was evaluated numerically.

The region 3 simulation was repeated using varying values of $G$. Figure 3.6 contains plots of the simulation results and the theoretical curve with $G = 0.724$. As can be seen, the simulated and theoretical curves now match well.
3.2 Inclusion of Angle Track Loop

This section includes the overall angle track loop in the simulation. A block diagram for this series of simulations is shown in Figure 3.7. The output of the discriminator, $e(t)$, is processed by an angle track filter, resulting in an estimated angle track, $\theta_{nk}(t)$, that is fed back to the antenna model creating a closed-loop configuration.
The angle track filter was configured to be a Type II servomechanism by selecting \( G(s) \) of the form

\[
G(s) = \frac{4\pi f_c \zeta s + (2\pi f_c)^2}{s^2}
\]

where

\[
f_c \text{ is the angle track loop bandwidth} = 10 \text{ Hz.}
\]

\[
\zeta \text{ is the damping coefficient} = 1 / \sqrt{2}.
\]

The discriminator verification tests from Section 3.1 were repeated beginning with a target Doppler frequency separation of 0 Hz. Results are shown in Figure 3.8 and again closely match the theoretical prediction of Equation (3.2).
Following the same approach as Section 3.1, the target Doppler frequency separation was increased to a non-zero value less than the AGC bandwidth (region 2 of Figure 1.1). Simulation results for a target Doppler frequency separation of 1 Hz are shown in Figure 3.9. The experimental angle track matches the theoretical prediction of Equation (2.48) whereby the angle tracker immediately transitions to the larger target. When the two target amplitudes are equal ($R = 0$ in dB), the tracked angle is the centroid of the two target angles.
Results for the final region tested are shown in Figure 3.10 with a configured target Doppler frequency separation of 30 Hz (region 3 of Figure 1.1). Since the two targets do not fall within the AGC bandwidth, the ACG filter responds to the RMS value of the received signal causing the experimental angle error to follow an elliptic centroid track between the two targets (see Equation (2.68)). Similar to the previous cases, when the two target amplitudes are equal ($R = 0$ in dB), the angle track is the centroid of the two target angles. Even with the inclusion of the closed-loop angle tracker, the simulated results follow the overall trend of the theoretical predictions. The theoretical prediction shows the output of the unified expression for the discriminator output (see Equation (3.5)) with the same $G = 0.724$ as used in the previous section (see Figure 3.6).
In this case, the curves do not match, which seems to imply the angle track filter is influencing the steady-state tracking behavior. As an experiment, $G$ was reduced to a value of 0.3. The resulting plot is shown in Figure 3.11. The fact that $G = 0.3$ improved the match between the theoretical and simulated results makes intuitive sense considering the overall behavior of the closed-loop configuration is influenced by both the AGC and the angle track filter.
3.3 Inclusion of Realistic Antenna Patterns

The final series of simulations introduced a realistic antenna pattern with a 3 dB beamwidth of 1.66°. The antenna beamwidth was chosen such that the two targets would be unresolvable in angle, given an angular separation of ±0.25° from the seeker’s reference frame (see Figure 3.1). The receive antenna patterns for the sum and difference channels are shown in Figures 3.12 and 3.13, respectively.
Figure 3.12 Normalized Sum Channel Antenna Gain
Figure 3.14 shows the simulation results for the case of identical target Doppler frequencies. The experimental angle track is characterized by a power centroid between the two targets. As indicated by the plot, the experimental results agree well with the theoretical results.
For the next case the target Doppler frequencies were chosen to provide a separation of 1 Hz. The simulation results are contained in Figure 3.15. While the overall trend follows that of a dichotomous tracker, the simulation shows a deviation from the theoretical predictions in that the experimental angle track does not abruptly transition between the two targets as expected (see Figure 3.9). Instead, the angle track begins transitioning somewhat before the relative target amplitudes reach 0 dB.
The gradual transition in the experimental results shown in Figure 3.15 was not observed using the closed-loop tracker with the ideal antenna pattern. This led to the conjecture the change in behavior is an artifact of the realistic antenna. Figure 3.16 shows a comparison between the instantaneous angle track when using an ideal (black curve) and realistic (red curve) antenna during the first fifteen seconds of simulation when the targets had 10 to 8 dB amplitude separation. As can be seen, the response is a sinusoidal that is close to being centered about 4.36°. Thus, for both the ideal and realistic antenna, the tracker tracks target 1.
Figure 3.16 Angle Track Comparison (Large Amplitude Separations)

Figure 3.17 shows the instantaneous angle track when the targets were separated by 4 dB to 1 dB. As the two targets get closer in amplitude, the angle track begins to exhibit an asymmetric oscillation. For the ideal antenna, the tall, narrow positive peak (black curve) averages with the short, wide negative peak to keep the average track point on target 1. When the realistic antenna is included, the height of the positive peak (red curve) is shortened because it is reduced by the sum channel antenna pattern. Because of this, the average track point moves away from target 1 toward the centroid of the two targets.
For equal amplitudes, the experimental angle track point is not the centroid of the two target angles (see Figure 3.15 when $R = 0$ dB). Figure 3.18 shows the signal levels during the 5 second interval when the two targets have equal amplitudes. When the target amplitudes are equal, the signals are periodically $180^\circ$ out of phase. As a result, the sum channel voltage drops to a small value causing the AGC to increase the IF amplifier gain (shown as a red line in Figure 3.18) for both the sum and difference channels. The increase in amplifier gain manifests as an increase in the discriminator gain. From servo theory, the gain increase causes the closed-loop poles to move into the left-half plane causing an oscillation of the steady-state angle track filter output [13]. As a result, the
angle track error at the output of the IF amplifier (shown as a black line in Figure 3.18) begins to oscillate.

Figure 3.18 Signal Levels – Equal Amplitudes, 1 Hz Doppler Separation

It is conjectured that this behavior will not be present in an actual seeker. The IF amplifier output would be limited to the thermal noise floor of the hardware, preventing the signal level from dropping as low as the simulated value. A thermal noise floor was not modeled as part of the simulation, but is an area that could be investigated in future studies. Also, practical AGCs are designed to limit the maximum IF amplifier gain. As a note, it is unlikely two complex targets would produce exactly the same amplitude return signal and be 180° out of phase. Thus, it is expected the behavior illustrated in the Figure 3.18 simulation results will very seldom be observed in an actual seeker.
For the final simulation, the Doppler frequency separation was set to 30 Hz, which is greater than the 20 Hz bandwidth of the AGC filter. Figure 3.19 shows the simulation results for this case. As shown, the experimental results match the theoretical results (using $G = 0.657$) very well.

Figure 3.19 Angle Track (Unified Equation, $G = 0.657$) – 30 Hz Doppler Separation
CHAPTER 4

SUMMARY AND FUTURE WORK

The theoretical development described two primary angle track regions delineated by the Doppler frequency separation between two targets relative to the AGC filter bandwidth. A target Doppler frequency separation less than the AGC bandwidth results in a dichotomous track where the angle track immediately transitions to the larger target. A special case of the dichotomous tracking region, defined by identical target Doppler frequencies, is characterized by a power centroid angle track between the two targets. A target Doppler frequency separation greater than the AGC bandwidth defines the region of an elliptic centroid track. That is, the angle track does not abruptly transition between targets, but follows a centroid that is defined in terms of elliptic integrals.

The first series of simulations verified the theoretical results using an ideal antenna. While experimental results matched the theoretical predictions for both regions 1 and 2 (see Figure 1.1), the region 3 behavior showed a slight deviation that was shown to be caused by finite AGC bandwidth. When the finite AGC bandwidth characteristic was included in the theoretical development, the experimental results matched the theoretical prediction.

In a second set of tests an angle track loop was included in the simulation. While the simulation matched the first two angle track regions, the theoretical results for the
region 3 angle track behavior again exhibited a deviation from the theory. This was determined to be due to the interaction between the AGC and the angle track loop. Adapting the theoretical prediction to the change in the closed-loop configuration with the addition of the angle track loop caused the experimental results to again match the theory.

The closed-loop simulation was further extended to include a realistic antenna model. The effect of the realistic antenna model was observed in all angle tracking regions. While the region 1 behavior closely followed the theory, region 2 showed a gradual angle track transition between the two targets rather than the dichotomous transition observed in previous simulations. An investigation into this change determined the deviation was an artifact of the realistic antenna pattern due to a reduction in the received sum channel voltage. Region 3 exhibited a similar change in the received sum channel signal level leading to a slight deviation from the theoretical prediction.

An extension of this thesis would include the Doppler tracking unit to provide the heterodyning signal, $h(t)$, shown in the theoretical development (see Equation (2.3)). Simulation analysis could determine the interaction between the angle and Doppler track loops. If the two track loops have conflicting closed-loop behavior, the overall system response may become unpredictable.

Further, an investigation into the $G$ parameter of Equation (3.5) could provide better insights into more realistic system behavior. Section 3.1 conjectured the equations describing both the dichotomous and elliptic centroid tracking regions could be combined into a single equation by means of a term describing the AGC bandwidth. This would
allow a more quantitative investigation into the transition between the two tracking regions (see Figure 1.1) and a better understanding of the impact of a non-zero finite AGC bandwidth.
APPENDIX A.

REGION 2 STEADY-STATE ERROR DERIVATION

This appendix contains the mathematics behind the steady-state tracking error for the dual target case where the Doppler separation between the targets is less than the AGC bandwidth (defined as region 2 in Figure 1.1); that is, the instantaneous AGC case. That error is given by Equation (2.41). The integral of Equation (2.41) is solved by analyzing three distinct cases based on regions of target amplitudes. Equation (2.41) is

\[
\frac{e_{DC}}{V_{lim}} = \frac{V_{lim} V}{4\pi} \theta_d \left[ G_{\Delta 1} \int_0^{2\pi} \frac{1 + R \cos(\Delta \varphi(t))}{1 + R^2 + 2R \cos(\Delta \varphi(t))} d\Delta \varphi(t) \right] 
+ \frac{G_{\Delta 2}}{G_{\Delta 2}} \int_0^{2\pi} \frac{R + \cos(\Delta \varphi(t))}{1 + R^2 + 2R \cos(\Delta \varphi(t))} d\Delta \varphi(t) \right]
\]

\[
= \frac{V_{lim} V}{4\pi} \theta_d \left[ G_{\Delta 1} \int_0^{2\pi} \frac{1 + R \cos(\Delta \varphi(t))}{1 + R^2 + 2R \cos(\Delta \varphi(t))} d\Delta \varphi(t) \right] 
+ \frac{G_{\Delta 2}}{G_{\Delta 2}} \int_0^{2\pi} \frac{R + \cos(\Delta \varphi(t))}{1 + R^2 + 2R \cos(\Delta \varphi(t))} d\Delta \varphi(t) \right]
\]

\[
= \frac{V_{lim} V}{4\pi} \theta_d \left( e_{DC1} + e_{DC2} \right) .
\]

The first integral can be written as
\[ e_{DCI} = \frac{G_{AI}}{G_{xi}} \int_{-\pi}^{\pi} \frac{1}{1 + R^2 + 2R \cos(\Delta \varphi(t))} d\Delta \varphi(t) \]

\[ = \frac{G_{AI}}{G_{xi}} \left[ \int_{-\pi}^{\pi} \frac{1}{1 + R^2 + 2R \cos(\Delta \varphi(t))} d\Delta \varphi(t) \right. \]
\[ + R \int_{-\pi}^{\pi} \frac{\cos(\Delta \varphi(t))}{1 + R^2 + 2R \cos(\Delta \varphi(t))} d\Delta \varphi(t) \].

(A.4)

Since the integrand is a periodic function of \( \Delta \varphi(t) \) with a period of \( 2\pi \), the integral can be evaluated over any \( 2\pi \) interval. Thus, the integral can be evaluated over \(-\pi \) to \( \pi \).

With this,

\[ e_{DCI} = \frac{G_{AI}}{G_{xi}} \left[ 2 \int_{0}^{\pi} \frac{1}{1 + R^2 + 2R \cos(\Delta \varphi(t))} d\Delta \varphi(t) \right. \]
\[ + R \int_{0}^{\pi} \frac{\cos(\Delta \varphi(t))}{1 + R^2 + 2R \cos(\Delta \varphi(t))} d\Delta \varphi(t) \].

Further, since the integrand is an even function of \( \Delta \varphi(t) \), the integral can be written as twice the integral from 0 to \( \pi \).

\[ e_{DCI} = \frac{G_{AI}}{G_{xi}} \left[ 2 \int_{0}^{\pi} \frac{1}{1 + R^2 + 2R \cos(\Delta \varphi(t))} d\Delta \varphi(t) \right. \]
\[ + 2R \int_{0}^{\pi} \frac{\cos(\Delta \varphi(t))}{1 + R^2 + 2R \cos(\Delta \varphi(t))} d\Delta \varphi(t) \].

(A.5)

These integrals can be solved by using the identity [14]

\[ \int_{0}^{\pi} \frac{\cos(nx)}{1 + a^2 - 2a \cos(x)} dx = \frac{\pi a^n}{1 - a^2}, \quad a^2 < 1. \]

(A.6)
Specifically, with $n = 0$ for the first integral and $n = 1$ for the second, and $a = -R$ for both integrals, $e_{\text{dc}1}$ becomes

$$e_{\text{dc}1} = \frac{G_{\text{A1}}}{G_{\Sigma 1}} \left[ 2 \left( \frac{\pi}{1-R^2} \right) + 2R \left( \frac{\pi(-R)}{1-R^2} \right) \right]$$

$$= 2\pi \frac{G_{\text{A1}}}{G_{\Sigma 1}}, \quad R^2 < 1. \quad (A.7)$$

Similarly, $e_{\text{dc}2}$ can be written as

$$e_{\text{dc}2} = \frac{G_{\text{A2}}}{G_{\Sigma 2}} R \int_0^{2\pi} \frac{R + \cos(\Delta \varphi(t))}{1 + R^2 + 2R \cos(\Delta \varphi(t))} \, d\Delta \varphi(t)$$

$$= \frac{G_{\text{A2}}}{G_{\Sigma 2}} \left[ R \int_0^{2\pi} \frac{1}{1 + R^2 + 2R \cos(\Delta \varphi(t))} \, d\Delta \varphi(t) \right.$$  
$$+ \left. \int_0^{2\pi} \frac{\cos(\Delta \varphi(t))}{1 + R^2 + 2R \cos(\Delta \varphi(t))} \, d\Delta \varphi(t) \right]. \quad (A.8)$$

This is similar to Equation (A.4) except for the $R$ multiplier on the first integral. Thus, following the procedure used to evaluate Equation (A.4), the solution of Equation (A.8) becomes

$$e_{\text{dc}2} = \frac{G_{\text{A2}}}{G_{\Sigma 2}} R \left[ 2R \left( \frac{\pi}{1-R^2} \right) + 2 \left( \frac{\pi(-R)}{1-R^2} \right) \right]$$

$$= 0, \quad R^2 < 1. \quad (A.9)$$

Combining Equations (A.7) and (A.9) gives
\[ e_{dc} = \frac{V_{\text{lim}} V}{4\pi} \theta_d \left( 2\pi \frac{G_{\Delta 1}}{G_{\Sigma 1}} + 0 \right) \]

\[ = \frac{V_{\text{lim}} V}{2} \frac{G_{\Delta 1}}{G_{\Sigma 1}} \theta_d, \quad R^2 < 1 \text{ or } A_1 > A_2. \quad (A.10) \]

For the case where \( A_2 > A_1 \) we let \( Q = \frac{1}{R} = \frac{A_1}{A_2} \) and rewrite Equation (A.2) as

\[ e_{dc} = \frac{V_{\text{lim}} V}{4\pi} \theta_d \left( 2\pi \frac{G_{\Delta 1}}{G_{\Sigma 1}} \int_0^{2\pi} \frac{Q + \cos(\Delta \varphi(t))}{1 + Q^2 + 2Q \cos(\Delta \varphi(t))} \right) \]
\[ + \frac{G_{\Delta 2}}{G_{\Sigma 2}} \left( 1 + Q \cos(\Delta \varphi(t)) \right) \int_0^{2\pi} \frac{d\Delta \varphi(t)}{1 + Q^2 + 2Q \cos(\Delta \varphi(t))} \]

\[ = \frac{V_{\text{lim}} V}{4\pi} \theta_d \left( e_{dc1} + e_{dc2} \right). \quad (A.11) \]

Following a procedure similar to the one that led to Equation (A.10) yields

\[ e_{dc} = \frac{V_{\text{lim}} V}{2} \frac{G_{\Delta 2}}{G_{\Sigma 2}} \theta_d, \quad Q^2 < 1 \text{ or } A_1 < A_2. \quad (A.12) \]

A final case exists such that \( A_1 = A_2 \). For this case, \( R = 1 \) and Equation (A.2) reduces to

\[ e_{dc} = \frac{V_{\text{lim}} V}{8\pi} \theta_d \left[ \frac{G_{\Delta 1}}{G_{\Sigma 1}} \int_0^{2\pi} d\Delta \varphi(t) + \frac{G_{\Delta 2}}{G_{\Sigma 2}} \int_0^{2\pi} d\Delta \varphi(t) \right] \]

\[ = \frac{V_{\text{lim}} V}{4} \left( \frac{G_{\Delta 1}}{G_{\Sigma 1}} + \frac{G_{\Delta 2}}{G_{\Sigma 2}} \right) \theta_d, \text{ when } R = 1 \text{ or } A_1 = A_2. \quad (A.13) \]

The three cases from Equations (A.10), (A.12), and (A.13) can be combined to give
\[ e_{DC} = \begin{cases} 
\frac{V_{\text{lim}} V G_{\Delta 1}}{2 G_{\Sigma 1}} \theta_d, & \text{when } A_i > A_2 \\
\frac{V_{\text{lim}} V G_{\Delta 2}}{2 G_{\Sigma 2}} \theta_d, & \text{when } A_2 > A_i \\
\frac{V_{\text{lim}} V}{4} \left( \frac{G_{\Delta 1}}{G_{\Sigma 1}} + \frac{G_{\Delta 2}}{G_{\Sigma 2}} \right) \theta_d, & \text{when } A_i = A_2 
\end{cases} \] (A.14)
APPENDIX B.

REGION 3 STEADY-STATE ERROR DERIVATION

This appendix describes the mathematics behind the steady-state tracking error for the dual target case where the Doppler separation between the targets is greater than the AGC bandwidth (defined as region 3 in Figure 1.1). The resultant error signal is defined by examining the DC component of the error signal from Equation (2.64). Specifically,

\[ e_{DC} = \frac{V_{lim} V_A}{4\pi \Sigma_{RMS}} \theta_d \left[ \frac{G_{A1}}{G_{S1}} \left( \frac{1 + R \cos(\Delta \phi(t))}{\sqrt{1 + R^2 + 2R \cos(\Delta \phi(t))}} \right) \right. 
+ \left. \frac{G_{A2}}{G_{S2}} R \left( \frac{R + \cos(\Delta \phi(t))}{\sqrt{1 + R^2 + 2R \cos(\Delta \phi(t))}} \right) \right] d\Delta \phi(t) \]

\[ = \frac{V_{lim} V_A}{4\pi \Sigma_{RMS}} \theta_d \left[ \frac{G_{A1}}{G_{S1}} \left[ \int_0^{2\pi} \frac{1 + R \cos(\Delta \phi(t))}{\sqrt{1 + R^2 + 2R \cos(\Delta \phi(t))}} d\Delta \phi(t) \right] 
+ \frac{G_{A2}}{G_{S2}} R \left[ \int_0^{2\pi} \frac{R + \cos(\Delta \phi(t))}{\sqrt{1 + R^2 + 2R \cos(\Delta \phi(t))}} d\Delta \phi(t) \right] \right] \]
In Equation (B.1),

\[ e_{DC1} = \int_{0}^{2\pi} \frac{1}{\sqrt{1+R^2 + 2R \cos(\Delta \phi(t))}} d\Delta \phi(t) . \]  

(B.2)

Since the integrand is a periodic function of \( \Delta \phi(t) \) with a period of \( 2\pi \), the integral can be evaluated over any \( 2\pi \) interval. Thus, the integral can be evaluated over \( -\pi \) to \( \pi \).

With this,

\[ e_{DC1} = \int_{-\pi}^{\pi} \frac{1}{\sqrt{1+R^2 + 2R \cos(\Delta \phi(t))}} d\Delta \phi(t) . \]  

(B.2)
Further, since the cosine integrand is an even function of $\Delta\varphi(t)$, the integral can be written as twice the integral from 0 to $\pi$.

$$e_{DC1} = 2 \int_{0}^{\pi} \frac{1}{\sqrt{1 + R^2 + 2R \cos(\Delta\varphi(t))}} \, d\Delta\varphi(t).$$

Using the trigonometric identity

$$\cos(\Delta\varphi(t)) = 1 - 2\sin^2\left(\frac{\Delta\varphi(t)}{2}\right)$$

and the change of variable of $\alpha = \Delta\varphi(t)/2$, Equation (B.2) becomes

$$e_{DC1} = 4 \int_{0}^{\pi/2} \frac{1}{\sqrt{1 + R^2 + 2R\left[1 - 2\sin^2(\alpha)\right]}} \, d\alpha$$

$$= 4 \int_{0}^{\pi/2} \frac{1}{\sqrt{(1 + R)^2 - 4R\sin^2(\alpha)}} \, d\alpha$$

$$= \frac{4}{(1 + R)} \int_{0}^{\pi/2} \frac{1}{\sqrt{1 - \frac{4R}{(1 + R)^2}\sin^2(\alpha)}} \, d\alpha.$$

Using $k^2 = \frac{4R}{(1 + R)^2}$ results in

$$e_{DC1} = \frac{4}{(1 + R)} \int_{0}^{\pi/2} \frac{1}{\sqrt{1 - k^2\sin^2(\alpha)}} \, d\alpha. \quad (B.3)$$
The integral of Equation (B.3) is an elliptic integral of the first kind, \( K(k) \) [14]. With this, Equation (B.3) becomes

\[
e_{DC1} = \frac{4}{(1+R)} K(k)
\]  

(B.4)

where

\( K(k) \) is the elliptic integral of the first kind.

\( k \) is the elliptic integral parameter \( = \sqrt{4R/(1+R)} \).

Referring to Equation (B.1), \( e_{DC2} \) is

\[
e_{DC2} = \int_{0}^{2\pi} \frac{\cos(\Delta \varphi(t))}{\sqrt{1+R^2 + 2R \cos(\Delta \varphi(t))}} d\Delta \varphi(t).
\]  

(B.5)

The numerator cosine signal can be modified to place it into a more manageable form in relation to the denominator term. This change multiplies and divides the original cosine term by \( 2R \) while also adding and subtracting a \( 1+R^2 \) term. Mathematically,

\[
\cos(\Delta \varphi(t)) = \frac{(1+R^2) + 2R \cos(\Delta \varphi(t)) - (1+R^2)}{2R}.
\]

With this, Equation (B.5) can be written as

\[
e_{DC2} = \int_{0}^{2\pi} \left( \frac{(1+R^2) + 2R \cos(\Delta \varphi(t)) - (1+R^2)}{2R \sqrt{1+R^2 + 2R \cos(\Delta \varphi(t))}} \right) d\Delta \varphi(t)
\]
\[
\begin{align*}
&= \frac{1}{2R} \int_0^{2\pi} \sqrt{1+R^2 + 2R \cos(\Delta \varphi(t))} \, d\Delta \varphi(t) \\
&\quad - \frac{(1+R^2)}{2R} \int_0^{2\pi} \frac{1}{\sqrt{1+R^2 + 2R \cos(\Delta \varphi(t))}} \, d\Delta \varphi(t).
\end{align*}
\]

The second integral is recognized as \( e_{DC1} \) (see Equations (B.2) - (B.4)). Thus,

\[
e_{DC2} = \frac{1}{2R} \int_0^{2\pi} \sqrt{1+R^2 + 2R \cos(\Delta \varphi(t))} \, d\Delta \varphi(t) - \frac{2(1+R^2)}{R(1+R)} K(k).
\]

Since the integrand of the first term is a periodic function of \( \Delta \varphi(t) \) with a period of \( 2\pi \), the integral can be evaluated over any \( 2\pi \) interval. Thus, the integral can be evaluated between \(-\pi\) and \( \pi \). With this,

\[
e_{DC2} = \frac{1}{2R} \int_{-\pi}^{\pi} \sqrt{1+R^2 + 2R \cos(\Delta \varphi(t))} \, d\Delta \varphi(t) - \frac{2(1+R^2)}{R(1+R)} K(k).
\]

Further, since the integrand is an even function of \( \Delta \varphi(t) \), the integral can be written as twice the integral from 0 to \( \pi \). Thus,

\[
e_{DC2} = \frac{1}{R} \int_0^{\pi} \sqrt{1+R^2 + 2R \cos(\Delta \varphi(t))} \, d\Delta \varphi(t) - \frac{2(1+R^2)}{R(1+R)} K(k).
\]

A trigonometric identity allows the cosine term in the first integral to be expanded as

\[
\cos(\Delta \varphi(t)) = 1 - 2\sin^2\left(\frac{\Delta \varphi(t)}{2}\right).
\]

A change of variable of \( \alpha = \frac{\Delta \varphi(t)}{2} \) allows the first integral to be written as
\[ e_{DC2} = \frac{2}{R} \int_{0}^{\frac{\pi}{2}} \sqrt{1+R^2 + 2R[1-2\sin^2(\alpha)]} \, d\alpha - \frac{2(1+R^2)}{R(1+R)} K(k). \]

Multiplying by \( \frac{1+R}{1+R} \) and using \( k^2 = \frac{4R}{(1+R)^2} \) reduces \( e_{DC2} \) to

\[ e_{DC2} = \frac{2(1+R)}{R} \int_{0}^{\frac{\pi}{2}} \sqrt{1-k^2 \sin^2(\alpha)} \, d\alpha - \frac{2(1+R^2)}{R(1+R)} K(k). \]

(B.6)

The first integral is an elliptic integral of the second kind, \( E(k) \) [14]. With this, Equation (B.6) becomes

\[ e_{DC2} = \frac{2(1+R)}{R} E(k) - \frac{2(1+R^2)}{R(1+R)} K(k) \] (B.7)

where

\[ E(k) \] is the elliptic integral of the second kind.

\( k \) is the elliptic integral parameter = \( \sqrt{4R/(1+R)} \).

Using \( e_{DC1} \) and \( e_{DC2} \) in Equation (B.1) yields

\[ e_{DC} = \frac{V_{lim} \, V_A}{4\pi \Sigma_{RMS}} \theta_d \left[ \left( \frac{G_{\Delta 1} + G_{\Delta 2}}{G_{\Sigma 1} + G_{\Sigma 2}} \right) \frac{4}{(1+R)} K(k) \right. \]

\[ + \left. R \left( \frac{G_{\Delta 1} + G_{\Delta 2}}{G_{\Sigma 1} + G_{\Sigma 2}} \right) \left( \frac{2(1+R)}{R} E(k) - \frac{2(1+R^2)}{R(1+R)} K(k) \right) \right]. \]

After considerable algebraic manipulation, \( e_{DC} \) can be written as
\[ e_{DC} = \frac{V_{hm} V_A}{2 \pi \Sigma_{RMS}} \theta_j \left[ \left( \frac{G_{\Delta_1}}{G_{\Sigma_1}} - \frac{G_{\Delta_2}}{G_{\Sigma_2}} \right)(1 - R) K(k) + \left( \frac{G_{\Delta_1}}{G_{\Sigma_1}} + \frac{G_{\Delta_2}}{G_{\Sigma_2}} \right)(1 + R) E(k) \right]. \] (B.8)
APPENDIX C.

AGC SIGNAL PROCESSING

This appendix describes the AGC signal normalization process. A block diagram of the AGC model used in the simulation is shown in Figure C.1.

The magnitude of the complex sum signal, $\Sigma(t)$, is computed via the absolute value block and then converted to a decibel value using

$$\Sigma_{dB}(t) = 20\log_{10} \left( \frac{1}{\Sigma_{AGC}} \sum |\Sigma(t)e^{j\phi}| \right).$$
This decibel sum signal, $\Sigma_{db}(t)$, drives a Type I servo loop with a bandwidth of $f_c$. Since the servo loop is Type I, its output, $\Sigma_{AGC,db}$, will approach $\Sigma_{db}(t)$ in steady-state. This means the output of the -1 gain block in the servo feedback path will approach $-\Sigma_{db}(t)$ in steady-state. Because of the sign reversal on $\Sigma_{AGC,db}$, the output of the “dB to magnitude” block will be

$$G_{AGC} = \frac{1}{\Sigma_{AGC}} \approx \frac{1}{\Sigma(t)}.$$

Thus, the AGC has the desired effect of producing a gain, $G_{AGC}$, that is approximately the inverse of $\Sigma(t)$, the magnitude of the sum signal. The AGC gain is then used to normalize the complex sum and difference signals via the multiplier blocks shown in the lower part of Figure C.1.
REFERENCES


