Design and testing of a Hall effect thruster with additively manufactured components

Ethan Hopping

Follow this and additional works at: https://louis.uah.edu/uah-theses

Recommended Citation
Hopping, Ethan, "Design and testing of a Hall effect thruster with additively manufactured components" (2017). Theses. 225.
https://louis.uah.edu/uah-theses/225

This Thesis is brought to you for free and open access by the UAH Electronic Theses and Dissertations at LOUIS. It has been accepted for inclusion in Theses by an authorized administrator of LOUIS.
DESIGN AND TESTING OF A HALL EFFECT THRUST WITH ADDITIVELY MANUFACTURED COMPONENTS

by

ETHAN HOPPING

A THESIS

Submitted in partial fulfillment of the requirements for the degree of Master of Science in Engineering in The Department of Mechanical and Aerospace Engineering to The School of Graduate Studies of The University of Alabama in Huntsville

HUNTSVILLE, ALABAMA

2017
In presenting this thesis in partial fulfillment of the requirements for a master’s degree from The University of Alabama in Huntsville, I agree that the Library of this University shall make it freely available for inspection. I further agree that permission for extensive copying for scholarly purposes may be granted by my advisor or, in his/her absence, by the Chair of the Department or the Dean of the School of Graduate Studies. It is also understood that due recognition shall be given to me and to The University of Alabama in Huntsville in any scholarly use which may be made of any material in this thesis.

Ethan Hopping

08/18/17

(date)
THESIS APPROVAL FORM

Submitted by Ethan Hopping in partial fulfillment of the requirements for the degree of Master of Science in Engineering in Aerospace Systems Engineering and accepted on behalf of the Faculty of the School of Graduate Studies by the thesis committee.

We, the undersigned members of the Graduate Faculty of The University of Alabama in Huntsville, certify that we have advised and/or supervised the candidate of the work described in this thesis. We further certify that we have reviewed the thesis manuscript and approve it in partial fulfillment of the requirements for the degree of Master of Science in Engineering in Aerospace Systems Engineering.

Dr. Kunning G. Xu 8/14/17 Committee Chair

Dr. Robert Frederick 8/14/17

Dr. Jason Cassibry 8/14/17 Department Chair

Dr. Keith Hollingsworth 8/14/17

Dr. Shankar Mahalingam 08/15/17 College Dean

Dr. David Berkowitz 8/24/17 Graduate Dean
ABSTRACT

School of Graduate Studies
The University of Alabama in Huntsville

Degree Master of Science in Engineering
College/Dept. Engineering/Mechanical and Aerospace Engineering

Name of Candidate Ethan Hopping
Title Design and Testing of a Hall Effect Thruster
With Additively Manufactured Components

The UAH-78AM is a low-power Hall effect thruster developed at the University of Alabama in Huntsville to study the application of low-cost additive manufacturing in the design and fabrication of Hall thrusters. The goal of this project is to assess the feasibility of using unconventional materials to produce a low-cost functioning Hall effect thruster and consider how additive manufacturing can expand the design space and provide other benefits. The thruster features channel walls and a propellant distributor that were manufactured using 3D printing with a variety of materials including ABS, ULTEM, and glazed ceramic. A version of the thruster was tested at NASA Glenn Research Center to obtain performance metrics and to validate the ability of the thruster to produce thrust and sustain a discharge. The design of the thruster and the transient performance measurements are presented here. Measured thrust ranged from 17.2 mN to 30.4 mN over a discharge power of 280 W to 520 W with an anode I_sp range of 870 s to 1450 s. Temperature limitations of materials used for the channel walls and propellant distributor limit the ability to run the thruster at thermal steady-state. While the current thruster design is not yet ready
for continuous operation, revisions to the device that could enable longer duration tests are discussed.

Abstract Approval: Committee Chair
Dr. Kunning G. Xu

Department Chair
Dr. Keith Hollingsworth

Graduate Dean
Dr. David Berkowitz
This document, as with any piece of literature, ultimately tells a story. As scientists and engineers, we often focus on the technical merit of our work, and consequently overlook the story behind the results. But as authors, we are shaped by our works inasmuch as our works shape our community. The following pages are dedicated to the people who played crucial roles in making this research possible as mentors, advisors, assistants, and friends. In doing so, they not only had a hand in shaping this thesis, but also in shaping the author’s future. For the majority of you interested in the thesis for its technical content, I apologize in advance for this section’s wordiness!

My greatest thanks goes to my research advisor, Dr. Gabe Xu. I certainly wasn’t convinced that building a Hall thruster was a good idea the first time we sat down in your office and discussed research topics! But I took a leap of faith, and you’ve been there to provide guidance and direction the rest of the way. In the end, I feel it was a very good decision, and I’m proud of what has been accomplished. Thank you for challenging me to keep going, especially in the beginning, and to do better every step of the way. I hope the following pages are a good testament to your mentorship as an advisor. Thank you for giving me the opportunity to dive deep into the world of electric propulsion.

Perhaps the most substantial impact on the author and this research came from the brief 4-months spent at NASA GRC. Wensheng, thank you for opening the
door for me to spend time at the center, and for serving as a second mentor during my tenure there. I’m still not sure if I should be concerned or happy with the number of times you said “just do it” while I was there, but thank you for keeping me on my toes! I think those 4 months did much more to advance my understanding and experience with Hall thrusters than my previous year of research on the subject.

In addition, I have many people to thank at Glenn for helping me get to testing. Hani, thank you for loaning me that BaO cathode, and teaching me how to use it! To Jon Macky and Tom Haag: Thank you for exposing me to heritage of Glenn EP thrust stands. Jon, I never would have figured out how the controller for the VF-8 stand works without your help (and I still think that piezo element only works for you)! To Jon Yim and Maria Choi: thank you for your guidance with the plenum calculations and a few other analysis tasks over the semester, and for letting me share an office with you (though I admittedly spent little time there)! I still laugh about how I mistook both of you for grad students my first day at GRC.

To Kevin Blake, Tom, Taylor Seablom, Danis, and other GRC techs: Thank you for all your help with the myriad of integration and facility tasks required for this testing to be successful. I’ve enjoyed working with all of you, and learned a lot as well. Thank you for taking time from your busy schedules working on the big projects to help a student with his research.

A special thanks to Thomas DeMichael for helping us reprint the thruster outer channel in ULTEM. Most of the results presented in this thesis would not have been possible without your support.
Finally, I’d like to thank Todd Tofil, who provided funding for our testing at GRC. Thank you for providing me with the opportunity to do research at the center.

I also owe thanks to many people at UAH. One of my biggest objectives in continuing with my master’s degree at UAH was to gain some manufacturing experience—something that I did not dedicate enough time to as an undergraduate. Still, when it came time to start building things I did not know how to begin. A special thanks to Adam Bower for manufacturing the magnetic circuit, and allowing me to look over his shoulder and ask lots of questions along the way. I still have a lot to learn about manual machining and CNC, but I know much more than I did a year ago, much in thanks to you! In the same vein, I’d like to thank Steve Collins for allowing us to use the machine shop and providing guidance to ensure fabrication proceed safely and was done correctly.

Before I tested at Glenn, I spent a year building and testing the thruster in facilities at UAH. Thanks is due to Tony Hall, Dr. Lineberry, and Dr. Frederick for first allowing me to test at the Propulsion Research Center and also assisting with the facility set up by answering my questions and guiding me to the tools I needed for the experiment.

In addition, I’d like to thank Claudia Meyering and Melissa Brown for their assistance with the myriad of paperwork and “school questions” associated with taking classes and pursuing a graduate degree. Your support was critical to the success of this research by enabling me as a student.

Ryota Nakano, I pass the baton to you if you choose to accept it! Thank you for wearing many hats while assisting me in the lab: an extra pair of hands when
assembling the thruster, someone to talk to while collecting those b-field maps, and a 3D printer master-in-training. I hope you learned a little from my many ramblings on how I think Hall thrusters work! Believe it or not, after I leave I think you will have the most technical expertise with Hall thrusters (minus Dr. Xu) of all of us. I look forward to where your future research takes you. From the brief time we spent working together, I know that you will be very successful with your future endeavors.

Finally, I’ll close by thanking the organization and it’s leaders that provided financial support for this research. The Alabama Space Grant Consortium was supporting me as an engineer and researcher before I knew who they were. Through Space Hardware Club, I was given the opportunity to practice and develop skills that would ultimately lead me to this opportunity. Most of the projects and achievements of my undergraduate and master’s career are attributable in some way to the support of Alabama Space Grant. Dr. John Gregory, Dr. Dale Thomas, Deborah Nielson, and Rachel Damiani: thank you for leading an organization that goes above and beyond its goal to expand opportunities for students and citizens to support NASA’s mission. As a citizen, I promise that I will continue to be a strong advocate for the Space Grant program and the benefits it provides for future generations of American scientists and engineers.
# TABLE OF CONTENTS

List of Figures xlv

List of Tables xvii

List of Symbols xviii

Chapter

1 Introduction 1

1.1 Problem Statement .............................................. 2

1.2 Research Contributions ......................................... 3

1.3 Motivation .......................................................... 4

2 Background 7

2.1 Physics of Rocket Propulsion ................................... 8

2.1.1 Tsiolkovsky’s Rocket Equation ............................... 8

2.1.2 The Role of Specific Impulse ................................. 10

2.1.3 Comparison of Propulsion Technologies ...................... 12

2.2 Physics of the Hall effect thruster .............................. 14

2.2.1 Basic Principles ................................................ 15

2.2.2 Material Requirements ........................................ 20

2.2.2.1 Discharge Channel ....................................... 20
2.2.2.2 Magnetic Circuit ........................................... 22
2.2.2.3 Anode/Propellant Distributor .......................... 23

2.2.3 Research Interests ........................................... 24
  2.2.3.1 Anomalous Diffusion ................................... 24
  2.2.3.2 Facility Effects ......................................... 26
  2.2.3.3 Lifetime Limitations ................................... 28
  2.2.3.4 Small Thrusters ........................................ 30

2.3 3D Printing Processes ......................................... 31
  2.3.1 Fused Deposition Modeling/Fused Filament Fabrication ... 32
  2.3.2 Powder Bed Fusion ......................................... 33
  2.3.3 Binder Jetting ............................................. 36

2.4 Closing Remarks .............................................. 37

3 Methods .......................................................... 38
  3.1 Development of the UAH-78AM .............................. 38
    3.1.1 Preliminary Design and Scaling .......................... 38
    3.1.2 Design Features ......................................... 41
    3.1.3 Performance Prediction ................................. 45
    3.1.4 Materials Selection .......................... 47
      3.1.4.1 Magnetic Circuit and Anode ......................... 47
      3.1.4.2 Discharge Chamber and Propellant Distributor ... 48
      3.1.4.3 Magnetic Screens .................................. 51
      3.1.4.4 Electromagnets ................................... 51


3.1.5 Magnetic Circuit ................................. 52
3.1.6 Manufacturing ................................. 53
  3.1.6.1 Tolerances ............................... 53
  3.1.6.2 Cost ................................... 54
  3.1.6.3 Turnaround Time ......................... 56
3.1.7 Cathodes ..................................... 57
3.2 Experimental Facilities ......................... 58
  3.2.0.1 UAH Johnson Research Center .......... 58
  3.2.0.2 NASA Glenn Research Center .......... 59

4 Results 62
  4.1 Johnson Research Center ..................... 62
  4.2 Glenn Research Center ....................... 63
    4.2.1 Thrust .................................. 66
    4.2.2 Anode Specific Impulse ................. 67
    4.2.3 Anode Efficiency ....................... 68
    4.2.4 Thrust to Power Ratio ................. 69

5 Discussion 71
  5.1 Performance Predictions ....................... 71
  5.2 Comparison with Other Low-Power Hall thrusters 72
  5.3 Outgassing .................................. 72
  5.4 Spot Formation ............................... 74
  5.5 Repeatability ............................... 76
5.6 Material Longevity ........................................... 78

6 Conclusions .................................................. 83

6.1 UAH-78AM Performance .................................... 83

6.2 Design Improvements ....................................... 84

6.3 Future Work ................................................ 85

APPENDIX A: Performance Prediction Detailed Calculations 88

APPENDIX B: Thrust Stand Uncertainty Analysis 91

B.0.1 Approach .................................................. 91

B.0.2 Example .................................................. 96

REFERENCES .................................................. 102
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>FIGURE</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1 UAH78-AM plume</td>
<td>3</td>
</tr>
<tr>
<td>2.1 ΔV of various deep space missions</td>
<td>10</td>
</tr>
<tr>
<td>2.2 Hall thruster components</td>
<td>14</td>
</tr>
<tr>
<td>2.3 Ionization and propellant acceleration in the Hall thruster</td>
<td>16</td>
</tr>
<tr>
<td>2.4 Electron motion</td>
<td>19</td>
</tr>
<tr>
<td>2.5 FDM/FFF build process</td>
<td>32</td>
</tr>
<tr>
<td>2.6 Powder bed fusion build process</td>
<td>34</td>
</tr>
<tr>
<td>3.1 P5 Hall thruster</td>
<td>39</td>
</tr>
<tr>
<td>3.2 UAH-78AM rendering</td>
<td>42</td>
</tr>
<tr>
<td>3.3 Propellant distributor cutaway</td>
<td>43</td>
</tr>
<tr>
<td>3.4 Channel and propellant distributor revisions</td>
<td>45</td>
</tr>
<tr>
<td>3.5 Secondary electron emission of polymers</td>
<td>49</td>
</tr>
<tr>
<td>3.6 JRC small vacuum facility</td>
<td>59</td>
</tr>
<tr>
<td>3.7 VF-8</td>
<td>60</td>
</tr>
<tr>
<td>3.8 VF-8 thrust stand</td>
<td>61</td>
</tr>
<tr>
<td>4.1 JRC testing</td>
<td>63</td>
</tr>
<tr>
<td>4.2 GRC testing</td>
<td>64</td>
</tr>
<tr>
<td>4.3 Sample thrust curve</td>
<td>65</td>
</tr>
</tbody>
</table>
4.4 Thrust .................................................. 66
4.5 Anode $I_{SP}$ ............................................. 68
4.6 Anode Efficiency ................................. 68
4.7 Thrust/Power ......................................... 70

5.1 30 second thrust test ................................. 73
5.2 Hotspot in V5 ........................................... 75
5.3 Hotspot in V2 .......................................... 76
5.4 Repeatability ............................................ 78
5.5 ABS channel wear ................................. 80
5.6 ULTEM channel wear ............................. 81
5.7 3D printed ceramic wear ....................... 82

B.1 Matlab data reduction interface .............. 92
B.2 Test stand noise ...................................... 95
# List of Tables

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Comparison of electric propulsion technologies</td>
<td>12</td>
</tr>
<tr>
<td>3.1</td>
<td>UAH-78AM scaling</td>
<td>40</td>
</tr>
<tr>
<td>3.2</td>
<td>UAH-78AM versions</td>
<td>44</td>
</tr>
<tr>
<td>3.3</td>
<td>UAH-78AM cost</td>
<td>54</td>
</tr>
<tr>
<td>3.4</td>
<td>UAH-78 cost (conventional materials)</td>
<td>56</td>
</tr>
<tr>
<td>5.1</td>
<td>Comparison of measured and predicted performance</td>
<td>71</td>
</tr>
<tr>
<td>5.2</td>
<td>Comparison to other thrusters</td>
<td>72</td>
</tr>
<tr>
<td>B.1</td>
<td>Sample Calibration Data for Example</td>
<td>96</td>
</tr>
<tr>
<td>B.2</td>
<td>Line of best fit estimates</td>
<td>97</td>
</tr>
<tr>
<td>B.3</td>
<td>Residual standard deviation</td>
<td>98</td>
</tr>
<tr>
<td>B.4</td>
<td>Example thrust data</td>
<td>99</td>
</tr>
<tr>
<td>B.5</td>
<td>Standard error</td>
<td>99</td>
</tr>
</tbody>
</table>
## LIST OF SYMBOLS

<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>DEFINITION</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>Thrust correction factor</td>
</tr>
<tr>
<td>$B$</td>
<td>Magnetic field</td>
</tr>
<tr>
<td>$B_r$</td>
<td>Radial magnetic field</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Beam divergence and multiply charged ion thrust correction factor</td>
</tr>
<tr>
<td>$\Delta V$</td>
<td>Vehicle velocity change</td>
</tr>
<tr>
<td>$E$</td>
<td>Electric field</td>
</tr>
<tr>
<td>$F$</td>
<td>Force</td>
</tr>
<tr>
<td>$F_t$</td>
<td>Beam divergence correction factor</td>
</tr>
<tr>
<td>$g_0$</td>
<td>Gravitational constant</td>
</tr>
<tr>
<td>$\eta_a$</td>
<td>Anode efficiency</td>
</tr>
<tr>
<td>$I_b$</td>
<td>Beam current</td>
</tr>
<tr>
<td>$I_d$</td>
<td>Discharge current</td>
</tr>
<tr>
<td>$I_H$</td>
<td>Hall current</td>
</tr>
<tr>
<td>$I_{SP}$</td>
<td>Specific Impulse</td>
</tr>
<tr>
<td>$J_H$</td>
<td>Azimuthal current density</td>
</tr>
<tr>
<td>$I^+$</td>
<td>Singly charged ion current</td>
</tr>
</tbody>
</table>
$I^{++}$  Doubly charged ion current
$L$  Plasma length
$M$  Propellant/ion mass
$M_f$  Final propellant mass
$M_i$  Initial propellant mass
$m$  Electron mass
$\dot{m}$  Propellant mass flow
$\dot{m}_a$  Anode mass flow
$n_e$  Electron number density
$\omega_c$  Cyclotron frequency
$P$  Power
$P_d$  Discharge power
$q$  Charge
$q_e$  Electron charge
$R$  Mean channel radius
$r_{L,e}$  Electron Larmor radius
$r_{L,i}$  Ion Larmor radius
$\langle \sigma_i v_e \rangle$  Ionization reaction rate coefficient for Maxwellian electrons
$T$  Thrust
$T_e V$  Electron temperature \\
$u_e$  Propellant exhaust velocity \\
$V_b$  Beam voltage \\
$V_d$  Discharge voltage \\
$v$  velocity \\
$v_e$  Electron velocity \\
$v_i$  Ion thermal velocity \\
$v_n$  Neutral propellant velocity \\
$v_{th}$  Electron thermal velocity \\
$w$  Plasma width
For Paul and Jane Hopping.

May my life, and all that I do, be a living legacy of your unconditional love and support. I love you both dearly.
CHAPTER 1

INTRODUCTION

The universe is probably littered with the one-planet graves of cultures which made the sensible economic decision that there’s no good reason to go into space—each discovered, studied, and remembered by the ones who made the irrational decision.

—Randall Munroe

Hall effect thrusters are a class of electrostatic propulsion devices that ionize and accelerate propellant in crossed electric and magnetic fields. The thrusters derive their name from the Hall effect, an azimuthal current that develops as electrons provided by an external cathode drift around the channel. The magnetic field induced by the Hall current couples with the magnetic circuit of the thruster and transfers the reactive force from ion acceleration to the thruster body. While the efficiency and specific impulse of the thrusters is lower than the comparable gridded ion engines, the plasma in the channel is not space charge limited which enables higher thrust-to-power and thrust-to-size ratios in addition to reduced power supply complexity [1,2].
1.1 Problem Statement

Additive manufacturing, or colloquially 3D printing, is being leveraged by the aerospace industry to dramatically reduce the cost of component fabrication [3] and provide other system and performance benefits. Chemical air breathing and rocket propulsion systems are particularly well suited to benefit from additive manufacturing processes due to the complex geometries and low-volume production rates encountered with these systems.

The question this thesis attempts to answer is whether additive manufacturing can be used in a similar manner to reduce the cost of Hall effect thruster fabrication and provide other benefits for test and flight hardware. As with chemical propulsion systems, electric propulsion systems are manufactured in low volumes and include components with complex internal geometries. It stands to reason that there may be cost or performance benefits that can be realized through the application of additive manufacturing to electric propulsion, as with chemical propulsion systems. Furthermore, additive manufacturing may offer additional benefits in a laboratory or test environment, such as the ability to rapidly iterate on a design or easily modify component geometries to incorporate sensors and transducers. This research is focused on low-cost, fast-turnaround, and highly-available 3D printing processes, such as fused-filament fabrication (FFF) and some powder sintering processes such as those that are currently available at many academic institutions.
1.2 Research Contributions

The research contribution of this work is a detailed investigation of the applicability of low-cost additive manufacturing to the design of Hall effect thrusters. To investigate applications for additive manufacturing in the design of Hall effect thrusters, we built and tested a small laboratory Hall thruster using 3D printed components to replace the channel and propellant distributor. The thruster has been titled the UAH-78AM, where 78 is representative of the outer channel diameter (in mm), and AM conveys that the thruster includes components that were produced using additive manufacturing. It is the first Hall effect thruster designed and manufactured at the University of Alabama in Huntsville (UAH). The performance of the thruster was measured in testing at NASA Glenn Research Center to determine
if the 3D printed components were having a significant impact on the performance of the thruster as compared to conventional designs. An image from this testing is provided in Figure 1.1. The experience designing and testing the thruster will be used to inform future use of the selected materials and unconventional geometry in future Hall thruster designs.

1.3 Motivation

There are many reasons we choose to explore space. However, as captured by Randall Munroe’s quote, few of them are currently motivated by economic merit. Access to space is expensive, and the cost of a mission is directly tied to the distance to the destination. Space travel is so expensive that only 9 of our planet’s 196 nations have orbital launch capability, and fewer still are the number of nations that have successfully explored other worlds in our solar system. While progress is being made to reduce the cost of space access, the physics of our current propulsion technology will continue to limit our ability to economically explore many destinations—unless we improve our technology.

The Hall thruster provides a capable tool that we can use to expand our reach within the solar system. In short, it enables us to reach destinations using much less propellant than possible with a chemical propulsion system; this translates to significant cost reductions which enable missions that would otherwise be economically or physically unfeasible. The disadvantage of Hall thrusters and other forms of electric propulsion are the low thrust and high power requirements. A Hall thruster must run for a long time to produce a total impulse comparable to a chemical propulsion
system, and while running the Hall thruster will consume a lot of energy. However, spacecraft power availability continues to increase, and the lifetime limitations that previously prevented Hall thrusters from being used for deep space missions have largely been resolved thanks in part to the advent of magnetic shielding [4, 5]. Therefore, as electric propulsion technology continues to mature, it is likely we are approaching an era where the Hall thruster and other electric propulsion technologies will be used to enable previously impossible deep space missions and new exploration opportunities.

In addition to potential cost and performance benefits, 3D printing may someday enable propulsion systems for spacecraft to be manufactured in space. This would provide a variety of benefits for near-Earth and deep space missions. For near-Earth missions, the ability to 3D print thrusters on orbit would enable customers to build satellites in space from pre-launched stock materials, thus eschewing launch costs. In deep space, the ability to 3D print the propulsion system would provide mission security by enabling servicing of the propulsion system without carrying a finite number of replacement parts. Furthermore, it is possible that raw materials at the destination could be used as precursors for 3D printing, thus enabling the propulsion system to be tailored to new or changing mission objectives.

It is the hope of the author that the results presented in this thesis can be used to inform the design of future Hall thrusters at UAH and other institutions. The ability to rapidly iterate on thruster design and channel geometry provides a valuable tool for researching some of the remaining technical challenges in Hall thruster design, such as maintaining performance while scaling to small dimensions. Further-
more, reducing cost and increasing awareness of Hall thruster capabilities is critical to encouraging adoption of the technology, especially on small satellites. It is the author’s hope that this thesis can prove beneficial on that front as well. Let’s keep exploring!

From here an overview of electric propulsion, the physics of the Hall Effect Thruster, and a review of literature are provided in Chapter 2. Focus then shifts to the development of the UAH-78AM test article and facilities used for data collection in Chapter 3. Presentation and discussion of the data are provided in Chapters 4 and 5, respectively. Finally, the text closes with conclusions from the results in Chapter 6.
CHAPTER 2

BACKGROUND

Professor Oberth has been right with so many of his early proposals; I wouldn’t be a bit surprised if one day we flew to Mars electrically!

—Wernher von Braun

The motivation for development and application of the Hall effect thruster arises from the physics of rocket propulsion. This chapter begins with a review of the general physics of rockets, electric propulsion systems, and Hall effect thrusters. We then transition to a review of challenges faced by government and industry that make Hall effect thrusters a technology of research interest within the academic community. Finally, the chapter closes with an overview of common additive manufacturing technologies employed in the growing 3D printing industry. The purpose in reviewing these somewhat disparate topics is to provide the necessary framework for informing the future discussion on the design and testing of the UAH-78AM.
2.1 Physics of Rocket Propulsion

2.1.1 Tsiolkovsky’s Rocket Equation

Almost every rocket propulsion text begins with a description of Tsiolkovsky’s rocket equation—with good reason. The simple equation captures the basic physics that govern all rocket propulsion systems. In addition, the equation shown in 2.1 provides the cornerstone for our explanation of the merit of electric propulsion systems:

\[
\frac{M_f}{M_i} = e^{-\frac{\Delta V}{u_e}} \tag{2.1}
\]

In Equation 2.1, \(M_f\) is the final vehicle mass, \(M_i\) is the initial vehicle mass, \(\Delta V\), is the vehicle velocity change, and \(u_e\) is the propellant exhaust velocity. \(M_i\) includes all components of the vehicle, such as the payload, propellant, and inert masses such as the structure. \(M_f\) equals the initial vehicle mass minus the propellant expended in the maneuver. It can be seen from Equation 2.1 that the ratio of final to initial vehicle mass decreases as required \(\Delta V\) increases. Since the objective of rocketry is to deliver some payload to a destination, this is a most unfortunate relation. It implies that vehicle propellant mass fraction must increase with mission \(\Delta V\)—limiting the amount of payload we can transport to distant destinations. The intuitive explanation for the relation is that the propellant itself represents additional vehicle mass. Therefore, the addition of more propellant to reach distant destinations requires additional propellant to accelerate the propellant brought along for the mission, hence the exponential relation.
The only mechanism physics provides for us to manage the effects of high mission $\Delta V$ is to increase the propellant exhaust velocity, $u_e$. An increase in $u_e$ works to reduce the magnitude of the exponential term, keeping the mass ratio in check. As we will discuss in greater detail, electric propulsion provides a mechanism that we can use to significantly increase the exhaust velocity relative to chemical propulsion systems. Remarkably, Tsiolkovsky himself was enough of a visionary to foresee the merit of using electric propulsion to attain higher exhaust velocities [6]. While Tsiolovosky incorrectly focused on the acceleration of electrons in his description, the suggestion that electric means can be used to accelerate propellant certainly motivated more extensive research on electric propulsion in the latter half of the 20th century.

How significant is the exhaust velocity improvement we can achieve with electric propulsion? An excellent example is provided by Richard Hofer in Figure 2.1

In Figure 2.1, the green section of the bars represents the $\Delta V$ capacity of the vehicle on-board propulsion system. The red bars represent the $\Delta V$ provided by the rocket. It is immediately clear that the two deep-space electric propulsion missions have significantly more $\Delta V$ capacity on board the spacecraft than similar chemical missions. However, some context relating to the mass of propellant required for the missions makes the plot much more meaningful. The Dawn Mission used 3 NSTAR Ion Engines and carried 358 kg of Xenon propellant to achieve a mission $\Delta V$ of 11 km/s while visiting the asteroid Vesta and dwarf planet Ceres [8]. To achieve the same $\Delta V$ using a traditional chemical propulsion system, Serak calculated that over 17,000
kg of propellant would be required\footnote{Sekerak doesn’t provide the $I_{sp}$ used for his calculation, but about 370 s is estimated by this author from the mass ratios. This is an optimistic $I_{sp}$ for storable chemical propellants.} \cite{9}! The Dawn Mission would not be possible without electric propulsion, which enabled the mission to reach its destination while maintaining a vehicle mass that can be launched by today’s rockets.

\subsection{2.1.2 The Role of Specific Impulse}

To compare the attainable exhaust velocities with different propulsion systems, we introduce the concept of specific impulse:

$$I_{sp} = \frac{T}{m_0 g_0} = \frac{u_e}{g_0}$$  (2.2)
where $T$ is thrust, $\dot{m}$ is the propellant mass flow rate, and $g_0$ is the acceleration due to gravity. Note the second relation, $I_{SP} = \frac{u_e}{g_0}$, is only true if pressure at the engine exit is equal to the ambient pressure. This is the case for electric propulsion systems which generally operate at low pressures; however, it is not necessarily a correct relation for chemical propulsion systems, especially those that are used in the atmosphere.

The $I_{SP}$ of a given propulsion system is directly related to the propellant exhaust velocity. Engines that produce higher exhaust velocities have higher specific impulses, and the $I_{SP}$ attainable by a given design is limited by the mechanism the engine uses to accelerate propellant. Chemical propulsion systems are limited to specific impulses of approximately 450 seconds by the amount of energy stored in chemical bonds, since the energy for propellant acceleration is provided by the propellant itself. The objective of electric propulsion then is to decouple the propellant acceleration mechanism from the propellant, enabling us to control the exhaust velocity and the mechanism used for acceleration. Therefore, we arrive at the definition of electric propulsion provided by Jahn:

\[\text{Electric propulsion is} \quad \text{The acceleration of gases for propulsion by electrical heating and/or by electric and magnetic body forces.} \quad [10]\]

Jahn’s definition captures the three mechanisms available to accelerate propellant outside of chemical means: electrothermal, electromagnetic, and electrostatic. All electric propulsion technologies fall into one of these three categories. In short, electrothermal propulsion systems directly heat the working propellant, while electrostatic and electromagnetic propulsion systems rely on electric and magnetic fields for propellant acceleration, respectively.
2.1.3 Comparison of Propulsion Technologies

The Hall effect thruster is one of many existing electric propulsion systems. Table 2.1 provides a summary of thrust and $I_{sp}$ of various electric propulsion engines, categorized by propellant acceleration method. The Hall effect thruster fills a unique position in the table. While the $I_{sp}$ is lower than the comparable Ion Engine, the Hall Thrusters can produce more thrust for a given engine size and power level. This makes the Hall effect thruster attractive for future deep space missions where the thrust requirements would demand an unsuitably large ion engine. Furthermore, the Hall effect thruster is simpler than the ion engine and requires only a single cathode and reduced number of power supplies.

Table 2.1: Comparison of electric propulsion technologies, compiled from [11, 12]

<table>
<thead>
<tr>
<th>Engine</th>
<th>Electrothermal</th>
<th>Electrostatic</th>
<th>Electromagnetic</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$I_{sp}$ (s)</td>
<td>Thrust (mN)</td>
<td>P (kW)</td>
</tr>
<tr>
<td>Resistojet</td>
<td>200-350</td>
<td>200-300</td>
<td>0.5-1</td>
</tr>
<tr>
<td>Arcjet</td>
<td>400-1000</td>
<td>200-1000</td>
<td>0.9-2.2</td>
</tr>
<tr>
<td>Ion Engine</td>
<td>1500-3600</td>
<td>0.01-500</td>
<td>0.1-200</td>
</tr>
<tr>
<td>Hall Thruster</td>
<td>1500-2000</td>
<td>0.01-2000</td>
<td>0.1-200</td>
</tr>
<tr>
<td>FEEP</td>
<td>8000-12000</td>
<td>0.001-1</td>
<td>0.01-0.15</td>
</tr>
<tr>
<td>Colloid</td>
<td>500-1500</td>
<td>.001-1</td>
<td>0.005-0.05</td>
</tr>
<tr>
<td>MPD Thruster</td>
<td>2000-5000</td>
<td>1-100000</td>
<td>10-1000</td>
</tr>
<tr>
<td>Pulsed Plasma</td>
<td>850-1200</td>
<td>0.05-10</td>
<td>&lt;0.2</td>
</tr>
</tbody>
</table>
A trend that can be discerned from Table 2.1 is that higher specific impulse is generally associated with lower thrust and higher power. This points to an important physical relation regarding power consumption in electric propulsion systems,

$$P = \frac{1}{2} T \cdot I_{SP} \cdot g_0$$  \hspace{1cm} (2.3)

Because power is supplied by the spacecraft for electric propulsion systems, the left side of Equation 2.3 is fixed. Therefore, a trade-off is revealed where higher \( I_{SP} \) (or exhaust velocity) must result in lower thrust at a given power level. Mission designers select the propulsion system based on the need to balance efficiency \( (I_{SP}) \) and thrust within a given power envelope. Another significant advantage of some electric propulsion systems for flight applications is the ability to throttle between high thrust and high \( I_{SP} \) operating modes. This enables electric propulsion systems to operate in high thrust modes for getting out of gravity wells then transition to high \( I_{SP} \) modes for longer duration cruise.

Electric propulsion engines flown to date produce much lower thrust than equivalent chemical systems. This is because the spacecraft must provide the power for propellant ionization and acceleration. Due to the energy intensive nature of these processes, electrically propelled spacecraft of today lack the power to expend propellant at flow rates that would produce comparable thrust to chemical propulsion systems. Therefore, electric propulsion systems must operate for longer durations to produce total impulses comparable to chemical propulsion systems. This introduces
unique materials and lifetime challenges for electric propulsion systems, which sometimes must operate for thousands of hours to achieve the required $\Delta V$.

2.2 Physics of the Hall effect thruster

Mechanically, the Hall effect thruster is a simple device. The assembly consists of an external hollow cathode and the main thruster body, as shown in Figure 2.2. Depending on the size and type of Hall thruster, the hollow cathode can be mounted externally or in the center of the thruster inner magnetic pole.

![Figure 2.2: Major components of a Hall effect thruster](image)

The thruster body consists of a toroidal discharge channel which can be lined with a conductive or dielectric material. The material selection has a dramatic impact
on the plasma properties in the channel. This is discussed in greater detail later in the chapter. Behind the channel, a soft-magnetic material such as carbon steel is used to create a magnetic circuit. Electromagnets are positioned around the perimeter of the discharge channel and in the center of the device to establish the magnetic field in the circuit. The objective is to project a primarily radial magnetic field into the discharge channel and across its exit plane. At the base of the channel rests a metal ring, which is positively biased and serves as the thruster anode. Propellant is distributed through a baffle and orifice assembly which can either be integrated into the anode or installed at the base of the channel.

2.2.1 Basic Principles

Thruster operation begins with the hollow cathode. The hollow cathode is responsible for providing free electrons by ionizing a small amount of thruster propellant through thermionic emission from a low work function emitter material. The electrons serve dual purposes of ionizing propellant in the thruster and neutralizing the stream of positively-charged ions in the plume, as shown in Figure 2.3. The physics of hollow cathodes is quite complex and continues to be an area of active research. The curious reader may find a more in depth discussion in [1]. Some of the electrons produced by the hollow cathode see the electric field established by the positively-biased anode and are drawn into the thruster. However, the radial magnetic field established at the channel exit plane imparts a Lorentz force on the moving
The purpose of the magnetic field is to increase the electron number density in the channel region and limit electron migration to the anode, thus promoting efficient propellant ionization.

$$F = q(E + v \times B)$$  \hspace{1cm} (2.4)

where $F$ is the force, $q$ is the particle charge, $E$ is the electric field, $v$ is the particle velocity, and $B$ is the magnetic field. The cross product in Equation 2.4 causes electrons to travel helically around radial magnetic field lines in the channel due to the force moving charges experience in a magnetic field.
Meanwhile, neutral propellant atoms, typically Krypton or Xenon, are injected into the thruster at the base of the channel. The neutrals diffuse into the ionization region where energetic electrons are confined in the thruster magnetic field. Collision events between the fast-moving electrons and slow neutral atoms cause the neutrals to lose electrons and become positively ionized. The ionization events release more free electrons which continue to ionize propellant neutrals. Once the neutrals are ionized, they are accelerated by the electric field between the anode and cathode and leave the thruster at velocity dependent on the anode voltage. Finally, charge neutrality of the system is maintained by a stream of electrons that are drawn from the cathode to neutralize the propellant plume.

Since propellant ions are charged, the ions also feel a force from the magnetic field established in the discharge channel. However, the ion cyclotron radius is much larger than the electron cyclotron radius,

\[
r_{L,i} = \frac{1}{B} \sqrt{\frac{2M}{q_e} V_b}
\]

(2.5)

\[
r_{L,e} = \frac{1}{B} \sqrt{\frac{8mT_{ev}}{\pi q_e}}
\]

(2.6)

In Equation 2.5 \(V_b\) is the beam voltage, \(M\) is the propellant mass, \(B\) is the magnetic field, \(q_e\) is the electron charge, and \(r_{L,i}\) is the ion radius of gyration. In Equation 2.6, \(r_{L,e}\) is the electron radius of gyration, \(m\) is the electron mass, and \(T_{eV}\) is the electron temperature. A Xenon ion is approximately 239,000 times more massive than an electron, resulting in a cyclotron radius that is larger than the size of the thruster.
Consequently, ions escape the magnetic field range-of-influence well before trajectories are significantly affected. This principle allows the thruster to efficiently confine electrons without confining ions. However, the magnetic field does impart a small angular velocity on ions. While this velocity is generally trivial, it does produce a “swirl torque” that has been measured on flight thrusters and must be accounted for during maneuvering [13].

Hall effect thrusters are classified as electrostatic accelerators due to the axial electric field established in the discharge channel to accelerate ions. However, thrust is transferred to the device through the magnetic circuit—not the electric field. This is best explained through the introduction of the Hall current, from which the thrusters derive their name.

Electrons confined in the channel magnetic field experience a net azimuthal drift induced by the axial electric field produced by the anode. The electron drift velocity is defined by Equation 2.7:

\[ v_e = \frac{E \times B}{B^2} \]  \hspace{1cm} (2.7)

Where \( v_e \) is the electron velocity. The azimuthal drift coupled with the magnetic field confinement causes electrons to migrate around the channel in a complex helical pattern visualized in Figure 2.4.

Since current is defined as moving charge, the total current is the integral of the electron velocity over the 2-D plasma area multiplied by the number of electrons,
as shown in Equation 2.8:

\[ I_H = n_e q_e w \int_0^L u_e dz = n_e q_e w \frac{V_d}{B} \]  

(2.8)

Where \( I_H \) is the Hall Current, \( n_e \) is the electron number density, \( w \) is the width of the plasma in the channel, \( L \) is the plasma length, and \( V_d \) is the discharge voltage. The second expression in Equation 2.8 provides an insightful relation by indicating that the Hall current is proportional to the ratio of discharge voltage to the applied magnetic field.

The magnitude of the Hall current in running thrusters varies with the discharge voltage and magnetic field settings, but is generally significantly higher than the discharge current. The Hall current induces a magnetic field in the plasma region that couples with the thruster magnetic circuit and imparts a force on the thruster body.
2.2.2 Material Requirements

The significant challenge of applying additive manufacturing to the design of Hall effect thrusters is accommodating the stringent and unconventional material requirements. The thruster itself can be divided into three general components: the magnetic circuit, the discharge channel, and the anode/propellant distributor. The following section defines the unique material requirements for each of these subsystems.

2.2.2.1 Discharge Channel

The discharge channel plays a variety of critical roles in the Hall effect thruster. Perhaps the most important purpose is to protect the magnetic circuit from sputter erosion by high energy ions. Most of the high energy ions produced in a Hall thruster are accelerated out of the discharge channel and into free space. However, some ions collide with surfaces inside the discharge channel and sputter erode the channel material. If the magnetic circuit were left directly exposed to the plasma, ion erosion would damage the electromagnets and flux guide and eventually cause either an electrical short or alteration of the magnetic field topography in the channel.

The discharge channel can be made with a metal or dielectric material. However, we will limit our future discussion to Hall effect thrusters with dielectric channel walls, commonly referred to as stationary plasma thrusters (SPT) in literature. Until the advent of magnetic shielding, implementation of Hall effect thrusters with metal channels was only efficient if the channel walls were biased to near cathode potential
to repel plasma electrons, as in the Thruster with Anode Layer (TAL) [1,14]. SPT-type thrusters with conducting walls have higher anode leakage currents and lower efficiencies than SPT’s with dielectric walls because the conductive channel shorts the plasma in the discharge channel and provides an additional path for cathode electrons to reach the anode [15,16]. Magnetic shielding may provide a mechanism to reduce the efficiency losses associated with conductive walls by reducing plasma-wall interaction; this is presently a topic of active research [17]. Most recent literature in the United States appears to be focused on the SPT-type thruster, and the majority of TAL research occurs in Russia.

In conventional SPT thrusters with dielectric walls, the channel material plays a critical role in modifying plasma properties and thruster performance. Some energetic electrons and ions possess enough kinetic energy to pass through the plasma sheath established at the channel wall and bombard the channel surface. The bombardment causes the channel wall material to emit secondary electrons at lower energy than the primary electrons [14,18,19]. The lower temperature plasma reduces the production of multiply-charged ions, which improves efficiency for fixed beam currents [20]. Furthermore, plasma-wall interactions allow electrons to diffuse across radial magnetic field lines towards the anode. The resulting near-wall conductivity is believed to be a contributing mechanism to cross-field electron transport in Hall effect thrusters, a process which continues to be poorly understood [9].

Power loss to the channel walls represents a significant source of efficiency reduction in SPT thrusters. Electron bombardment is the primary source of heating in the channel [21]. Due to operation in a vacuum environment at high plasma
temperatures, the steady state operating temperature of the channels can be on the order of 600-800K in the unshielded SPT [21]. Magnetically shielded Hall thrusters have lower channel temperatures compared to unshielded thrusters at the same power levels, but temperatures are still on the order of 500-800K [17].

Due to the high steady state operating temperatures and dielectric material requirements, most Hall thrusters use refractory ceramics such as boron nitride or borosil for the channel materials. Polymers are believed to have secondary electron emission characteristics comparable to ceramics, but melting temperatures are generally too low for steady state thruster operation.

2.2.2.2 Magnetic Circuit

The magnetic circuit is responsible for projecting the radial magnetic field across the discharge channel exit plane. Permanent magnets or electromagnets may be used for producing the field, and both approaches have their advantages and disadvantages. Permanent magnets do not require power from the spacecraft in order to operate, therefore reducing thruster power requirements and increasing the thrust-to-power ratio. However, permanent magnets fix the magnetic field to a single configuration and limit the ability to optimize the magnetic field at different discharge voltages. Care must also be taken in the design of permanent magnet thrusters to ensure steady-state operating temperature is below the Curie Temperature of the magnetic material to avoid demagnetization.

In general, a soft magnetic material is used in tandem with the magnetic material to act as a flux guide for shaping the magnetic field in the channel. Soft magnetic
materials conduct magnetic field lines more effectively than free space, but do not produce a magnetic field of their own. Materials with high magnetic permeability are favorable for this task, such as low-carbon steels and nickel-iron alloys. An important design consideration with the magnetic circuit is ensuring that the magnetic flux density in the material does not approach saturation. If the magnetic circuit saturates, magnetic efficiency is decreased and further increases in magnetic field intensity at the electromagnets are not properly conveyed to the channel exit plane.

2.2.2.3 Anode/Propellant Distributor

The anode is traditionally fabricated from non-magnetic material in order to avoid interference with the magnetic circuit. Depending on thruster geometry and operating point, the anode can reach temperatures on the order of 700-1100K due to electron bombardment and radiative heating from the plasma [22, 23]. The location of the anode at the base of the channel and need for electrical isolation limits cooling mechanisms. Because of this, stainless steels are commonly used as anode materials for their high service temperatures and nonmagnetic properties.

Propellant is frequently distributed through a baffle and orifice assembly integrated into the anode. The baffle and orifice assembly is intended to maximize neutral residence time in the channel which improves ionization efficiency. Integration of the propellant distributor into the anode simplifies manufacturing of the channel; however, anode fabrication becomes costly due to the welding processes required to integrate the distributor and baffle assembly.
2.2.3 Research Interests

Hall effect thrusters trace their origin to research on magnetrons and other cross field plasma sources in the 1960’s [20]. While the technology was first demonstrated in working devices by American scientists, development shifted to the Soviet Union as Americans focused on the higher specific impulse ion engine [9, 24]. At the end of the Cold War, Hall thruster research in the US was rekindled by technology developments made in the Soviet Union. Subsequent research efforts in the US focused on increasing the specific impulse and lifetime of the Hall thruster as compared to Soviet designs. As of 2008, over 140 Hall effect thrusters have been operated in space, most of Soviet design heritage [9]. This demonstrates that the Hall effect thruster has successfully transitioned from an experimental propulsion technology to a flight proven technology. However, technical challenges remain with adapting Hall effect thrusters to new applications, namely deep space exploration and small satellites. These challenges arise from a deficiency of understanding of some physical mechanisms governing Hall thruster behavior; therefore, the thrusters continue to be of research interest in the academic community. This section will focus on four frontiers in Hall effect thruster research. A significant portion of present work in the industry is directed towards addressing one of these challenges.

2.2.3.1 Anomalous Diffusion

A curious paradox of Hall effect thrusters is that a current of electrons must reach the anode for the thruster to operate, yet high efficiency operation is associated
with minimizing this anode leakage current. This odd relationship can be seen by examining the equation for anode efficiency:

\[ \eta_a = \frac{1}{2} \frac{T^2}{m_a P_d} \]  

(2.9)

Where \( P_d \) is defined as:

\[ P_d = I_d V_d \]  

(2.10)

Where \( I_d \) is the discharge current and \( V_d \) is the discharge voltage. It can be seen from Equation 2.9 and Equation 2.10 that an increase in \( I_d \) results in an increase in \( P_d \) which reduces anode efficiency assuming the thrust remains constant. Furthermore, minimizing \( I_d \) for a fixed discharge voltage improves efficiency.

The purpose of this introduction is to highlight the need for an explanation of the physical mechanism by which electrons reach the anode. Electrons are confined by the radial magnetic field to the ionization and acceleration regions. Yet, several proposed mechanisms contribute to electron migration towards the anode. These mechanisms include electron-wall, electron-neutral, and electron-ion collisions, as well as turbulent plasma fluctuations. The diffusion contribution from electron-neutral and electron-ion collisions is well understood, but a first principles theory to predict contributions from electron-wall and turbulent plasma fluctuations continues to elude researchers [9]. The challenge is exasperated by the fact that the contribution of electron-wall and turbulent plasma fluctuations is likely strongly dependent on thruster geometry and magnetic field topography, limiting the ability to compare
behavior between devices. This is highlighted in recent research that suggests plasma oscillations differ for magnetically shielded and unshielded thrusters [25–27].

The consequence of a missing first principles theory for electron wall and turbulent plasma diffusion is a limited ability to predict thruster performance without experimental testing. Currently, this increases Hall thruster development time and cost because prototypes must undergo intensive test programs to characterize how plasma oscillations and wall interactions impact performance. The capability to predict anomalous diffusion mechanisms in a thruster would simplify test programs and allow designers to optimize thrusters based on these performance parameters.

The anomalous diffusion problem is one of the oldest mysteries of Hall thruster physics, and researchers have been trying to understand the mechanism since the 1960’s [28]. The advent of modern modeling and simulation tools including particle-in-cell and fluid/hybrid codes have helped researchers gain insight into turbulent diffusion, but a solution remains elusive [29]. While predicting the future is seldom a wise endeavor, the author suspects that there is enough interest in solving the anomalous diffusion problem that an adequate solution will be found in the next several decades as modeling and simulation tools continue to mature.

2.2.3.2 Facility Effects

Closely tied to the anomalous diffusion problem is impact of facility effects on Hall thruster performance. Hall thrusters are currently tested on the ground in large vacuum chambers designed to duplicate the environment of space. However, even the best facilities cannot duplicate the low base pressures of space. In addition, thrusters
in ground test environments have been shown to electrically couple with the chamber facilities [30–33].

The elevated base pressures in Hall thruster test facilities are caused by neutral propellant expelled by the thruster residing in the vacuum chamber before being collected by pumping facilities. During this time, the propellant can re-enter the discharge chamber and become re-ionized, contributing to artificially high thrust and discharge currents not observed in flight applications [34–37]. A variety of facility implementation strategies have been developed to minimize neutral ingestion, including positioning of cryopump and beam dump surfaces to minimize the quantity of plume neutrals reflected back at the thruster discharge chamber. However, no pumping facility will ever perfectly match the low base pressures of space. Therefore, development of models to predict the impact of facility effects are of significant interest, especially as thrusters scale up to higher powers and larger propellant flow rates. For example, there are few vacuum facilities in the world with the pumping speed to test 100 kW+ Hall thrusters such as the X3 and maintain acceptable background pressures [38,39].

The electrical interactions of the thruster with the chamber are more subtle and still not fully understood. Frieman, Walker, et. al. have shown that the chamber walls conduct current and are a recombination site for plume ions [30]. This current return path does not exist in space and could change plasma properties, thruster efficiencies, and plume behaviors as measured on the ground in comparison to on orbit operation. In particular, there is some evidence that plume divergence angles are affected [40], which limits the ability of spacecraft designers to predict interactions of the thruster plume with other spacecraft components such as solar panels.
Both electrical and pressure facility effects are coupled to anomalous diffusion because they may affect plasma instability modes and turbulent diffusion in the discharge chamber. Therefore, a better understanding of facility effects also provides insight concerning anomalous diffusion, and vice-versa.

2.2.3.3 Lifetime Limitations

Considerable effort has been directed towards extending the operational lifetime of Hall effect thrusters since the resurgence of US interest in the devices at the end of the Cold War. The lifetime of conventional SPT-type Hall effect thrusters is limited by the lifetime of the insulating discharge channel material. This material is subjected to erosion by energetic ions, eventually exposing the magnetic circuit and causing the thruster to fail.

Two approaches to address the erosion problem can be considered. The simplest is to select materials with increased resistance to the ion sputtering process. However, the material requirements in the SPT are already limited due to the SEE and thermal requirements, and early designs converged on boron nitride and borosil as optimal solutions. Therefore, most research efforts have focused on eliminating the source of the ion sputtering through manipulation of the plasma location in the channel.

It has been shown that the magnetic field topology in the channel affects the performance and efficiency of a Hall thruster [1, 20, 24]. Electron mobility is limited across magnetic field lines but is uninhibited parallel to the field. This causes the electron thermalized potential to be constant along magnetic field lines to the first
order [1, 34]. Therefore, the plasma potential along a field line is only a function of the plasma density, and the magnetic field lines in the channel are representative of equipotential lines of the electric field to an accuracy on the order of the electron temperature [34]. The consequence of this important relationship is that magnetic field topology in the channel controls the shape of the electric field and the direction in which ions are accelerated.

Between 2007 and 2009, a qualification model Aerojet BPT-4000 demonstrated a lifetime surpassing 10,400 hours [41]. Remarkably, channel erosion was observed to cease in the thruster after approximately 5,600 hours. Subsequent analysis by JPL revealed that the shape of the magnetic field topology in the channel was responsible for preventing further channel erosion [4]. In summary, the magnetic field topology in the channel featured equipotential lines that ran nearly parallel to the discharge channel walls in the acceleration region once the walls had partially eroded away. Since electron temperature is roughly constant along field lines, this resulted in a shield of low temperature electrons at the discharge channel wall and also eliminated the electric fields responsible for accelerating high-energy ions into the walls. The field topology has been termed magnetic shielding and is considered by many to be a breakthrough in extending the lifetime of Hall effect thrusters.

Subsequent research on magnetic shielding has extended application of the technology to thrusters of various scales including the NASA-300MS [42], H6MS [43], HERMeS [44], and MaSMi [45, 46]. Research efforts are now focused on characterizing the secondary effects of magnetic shielding to bring the technology to the required maturity for flight applications. Magnetic shielding is not without disadvantages.
The shielded field topology shifts the acceleration region out of the discharge channel relative to conventional thrusters—increasing plume divergence and making pole piece covers more susceptible to erosion [44]. However, it is likely that these technical challenges can be managed once appropriately characterized, and magnetic shielding will prove a valuable tool for bringing the lifetime of Hall thrusters to durations acceptable for deep space applications.

It should also be noted that magnetic shielding is not the only proposed magnetic field topology modification that can be employed to extend the life of Hall effect thrusters. CNRS has proposed a wall-less field topology which shifts the ionization and acceleration plasma regions outside the channel entirely [47,48]. Yongjie Ding et. al. have proposed unique "push down" field topology which also claims to reduce ion sputtering of the channel walls [49].

2.2.3.4 Small Thrusters

The scaling of Hall thrusters to small sizes presents unique challenges that limit thruster performance and lifetime. A significant factor is the increase in surface-to-volume ratio of the channel as thruster size is decreased [45]. The increase in surface-to-volume ratio results in greater wall power losses and reduced thruster lifetime. Therefore, many small Hall thruster designs adopt geometries that attempt to minimize the surface-to-volume ratio by recessing the center pole or adjusting the channel dimensions. These unconventional geometries include the Cylindrical Hall thruster (CHT) [50] and Fully Cylindrical Hall thruster (FCHT) [51]. The consequence of geometry changes to minimize surface-to-volume ratio is increased complexity of the
magnetic field topology in the channel which can negatively impact performance by increasing beam divergence.

Equation 2.6 presents an additional challenge for reducing the size of Hall thrusters. The magnetic field strength required to maintain an appropriate electron-Larmor radius is inversely proportional to thruster size. Therefore, smaller thrusters require stronger magnetic fields in the channel; yet less space is available to integrate the magnetic circuits necessary for these higher field strengths [50]. Developing the magnetic circuit for the center pole becomes especially challenging due to the increased thermal load and potential for magnetic saturation.

For these reasons, commercially available Hall effect thrusters with discharge powers below 100 W are rarely seen, and thrusters with discharge power of 200-500 W have shorter lifetimes and poor performance as compared to conventional thrusters [45]. Therefore, the development of strategies to improve thruster performance and lifetime at these scales remains an area of active research.

2.3 3D Printing Processes

The 3D printing industry continues to be volatile and competitive as disruptive technology improvements redefine the role of additive manufacturing in both commercial and consumer spaces. A comprehensive survey of 3D printing technologies, if provided in this thesis, would likely be outdated in only a couple of years. However, there is a need to review 3D printing technologies as they relate to materials selection and design of Hall thrusters. We review the principles of three well-established 3D printing technologies which use materials that have applications in Hall thrusters.
2.3.1 Fused Deposition Modeling/Fused Filament Fabrication

Fused Deposition Modeling, (the term trademarked by Stratasys) or Fused Filament Fabrication, refers to a class of 3D printing processes that build parts from melted and extruded filament that fuses to preceding layers as the build material cools, as shown in Figure 2.5. The invention of FFF in the United States is credited to S. Scott Crump, whose patent for the process was accepted in 1989 [52]. Crump proceeded to co-found Stratsys Ltd., which continues to be one of the market leaders in the additive manufacturing industry. The patent on the FFF processes expired in 2009, resulting in a dramatic decrease in cost of FFF parts and a significant increase in process availability. FFF 3D printers are now available to businesses and consumers from a variety of manufactures at price points well below one-thousand dollars.

![Figure 2.5: FDM/FFF build process](image)

Figure 2.5: FDM/FFF build process
FFF primarily uses polymers such as ABS, PLA, Polycarbonate, Nylon, and ULTEM as build materials, although metals and powdered ceramics may also be used. Polymers are favorable for the process due to their low melting and glass transition temperatures. Since the expiration of the 2009 patent, the number of available build materials has expanded substantially. In addition, the practice of including additives in polymers to change material properties such as tensile strength, conductivity, or surface finish has become increasingly common. FFF is perhaps the most versatile form of 3D printing on the market today due its broad availability and low cost.

However, the process is not without limitations. Resolution is typically limited to a few hundred microns because of the increasing surface to volume ratio at smaller nozzle diameters. In addition, print times increase substantially for small nozzles as diameters continue to decrease. Parts are susceptible to failure by delamination at the interface between layers, and this contributes to anisotropic strength properties in finished components. Furthermore, application of the process to higher temperature materials such as metals is rarely seen, because the nozzle must remain solid at temperatures above the melting point of the build material and not erode during the printing process. Surface finish is also poor, and as seen in Figure 2.5, parts always have a ribbed surface finish due to the layering process.

2.3.2 Powder Bed Fusion

Powder bed fusion is used to refer to a broad class of additive manufacturing processes that use some kind of energetic beam to fuse a powder into a solid object, as shown in Figure 2.6. Specific technologies include Direct Metal Laser Sin-
tering (DMLS), Selective Laser Sintering (SLS), Selective Laser Melting (SLM), and Electron Beam Melting (EBM). The processes differ in terms of the energy transfer mechanism, material properties of the finished part, and build environment—but the fundamentals of the processes are the same. The invention of powder bed fusion predates FDM, and the earliest patent on the processes using a laser was filed by Housholder in 1979 [53].

Figure 2.6: Powder bed fusion build process

1 Powder chamber (raw material)
2 Roller deposits new layers of powder in build chamber
3 Laser/electron beam/plasma beam sinters particulates
4 Piston lowers finished layers

Powder bed fusion has the benefit of being able to work with almost any material that can be atomized or reduced to a powdered precursor. Because melting occurs external to a restive heater or nozzle, the process can occur at temperatures above what would be achievable in an FFF-type heater and extrusion system. Build
materials include titanium, superalloys such as inconel, copper, aluminum, polymers, and ceramics. In addition, parts have superior material properties as compared to FFF. Material properties of parts are much closer to isotropic than with FFF, and part densities can approach those of cast or machined components. Furthermore, part accuracy is much higher than FFF since resolution is only limited by powder and laser/electron beam spot size.

Compared to FFF, the cost of fused components is significantly higher in part due to the high cost of the equipment required for the processes. The expensive laser or electron beam equipment currently limits the market for powder bed fusion 3D printers to commercial customers. Furthermore, build parameters and part post-processing are very material specific and continue to be an area of active research. Parts must frequently undergo a bake-out processes after printing to remove residual internal stresses that are incurred as part of the fabrication processes. Parts also have a rough surface finish.

Ceramics can be processed in powder bed fusion systems, but present unique challenges. Ceramics do not tolerate high residual stresses, increasing the risk of fractures or cracks as compared to metals. Additional challenges are discussed in [54]. The implementation challenges are certainly not insurmountable, but have limited commercial availability for ceramic processes as compared to powder bed fusion of metals.
2.3.3 Binder Jetting

The binder jetting processes is similar to powder bed fusion in that parts are built from a powder precursor material. However, instead of using an energy source to directly fuse powder particles, an adhesive binder material is used to glue particles together. Depending on the build material and process, the part can be post-processed to remove the binder material and fuse the primary powder into a solid. The binder may also be left in the part with no finishing processes.

The binder jetting processes can work with a variety of materials including metals, ceramics, plastics, and plasters. A significant benefit of the processes is that a heat source is not required during the manufacturing process. Therefore, parts do not have residual stresses that can be encountered with FFF and powder bed fusion processes. In addition, machine cost is reduced relative to powder bed fusion due to elimination of the directed energy source and build chamber environmental controls required for those systems. Hot isostatic pressing is used with metals and ceramics to remove the binder material and fuse the powder, producing parts that are nearly solid.

Binder jetting is included in this chapter because the ceramic inner channel used in the thruster was produced using a binder jetting process. In this process, a “green” ceramic is 3D printed using the binder material to hold the ceramic powder together. The ceramic is then glazed and fired in a conventional kiln process, and the binder decomposes and burns off. The finished part has the physical properties of a ceramic, but lacks the density or dimensional accuracy of a part produced through
hot pressing and conventional machining. This highlights the significant limitation of
the binder jetting processes: parts must undergo secondary processing to remove the
binder, and the processing may change part geometry or fail to produce fully dense
parts.

2.4 Closing Remarks

The results presented in this thesis do not directly address any of the preceding
research topics. However, application of additive manufacturing in the design of Hall
effect thrusters could expand the design space in a manner that provides researchers
with new tools to address all of the research challenges outlined in this chapter. For
example, the ability to adjust channel wall surface geometry with fine detail may
provide insight into the role of wall effects in anomalous diffusion, and reducing the
cost of a single laboratory Hall thruster enables researchers to put more resources
towards test programs. If additive manufacturing ever proves to have application for
flight hardware, the associated cost reductions could increase the availability of Hall
thrusters for the commercial space industry. Therefore, our objective in this thesis
is to assess the feasibility of using 3D printing as a tool to advance Hall Thruster
research activities and the development of flight hardware.
CHAPTER 3

METHODS

*Everything should be made as simple as possible, but not simpler.*

—Albert Einstein

To research applications for the use of low-cost additive manufacturing in the design of Hall thrusters, we designed an SPT-type Hall thruster to use as a test bed for different channel designs and materials. The UAH-78AM is the first Hall Thruster to be developed at UAH. A review of the design process, materials selection, and manufacturing of the UAH-78AM is provided as it is the main focus of our experimental methods and results. Other aspects of the experiment setup are also addressed, such as the facilities and diagnostic tools.

3.1 Development of the UAH-78AM

3.1.1 Preliminary Design and Scaling

The channel geometry of the UAH 78-AM is roughly scaled from the P5-Hall thruster. The P5 is a 5-kW laboratory Hall thruster jointly developed by the University of Michigan and Air Force Research Laboratory in the early 2000’s for research and diagnostic testing [24]. An image is provided in Figure 3.1. Research
with the thruster later inspired high specific impulse thrusters such as the NASA-173Mv1 [20]. We selected the P5 for scaling because it is a well-characterized thruster and dimensions of the magnetic circuit and channel are published in the literature [24].

![P5 Hall thruster](image.png)

**Figure 3.1**: P5 Hall thruster [55]

The UAH-78AM was designed to fit in CubeSat dimensions (10 x 10 x 10 cm) for demonstration purposes and to facilitate testing in small vacuum facilities. Dimensions were linearly scaled from the P5, as shown in Table 3.1. The scaling factor used to bring geometry into the CubeSat form factor was approximately 1/2.15.

More rigorous scaling methodologies exist in literature that take into consideration plasma physics in the channel while scaling the thruster [56]. We decided to forgo an analysis with these methodologies because our scale factor is relatively low and lots of thrusters already exist in this size class. If we were scaling to high or low powers where few operational Hall thrusters exist, we would have applied more
Table 3.1: Comparison of P5 and UAH-78AM dimensions (mm)

<table>
<thead>
<tr>
<th></th>
<th>P5</th>
<th>UAH-78AM</th>
<th>Scale Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outer Channel Diameter</td>
<td>170</td>
<td>78</td>
<td>2.18</td>
</tr>
<tr>
<td>Channel Width</td>
<td>25.4</td>
<td>11.6</td>
<td>2.19</td>
</tr>
<tr>
<td>Depth to Anode Face</td>
<td>38</td>
<td>18.2</td>
<td>2.09</td>
</tr>
</tbody>
</table>

rigor to the scaling methodology to ensure the channel geometry produced plasma properties conducive to efficient Hall thruster operation.

The thruster has a discharge power in the range of 300-500W, which is well above the power supply capabilities of most CubeSats. The higher discharge power was chosen because Hall thrusters small enough to be powered by CubeSats currently have significant channel wall erosion and electron losses due to the increased surface to volume ratio [45]. Our research objective is to explore opportunities for the use of additive manufacturing in the development of Hall thrusters, and the small dimensions and high wall power losses seen in smaller thrusters would interfere with this objective.

There are several verification checks that can be completed with a Hall thruster geometry to ensure plasma parameters are acceptable for normal operation. Perhaps the most important criteria when scaling a Hall thruster are ensuring that electron Larmor radius remains smaller than the plasma depth in the channel, and ion Larmor radius remains larger than the channel diameter,

\[ r_{L,e} = \frac{v_{th}}{\omega_c} = \frac{m}{q_e B} \sqrt{\frac{8kT_e}{\pi m}} \ll L \tag{3.1} \]
where $r_e$ and $r_i$ are the gyration radii of the electrons and ions, respectively, and $\omega_c$ represents the cyclotron frequency. $v_{th}$ represents the thermal velocity of electrons in the channel, and $v_i$ represents ion velocity. $L$ represents the magnetized plasma depth in the channel, which can be approximated using Equation 3.3,

$$L = \frac{3v_n}{n_e\langle \sigma_i v_e \rangle}$$

where $\langle \sigma_i v_e \rangle$ represents the ionization reaction rate coefficient for Maxwellian electrons, which can be found in Appendix E of [1] for Xenon. $v_n$ represents the neutral velocity of propellant entering the plasma region.

Without knowledge of the electron temperature in the channel, some assumptions are made in order to apply the preceding equations to Hall thruster scaling. An electron temperature $T_e$ of approximately 25 eV can be assumed to first order from measurements made in other Hall thrusters. The UAH-78AM is found to satisfy the requirements of Equation 3.1 and Equation 3.2 with $T_e = 25$ eV. Detailed calculations are provided in Appendix A.

### 3.1.2 Design Features

A rendering of the UAH-78AMv5 as tested at NASA GRC is provided in Figure 3.2.
Unlike the P5, the UAH-78AM was designed to use 4 outer electromagnets (as opposed to 8) due to its smaller size and the constraint of fitting in the CubeSat form factor. In versions 4 and 5, these electromagnets were replaced by a single coil wrapped around the outer magnetic screen to increase the strength of the outer electromagnet, as shown in Figure 3.2. The original pole pieces were kept in the design for the role they play in connecting the front and back plates of the flux guide. A single electromagnet is used in the center of the thruster, and 1/4-20 carbon steel screws are used as flux guides to connect the inner and outer magnetic circuit components. Like the P5, the UAH-78AM is designed to use an externally-mounted hollow cathode primarily due to the challenge of incorporating a central hollow cathode into the magnetic circuit design in smaller thrusters.

The design process for the UAH-78AM was unconventional due to the use of additively manufactured components, which enable an incremental design and test approach because of the low-cost and fast manufacturing time. To date, the thruster
has gone through 5 incremental revisions. Changes were made to the design with each version as testing revealed new opportunities for improvement. The changes associated with each version are summarized in Table 3.2.

Components selected for 3D printing included the channel and propellant distributor. Initially, these two parts were printed as a monolithic component to reduce part count. The propellant distributor was integrated into the base of the channel, as shown in Figure 3.3. The printed ABS propellant distributor allowed us to use a simple stainless steel ring as the anode, thus separating propellant injection from the anode. Initial testing revealed that the lifetime limiting component in the thruster was the channel, which would degrade due to heating near the channel exit plane. Therefore, later versions of the thruster separated the channel from the propellant
Table 3.2: UAH-78AM changes with each revision

<table>
<thead>
<tr>
<th>Version</th>
<th>Features</th>
</tr>
</thead>
<tbody>
<tr>
<td>V1</td>
<td>Baseline design with five (5) 1018 steel-core solenoids for the magnetic circuit. The channel and propellant distributor were manufactured as a single ABS part, and included unnecessary features such as covers for the magnetic solenoids that were removed in later revisions to simplify assembly and disassembly.</td>
</tr>
<tr>
<td>V2</td>
<td>Redesigned channel to break into three pieces: inner channel, outer channel, and rear propellant distributor. All pieces were threaded for assembly. The outer channel included an outer pole cover.</td>
</tr>
<tr>
<td>V3</td>
<td>Added magnetic screens fabricated from 1010 carbon shim stock to reduce anode current losses. Increased gap clearance between the anode and the surface of the propellant distributor.</td>
</tr>
<tr>
<td>V4</td>
<td>Replaced the four separate outer solenoids with a single coil wrapped around the outer magnetic screen. Separated inner and outer electromagnets, allowing the inner and outer magnet current to be set with two different supplies. Increased gap clearance between the anode and bottom of the propellant distributor. Replaced inner ABS channel with a 3D printed-glazed ceramic. Removed pole covers to accommodate ceramic 3D printing. This version was tested at NASA GRC</td>
</tr>
<tr>
<td>V5</td>
<td>Replace ABS outer channel with ULTEM material. This version was tested at NASA GRC</td>
</tr>
</tbody>
</table>

distributor, simplifying thruster disassembly and enabling the channels to be replaced without reprinting the propellant distributor. This is shown in Figure 3.4, where blue components represent the propellant distributor and red components represent channels. Separating the channel from the propellant distributor also allowed the channel to be printed from a higher-temperature thermoplastic such as ULTEM in order to improve lifetime.
Figure 3.4: Channel and propellant distributor revisions. Blue and red components are 3D-printed

3.1.3 Performance Prediction

A first order prediction of thrust and $I_{SP}$ for a design can be made from the channel geometry. However, assumptions must be made regarding the electron temperature, number density, and beam divergence angles unless informed by measurements. These properties can all be approximated from Hall thrusters with similar performance characteristics. The Hall current is first estimated from plasma properties and channel dimensions, using Equation 2.8. The value of $n_e$ must be estimated from other Hall thrusters, and $B$ can be approximated from magnetostatic simulations of the magnetic circuit. For the UAH-78AM, we assume an electron number density of $1.6 \cdot 10^{17}$ which is used by Goebel and Katz as the plasma density at the thruster exit plane of the SPT-100 [1]. Equation 2.8 yields a Hall current of 2.97 A at 200 V discharge. The Hall current is then used to calculate the beam current,

$$I_b = \frac{I_H 2\pi RB}{\sqrt{\frac{MV_d}{2q_e}}}$$

(3.4)
where $R$ is the mean channel radius. Finally, the beam current is used to calculate thrust and $I_{SP}$ according to Equation 3.5 and Equation 3.6, respectively.

$$T = \gamma \sqrt{\frac{2M}{q_e} I_b \sqrt{V_d}}$$  \hspace{1cm} (3.5)

$$I_{SP} = \frac{T}{I_b M g_0}$$  \hspace{1cm} (3.6)

$\gamma$ is a thrust correction factor for beam divergence and multiply charged ions.

$$\gamma = \alpha F_t$$  \hspace{1cm} (3.7)

In Equation 3.7, $\alpha$ equals,

$$\alpha = \frac{I^+ + \frac{1}{\sqrt{2}} I^{++}}{I^+ + I^{++}}$$  \hspace{1cm} (3.8)

and $F_t$ is the beam divergence correction factor, which equals,

$$F_t = \cos \theta$$  \hspace{1cm} (3.9)

Finally, $\theta$ is the average half-angle divergence of the beam. Additional estimates must be made for the beam divergence angle and multiply charged ion populations. As with plasma number density, these can be estimated from thrusters of similar scale. We assume a $\gamma$ of 0.92 in our calculations, which arises from a doubly charged ion population of 7% and a beam divergence angle of 20°. This results in an estimated thrust of 23 mN and $I_{SP}$ of 1609 s for the UAH-78AM at a 200 V discharge current.
This is expected to be an over-estimate of true performance since power and efficiency losses are not factored into the analysis.

This first order estimate was used to size the calibration weights for the test stand. For test stand calibration, it is important to select weights that displace the stand on a comparable scale to the thrust produced by the device. The first order analysis allowed for sizing of the calibration string without knowing exactly how much thrust the device would produce.

### 3.1.4 Materials Selection

#### 3.1.4.1 Magnetic Circuit and Anode

1018 carbon steel was selected for the magnetic circuit for its low cost and high availability. The magnetic permeability is inferior to specialized high-permeability metals such as Mu-Metal and Permalloy. However, the permeability is adequate for our purposes of demonstration and short duration test. Furthermore, disadvantages associated with the lower permeability can be mitigated by ensuring enough material is present in the magnetic circuit to prevent saturation. In mass constrained systems such as flight thrusters, higher permeability metals enable the design of more compact magnetic circuits which reduce thruster mass. However, for ground testing where weight is not a significant concern, 1018 low carbon steel performs adequately.

316 stainless steel was selected for fabrication of the anode. 316 stainless steel is not magnetically permeable, ensuring the anode would not interfere with the field projected into the channel by the magnetic circuit. In addition, the melting
temperature of 316 stainless is on the order of 1375 °C, ensuring that the anode will survive the operating temperatures in the channel, especially in short duration testing. 316 also has a history of use as an anode material in other Hall thrusters [22].

### 3.1.4.2 Discharge Chamber and Propellant Distributor

More consideration was given to materials selection for the channels and propellant distributor. These components were selected for 3D printing in order to reduce cost and component count. In conventional Hall thrusters, refractory ceramics such as boron nitride are ideal materials for the discharge chamber. However, 3D printing of refractory ceramics is presently of limited availability especially at larger build volumes due to the challenging material properties. Furthermore, the costs of such processes would be prohibitive and conflict with our objective of reducing the cost of Hall thruster fabrication. Therefore, we decided to consider materials other than the refractory ceramics.

Two options were considered: polymers and powdered ceramics. 3D printing of polymers, such as ABS, PLA, Nylon, and Polycarbonate, is broadly available at very low cost. The challenge of working with these materials is low melting temperatures on the order of 200-300 °C. To enable steady state operation at these temperatures, either a heat dissipation or thermal isolation mechanism would have to be implemented to prevent the channel from approaching glass transition temperature. Higher melting temperature thermoplastics, such as PEEK and ULTEM, provide melting temperatures in the range of 300-400 °C but are less common.
Furthermore, there is little information in literature on the SEE characteristics of polymers in the incident electron temperature range of Hall effect thrusters. The author suspects this is because most existing SEE data is intended for scanning electron microscope applications, where minimum primary electron energy is on the order of 100 eV or more. Therefore, our best estimate for the SEE characteristics of polymers in the 0-100 eV range is based on extrapolating the attenuation model provided by [57]. The results are presented in Fig 5 for Nylon-12 and PTFE. These are compared to the SEE of ceramics provided by [1].

![Figure 3.5](image)

**Figure 3.5:** Extrapolated secondary electron emission for selected polymers below 100eV

SEE for both Nylon-12 and PTFE are comparable to the values for boron nitride and borosil from approximately 40 to 100 eV. At energies below 40 eV, the ceramics appear to have higher SEE. However, this conclusion is based on model extrapolation for the polymers and is not supported by physical measurements. It is likely that a model fit to higher-energy data will not accurately predict SEE at the
low energies of interest in Hall effect thrusters. However, the results are encouraging in that they indicate SEE profiles are comparable as energies approach 100 eV, where data are available from electron microscope research. In addition, ceramics and poly- mers are both insulating materials. As compared to conductors, insulators generally have higher SEE yields due to the lack of a highly populated conduction band which reduces the SEE in metals [14]. While the discussion cannot be further informed without data, it is expected that the secondary electron emission of polymers such as ABS and ULTEM to be higher than metals and comparable to the ceramics frequently used in Hall thrusters.

Powder-based 3D printing of non-refractory ceramics is also now available at relatively low cost. Note that these are not hot-pressed ceramics; they are porous in the finished state unless a glaze is applied. These ceramics can be printed using either FFF or the binder jetting processes. The final green ceramic must then be fired to produce a finished product. This is a significant disadvantage of the process. Since the part must undergo firing, the printed geometry must be able to survive the thermal stresses of firing. In addition, dimensions shrink as the part cures. To some extent the design can be adjusted to accommodate these limitations, but fine detail and tight tolerances are not feasible.

Secondary electron characteristics of components are expected to depend on the glazing material applied to the part. In general, ceramic glazes are similar to glass but include a flux material to lower the melting temperature. Some data is available on the secondary electron characteristics of glass-coated ceramics in literature [58].
From the results provided by [58], it appears that the secondary electron properties of these materials are comparable to ceramics and other electrical insulators.

Unlike polymers, the ceramic components can operate at temperatures up to 600 C, which is within the steady state operating temperature range of most Hall thrusters.

A benefit of the design of the UAH-78AM is the ability to change the propellant distributor and discharge channel material with relative ease. We decided to test with both ceramic components and polymer components in the thruster to assess performance of both materials.

3.1.4.3 Magnetic Screens

Magnetic screens were installed in the UAH-78AM starting with version 3. These were created from 0.031 in. 1008-1010 carbon shim stock due to its low cost and high availability. The objective was to reduce the magnitude of the radial magnetic field in the anode region. Strong magnetic fields near the anode can enhance anode heating and reduce efficiency.

3.1.4.4 Electromagnets

20 AWG enamel-coated magnet wire was used for the electromagnets. The wire is available at low cost and is primarily intended for motor and sound system applications. The enamel coating on the wire we selected is rated for service temperatures up to 200 C. While the steady state operating temperatures in Hall thrusters can easily exceed 200 C, the thermal isolation of the electromagnets from the discharge
channel and plasma provides a significant time delay before the electromagnets approach these high temperatures.

For flight thrusters and laboratory thrusters undergoing long duration testing, wires with fiberglass insulation are used for the electromagnets. The fiberglass can sustain much higher service temperatures, enabling the thruster to run at thermal steady-state.

### 3.1.5 Magnetic Circuit

A model of the magnetic circuit was developed in several programs including FEMM, ANSYS, and ESI CFD-ACE to verify that magnetic saturation was not reached in the 1018 carbon steel and that the desired field intensities could be produced in the channel. The magnetic circuit was simulated with a breadth of coil configurations and currents. The design was first simulated in 2D using FEMM, then translated to 3D using ESI CFD-ACE and ANSYS to check for nonuniformities. Since the design initially incorporated four outer coils, there were concerns that the radial magnetic field between coils would be significantly weaker than the field at the corners, or that saturation would occur in the narrower carbon steel regions between poles. 3D magnetostatic simulations verified that the differences in field uniformity throughout the channel were acceptable. The acceptance criteria was that variations were not significant enough to result in demagnetization of electrons in the channel.

The UAH-78AM is not a magnetically shielded thruster due to the design heritage from the P5. Initial versions of the thruster lacked the magnetic screens later installed in the P5 to reduce electron current losses to the anode [24]; these
were later added to improve the performance of the thruster. To limit independent variables in testing at GRC, the inner and outer magnet currents were held fixed at 4.09 A for all tests. It is likely that thruster performance and longevity can be improved through optimization of the magnetic circuit or a switch to magnetically shielded field topology.

3.1.6 Manufacturing

3.1.6.1 Tolerances

The additive manufacturing process presents challenges in terms of part tolerance and minimum feature size. Part tolerances and accuracy are difficult to quantify with many 3D printing processes because they are frequently geometry dependent. Accuracy limitations were most apparent for the 3D printed glazed ceramic. The firing process induces part shrinkage on the order of 3% of total part size and must be accounted for in the design. In addition, minimum feature size is limited to approximately 2 mm. The ceramic process was not considered for the propellant distributor due to the more stringent tolerances required for the part.

Separate 3D printers were used for the ULTEM outer channel and ABS propellant distributor. Part accuracy for the ULTEM printer is 0.130 mm or better. Accuracy for the ABS 3D printer is more difficult to predict as this was not a commercial 3D printer. Therefore, factors such as belt backlash and part shrinkage are not taken into account when quoting accuracy. However, axis resolution is 0.01 mm, and since the 3D printing technology is functionally identical to the ULTEM 3D
printer, part accuracy is likely comparable. The smallest features in our parts were the propellant distributor orifices, which had a diameter of .01 in (0.254 mm). Light sanding was used for part cleanup on polymer components in some areas to improve fit.

3.1.6.2 Cost

Table 3.3 provides a cost breakdown for the UAH-78AM, in USD. All materials for the thruster in the United States can be procured for a total of $300 or less. This cost assumes the availability of 3D printers and other equipment. The low cost makes manufacturing the thruster accessible to most education and research programs.

<table>
<thead>
<tr>
<th>Material</th>
<th>Component</th>
<th>Cost</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1018 Carbon Steel</td>
<td>Magnetic Circuit</td>
<td>$ 57</td>
<td></td>
</tr>
<tr>
<td>1010 Carbon Shim Stock</td>
<td>Magnetic Circuit</td>
<td>$ 15</td>
<td></td>
</tr>
<tr>
<td>Fasteners</td>
<td>Magnetic Circuit</td>
<td>$ 30</td>
<td></td>
</tr>
<tr>
<td>Magnet Wire</td>
<td>Magnetic Circuit</td>
<td>$ 30</td>
<td></td>
</tr>
<tr>
<td>316 Stainless Steel</td>
<td>Anode</td>
<td>$ 4</td>
<td>Fabricated from stainless steel washer</td>
</tr>
<tr>
<td>3D printed glazed ceramic</td>
<td>Inner Channel</td>
<td>$ 21</td>
<td>Quote from manufacturer</td>
</tr>
<tr>
<td>ULTEM</td>
<td>Outer Channel</td>
<td>$ 97</td>
<td>Quote from manufacturer</td>
</tr>
<tr>
<td>ABS</td>
<td>Propellant Distributor</td>
<td>$ 2</td>
<td>By volumetric material cost</td>
</tr>
<tr>
<td><strong>Material Total</strong></td>
<td></td>
<td><strong>$ 256</strong></td>
<td></td>
</tr>
<tr>
<td>Labor</td>
<td></td>
<td>$ 560</td>
<td>35 $/hr for 16 hrs</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td><strong>$ 816</strong></td>
<td></td>
</tr>
</tbody>
</table>

Costs for the ULTEM outer channel and glazed ceramic inner channel are based on quotes directly from 3D printer suppliers. Consequently, these costs are significantly inflated as compared to the true costs of materials and print time. It is
increasingly common for academic institutions to have access to 3D printing services on campus or through business partnerships. These services frequently provide print services at material cost or less, so it is possible that the inner and outer channel components could be procured for much lower cost. ABS 3D printing is so broadly available through professional and hobbyist services that we provide the component price based on volumetric material cost.

The most significant labor costs are in fabrication of the magnetic circuit, which is cut from 1018 carbon steel stock using a conventional machining process. However, significant efforts were made in the design of the thruster to simplify machining operations as much as possible. Machining for the magnetic circuit is dominated by hole processes. While access to CNC machining simplifies manufacturing, all parts could be produced with relative ease using manual machines. The authors estimate that total labor time for a skilled machinist on magnetic circuit fabrication would be a day or two. However, labor remains the costliest portion of UAH-78AM procurement assuming a machinist pay of $35 per hour.

In comparison, a first order cost estimate is provided for producing the UAH-78 using conventional methods in Table 3.4. The drivers of cost in this estimate are the labor costs associated with anode fabrication and the boron nitride channel. For the anode, more time is required for the machining and welding associated with integrating the propellant distributor. Likewise, a significant increase in machining time is incurred for fabrication of the channel. The requirement for more skilled labor time causes the conventionally manufactured UAH-78AM to be more expensive than the 3D printed thruster by over a factor of 6. This example suggests that a significant
Table 3.4: UAH-78 cost breakdown (conventional materials)

<table>
<thead>
<tr>
<th>Material</th>
<th>Component</th>
<th>Cost</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1018 Carbon Steel</td>
<td>Magnetic Circuit</td>
<td>$ 57</td>
<td></td>
</tr>
<tr>
<td>1010 Carbon Shim Stock</td>
<td>Magnetic Circuit</td>
<td>$ 15</td>
<td></td>
</tr>
<tr>
<td>Fasteners</td>
<td>Magnetic Circuit</td>
<td>$ 30</td>
<td></td>
</tr>
<tr>
<td>Magnet Wire</td>
<td>Magnetic Circuit</td>
<td>$ 30</td>
<td></td>
</tr>
<tr>
<td>316 Stainless Steel</td>
<td>Anode</td>
<td>$ 50</td>
<td>Thicker stock material to incorporate distributor geometry</td>
</tr>
<tr>
<td>Boron Nitride Channel</td>
<td>Discharge Channel</td>
<td>$ 1060</td>
<td>Scaled from larger thruster. Includes labor</td>
</tr>
</tbody>
</table>

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><strong>Material Total</strong></td>
<td><strong>$ 1242</strong></td>
<td></td>
</tr>
<tr>
<td>Labor</td>
<td>$ 3800</td>
<td>35 $/hr for 10 days, anode and magnetic circuit fabrication + $1000 for orifice drilling</td>
<td></td>
</tr>
</tbody>
</table>

Total $ 5042

cost reduction associated with 3D printing is incurred by reducing the skilled labor costs associated with manufacturing the thruster.

3.1.6.3 Turnaround Time

Because the channel and propellant distributor are produced additively, all other labor costs are associated with manual assembly. Experience from testing demonstrates that the thruster can be assembled from raw components in a week or less. Most of this time is associated with winding and securing the electromagnets. Since electromagnets are secured with epoxy to prevent unwinding, setting time is required for the epoxy to cure. The remaining time is dedicated to assembly of other thruster components using fasteners.

The turnaround time for servicing between tests is on the order of a couple of days. This time is used to remove and replace worn channel and propellant distributor
components. The magnetic circuit and electromagnets are undamaged between short duration tests and therefore require little service.

3.1.7 Cathodes

A Lanthanum Hexaboride cathode loaned by the Georgia Tech HPEPL was used for testing at the UAH Johnson Research Center. Lanthanum Hexaboride refers to the emitter material used by the cathode to produce free electrons through thermionic emission. There are two main competing emitter material technologies: Lanthanum Hexaboride (LaB$_6$) and Barium Oxide (BaO). LaB$_6$ must be heated to higher temperatures than BaO for thermionic emission, but the emitter is much less susceptible to oxygen poising. This presents some advantages because cathode conditioning and operating procedures are simplified as compared to BaO. BaO cathodes must run through a conditioning process to prevent oxygen poisoning, but can be started with lower heater powers and operate at lower temperatures, which are advantages in flight applications. A more thorough description of the emitter materials is available in [1].

A BaO cathode was used with a fixed flow rate of 0.5 mg/s for all tests at Glenn Research Center. Both cathodes were oversized for the anode current required by the thruster to sustain plasma discharge. Consequently, the discharge current was too low for either cathode to operate in a self-heating mode. If the discharge current drawn from a hollow cathode is sufficiently high, self heating of the emitter material can be used to keep the emitter above its thermionic emission temperature. This operational mode is used in flight hardware to eliminate the need to power the
cathode heater after thruster ignition. At GRC, the cathode heater was kept on at half power during testing to help the emitter material remain at the appropriate temperature.

Since cathode flow is not optimized, all specific impulses in the results are presented in terms of anode flow. This is done because the cathode flows were higher than needed to run the thruster in order to keep the oversized cathodes operating. If the cathode flow were factored into the thruster $I_{SP}$, the performance would appear low because little thrust is produced from propellant flow through the cathode. It is likely that cathode flow could be reduced to 6-7% of anode flow while maintaining a stable discharge in flight applications. Therefore, cathode flow is neglected in the $I_{SP}$ results. This presents an optimistic view of performance, but makes it easier to compare thruster performance to other devices because cathode settings do not directly factor into the results. Likewise, the anode efficiency is presented instead of the total efficiency in order to neglect the effects of cathode flow and magnetic circuit power requirements on efficiency, which can vary between test setups.

3.2 Experimental Facilities

3.2.0.1 UAH Johnson Research Center

Initial testing was conducted at the UAH Johnson Research Center, which provides a variety of facilities for propulsion system research and test. An image of the JRC Small Vacuum Facility used for our first tests is provided in Figure 3.6.
The chamber uses a large turbomolecular pump backed by a conventional roughing pump to achieve high-vacuum. However, the flow rate required to run the thruster and cathode exceeds the pumping speed of the chamber. This limits test durations to no more than a few minutes before chamber pressures become unacceptably high for testing. Furthermore, the chamber dimensions of 40 cm diameter and 70 cm length are too small to accommodate the thruster plume, causing the device performance to be modified by neutral ingestion.

The thruster was tested using Krypton at JRC due to the significantly lower cost as compared to Xenon. Anode and cathode propellant flow were controlled by 100 and 10 sccm mass flow controllers manufactured by MKS Instruments.

3.2.0.2 NASA Glenn Research Center

Performance measurements were taken in Vacuum Facility 8 at NASA Glenn Research Center. The main chamber of VF-8 has a diameter of 1.5m and a length of 4.5m. The chamber was manufactured by Steel and Alloy Tank co. in 1962, and as a
volume of 300 \( ft^3 \). Pumping is provided by four oil-diffusion pumps with a speed of \( 1.2 \times 10^5 \) liters per second with air at \( 10^{-5} \) torr [59]. VF-8 features two bell-jars that can be independently isolated from the main chamber using gate valves. Images of VF-8 are provided in Figure 3.7.

The thruster was mounted on an inverted-pendulum thrust stand attached to the vacuum flange of the primary bell jar, as shown in Figure 3.8. The thruster assembly is supported at the base by low-friction flexures. When thrust is produced, the thruster and cathode assembly is deflected. The deflection is recorded by a LVDT, and the voltage output is correlated to the amount of force produced on the stand by the thruster. In addition, the thrust stand includes a damper assembly to reduce oscillations in the thrust output during transient events.
Leveling of the test stand is accomplished using the stepper motor and piezo assembly mounted at the back of the stand. The stepper motor allows for coarse adjustment of the stand pitch, and the piezo element attempts to counteract thermal and other long duration effects that gradually impact the pitch of the stand. Calibration of the stand is accomplished by loading and unloading fishing weights which pull on the flexure assembly, simulating the reaction force due to thrust. The design and operation of this type of thrust stand is well established in literature, and further details on the design of similar stands at Glenn are provided in [60].

Anode and cathode propellant flow were provided by 100 sccm and 25 sccm mass flow controllers manufactured by Celerity, and all tests were run using Xenon. No ion or plume data were collected due to the short duration of tests.
CHAPTER 4

RESULTS

I am mindful that scientific achievement is rooted in the past, is cultivated to full stature by many contemporaries and flourishes only in favorable environment. No individual is alone responsible for a single stepping stone along the path of progress, and where the path is smooth progress is most rapid.

—Ernest Lawrence

4.1 Johnson Research Center

Testing at the UAH Johnson Research Center was focused on design optimization to enable quick thruster servicing and replacement of components damaged during testing. Due to the lack of a thrust stand or vacuum facilities large enough to run the thruster without neutral re-ingestion, results from this testing are mostly qualitative. However, we were able to verify that the discharge power and propellant flow requirements were in the range expected from performance predictions. Some images from testing at the JRC are provided in Figure 4.1.
4.2 Glenn Research Center

Testing at NASA GRC was focused on collecting thrust data due to access to this instrumentation at the facility. Thrust data were used to fix $\text{I}_{\text{sp}}$ and anode efficiency, allowing us to compare performance of the UAH-78AM to predictions. Images from testing at NASA GRC are provided in Figure 4.2.

Two thruster configurations were tested at GRC—one with an ABS outer Channel and one with an ULTEM outer channel. These are V4 and V5 of the UAH-78AM, respectively. The V4 ABS channel was damaged due to spotting within the first 10 seconds of testing. After spot formation, all subsequent attempts to start the
thruster resulted in plasma attachment at the spot location and unacceptably high anode leakage currents. Therefore, we only report on performance measurements from the ULTEM channel here, and a more in-depth report on the spotting behavior is provided in the discussion.

Without the ability to operate the thruster at steady-state, we choose to characterize performance by comparing thrust and specific impulse at fixed times after ignition. The ignition event is identified through a derivative approach. The derivative is taken of the thrust trace and time zero is identified as the location where the thrust rate of change exceeds 20 mN/sec, as this behavior is only seen during thruster

Figure 4.2: Testing at NASA GRC
ignition. For the 5 second tests, thrust and specific impulse are measured 4 seconds after the ignition event. For the 15 second tests, measurements are taken 14 seconds after ignition. A graphical example on a demonstration thrust curve is provided in Figure 4.3. We expect thruster thermal conditions to be similar at fixed times after ignition, enabling comparison across different discharge voltages and flow rates. However, the heating rate and thermal condition of the channel likely varies between tests, resulting in some error in repeatability. We attempt to characterize this uncertainty through repeated tests. Calibration and measurement uncertainties associated with the equipment are calculated and reported according to the best practices identified in Ref [61, 62]. A detailed discussion of the uncertainty analysis is provided for the reader in Appendix B. While thrust and I$_{SP}$ uncertainty vary slightly with the calibration for each test run, the average thrust uncertainty at 95% confidence is ±0.72 mN, and I$_{SP}$ uncertainty is ±40 sec.

![Thrust Trace](image)

**Figure 4.3**: 15-second thrust trace showing ignition and measurement locations
4.2.1 Thrust

Figure 4.4: Thrust as a function of discharge voltage from 5 second tests (left) and 15 second tests (right)

Figure 4.4 provides measured thrust as a function of discharge voltage for the 5 and 15 second test runs. General trends are as expected for conventional Hall thrusters, with thrust increasing with both discharge voltage and flow rate. Measured thrust ranges from 17.2 mN to 30.4 mN. The 15 second thrust measurements are higher than their 5 second counterparts at most operating points. It is suspected that this result is due to a larger thrust contribution from outgassing of the polymer components, since the channel and propellant distributor reach higher temperatures in the 15 second tests than in the 5 second tests. A more thorough explanation of this behavior is provided in the Discussion.

Repeat tests are visible in the 5 second data at 180, 200, and 220 V operating points. At 180 V, the 2.00 mg/s repeat tests display a vertical spread of approximately 5 mN, and the error bounds do not account for the differences in measured thrust. During testing it was noted that the thruster took longer than normal to start and

66
for the discharge to settle at this operating point. It is suspected that the 180 V operating point is at the lower limit of jet-mode discharge with our magnet settings for 2.00 mg/s flow rate, resulting in poor stability. Repeat tests at 200 and 220 V fall within the uncertainty of our equipment, demonstrating that the short duration tests can yield consistent measurements at higher voltages. The thruster could not be started at 1.64 mg/s and 180 or 200 V without adjusting magnet settings, thus no data were collected at these operating points.

While collecting 15 second data at 220 V and 2.00 mg/s, a spot formed and attached to the outer channel wall. Affected tests are labeled in all plots with hollow circles. The damage associated with spot formation on the outer channel could change the efficiency of the thruster by increasing anode leakage current, limiting comparisons with preceding operating points. However, the data points are included for completeness since we were able to start and run the thruster without visible spotting behavior after the channel had been allowed to cool. Furthermore, no 15 second measurements were made at 2.18 mg/s due to the spot formation. The channel wall heating rate increases at higher discharge powers, which are associated with higher flow rates [21]. To avoid further spotting damage to the outer channel, we decided to forgo longer duration testing at the higher discharge powers associated with the 2.18 mg/s flow rate.

4.2.2 Anode Specific Impulse

Figure 4.5 provides anode specific impulse as a function of discharge voltage. The anode specific impulse ranges from 870 to 1,450 seconds and increases with
Figure 4.5: Anode specific impulse as a function of discharge voltage for 5 second tests (left) and 15 second tests (right)

discharge voltage, which is a normal behavior for Hall thrusters. The data also suggest that specific impulse increases with flow rate in our test matrix; however, due to equipment uncertainty this correlation cannot be proven from the data.

4.2.3 Anode Efficiency

Figure 4.6: Anode efficiency as a function of discharge voltage for 5 second tests (left) and 15 second tests (right)
Figure 4.6 provides anode efficiency as a function of discharge voltage. Anode efficiency generally increases as a function of discharge voltage and ranges from 27.8% to 42.2%. At 260 V and anode flow of 2.00 mg/s, the anode efficiency decreases relative to the 240 V operating point in the 15 second tests. This measurement was taken after initial formation of the spot on the outer channel wall. The decrease in anode efficiency may be a product of increased anode leakage current due to the damage to the outer channel wall. It was noted in testing that discharge current increased relative to the 5 second tests after formation of the spot, which would be expected if the damage from spotting were reducing thruster performance.

The effect of anode flow on thruster efficiency is unclear from the data due to uncertainty. In the 15 second tests, it appears that the 1.64 mg/s flow rate results in higher efficiencies at discharge voltages above 200 V, while 2.00 mg/s is more efficient at the lower voltages. The 5 second data suggest that 2.18 mg/s flow results in the highest anode efficiency at all operating points except 180 V. No pattern is clearly discernible from these results, and the measurement uncertainties limit the significance of any identified trends with respect to flow rate.

4.2.4 Thrust to Power Ratio

Figure 4.7 provides the thrust to power ratio as a function of discharge voltage. The thrust to power ratio is generally seen to decrease with increasing discharge voltage. This is the expected relationship in electric propulsion systems, since accelerating ions to higher velocities requires more power. As with anode efficiency,
Figure 4.7: Thrust/Power as a function of discharge voltage for 5 second tests (left) and 15 second tests (right)

uncertainties limit the ability to draw conclusions about the relationship between anode flow and thrust to power ratio.
CHAPTER 5

DISCUSSION

"Science cannot solve the ultimate mystery of nature. And that is because, in the last analysis, we ourselves are a part of the mystery that we are trying to solve."
—Max Planck

5.1 Performance Predictions

Table 5.1 compares the UAH-78AM measured performance at 200V with the predicted performance.

Table 5.1: Comparison of predicted and measured performance at $V_d=200V$

<table>
<thead>
<tr>
<th></th>
<th>Predicted</th>
<th>Measured</th>
<th>Percent Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thrust (mN)</td>
<td>23</td>
<td>19.7</td>
<td>15.5</td>
</tr>
<tr>
<td>$I_{SP}$ (s)</td>
<td>1609</td>
<td>1094.7</td>
<td>38</td>
</tr>
<tr>
<td>Anode Efficiency, theoretical max (%)</td>
<td>84</td>
<td>34.1</td>
<td>84.5</td>
</tr>
</tbody>
</table>

The predicted and measured performance are in good agreement with the exception of anode efficiency. As expected, measured performance is lower than the predicted performance due to efficiency losses. This is especially true for the anode efficiency, which is derived from a discharge current that is unrealistically low due to
omission of wall, anode, radiative, and ionization losses. The differences in thrust and $I_{SP}$ can likely be attributed to differences in electron temperature, beam divergence, and multiply charged ion populations.

5.2 Comparison with Other Low-Power Hall thrusters

Table 5.2: Comparison of thrusters in the UAH-78AM power class at similar discharge voltages

<table>
<thead>
<tr>
<th>Thruster</th>
<th>$V_d$ (V)</th>
<th>Power (W)</th>
<th>Thrust (mN)</th>
<th>Anode $I_{SP}$ (s)</th>
<th>Anode Efficiency %</th>
</tr>
</thead>
<tbody>
<tr>
<td>MaSMi [46]</td>
<td>200-250</td>
<td>160-747</td>
<td>8.8-33</td>
<td>775-1321</td>
<td>21-29</td>
</tr>
<tr>
<td>SPT-50 [63]</td>
<td>199-282</td>
<td>210-389</td>
<td>12.9-18.9</td>
<td>1160-1524</td>
<td>35-41</td>
</tr>
<tr>
<td>BHT-200 [64]</td>
<td>200-275</td>
<td>200</td>
<td>11.5-12.5</td>
<td>—</td>
<td>35-42</td>
</tr>
<tr>
<td>UAH-78AM</td>
<td>180-260</td>
<td>280-520</td>
<td>17-30</td>
<td>870-1450</td>
<td>27-42</td>
</tr>
</tbody>
</table>

Table 5.2 provides a comparison of the performance parameters collected from the UAH-78AM to other Hall thrusters of similar power levels. The data are provided not to compare thruster performance, but to demonstrate that performance measurements of the UAH-78AM are comparable to experimental measurements of other 300-500 W Hall thrusters. Anomalous performance results would indicate that the unconventional design features of the UAH-78AM were changing the performance in a manner that requires further testing and research to understand.

5.3 Outgassing

A characteristic raw thrust curve is presented in Figure 5.1. This thrust measurement is from a 30-second test at a 200 V discharge and anode flow of 1.82 mg/s.
Figure 5.1: Thrust as a function of time at 200V discharge voltage and anode flow of 1.82mg/s.

Thrust stabilizes at approximately 19 mN, but gradually climbs until test conclusion. The increase in thrust is likely due to outgassing that increases in rate as the polymer thruster components are heated. Similar behavior is seen in conventional thrusters during first start, as water vapor and other volatile compounds outgas from the channel walls. The outgassing process is thought to modify the secondary electron behavior of the channel walls in boron nitride thrusters, resulting in artificially high discharge currents [65]. In an effort to reduce moisture content, we performed a 12 hour low-temperature bake out on the ABS components. The ULTEM outer channel was manufactured in a heated build chamber with temperatures approaching 200 C, so it is expected that the moisture content in this component was low.

The outgassing processes may also be more complex in the UAH-78AM than evaporation of surface water or other contaminants from exposure to ambient air. What is referred to as outgassing may be a combination of sublimation and/or ab-
lation processes that occur as the channel wall surface is heated beyond the glass transition temperature of the polymer. Chemical processes could also be playing a role, as the polymer gases could decompose into their constituent atoms as they diffuse into the channel. For lack of a better term, we will continue to refer to the process as outgassing; however, the physical process may be more complex than what is encountered in conventional Hall thrusters.

5.4 Spot Formation

The presence of polymer components in our thruster also presents unique challenges for quantifying baseline performance. Not only were we unable to run the thruster long enough to get through the transitional regime associated with conventional thruster start-up, but the heating of the polymer components limited testing duration to approximately 30 seconds. Beyond 30 seconds, a failure mode is observed where a hotspot attaches to the outer ULTEM discharge channel wall and the thruster enters a current-limited mode of operation. Hotspot formation can be a problem in conventional Hall thrusters [38], but the cause is different as compared to the UAH-78AM. Hotspots are especially troubling with our thruster since the hotspot increases the polymer outgassing rate which encourages continued spot formation and growth. If left unchecked, the hotspot permanently damages the discharge channels and the polymer components must be replaced. An image of the spotting behavior is provided in Figure 5.2.

The spotting behavior was first observed in testing at the JRC, as shown in Figure 5.3. It was suspected that the spots could be the product of poor cathode po-
positioning or a defect in the magnetic circuit because they seemed to favor attachment at the same location in the discharge channel. Instead, testing at GRC confirmed that the spotting behavior was associated with the heating of the channel walls and was not directly associated with cathode positioning or the magnetic circuit. This confirmation was made possible by a significant change in the thruster starting procedures for testing at GRC. Instead of starting in a glow discharge then gradually increasing magnetic field strength, the UAH-78AM was started at GRC by setting the magnets and anode at fixed settings then initiating propellant flow. This enabled the thruster to start directly with a Hall discharge—limiting the channel heating that occurred in previous tests while the magnets were brought up to full strength. The ability to operate the thruster for a period of time without spotting in jet-mode confirmed that spot formation was thermally induced. Furthermore, no spotting was observed on the inner channel wall during testing at GRC. The inner channel was manufactured

Figure 5.2: Image of hotspot formation and corresponding decrease in thrust
from the 3D printed glazed ceramic, which can withstand higher temperatures than ULTEM.

Figure 5.3: Hotspot observed while testing V2 at JRC

5.5 Repeatability

While measurement uncertainty due to equipment error is quantified, a more significant challenge with transient testing is verifying that our methodology for comparing performance at different operating points is valid. The transient performance of the thruster is sensitive to the thermal condition of the channel; therefore, we chose to compare performance at points where the thermal condition of the channel should be similar by looking at a fixed time after ignition. However, the heating rate and wall power losses in Hall thrusters also vary depending on the discharge voltage, since
this affects electron temperature and consequently channel wall losses [1, 21]. This effect could be a source of error since it could cause the outgassing to vary between tests.

Furthermore, the vacuum environment limits heat dissipation mechanisms, causing the channel to take a long time to return to ambient temperature after a test. In order to minimize the amount of propellant used for running the cathode, tests were run in sequences of 4 or 5 with a few minutes between tests to allow the thruster to cool. For the 5 second tests, the pauses were adequate to allow the channel to cool between tests. However, the pauses were not adequate during the 15 second tests, and the cumulative heating from running at higher discharge powers eventually led to spot formation on the outer channel wall. Future test programs with the UAH-78AM should make a better effort to track the temperature of the outer channel to ensure the temperature of the channel is not an uncontrolled variable in testing.

In an effort to characterize the impact channel temperature may have on results, we repeated tests at several operating points. Figure 5.4 shows repeat thrust measurements at 200 V and an anode mass flow of 1.82 mg/s, which was the most-tested operating point.

Initial thermal condition of the channel was different for each of the repeat tests. For example, Test 2 was the last of a sequence of four 5-second tests, so the channel temperature should have been elevated above ambient conditions. Tests 1-3 show strong agreement and converge on a thrust of approximately 19.5 mN. Test 4 is noticeably shifted down relative to the first three tests. However, test 4 is the 30 second run, and the thrust measurement eventually converges on a value within 0.2
mN of the first 3 tests. The behavior suggests that the outgassing contribution to thrust approaches a steady-state value that is stable over our short duration tests before spotting occurs. Similar behavior is seen with other repeat tests, providing some confidence that transient thruster performance can be compared across operating points.

5.6 Material Longevity

ABS, ULTEM, and the 3D printed glazed ceramic were all exposed to plasma as part of testing in the UAH-78AM. The three materials displayed different wear mechanisms and tolerances to the thermal load induced by the plasma.

ABS was used for the channel and propellant distributor starting with Version 1 of the thruster. Of the three materials tested, ABS seemed to be the most susceptible
to damage from spot formation. This is an expected outcome given the relatively low glass transition and melting temperatures of 100 and 230 C, respectively. When a spot forms on an ABS component, the plasma causes the surface layer of plastic to char and decompose. In addition, the heating causes the filaments that comprise the 3D printed part to expand along their length, encouraging layer separation. This can be seen in Figure 5.5, where the damaged outer wall appears to bulge in to the discharge channel. Unfortunately, the layer expansion and separation further encourages spot formation by pushing plastic closer to the plasma. When working with 3D printed components, design decisions can be made to reduce the tendency of the wall material to bulge into the discharge channel. For example, the pole covers were removed from V2 of the UAH-78AM in part to remove hollow air pockets formed as part of the 3D printing infill process. These pockets expand as the 3D printed components heat up, and encourage the wall material to bulge into the discharge chamber.

In a normal Hall discharge, the maximum heat load from the plasma occurs approximately where the radial magnetic field is most intense at the discharge channel exit plane [21]. From our testing with the UAH-78AM, it appears unlikely that ABS can survive the normal thermal load at the exit plane for more than a few seconds in a non-magnetically-shielded Hall thruster. However, ABS demonstrates potential utility for transient testing in cooler parts of the thruster, such as the propellant distributor located at the base of the channel.

As a polymer, ULTEM displays similar wear characteristics to ABS. However, due to the significantly higher glass transition and melting temperatures (approximately 186 and 350 C, respectively) ULTEM is able to withstand the plasma heating
Figure 5.5: Testing damage to ABS outer channels

for longer durations than ABS. Furthermore, the plastic seems to better tolerate spot formation with regard to part deformation and charring. After spot formation during 15 second testing, we were able to restart the thruster and run with a normal Hall discharge after allowing the channel to cool. An image of this spot damage is provided in Figure 5.6. While ULTEM is still damaged by the plasma, the higher melting and glass transition temperatures make it a good candidate for replacing ABS components in the thruster.

The glazed 3D printed ceramic demonstrated the best resistance to the plasma heating. A post test image of the ceramic inner channel is provided in Figure 5.7. Interestingly, the discoloration only occurred on the region of the inner channel nearest to where the spot attached to the outer channel. This suggests that the discoloration could be a sputtered decomposition product from melting and outgassing of the outer
Figure 5.6: ULTEM spot damage at channel exit plane, circled in red

channel. The discoloration could not be removed with ethyl alcohol and appears to be deposited into the glaze surface.

Beyond the discoloration, there is no evidence of structural damage to the glazed ceramic due to testing. This suggests that the ceramic component would be able to endure longer test durations than the polymer components due to the higher service temperatures. However, the limited part tolerances present a challenge for using the ceramic 3D printing process for anything other than simple shapes. These limitations become more restrictive as thrusters are scaled to smaller dimensions.
Figure 5.7: 3D Printed Ceramic Post Testing
CHAPTER 6

CONCLUSIONS

The greatest gain from space travel consists in the extension of our knowledge. In a hundred years this newly won knowledge will pay huge and unexpected dividends.

—Wernher von Braun

6.1 UAH-78AM Performance

The main goal of this work was to assess whether low-cost additive manufacturing processes such as FFF or 3D printing of glazed ceramics can be used in the fabrication of Hall effect thrusters. While the data collected cover only short duration testing, our results demonstrate that the UAH-78AM is capable of operating with a normal jet-mode Hall discharge with performance comparable to other thrusters of a similar power and size. Therefore, by the most basic definition, the UAH-78AM is a fully functioning Hall thruster, and FFF and other low-cost additive manufacturing technologies can be used to build Hall thrusters.

However, for a Hall thruster to be useful, it must be capable of sustaining a Hall discharge for long enough duration to collect meaningful data or provide sustained thrust for satellites in flight applications. The acceptable running duration is dependent on the type of data being collected. In the case of the UAH-78AM,
current test durations are too short to collect plume data or steady-state thrust and temperature data using conventional methods. The only measurements that can be collected with the UAH-78AM are transient, since the steady state thermal operating condition for the thruster is beyond the temperature limits of the materials used for the channel. While transient data might be insightful to baseline the performance of a thruster, steady state data is ultimately needed for the development of flight hardware.

6.2 Design Improvements

We remain optimistic because unexplored design strategies remain for improving the runtime of the UAH-78AM. Channel wall heating and erosion in Hall effect thrusters are dependent on the magnetic field topology in the channel. Magnetically shielded field topologies have been found to reduce channel wall heating by reducing the contact between the plasma and the wall [5, 17] Our magnetic circuit design is based on an unshielded field topology for the purpose of simplicity; however, the unshielded design only increases thermal losses to the channel walls as compared to shielded designs. Furthermore, thermal damage in the UAH-78AM remains localized to the discharge plane. It seems likely that material modifications through more gregarious use of high temperature thermoplastics and ceramic materials in the right places could improve the lifetime of the UAH-78AM to be on the order of minutes or hours. Longer test durations open up possibilities for more extensive research.
6.3 Future Work

The heating and outgassing behavior of the polymer components have a distinct impact on the performance of the thruster and lead to a unique failure mechanism. The data suggest, but do not prove, that polymer outgassing contributes to an increase in thrust until spot formation. However, an analysis of species in the plume would be necessary to experimentally verify if polymer heating and outgassing are contributing to the positive thrust drift over the test duration. Plume properties and plasma-wall interactions in the thruster may be interesting areas for future research, since the 3D printed components in the thruster likely modify the channel wall sheaths and plume properties relative to conventional thrusters. It is likely that such modifications have a discernible impact on thruster performance but may be challenging to identify in transient testing.

The development of a UAH-78 with conventional materials may be advantageous to provide a direct baseline for comparison of thruster performance with 3D printed materials. The data from a thruster with conventional materials could be compared to the data from the 3D printed version in order to provide a direct comparison of the impact of material changes on device performance. This data may be insightful as thruster lifetime is increased and material longevity plays a more significant role in device performance.

A model of the plasma wall interaction with the polymer channel would prove insightful for better understanding the spotting failure mechanism. Photographic and physical evidence from testing indicate that spot formation significantly modifies
electron transport and plasma location in the channel. It may prove insightful to gain a better understanding of how the channel environment is modified when a spot forms, as this may improve our understanding of anomalous diffusion mechanisms in the channel with a normal Hall discharge.

Our ultimate goal is to leverage the benefits of additive manufacturing as part of more fundamental research. Topics that seem particularly well suited to this include studies of neutral flow dynamics and in-situ measurement of the Hall current and other plasma properties in the channel. With regard to neutral flow dynamics, it is particularly simple to modify parameters such as orifice size location, and channel geometry with additive manufacturing—perhaps collecting experimental data on some of the injection schemes outlined in [23]. Embedding of transducers in the discharge channel is also of particular interest since this is a relatively simple task with 3D printing.
APPENDICES
APPENDIX A

PERFORMANCE PREDICTION DETAILED CALCULATIONS

\[ eV = 1.60219 \cdot 10^{-19} \text{ J} \quad q_e = 1.60219 \cdot 10^{-19} \text{ C} \quad m_e = 9.10938 \cdot 10^{-31} \text{ kg} \]
\[ m_{Xe} = 2.18017 \cdot 10^{-25} \text{ kg} \quad T_n = 293 \text{ K} \quad B = 200 \text{ G} \quad n_i = 1.6 \cdot 10^{15} \text{ m}^{-3} \]
\[ T_e = 25 \text{ eV} \quad V_d = 200 \text{ V} \quad A_i = 3.1871 \text{ in}^2 \quad w = 11.6 \text{ mm} \]
\[ MCD = 33.2 \text{ mm} \quad I_2 = 0.07 \]

a) Plasma Region Length

\[ \sigma_i = 3.81 \cdot 10^{-20} \text{ m}^2 \]

\[ v_e = \sqrt{\frac{8 \cdot T_e \cdot q_e \cdot B}{\pi \cdot m_e}} \text{ m s}^{-1} \]
\[ v_n = \sqrt{\frac{8 \cdot k \cdot T_n}{\pi \cdot m_{Xe}}} = 217.37047 \text{ m s}^{-1} \]
\[ L = \frac{3 \cdot v_n}{n_e \cdot \sigma_i \cdot v_e} = 3.20 \text{ cm} \]

b) Electron-ion motion margins

Electron Larmor Radius

\[ r_{Le} = \frac{m_e}{q_e \cdot B} \left( \frac{8 \cdot T_e}{\pi \cdot m_e} = 0.1 \text{ cm} \right) \]
\[ \text{margin} = \frac{L}{r_{Le}} = 33.61 \]

Ion Larmor Radius

\[ r_{Li} = \frac{m_{Xe}}{q_e \cdot B} \left( \frac{2 \cdot q_e \cdot V_d}{m_{Xe}} = 1.17 \text{ m} \right) \]
\[ \text{margin} = \frac{r_{Li}}{L} = 36.49 \]
Hall current

\[ I_H = n_e q_e \cdot \frac{V_d}{B} = 2.97 \text{ A} \]

c) Ion Beam Current

\[ I_b = \frac{I_H \cdot 2 \cdot \pi \cdot MCD \cdot B}{\sqrt{m_{Xe} \cdot V_d / 2 \cdot q_e}} = 1.06 \text{ A} \]

d) Thrust

\[ I_1 + \left( \frac{1}{\sqrt{2}} \right) I_2 \]

\[ \alpha = \frac{I_1 + I_2}{I_1 / I_2} = 0.98 \]

\[ \gamma = \alpha \cdot F_t = 0.92 \]

\[ T = \gamma \cdot \frac{2 \cdot m_{Xe}}{q_e} \cdot I_b \cdot \sqrt{V_d} = (2.28 \cdot 10^{-2}) \text{ N} \]

d) Specific Impulse

\[ m_{\text{dot}} = \frac{I_b}{q_e} \cdot m_{Xe} = (1.45 \cdot 10^{-6}) \text{ kg} / \text{s} \]

\[ I_{sp} = \frac{T}{m_{\text{dot}} \cdot 9.81} = (1.61 \cdot 10^3) \text{ s} \]

d) Anode efficiency

\[ P_d = I_b \cdot V_d = 212.71 \text{ W} \]

\[ \eta_a = \frac{1}{2} \cdot \frac{T^2}{m_{\text{dot}} \cdot P_d} = 0.85 \]
B.0.1 Approach

Uncertainty quantification for the data collected at GRC was a major component of the data reduction process for the results presented in this thesis. Due to the significance of this analysis, a description is provided here as it will likely be relevant for future researchers at UAH.

Procedures for uncertainty analysis were followed according to the best practices identified in [61]. Matlab was used for all calculations, and a thrust conversion GUI was developed for processing the raw data. An image of the GUI with a raw LVDT trace is provided in Figure B.1.

The pyramid shapes seen in the LVDT trace of Figure B.1 are thrust stand calibrations. When a weight is loaded onto the stand, the deflection causes a change in the LVDT voltage that is associated with the applied force. The user brackets the horizontal regions of the pyramid by identifying the times associated with the loading or unloading of each weight. Matlab then calculates the average LVDT voltage for each horizontal region and uses this in deriving the calibration slope. In addition, the correlation coefficient and residual standard deviation are calculated to provide
the user with a metric for the quality of each calibration. Calibration pyramids were made before and after each set of thruster tests in order to account for long duration thermal drift. However, since thruster tests were so brief, it was found that the drift over our test durations were negligible, so only the starting calibration pyramid was used.

Before we introduce standard error, we discuss the residual standard deviation, which is a precursor for the standard error equation:

$$s(r) = \sqrt{\frac{\sum_{i=1}^{n} (y_i - \hat{y}_i)^2}{n - 2}}$$  \hspace{1cm} (B.1)

Where \((y_i - \hat{y}_i)\) is the residual between the predicted calibration curve and a measured calibration point at the corresponding x-value, and \(n\) is the number of
calibration points in the sample. The number of calibration points vary for each test sequence because the number of samples taken in each calibration pyramid is different for each test. Therefore, the residual standard deviation is calculated along with the calibration slope and correlation coefficient for each test sequence.

The central equation used in the uncertainty analysis is the standard error formula. The equation is introduced in [61] but described in much greater detail in [62].

\[
s_{x_0} = \frac{s(r)}{m} \sqrt{\frac{1}{N} + \frac{1}{n} + \frac{(\bar{y}_0 - \bar{y})^2}{m^2 \sum_{i=1}^{n} (x_i - \bar{x})^2}}
\]  

(B.2)

Where:

- \(s(r)\) is the residual standard deviation.
- \(m\) is the slope of the calibration equation.
- \(n\) is the number of calibration points.
- \(N\) is the repeat measurements of the sample.
- \(\bar{y}_0\) is the mean of the \(N\) repeat measurements for the sample.
- \(\bar{y}\) is the mean of the \(y\) values in the calibration dataset.
- \(x_i\) is an \(x\)-axis value.
- \(\bar{x}\) is the mean of the \(x_i\) values.

The standard error formula captures errors due to both calibration and measurement uncertainties. \(s_{x_0}\) can be used with the appropriate distribution for the sample size to calculate a confidence interval. The challenge of applying Equation B.2 is ensuring that the result is presented in the correct units. For example, if Equation B.2 is being used to derive thrust, the calibration slope must be presented in
units of $V/mN$ to ensure cancellation with the units of the residual standard deviation. If the calibration slope is derived in the opposite manner, $mN/V$, the slope must be inverted before it can be used in Equation B.2.

For this uncertainty analysis, we use $n = 4$ and $N = 40$ in our calculations. $n = 4$ is selected because the calibration pyramid includes LVDT voltages at 4 separate loading conditions. We choose to take an average of 40 samples when calculating a thrust measurement for our results to reduce the influence of electrical and mechanical noise on the measurement uncertainties. When choosing an average, we wish to strike a compromise between averaging away transient events and minimizing the influence of noise. A view of the noise profile during a typical thrust measurement is provided in Figure B.2. Selection of an appropriate value for $N$ is somewhat qualitative. At our sample rate of 250Hz, 40 samples are enough to capture several periods of the dominant noise frequency; yet, the averaged sample time is only 0.16 seconds, which preserves most transient events of interest. The appropriate value for any given test will depend on the noise environment and timescale of events of interest.

Once the standard error is calculated for thrust and anode flow measurements, Kline McClintock uncertainty analysis [66] can be used to propagate these errors into the $I_{sp}$ and anode efficiency. Resulting equations from taking the appropriate partials of $I_{sp}$ and anode efficiency equations are as follows:

$$u_{I_{sp}} = \sqrt{\left(\frac{u_T}{\dot{m}g_0}\right)^2 + \left(-\frac{T u_{\dot{m}}}{\dot{m}^2 g_0}\right)^2}$$ (B.3)
\[ u_{\eta_a} = \sqrt{\left( \frac{T u_T}{\dot{m}_a P_d} \right)^2 + \left( -\frac{T^2 u_{\dot{m}_a}}{2 \dot{m}_a^2 P_d} \right)^2 + \left( -\frac{T^2 u_p}{2 \dot{m}_a P_d^2} \right)^2} \] \tag{B.4}

Note that uncertainty in the discharge power is neglected, since it is likely small in relation to thrust and flow uncertainties. Furthermore, discharge voltage and current were recorded from the power supply as single measurements, and there are no statistical tools for uncertainty quantification with single measurements. \( \dot{m} \) in Equation B.3 can refer to either anode flow (\( \dot{m}_a \)) or total flow, depending on whether cathode flow was included in the \( I_{SP} \).

As with thrust and flow uncertainties, Equation B.3 and Equation B.4 can be used in conjunction with the appropriate distributions to calculate confidence intervals for the measurements.
B.0.2 Example

This example begins with sample calibration data, which is used for calculating the line of best fit and residual standard deviation. The calibration data are provided in Table B.1. Note that actual tests included thousands of repeat measurements for each calibration weight since the sample rate was 250 Hz. The number is reduced to simplify the example for the reader and the author. Weights presented are cumulative, so weight 2 = cal mass 2 + cal mass 1.

Table B.1: Sample calibration data

<table>
<thead>
<tr>
<th>Weight (mN)</th>
<th>LVDT (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.807</td>
<td>-9.545</td>
</tr>
<tr>
<td>4.807</td>
<td>-9.545</td>
</tr>
<tr>
<td>22.568</td>
<td>-9.157</td>
</tr>
<tr>
<td>22.568</td>
<td>-9.159</td>
</tr>
<tr>
<td>41.457</td>
<td>-8.744</td>
</tr>
<tr>
<td>41.457</td>
<td>-8.746</td>
</tr>
<tr>
<td>56.822</td>
<td>-8.34</td>
</tr>
<tr>
<td>56.822</td>
<td>-8.339</td>
</tr>
</tbody>
</table>

The line of best fit is calculated from Table B.1. The calculation can be done manually, but a variety of software packages including Excel, Matlab, Mathcad, and Igor provide the capability. For the sake of brevity, it is assumed that the reader has access to and familiarity with one of these tools. The line of best fit is presented in Equation B.5.

\[
y (V) = 0.0230x (mN) - 9.6706 \quad (B.5)
\]

The units of x and y are called out in parenthesis in order to emphasize the importance of arranging the equation in the correct form for the slope to be used in the standard error equation. It seems more intuitive to arrange the linear equation in the inverse
form, so that thrust can be determined from LVDT voltage. However, the units of the slope used in the standard error formula must be $\frac{V}{mN}$ to ensure standard error has units of force.

Using the line of best fit, the LVDT voltage is calculated for each calibration weight. The results are provided in Table B.2.

<table>
<thead>
<tr>
<th>Weight (mN)</th>
<th>LVDT (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.807</td>
<td>-9.560</td>
</tr>
<tr>
<td>22.568</td>
<td>-9.152</td>
</tr>
<tr>
<td>41.457</td>
<td>-8.717</td>
</tr>
<tr>
<td>56.822</td>
<td>-8.364</td>
</tr>
</tbody>
</table>

To determine the residuals, the results presented in Table B.2 are subtracted from the measured LVDT voltages presented in Table B.1. This process is tabulated as part of determining the residual standard deviation, which was introduced in Equation B.1. The result is presented in Table B.3.
Table B.3: Tabulated calculation of the residual standard deviation

<table>
<thead>
<tr>
<th>$x_i$</th>
<th>$y_i$</th>
<th>$\hat{y}_i$</th>
<th>$(y_i - \hat{y}_i)^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.807</td>
<td>-9.545</td>
<td>-9.560</td>
<td>2.250e-4</td>
</tr>
<tr>
<td>4.807</td>
<td>-9.545</td>
<td>-9.560</td>
<td>2.250e-4</td>
</tr>
<tr>
<td>41.457</td>
<td>-8.744</td>
<td>-8.717</td>
<td>7.290e-4</td>
</tr>
<tr>
<td>41.457</td>
<td>-8.745</td>
<td>-8.717</td>
<td>7.840e-4</td>
</tr>
<tr>
<td>56.822</td>
<td>-8.340</td>
<td>-8.364</td>
<td>5.760e-4</td>
</tr>
<tr>
<td>56.822</td>
<td>-8.339</td>
<td>-8.364</td>
<td>6.250e-4</td>
</tr>
<tr>
<td>56.822</td>
<td>-8.341</td>
<td>-8.364</td>
<td>5.290e-4</td>
</tr>
</tbody>
</table>

$\sum_{i=1}^{12} (y_i - \hat{y}_i)^2 = 4.873e-3$

$\sqrt{\frac{\sum_{i=1}^{12} (y_i - \hat{y}_i)^2}{12-2}} = 2.208e-2 \ (V)$

The residual standard deviation can be used to determine the uncertainty of a thrust measurement with the standard error formula presented in Equation B.2. While 40 repeat measurements were used in testing, the analysis will be simplified by only using 3 repeat measurements to make the process easier to follow for the reader. The thrust data are presented in Table B.4
Table B.4: Example thrust data with 3 repeat measurements (N=3)

<table>
<thead>
<tr>
<th>Uncalibrated Thrust, $y_0$ (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-9.106</td>
</tr>
<tr>
<td>-9.108</td>
</tr>
<tr>
<td>-9.101</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Average Uncalibrated Thrust, $\bar{y}_0$ (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-9.105</td>
</tr>
</tbody>
</table>

One tabular calculation remains for standard error. This is calculation of the $\sum_{i=1}^{12}(x_i - \bar{x})^2$ term. The calculation is provided in Table B.5

Table B.5: Tabulated calculation for the standard error

<table>
<thead>
<tr>
<th>$x_i$</th>
<th>$y_i$</th>
<th>$(x_i - \bar{x})^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.807</td>
<td>-9.545</td>
<td>707.9</td>
</tr>
<tr>
<td>4.807</td>
<td>-9.545</td>
<td>707.9</td>
</tr>
<tr>
<td>4.807</td>
<td>-9.548</td>
<td>707.9</td>
</tr>
<tr>
<td>22.568</td>
<td>-9.157</td>
<td>78.24</td>
</tr>
<tr>
<td>22.568</td>
<td>-9.159</td>
<td>78.24</td>
</tr>
<tr>
<td>22.568</td>
<td>-9.163</td>
<td>78.24</td>
</tr>
<tr>
<td>41.457</td>
<td>-8.744</td>
<td>100.9</td>
</tr>
<tr>
<td>41.457</td>
<td>-8.746</td>
<td>100.9</td>
</tr>
<tr>
<td>41.457</td>
<td>-8.745</td>
<td>100.9</td>
</tr>
<tr>
<td>56.822</td>
<td>-8.340</td>
<td>645.6</td>
</tr>
<tr>
<td>56.822</td>
<td>-8.339</td>
<td>645.6</td>
</tr>
<tr>
<td>56.822</td>
<td>-8.341</td>
<td>645.6</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\bar{x}$</th>
<th>$\bar{y}$</th>
<th>$\sum_{i=1}^{12}(x_i - \bar{x})^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>31.41</td>
<td>-8.948</td>
<td>4597</td>
</tr>
</tbody>
</table>

All values required for the standard error formula have now been calculated.

The calculation and result are presented in Equation B.6.
\[ s_{x_0} = \frac{2.208 \times 10^{-2} \text{V}}{0.0230 \text{V mN}^{-1}} \sqrt{\frac{1}{3} + \frac{1}{4} + \frac{(-9.105 - (-8.948))^2 \text{V}^2}{0.0230^2 \times 4597 \text{V}^2}} = 0.740 \text{mN} \quad (B.6) \]

Since only 3 thrust measurements were taken, the student’s t distribution is used in calculation of the 95% confidence interval for a single measurement. The corresponding 2-tailed t value is 4.30265.

\[ u_T = 4.30265 \times \frac{0.740}{\sqrt{3}} = \pm 3.18 \text{mN} \quad (B.7) \]

This uncertainty is much higher than the uncertainties presented in the results since only 3 samples were taken of the thrust measurement. The calibration slope may then be used to determine the thrust. Since the null weight (4.807 mN) was used as part of the calibration, this weight must be subtracted from the thrust predicted by the calibration slope to determine the thrust produced by the device. The null weight remains loaded on the thrust stand during testing to prevent the string from sliding off the pulley. The resulting thrust estimate is 19.78 ± 3.18 mN.

An identical procedure may be used for calculating the uncertainty in flow rate measurements. With thrust and flow uncertainties, Equation B.3 and Equation B.4 may be used to propagate the flow and thrust uncertainties into anode I_{SP} and anode efficiency. A flow uncertainty of ± 0.0167 mg/s is assumed, and the discharge power is 310W. The results are presented in Equation B.8 and Equation B.9.
\[ u_{ISP} = \sqrt{\left(\frac{3.18 \times 10^{-3} \text{N}}{1.82 \times 10^{-6} \text{kg s}^{-1} 9.81 \text{ms}^{-2}}\right)^2 + \left(\frac{-1.978 \times 10^{-2} \text{N} 1.67 \times 10^{-8} \text{kg s}^{-1}}{(1.82 \times 10^{-6} \text{kg s}^{-1})^2 9.81 \text{ms}^{-2}}\right)^2} = 178.4 \text{s} \quad (B.8) \]

\[ u_{\eta} = \sqrt{\left(\frac{1.978 \times 10^{-2} \text{N} 3.18 \times 10^{-3} \text{N}}{1.82 \times 10^{-6} \text{kg s}^{-1} 310 \text{W}}\right)^2 + \left(\frac{-(1.978 \times 10^{-2} \text{N})^2 1.67 \times 10^{-8} \text{kg s}^{-1}}{2(1.82 \times 10^{-6} \text{kg s}^{-1})^2 310 \text{W}}\right)^2} = 9.41\% \quad (B.9) \]

As with thrust uncertainty, anode ISP and anode efficiency uncertainties are higher than those presented in the results due to the reduced number of samples used for the thrust measurement. Of note in Equation B.9 is that the third power uncertainty term is neglected since no estimates of this uncertainty were collected.
REFERENCES


[36] Hani Kamhawi, Wensheng Huang, Thomas Haag, John Yim, Li Chang, Lauren Clayman, Daniel Herman, Rohit Shastry, Robert Thomas, Timothy Verhey, and others. Overview of the development of the solar electric propulsion technology demonstration mission 12.5-kW Hall thruster. 3898.


[40] David Manzella, Robert Jankovsky, Frederick Elliott, Ioannis Mikellides, Gary Jongeward, and Doug Allen. Hall thruster plume measurements on-board the Russian Express satellites.


[49] Yongjie Ding, Wuji Peng, Hezhi Sun, Liqiu Wei, Ming Zeng, Fufeng Wang, and Daren Yu. Visual evidence of suppressing the ion and electron energy loss on the wall in Hall thrusters. 56(3):038001.


