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Gravitational Wave Content of Spatial Hypersurfaces

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Introduction

The research is on the geometry of three dimensional hypersurfaces corresponding to perturbed Kerr geometries. These geometries approximate the intermediate states of the deformed black hole created by the collision of two astrophysical black holes, approaching a quiescent Kerr state. Thus, it attempts to show the difference in gravitational waves when two black holes collide and form one calm black hole. The gravitational waves that the final black hole emits, while in the process of calming down, are still detectable. Also, it can be determined from these waves whether or not the black hole was created from the collision of two black holes.

Background

Knowledge of black holes has come a long way over the years, from the first thought of black holes to black holes being accepted in the scientific community to the hopeful discovery of black holes in the near future. Since the idea of a black hole was generally accepted, many physicists have done mathematical computations to describe black holes and come up with various properties for black holes.

History of Black Holes

For many years black holes were simply constructs discussed by theoreticians [1]. It was not until the 1960s that physicist began to take black holes seriously [2]. Since then, the black hole has "become the object of intense astronomical study"[1].

Dark Stars

In the late eighteenth century, a natural philosopher, or physicist, by the name of John Michell "dared to combine the corpuscular description of light with Newton's gravitation laws and thereby predict what very compact stars should look like"[2]. These predictions led Michell to find the escape velocity, the minimum initial speed for escape, of a particle ejected from the star's surface. Upon further investigation into the escape velocity, he came up with the critical circumference for a star. The critical circumference is where the escape velocity is equal to the speed of light. So, for any star with a circumference smaller than the critical circumference for the mass of the star, light is prevented from escaping the gravitational pull of the black hole. Michell called these stars dark stars [2]. Now they are referred to as Newtonian black holes. Trajectories of light particles for different types of stars can be seen in Figure 1. As it is depicted in the figure, stars emit the most light, and black holes emit no light since all of the light particles from a black hole are pulled back in by the strong gravitational pull of the black hole.

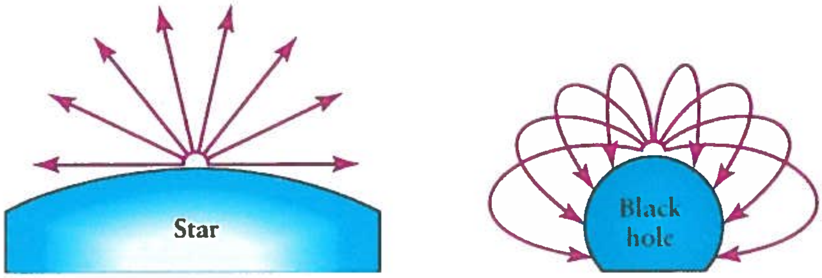

Figure 1: Particle trajectories of a star compared to a black hole

Source: MNSU. http://odin.physastro.mnsu.edu/~eskridge/astr101/kauf24_7.JPG. 11 April 2009

Schwarzschild Singularity

Michell's dark stars were based on the assumptions that space and time are absolute and the speed of light is relative [2]. However, by the early twentieth century there were other theories about black holes based on Einstein's general relativity rather than Newtonian physics. In 1916, Karl Schwarzschild was the first to give a theoretical description of a black hole using general relativity [1]. He proposed that near a black hole, the flow of time is dilated. Based on this, "if emitted from the critical circumference, the light must get shifted in wavelength an infinite amount, while traveling upward an infinitesimal distance" [2]. To make his calculations simpler, Schwarzschild only worked with stars that were spherical with no spin [2]. He also proposed that the radius of the star did not affect the curvature of space. Yet, if the star were smaller than a certain size then the star's gravity would be felt, but the star would not be seen. Schwarzschild referred to the boundary between visibility and invisibility as a horizon (now commonly referred to as the event horizon). Schwarzschild's horizon is the same as Michell's critical circumference; they both refer to the minimum size of a star where anything, including light particles, can escape. Now any spherical body smaller than the radius that corresponds to the horizon is called a Schwarzschild black hole or Schwarzschild singularity. Anything within the horizon will be drawn to a central point called a singularity where gravity is so strong that anything "would be crushed out of existence" [1]. The curvature of space and time surrounding a Schwarzschild black hole can be seen in Figure 2. The figure also shows where the singularity and horizon are on the curved space-time.

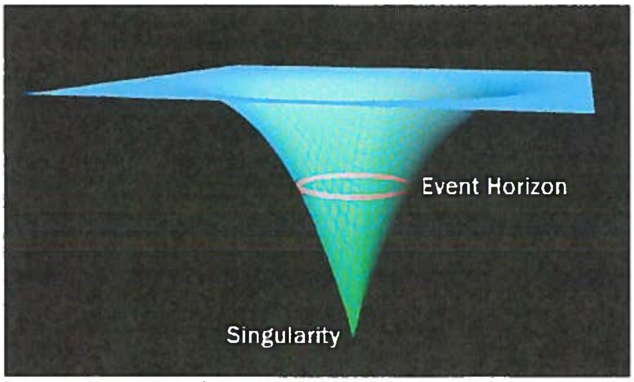

Figure 2: Curvature of spacetime around a Schwarzchild black hole

Source: http://www.odec.ca/projects/2007/joch7c2/Event_Horizon.html. 11 April 2009

Kerr Black Holes

As more time passed, many other physicists and mathematicians studied black holes. Then in 1963, Roy Kerr discovered a mathematical description for black holes that represents the gravitational field around a rotating collapsed object. This was more general and applicable than the solution Schwarzschild had given nearly 50 years earlier. This new solution is called the Kerr solution [1]. However, when Kerr first came up with this solution, it seemed to describe the spacetime curvature outside a spinning star. It was later discovered by Dennis Sciama that the solution did not describe all spinning stars. After this discovery, Brandon Carter, one of Sciama's students at Cambridge University, showed mathematically that Kerr's solution actually described any spinning black hole [2]. The Kerr solution led to the Kerr black hole, or a spinning black hole. Like the Schwarzschild black hole, the Kerr black hole has a singularity at its center. However, the Kerr black hole differs from Michell's dark star and Schwarzschild's black hole because it has two horizons. The outer horizon is the event horizon, and the inner horizon is the Cauchy horizon [3]. Also, since the Kerr black hole is spinning, it has an "equatorial bulge" [1]. Also, due to the spinning of the black hole, there is a region known as the ergosphere, bounded by the static limit, in which space spins so fast that light also spins around the black hole [1]. Figure 3 shows a Kerr black hole with the static limit, the ergosphere, the two horizons, the axis of rotation and the singularity.

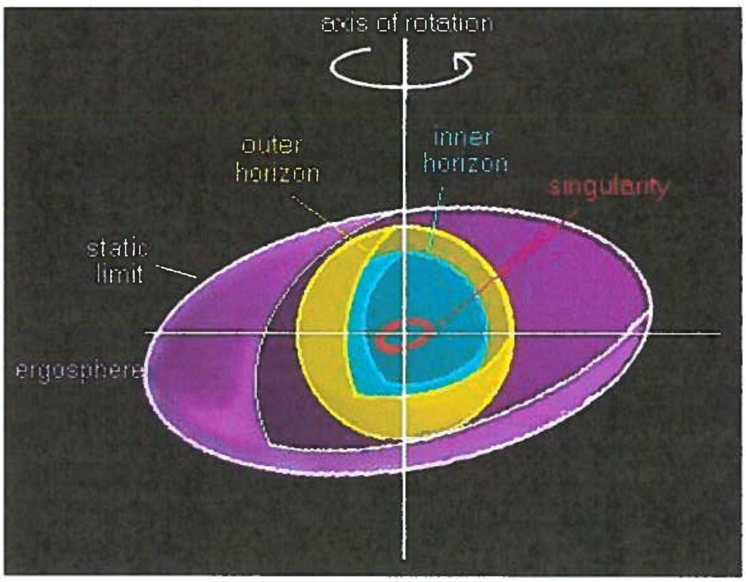

Figure 3: Kerr black hole

Source: Astronomy Café. http://www.astronomycafe.net/qadir/q1656.html. 11 April 2009

LIGO

The predictions from Michell and the calculations from Schwarzschild and Kerr have enabled many other physicists and astronomers to study black holes. Without Michell's predictions the scientific community may not have discovered black holes. Also, without the calculations of Schwarzschild and Kerr, black holes would still be strictly theoretical and would not be taken seriously by physicists. Due to their predictions and calculations, black holes have been studied by many people and will continue to be studied in the future. Currently, physicists are attempting to detect black holes using technologies that previously were not available. In 2002, the Laser Interferometer Gravitational-Wave Observatory (LIGO) made black hole detection a reality. LIGO has two observatories one in Louisiana and another in Washington state. The observatories consist of an L-shaped vacuum system that measures four kilometers on each side. Inside these vacuums, there are up to five interferometers with the primary interferometers located in the corners of the L-shaped vacuums. LIGO is extremely precise; it is able to detect gravitational waves that started tens of millions of light years away from Earth. Gravitational waves at this distance are estimated to distort the 4 kilometer mirror spacing by about 10^{-18} m. This distortion is about a thousandth the length of a proton. These distortions from gravitational waves will help to detect black holes [4].

Properties of Black Holes

The most basic concept surrounding black holes is that nothing can escape the gravitational pull of a black hole. Even light is unable to escape. As more and more physicists studied black holes, more properties were discovered. One such property is that black holes have the ability to spin, and as they spin the light near the horizon begins to spin in the curved spacetime surrounding the black hole. The area immediately outside the horizon has an abundant amount of energy. Due to this energy and the intense gravitational pull of the hole, black holes are always smooth. This means that they are always spheres or ellipsoids. If they are not smooth, then the imperfection is radiated away as gravitational radiation. Another property is that when stars, planets, or other smaller black holes are pulled into a black hole, the horizon of the black hole begins to pulsate. These pulsations create “ripples in the curvature of spacetime that propagate out through the Universe” [2].

Due to general relativity and the research done by Schwarzschild and Kerr, it was discovered that all the physical properties of black holes could be predicted using only three variables. The variables are the mass of the black hole, the rate of spin of the black hole, and the electric charge of the black hole. The physical properties that can be calculated include “the shape of the hole’s horizon, the strength of its gravitational pull, the details of the swirl of spacetime around it, and its frequencies of pulsatation” [2].

One of the most intriguing properties of black holes over the last few decades has been gravitational radiation, or energy attached to gravitational waves. Gravitational waves are created when two objects in space are orbiting one another or crash into each other. In 1974, Stephen Hawking realized that black holes actually radiate. “Hawking’s discovery revealed deep conceptual links between gravity, quantum theory, and thermodynamics”[1]. This discovery is now known as Hawking radiation. The basic concept of Hawking radiation is that the particles on the edge of the event horizon are in pairs where one is pulled into the black hole and the other is radiated out. When this happens, the total energy of the action has to be preserved. To do this, the particle that is pulled into the black holes has negative energy. This negative energy causes the black hole to lose mass. The loss of mass can lead to black hole evaporation depending on the original size of the black hole. Larger black holes take significantly longer to radiate the particle away which causes them to shrink much slower than the smaller black holes. Also, the larger black holes are much more likely to pull in other forms of energy from light near the black hole.

It is impossible to see a black hole using a telescope or any other device. This fact has caused many artists to come up with their interpretations of black holes. However, despite the fact that all the pictures of black holes are simply artist interpretations, some are better than others. Figure 4 shows a simulated view of a 10-solar-mass black hole 600 miles (900 km) away from the observer -- and against the plane of the Milky Way Galaxy.

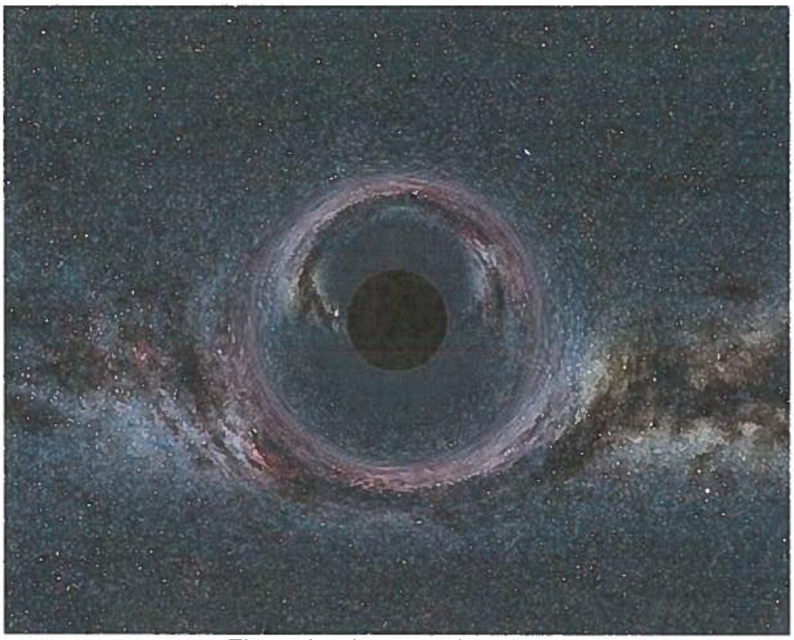

Figure 4: Black Hole Simulation

Source: Wikipedia. http://en.wikipedia.org/wiki/File:Black_Hole_Milkyway.jpg#filehistory. 11 April 2009.

Methodology

The study of black holes is part theory and part math. Due to the complexity of the math used to calculate the properties of black holes from the mass, rate of spin, and electric charge, the math is typically computed by computers.

Computer Programs

Much of the mathematical computations needed during the research and study of black holes is complex, tedious, and time consuming. This leads to the use of mathematical computer programs to aid in the calculations. In addition to the computer helping to save time and energy on behalf of the person doing the calculations, the computer also guarantees that no computational errors are made. Using a computer program, the only errors are input errors. Maple is one such computer program used in black hole calculations. In addition to Maple, a useful computer program is GRTensor. GRTensor was designed to do calculations concerning general relativity. It is a computer program that works with Maple and expands the functions of Maple to include differential geometry in order to calculate tensor components on curved spacetime. It does this by adding to the library of standard definitions contained in Maple. By using Maple in combination with GRTensor, complex black hole calculations can be done much quicker. Calculations that would take an entire day to do by hand can be done in just minutes using these tools [5].

Black Hole Mathematics

The subject of black holes is explored mostly through mathematics. There are certain mathematical subjects which show up often in the study of black holes that are not necessarily common to use in everyday mathematics. These include tensors, metrics, and Taylor expansions.

Taylor Series Expansions

A useful tool in mathematics is a Taylor series expansion. The Taylor expansion is more common in general mathematics than tensors and metrics. The Taylor series gets its name from Brook Taylor, an English mathematician from the late seventeenth and early eighteenth centuries. He published his work on series in his book *Methodus incrementorum directa et inversa* in 1715 [6]. The basic idea for a Taylor series is that any function can be written as a power series with the coefficients equal to the n^{th} derivative of the original function divided by n factorial. A power series expansion at a value, a, is a series from zero to infinity of a coefficient, c_n, times the independent variable minus the value, a, raised to the power n. This can be written mathematically as

$$f(x) = \sum_{n=0}^{\infty} c_n (x-a)^n$$

where the original function is written as $f(x)$ [6]. For a Taylor series expansion, the value c_n can be written mathematically as

$$c_n = \frac{f^{(n)}(x)}{n!}$$

where $f^{(n)}(x)$ is the n^{th} derivative of the function $f(x)$ [6]. Combining the Taylor series coefficient with the formula for the power series gives

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x)}{n!} (x-a)^n$$

The application of a Taylor series expansion for black hole mathematics does not require the entire Taylor series to be used. Instead, only a few terms need to be determined. The number of terms needed is determined by examining the original function and looking for the highest powered term that you are expanding. The power on this term is usually the highest order the Taylor expansion needs be taken to. Taking this into account, it can be useful to write the

Taylor series expansion out by terms instead of writing it as a summation. Written out by terms, the expansion looks like

$$f(x) = f(a) + \frac{f'(x)}{1!}(x-a) + \frac{f''(x)}{2!}(x-a)^2 + \frac{f'''(x)}{3!}(x-a)^3 + \cdots$$

By expanding a function into a Taylor series, the function can be easier to work with for future computations [6].

Tensors

The term tensor has different meanings depending on the context in which the tensor is found. Tensors are generally used in differential geometry and multilinear algebra. In general, a tensor is a convenient way of collecting sets of numbers together. A rank one tensor is common in many mathematical subject; it is known as a vector. An example of a rank one tensor follows.

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

Rank two tensors are matrices. An example of a rank two tensor follows.

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

Higher order tensors exist, but are more difficult to work with mathematically. A tensor can have two different types of indices: covariant and contravariant. Covariant indices are represented as subscripts on the variable used for the tensor, and contravariant indices are represented as superscripts. An example of this notation follows where the tensor is represented by the letter T, the contravariant indices are represented by the letter i, and the covariant indices are represented by the letter j [7].

$$T^{i_1 \cdots i_r}{}_{j_1 \cdots j_s}$$

Two tensors can be added together if both the covariant and contravariant indices are the same for the two tensors. The resulting tensor will also have the same indices. The following equation shows how this would look [7].

$$T_3{}^{i_1 \cdots i_r}{}_{j_1 \cdots j_s} = T_1{}^{i_1 \cdots i_r}{}_{j_1 \cdots j_s} + T_2{}^{i_1 \cdots i_r}{}_{j_1 \cdots j_s}$$

If two tensors are multiplied together, then the resulting tensor will have the covariant and contravariant indices of both of the original tensors. The following equation shows how this would look [7].

$$T_3{}^{i_1\cdots i_r k_1\cdots k_t}{}_{j_1\cdots j_s l_1\cdots l_u} = T_1{}^{i_1\cdots i_r}{}_{j_1\cdots j_s} T_2{}^{k_1\cdots k_t}{}_{l_1\cdots l_u}$$

A special type of tensor commonly used in general relativity is a metric tensor, also known as a metric. Metrics are rank two tensors that are symmetrical [7]. Since rank two tensors are matrices, a metric is a symmetric matrix, a matrix that is equal to itself transposed [7]. These are used in general relativity to show how the spatial and time components of calculations are distorted around black holes.

Research Procedure

Before beginning any of the actual research and calculations, time was spent studying the history of black holes and the mathematical basis of the research. This included reading various books, papers, and notes on the subject of black holes. To begin the actual research, a mathematical program had to be chosen to help with the calculations. The program chosen was GRTensor in conjunction with Maple. This allowed the computations to be done in minutes that otherwise could possibly take a day or more if done by hand. The current version of GRTensor was written to operate with Maple 6. This created a problem before the research could begin because the latest version of Maple is Maple 12. Because of this, the directions for how to properly install GRTensor and have it run through Maple did not work properly. It took a week to finally figure out how to get GRTensor installed with Maple 12 and functioning properly. After getting GRTensor running, the first step was to run some old calculations to verify that the program was in fact working correctly.

The research began with a specific type of Kerr black hole where ε is equal to zero, which is exact Kerr. If ε is equal to zero, then the amplitude of the gravitational waves is zero. Also, the research does not deal with the time elements of the Kerr black hole. So, the resulting metric is called the quasi-Kerr metric. The first step was to derive a way to calculate the electric and magnetic components of the black hole relating to the mass, the radius, and other variables with known values. These components came from the following metrics where E is the electric metric and B is the magnetic metric.

$$E = \begin{bmatrix} E_{rr} & E_{r\theta} & E_{r\phi} \\ E_{\theta r} & E_{\theta\theta} & E_{\theta\phi} \\ E_{\phi r} & E_{\phi\theta} & E_{\phi\phi} \end{bmatrix} \text{ and } B = \begin{bmatrix} B_{rr} & B_{r\theta} & B_{r\phi} \\ B_{\theta r} & B_{\theta\theta} & B_{\theta\phi} \\ B_{\phi r} & B_{\phi\theta} & B_{\phi\phi} \end{bmatrix}$$

Some of the electric and magnetic components calculated were zero, but the components that were not zero were all complicated. In order to make the future calculations simpler, the electric and magnetic components were simplified down as far as possible using Maple and then simplified further by hand. In these simplifications, $\rho = r^2 + a^2\cos^2\theta$, $\zeta = (r^2 + a^2)(r^2 + a^2\cos^2\theta) + 2a^2 mr\sin^2\theta$, and $\Delta = r^2 - 2mr + a^2$. The simplified components follow.

$$E_{rr} = \frac{-mr(r^2 - 3a^2\cos^2\theta)(a^2\Delta\sin^2\theta + 2(a^2 + r^2)^2)}{\Delta\rho^2\zeta}$$

$$E_{r\theta} = E_{\theta r} = \frac{-3a^2 m \sin\theta\cos\theta\,(a^2\cos^2\theta - 3r^2)(a^2+r^2)}{\rho^2\zeta}$$

$$E_{\theta\theta} = \frac{mr\,(r^2 - 3a^2\cos^2\theta)(2a^2\Delta\sin^2\theta + (a^2+r^2)^2)}{\rho^2\zeta}$$

$$E_{\phi\phi} = \frac{-mr\sin^2\theta\left(3a^2\cos^2\theta\left(\Delta\rho + 2mr(a^2+r^2)\right) - r^2\zeta\right)}{\rho^4}$$

$$B_{rr} = \frac{am\cos\theta\,(3r^2 - a^2\cos^2\theta)(a^2\Delta\sin^2\theta + 2(a^2+r^2)^2)}{\Delta\rho^2\zeta\sqrt{-1}}$$

$$B_{r\theta} = B_{\theta r} = \frac{-3amr\sin\theta\,(a^2+r^2)(3a^2\cos^2\theta - r^2)}{\rho^2\sqrt{-1}}$$

$$B_{\theta\theta} = \frac{-am\cos\theta\,(a^2\cos^2\theta - 3r^2)(-2a^2\Delta\sin^2\theta - (a^2+r^2)^2)}{\rho^2\sqrt{-1}}$$

$$B_{\phi\phi} = \frac{am\zeta\cos\theta\sin^2\theta\,(a^2\cos^2\theta - 3r^2)}{\rho^4\sqrt{-1}}$$

After the components were fully simplified, the equations were Taylor expanded about a to make them easier to work with for future computations. Although some of the Taylor expansions do not look any simpler than the simplifications above, they are much easier to work with. The simplifications above all include ρ, Δ, and ζ, and all three of these contain the variable a. To do the Taylor expansions, these three equations were substituted back into the simplified equations above. The Taylor expanded components follow.

$$E_{rr} = -\frac{2m}{r(r^2 - 2mr)} + \frac{m(\cos^2\theta(13-6m)(r-2m) - (r^2 - 12mr + 12m^2))}{r^5(-r+2m)^2}a^2 + O(a^4)$$

$$E_{r\theta} = E_{\theta r} = \frac{9m\sin\theta\cos\theta}{r^4}a^2 + O(a^4)$$

$$E_{\theta\theta} = \frac{m}{r} + \frac{m(-8r\cos^2\theta + 3r + 6m\sin^2\theta)}{r^4}a^2 + O(a^4)$$

$$E_{\phi\phi} = \frac{m\sin^2\theta}{r} - \frac{m\sin^2\theta\,(\cos^2\theta(6r+2m) - r - 2m)}{r^4}a^2 + O(a^4)$$

$$B_{rr} = \frac{6im\cos\theta}{r^3(2m-r)}a + \frac{im\cos\theta\,(\cos^2\theta(23r-18m)(r-2m)-(3r^2-36mr+36m^2))}{r^2(2m-r)^2}a^3 + O(a^5)$$

$$B_{r\theta} = B_{\theta r} = -3imr\sin\theta a + \frac{3im\sin\theta\,(5\cos^2\theta-1)}{r}a^3 + O(a^5)$$

$$B_{\theta\theta} = 3imr^2\cos\theta\,a + \frac{im\cos\theta\,(-13r\cos^2\theta+12r-12m\sin^2\theta)}{r}a^3 + O(a^5)$$

$$B_{\phi\phi} = -9im\cos\theta\sin\theta\,a + \frac{9im\cos\theta\sin\theta\,(\sin\theta\cos^2\theta\,(3r+2m)-\sin\theta\,(r+2m)-2r^3\cos^2\theta)}{r^3}a^3 + O(a^5)$$

While the simplifications and Taylor expansions were being done on the electric and magnetic components of the Kerr black hole, the next step in the research was started. This step was to repeat the same process done for the quasi-Kerr metric, but using a Schwarzschild metric that had the same components as the reduced Kerr metric that was used. In order to accomplish this, a new metric was written to accommodate the needs of the research. The new metric was simply the Schwarzschild metric without the time components. After writing the new metric, the electric and magnetic components were calculated. The magnetic components all reduced to zero, and unlike the components for the Kerr metric, the electric components were fairly simple. The electric metric for Schwarzschild follows.

$$E = \begin{bmatrix} \frac{2m}{r^2(-r+2m)} & 0 & 0 \\ 0 & \frac{m}{r} & 0 \\ 0 & 0 & \frac{m\sin^2\theta}{r} \end{bmatrix}$$

After finding the electric and magnetic components for the Schwarzschild metric, the next step in the research was to calculate ξ, the gravitational radiation scalar, for the Schwarzschild metric. The scalar follows.

$$\xi = 0$$

This as far as the research has progressed thus far. Currently, the value for β, a value used to determine the electric and magnetic components, is being checked. This value will also affect the gravitational radiation scalar for the Schwarzschild metric. If β ends up being incorrect as it is now, then the value of the gravitational radiation scalar will change. After verifying the value for the gravitational radiation scalar for the Schwarzschild metric, the next step will be to calculate the gravitational radiation scalar for the quasi-Kerr metric. Once this is found, the two scalars will be compared. They should be similar, but there will be definite differences between the two. These differences are the gravitational waves resulting from the black hole collision.

Applications

The research can be applied in the area of black hole detection. Currently LIGO is capable of detecting black holes, and soon there will be another experiment, LISA, capable of detecting black holes.

The Laser Interferometer Gravitational-Wave Observatory (LIGO) is one way physicists are attempting to discover black holes. LIGO uses interferometers to detect gravitational waves from as far as tens of millions of light years away. These gravitational waves would be able to tell physicists if there has been a distant collision of two stellar-mass black holes. So far LIGO has not detected a black hole, but it is hoped that one will be detected within the next few years [4].

The Laser Interferometer Space Antenna (LISA) is another experiment that will be detecting gravitational waves. This will be a joint venture between the European Space Agency and NASA. LISA will be in space for approximately two years with an expected launch date somewhere between 2018 and 2020. It will go into an orbit around the sun similar to that of the earth, but trailing behind by approximately 20 degrees. The antenna will have three pieces spaced several kilometers apart and aligned so that the antenna forms an equilateral triangle. The hope is that when a gravitational wave disturbs the space-time field between two of the pieces the small difference in length of one side of the triangle should be measurable. The antenna is expected to be able to measure a difference of 20 picometers (10^{-12} m) over a distance of 5 million kilometers. Thus, LISA will be more accurate than LIGO. LISA will not have to deal with the waves created by the earth and by objects on the earth. Also, because LISA will be in space as opposed to on the earth, it will be able to detect supermassive black hole collisions [8].

Conclusion

There is still a lot to be learned about black holes. The study of black holes is still relatively new compared to other subjects in science. Black holes were only first theorized about in the eighteenth century when Michell attempted to predict what compact stars would look like using Newtonian physics [2]. Then it was not until the early twentieth century that further significant advances were made concerning black holes. All of the newer theories were based on Einstien's general relativity instead of Newtonian physics [1]. Using general relativity as a basis for the computational research on black holes, Schwarzschild was able to theoretically describe black holes, and Kerr was able to come up with a generalized mathematical description of black holes [2]. Due to the work of Michell, Schwarzschild, and Kerr, the scientific community accepted the idea of black holes, and now physicists are attempting to discover black holes using experiments such as LIGO and LISA [4, 8]. These experiments work by detecting gravitational waves emitted by the collision of two black holes. The research on the geometry of three dimensional hypersurfaces approximates the intermediate states of the deformed black hole created by the collision of two astrophysical black holes. This is similar to what LIGO is currently doing and what LISA will be doing.

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Honors Research Project
Approval

Form 3 – Submit with completed thesis. All signatures must be obtained.

Name of candidate: Christin Cobb

Department: physics

Degree: math

Full title of project: Gravitational Wave Content of Spatial Hypersurfaces

Approved by:

Lior Burko 4/27/2009
Project Advisor Date

[signature] 4/27/09
Department Chair Date

John L. Nelson 11 May 2009
Honors Program Director for Honors Council Date