Development of an optimization framework for a circulation control airfoil with a morphing trailing edge

Micajah Schweikert

Follow this and additional works at: https://louis.uah.edu/uah-theses

Recommended Citation
Schweikert, Micajah, "Development of an optimization framework for a circulation control airfoil with a morphing trailing edge" (2022). Theses. 393.
https://louis.uah.edu/uah-theses/393

This Thesis is brought to you for free and open access by the UAH Electronic Theses and Dissertations at LOUIS. It has been accepted for inclusion in Theses by an authorized administrator of LOUIS.
DEVELOPMENT OF AN OPTIMIZATION FRAMEWORK FOR A CIRCULATION CONTROL AIRFOIL WITH A MORPHING TRAILING EDGE

by

MICAJAH SCHWEIKERT

A THESIS

Submitted in partial fulfillment of the requirements for the degree of Master of Science in Engineering in The Department of Mechanical and Aerospace Engineering to The School of Graduate Studies of The University of Alabama in Huntsville

HUNTSVILLE, ALABAMA

2022
In presenting this thesis in partial fulfillment of the requirements for a master’s de-
gree from The University of Alabama in Huntsville, I agree that the Library of this
University shall make it freely available for inspection. I further agree that permis-
sion for extensive copying for scholarly purposes may be granted by my advisor or,
in his/her absence, by the Chair of the Department or the Dean of the School of
Graduate Studies. It is also understood that due recognition shall be given to me
and to The University of Alabama in Huntsville in any scholarly use which may be
made of any material in this thesis.

Micajah Schweikert
Micajah Schweikert

10/30/2022
(date)
Submitted by Micajah Schweikert in partial fulfillment of the requirements for the degree of Master of Science in Engineering in Aerospace Engineering and accepted on behalf of the Faculty of the School of Graduate Studies by the thesis committee.

We, the undersigned members of the Graduate Faculty of The University of Alabama in Huntsville, certify that we have advised and/or supervised the candidate of the work described in this thesis. We further certify that we have reviewed the thesis manuscript and approve it in partial fulfillment of the requirements for the degree of Master of Science in Engineering in Aerospace Engineering.

Committee Chair
Dr. Chang-kwon Kang
10/30/2022

Advisor
Dr. Konstantinos Kanistras
10/30/2022

Department Chair
Dr. Naga Venkat Adurthi
10/31/2022

College Dean
Dr. Keith Hollingsworth
11/01/22

Graduate Dean
Shankar Mahalingam
Digitally signed by Shankar Mahalingam
Date: 2022.11.03 08:21:47 -05'00'

Graduate Dean
Dr. Jon Hakkila
Digitally signed by Jon Hakkila
DN: cn=Jon Hakkila, cn=UAP, ou=Associate Provost/Graduate Dean, email=jon.hakkila@uah.edu, c=US
Date: 2022.11.29 14:38:23 -06'00'
Adobe Acrobat version: 2020.005.30418
ABSTRACT

School of Graduate Studies
The University of Alabama in Huntsville

Degree Masters of Science       College/Dept. Engineering/Mechanical and
                   in Engineering         Aerospace Engineering

Name of Candidate Micajah Schweikert

Title Development of an Optimization Framework for a Circulation Control
   Airfoil with a Morphing Trailing Edge

This research investigates the impact of a circulation control airfoil with a
deformable trailing edge, on lift-to-drag ratio, to evaluate the use of deformation to reduce the momentum coefficient of blowing ($C_\mu$). A framework is developed for optimization of the deformable trailing edge on a NACA 0015 consisting of: a developed parametrization method (Chord Limited discontinuous Morphing Class Shape Transformation), Bayesian optimization, an objective function ($\frac{C_l}{C_d}$), a developed communication method, and a Computational Fluid Dynamics solver (Ansys Fluent). The optimization takes place over five blowing actuations ($C_\mu = 0, 0.05, 0.1, 0.15, \text{ and } 0.2$) and two angles of attack ($AoA = 0^\circ$ and $7^\circ$). The geometric results from the optimization show good convergence in terms of deflection and curvature. Results indicate that, for no blowing actuation, optimization at $AoA = 7^\circ$ performs better than $AoA = 0^\circ$. However, with blowing actuation, optimization results in significantly higher $\frac{C_l}{C_d}$ with $AoA = 0^\circ$, especially at blowing coefficient of $C_\mu = 0.05$. 

iv
ACKNOWLEDGMENTS

I am exuberantly grateful to all who have supported me. Dr. Kanistras for his tireless work, excellent guidance, and accurate feedback throughout my academic career; The support of the University of Alabama in Huntsville from the Mechanical and Aerospace Engineering department; and, finally my parents, for their continued moral support, direction, and patience.
# TABLE OF CONTENTS

List of Figures ........................................ x

List of Tables .......................................... xiii

List of Symbols ......................................... xiv

Chapter

1 Introduction .......................................... 1

1.1 Motivation ........................................ 4

1.2 Problem Statement ................................. 5

1.3 Methodology ....................................... 6

1.4 Contributions ...................................... 7

1.5 Organization ...................................... 8

2 Literature Review ................................... 9

2.1 Active Flow Control .............................. 9

2.1.1 Circulation Control ............................ 9

2.1.2 Morphing Wing Technology .................. 12

2.2 Parameterization and Optimization Techniques .... 14

2.2.1 Parameterization .............................. 14

2.2.2 Optimization ................................. 16
4 Results and Discussion

4.1 Framework Validation using Genetic Algorithms and dModCST

4.1.1 Genetic Algorithm Initialization-Xfoil

4.1.2 Xfoil Optimizationcoefficient

4.1.3 CFD Optimization

4.1.4 Review of Validation Case

4.2 Geometric Optimization at Fixed Blowing Coefficients

4.2.1 No Blowing Case ($C_{\mu} = 0$)

4.2.2 Low Blowing Case ($C_{\mu} = 0.05$)

4.2.3 High Blowing Cases ($C_{\mu} = 0.1, 0.15, 0.2$)

4.3 Comparison of Optimized Airfoils

4.4 Comparison of Performance of Optimized Trailing Edges over Various Blowing Conditions

4.5 Performance Comparison

5 Conclusion

REFERENCES
### LIST OF FIGURES

<table>
<thead>
<tr>
<th>FIGURE</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1 Communication method of the designed framework for shape optimization.</td>
<td>21</td>
</tr>
<tr>
<td>3.2 Effect of CST’s class function hyper parameters, $n_1$ and $n_2$, on a baseline, symmetric airfoil.</td>
<td>25</td>
</tr>
<tr>
<td>3.3 CST’s shape function with regions of Bézier function maximum effectiveness.</td>
<td>26</td>
</tr>
<tr>
<td>3.4 Effect of the TE control parameters $C_{o_t}$ on the morphed airfoil geometry. [1]</td>
<td>30</td>
</tr>
<tr>
<td>3.5 Effect of the TE control parameters $P_{o_t}$ on the morphed airfoil geometry. [1]</td>
<td>31</td>
</tr>
<tr>
<td>3.6 Effect of the morphing coefficient $C_{o_3}$ on a baseline airfoil for the dM-CST method.</td>
<td>36</td>
</tr>
<tr>
<td>3.7 Effect of the morphing power $P_{o_1}$ on a baseline airfoil for the dMCST method.</td>
<td>37</td>
</tr>
<tr>
<td>3.8 Genetic algorithm optimization flow chart.</td>
<td>40</td>
</tr>
<tr>
<td>3.9 Dynamic mutation and crossover values over generations in the genetic algorithm.</td>
<td>40</td>
</tr>
<tr>
<td>3.10 Bayesian optimization procedure flow chart.</td>
<td>42</td>
</tr>
<tr>
<td>3.11 Representative geometry generation of computational domain in SpaceClaim.</td>
<td>44</td>
</tr>
<tr>
<td>3.12 Depictions of the structured blocks used for mesh generation in Ansys Mechanical</td>
<td>45</td>
</tr>
<tr>
<td>3.13 Mesh convergence $C_l$ and $C_d$ values for the majority mesh.</td>
<td>48</td>
</tr>
<tr>
<td>3.14 Mesh convergence $C_l$ and $C_d$ values for the mesh of the Injection domain.</td>
<td>49</td>
</tr>
</tbody>
</table>
3.15 Overview of representative chosen mesh. .................................. 49
3.16 Representative chosen mesh near airfoil. ................................. 50
3.17 Representative chose mesh near the injection slot. .................... 50
3.18 Effect of the freestream turbulence intensity on the lift-to-drag ratio compared to the default settings in Ansys Fluent with constant turbulent viscosity ratio (10). ........................................ 52
3.19 Effect of the freestream turbulence viscosity on the lift-to-drag ratio compared to the default settings in Ansys Fluent with constant turbulence intensity (5%). ........................................ 53
3.20 y+ values over the airfoil domain calculated by Fluent over a geometry with maximum deflection and $C_p = 0$. ................................. 54
3.21 Boundary conditions of the simulated domain. ........................... 55

4.1 Analysis of the coefficients and powers of the deformation polynomial in Xfoil optimization [1]. ........................................ 62
4.2 Trailing edge geometries with the largest $C_l$ produced by MIGA Optimization. ......................................................... 63
4.3 Representation of the trailing edge deformation of the three best performing airfoils for the optimized geometry of cases 1 and 2. ...... 66
4.4 Representation of the trailing edge deformation of the three best performing airfoils for the optimized geometry of Cases 3 and 4. ...... 68
4.5 Trailing edge deformation of the best performing airfoils with higher blowing coefficients. .............................................. 70
4.6 Geometric comparison of trailing edge deformation and approximated separation points for all optimized cases at two angles of attack. ... 73
4.7 Lift-to-Drag ratio comparison of the optimized airfoils for $AoA = 0^\circ$ and $AoA = 7^\circ$. .............................................. 74
4.8 Coefficient of lift of the optimized fixed deformed airfoils over various blowing coefficients with different angles of attack. ............ 76
4.9 Coefficient of drag of the optimized fixed deformed airfoils over various blowing coefficients with different angles of attack 77

4.10 Lift-to-drag ratio of the optimized fixed deformed airfoils over various blowing coefficients with different angles of attack 78

4.11 Comparison of optimal deformation, a deflecting, and a fixed trailing edge 80
# LIST OF TABLES

<table>
<thead>
<tr>
<th>TABLE</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Comparison of common parameterization methods used for morphing wings [2].</td>
</tr>
<tr>
<td>3.1</td>
<td>Mesh discretization spacings and elements used for the mesh convergence study. Spacing 1 corresponds to the vertical spacing of the domain.</td>
</tr>
<tr>
<td>3.2</td>
<td>Mesh discretization of the slot geometry and corresponding mesh elements used for a mesh study concerning the Injection domain.</td>
</tr>
<tr>
<td>3.3</td>
<td>Simulation parameters which are held constant throughout the optimization process.</td>
</tr>
<tr>
<td>3.4</td>
<td>Description of the optimization cases and their variables.</td>
</tr>
<tr>
<td>4.1</td>
<td>Resulting parameters and aerodynamic coefficients of the optimization for Case 1 and 2 ($C_\mu = 0$).</td>
</tr>
<tr>
<td>4.2</td>
<td>Resulting parameters and aerodynamic coefficients of the optimization for Case 3 and 4 ($C_\mu = 0.05$).</td>
</tr>
<tr>
<td>4.3</td>
<td>Resulting parameters and aerodynamic coefficients of the optimization for cases 5-10.</td>
</tr>
</tbody>
</table>
### LIST OF SYMBOLS

<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>DEFINITION</th>
</tr>
</thead>
<tbody>
<tr>
<td>AFC</td>
<td>Active flow control.</td>
</tr>
<tr>
<td>CC</td>
<td>Circulation control, typically used to refer to trailing edge blowing.</td>
</tr>
<tr>
<td>GUI</td>
<td>Graphical User Interface</td>
</tr>
<tr>
<td>CFD</td>
<td>Computation Fluid Dynamics. Software capable of approximately solving the Navier-Stokes equations which govern the dynamics of fluids.</td>
</tr>
<tr>
<td>TE</td>
<td>The trailing edge of an airfoil defined as $0.8 &lt; x/c &lt; 1$</td>
</tr>
<tr>
<td>AoA</td>
<td>Angle of Attack</td>
</tr>
<tr>
<td>$c$</td>
<td>chord. The length between the leading and trailing edge of an airfoil. Defined as the average between the upper and lower surfaces.</td>
</tr>
<tr>
<td>$\delta$</td>
<td>A representative conventional deflection used for comparison for CLdMCST parameterized airfoils.</td>
</tr>
<tr>
<td>$\frac{C_l}{C_d}$</td>
<td>The lift-to-drag ratio enumerating the aerodynamic efficiency of an airfoil.</td>
</tr>
<tr>
<td>$C_\mu$</td>
<td>Momentum blowing coefficient which characterized the actuation pressure used in trailing edge blowing.</td>
</tr>
<tr>
<td>$C_\delta$</td>
<td>A constructed morphing parameter which encapsulates the deformation of the trailing edge geometry</td>
</tr>
<tr>
<td>$E$</td>
<td>The lift-to-drag ratio surface for a designed airfoil.</td>
</tr>
</tbody>
</table>
$\epsilon_{C_p}$ The efficiency of the blowing coefficient when compared to a fixed, conventional-like deflection.

$\epsilon_{C_s}$ The efficiency of morphing when compared to a fixed, conventional-like deflection.

PBO Population Based Optimization. A class of optimization methods which stochastically optimize the design space

GA Genetic Algorithms. A member of the PBO class which relies on ideas from evolution to optimize the design space.

SBO Surrogate Based Optimization. A class of optimization methods which used surrogate functions to aid in efficient optimization.

BO Bayesian optimization. A member of the SBO family.

PARSEC Parameterized Sections. A parameterization method.

CST Class Shape Transformation. A constructive parameterization method.

MCST Modified Class Shape Transformation. A parameterization method with greater control over the trialing edge of the airfoil.

dModCST discontinuous Modified Class Shape Transformation method. A modification to the CST parameterization method capable of injecting a slot used for trialing edge blowing into the airfoil geometry.

dMCST discontinuous Morphing Class Shape Transformation method. A modification to the CST parameterization method capable of deforming discrete components of the geometry (i.e. trialing edge) and introducing discontinuities into the airfoil.

CLdMCST Chord Limited discontinuous Modified Class Shape Transformation. A modification to the dMCST method which preserves the airfoils chord throughout deformation.

$\tilde{c}$ The coefficients used to determin an airfoil shape in the CST method.
$B_i$ A Bézier polynomial of index $i$.

$Co_i$ The coefficients used in the MCST, dModCST, dMCST, and CLdMCST parameterization methods for trailing edge deflection control.

$Po_i$ The powers used in the MCST, dModCST, dMCST, and CLdMCST parameterization methods for trailing edge curvature control.
To the shoulders of giants on which I stand both personally and scientifically.
CHAPTER 1

INTRODUCTION

Since the beginning of the aviation industry, engineers have endeavored to incrementally improve aerodynamic efficiency. While there have been significant improvements in understanding of the flow characteristics necessary for the development of technologies that would improve the performance of aircraft, there still remain key challenges in the world of aviation. Of particular interest is the wind design improvement for lift enhancement. The invention of modern, high performance computers has drastically increased the rate at which aircraft can be improved. Particularly, geometry optimization, which tailors the wing geometry and planform to typical operating conditions, has seen a surge in applications. The mathematical schema surrounding the optimization techniques has also seen rapid improvements. Specifically, the domain of machine learning, which would largely be intractable without current computational prowess, has spurred the development of problem specific optimization methods. These optimization techniques have since matured into methods which have significant application in geometric optimization. However, strict geometric optimization for a given flight condition does not provide the flexibility to efficiently operate during all flight conditions. For these conditions, it is necessary to dynami-
cally adapt the wing geometry to perform better at any given flight condition. The genesis of this flexibility began with flaps, which dynamically deflect into the flow field altering its effects. Yet, the inclusion and extension of trailing edge deflective surfaces (ailerons, flaps, slats, etc.) leads to inherent flow instabilities or losses in efficiency as the surface of the wing is non continuous. Morphing techniques may be able to overcome these challenges and have seen a rise in interest. Specifically, the introduction of compliant mechanisms, shape memory alloys, and other methods of continuous, reversible deformation have invigorated investigations in morphing techniques [3–5].

Morphing wing technology belongs to a class of methods known as Active Flow Control (AFC), which aim to affect the flow field around a body for an improvement in performance. Specifically, morphing wings aim to physically alter the geometry of a lifting body during flight to produce the optimum geometry for the instantaneous flow condition. The challenges associated with applying this technology to commercial aircraft have largely been relegated to the physical implementation of the actuation system, design of reliable deformative materials, and the transient effects of morphing [6]. For investigating the steady-state effects of a morphing wing on the flow field, it is appropriate to consider the wing cross section with the relevant trailing edge deformation. The shape optimization incorporated into morphing-like trailing edge deformation may be able to improve the technological readiness of another AFC method, Circulation Control (CC).

CC has seen a large effort on behalf of the research community to improve viability for commercial large-scale aircraft. In principle, this technique aims to inject momentum in the boundary layer of the wing to enhance its flight characteristics [7].
A similar, albeit passive, implementation of this technique was used to improve the aerodynamics of flapped wings. In this particular application, a small gap between the flap and the main wing body is used to allow the high pressure, lower surface of the wing to inject momentum onto the low-pressure upper surface; thus, improving the lifting capabilities of a wing. This technique does have some drawbacks as the loss of high pressure on the lower surface has some detrimental effects on the flow field. CC, on the other hand, does not suffer from the same issue, as the momentum injection comes from an external source. The cost of supplying the momentum, codified by the blowing coefficient \((C_{\mu})\), needed to actively actuate the flow field is the main restraint on adopting this technology. Specifically, the supply of momentum is typically ducted from the aircraft's power plant, potentially reducing efficiency. The infrastructure required to duct the flow to the wing increases weight and complexity [8]. Additionally, the ducting will also come with losses associated with the physical design. Once the momentum is effectively transferred to the wing there are three main locations to inject the momentum into the flow field. Fortunately, injecting the momentum over the upper-surface trailing edge of the wing has proven to be the most effective AFC method for lift enhancement [8].

Lift enhancement has direct dividends on decreasing aircraft noise and enabling shorter runways. Using CC, engineers are able to reduce or even eliminate conventional control surfaces which can reduce aircraft radar signatures [9]. Most prominently, CC has been proven to improve the aerodynamic efficiency or lift-to-drag ratio of a planform. Improvements to the lift-to-drag ratio correlate with a reduction in carbon emissions from aircraft engines, increased endurance, and may
ease the adoption of electric aircraft as the power-plant size can be reduced. The flexibility of CC makes it an attractive addition to commercial aircraft.

1.1 Motivation

CC is poised to be a revolutionary technology for current generation aircraft because of its ability to significantly improve lift and increase the aerodynamic efficiency of aircraft. Unfortunately, the cost associated with momentum injection has stymied its incorporation into modern aircraft. If the effect of blowing could be increased, the required $C_\mu$ to produce a desired effect would subsequently be reduced. With an optimized trailing-edge shape this challenge may be met. Furthermore, in addition to improving the efficiency of blowing, a CC enabled morphing-like airfoil would compound the benefits of both AFC methods.

To effectively investigate the impact of $C_\mu$ on the aerodynamic efficiency with an optimized geometry, a framework must be developed. First, an appropriate parameterization method must be developed to effectively reduce the trailing edge controlling parameters to a tractable state. Second, an optimization method must be selected capable of efficiently optimizing the geometry. Third, an objective function to quantify the quality of a particular result must be identified. Fourth, a communication method capable of interlocking the parameterization, optimization, and objective function methods with the simulation component must be generated. Finally, a simulation component which is able to produce results regarding the aerodynamics of a tested geometry must be generated. Therefore, the motivation for this work is
the implementation and the investigation of the optimal geometry configuration and blowing coefficient though a designed framework.

1.2 Problem Statement

This research is intended to investigate the effects of the geometry optimization and blowing on aerodynamic efficiency. Concurrent optimization of the geometry and blowing coefficient may prove to be intractable and may not be the best way to improve the technological readiness of CC for four reasons. First, the shape of the design space, within reasonable blowing coefficients, may not have an optima. As a result, the optimal $C_\mu$ could be located at an unreasonable $C_\mu$. Second, optimizing geometry and blowing at the same time does not provide any insights into how the geometry must deform to become optimal at various $C_\mu$ values. Third, the concurrent optimization would largely be limited to the baseline airfoil, parameterization method, and the freestream conditions. Finally, the optimal blowing coefficient on a realized system would also depend on the cost required to generate the $C_\mu$. For example, if the optimal $C_\mu$ was 0.1 for a lift-to-drag increase of 10%, but supplying this $C_\mu$ resulted in a decrease in power-plant efficiency of 50%, the resulting aircraft would be less efficient than a baseline aircraft.

It is, therefore, more widely applicable to provide some insight into the region of blowing which optimizes the lift-to-drag ratio. Generating a class of optimized airfoils at various blowing coefficients so that a morphing system can be designed to generate the optimal geometry for any $C_\mu$ during flight is equally important. If a specific geometry performs well over all $C_\mu$ values, this would necessarily indicate that
morphing is not a viable strategy for reducing $C_\mu$. Constructing the problem as a set of optimizations which take place with a fixed $C_\mu$ addresses each of these concerns. Subsequently, this analysis considers the geometric optimization of an airfoil with a deformable trailing edge at various, fixed blowing coefficients.

1.3 Methodology

To complete the geometric optimization of a deformable trailing edge airfoil at various fixed blowing coefficients a framework must be developed. The proposed framework consists of five components: a communication method, a parameterization method, an optimization method, an objective function, and a computational simulation package. The communication method facilitates the exchange of information between each of the framework’s components and is divided into two scripts: an Ansys wrapper and a Graphical User Interface (GUI). The Ansys wrapper is capable of communicating with any Ansys simulation package, in this analysis Fluent is used, through design parameters. The GUI collects the rest of the framework components into a single class enabling execution of name-protected methods within each component. Communications between methods are able to transfer a significant amount of data, however design point interactions within Ansys do not provide such freedom. In the case of geometric optimization, the number of parameters is theoretically infinite. As a result, a parameterization method must be utilized to significantly reduce the controlling design points of the geometry. For this analysis the discontinuous Modified Class Shape Transformation (dMCST) is augmented and deployed to reduce the number of necessary parameters for aircraft shape generation [2]. The control
over what specific parameter values are, is left to the optimization method. For efficient use of computational resources, a member of the Surrogate Based Optimization (SBO) methods, Bayesian Optimization (BO), is used [2]. To quantify the viability of a suggested airfoil, a function which specifies the objective of the optimization must be used. Since the goal is improvement in the lift-to-drag ratio, the chosen objective function is \( \frac{C_L}{C_d} \). The objective function requires information about the aerodynamic characteristics of an airfoil. To provide the lift and drag of the airfoil, a computational solver is used to generate a fully structured computational domain and solve the navier-stokes equations. The framework is then used to optimize the trailing edge geometry of a baseline airfoil at a specific \( C_L \) and the results are post-processed with MATLAB.

1.4 Contributions

The primary contribution of this work is the investigation into lift-to-drag optimal geometry of a trailing edge deformable, CC airfoil. The following summarizes the contributions of this work.

- The construction of a framework for optimization and its application to CFD.
- Augmentation of a previously described parameterization method and improvements on the parameterization method for feasible trailing edge deformation.
- An improved understanding between the optimum blowing coefficient for aerodynamic efficiency improvement. As well as the effect of deformation on the expected improvement in the lift-to-drag ratio gained from blowing.
• Approximating the impact of optimal deformation and blowing, respectively, on the lift-to-drag ratio. This also serves as an evaluation of morphing as a $C_\mu$ reduction technique. Specifically, if optimal deformation with blowing does not improve the impact of blowing on the lift-to-drag ratio, then morphing is not a suitable technique for $C_\mu$ reduction. The optimal deformation improved the lift-to-drag ratio, when compared to a conventional-like flap, by 23%.

1.5 Organization

The remainder of this work follows the following format: Chapter 2 reviews the literature regarding circulation control, morphing technologies, parameterization methods, and optimization methods. Chapter 3 identifies and constructs the framework necessary to complete the optimization. The formulation of each of the five framework components are provided. Chapter 4 presents and discusses the results of each optimization case. A comparison of the optimized geometry for each blowing coefficient and angle of attack is provided. Chapter 5 concludes the work and discusses some potential future research in this domain.
CHAPTER 2

LITERATURE REVIEW

This chapter provides a literature study of previous endeavors into AFC techniques, aerodynamic shape optimization, and methodologies therein. It begins by describing two AFC methods and their effects, motivations, and applications. The chapter concludes with a discussion of optimization and parameterization methods.

2.1 Active Flow Control

AFC is a class of methods to manipulate a flow-field through some form of actuation for a performance improvement [10]. Some methods in AFC have been shown to reduce flow separation, enhance lift, reduce drag, and suppress noise [11–14]. Methods of particular importance are Circulation Control (CC) and morphing wing technology.

2.1.1 Circulation Control

Circulation Control, which has proven to be the most effective AFC method for lift enhancement, affects the flow field by the injection of momentum into the flow field which improves performance [15]. The earliest investigations into CC for boundary
layer control was developed by Spence in 1961 where a theoretical solution to lift-enhancement was given for a thin-jet which was expelled over the flap of an airfoil [16]. The empirical solution, based largely on thin airfoil theory, was then validated against experimental results and showed reasonable agreement in the lift coefficient. This method of CC is known as trailing edge blowing; wherein, the injection of momentum over the flap reattaches the flow on the flap providing a substantive increase of the lift coefficient. However, Spence’s treatment of the airfoil ignores both the geometry of the flap, which has a direct and significant impact on flow turning and therefore lift, and the viscous effects of separation.

Englar’s landmark studies in 1972, 2000, and 2009 investigated the application of lifting bodies with circular trailing edges and upper and lower surface trailing edge blowing. This showed significantly enhanced lift and maneuverability capabilities [17–19]. However, in the non-blowing regime, blunted trailing edges show significant performance degradation.

Subsequent analyses have since aimed to quantify the improvement of a more traditionally flapped airfoil. In 2010, Golden and Marshall, for example, began to tackle the issue by conducting a parametric investigation of a so called dual-radius flap. Here, the flap is constructed by two arcs. The first is constructed locally tangent to the slot and the second constructed locally tangent to the first providing a variety of local curvature to the flap [20]. Such a curved flap is termed Coandă flap, where the surface curvature accentuates the Coandă effect through the application of blowing. Kanistras et al. expanded on the work of Golden using wind-tunnel analyses in lower Reynolds numbers [21].
Several groups have aimed to improve fundamental understanding of Coandă flow through the use of mathematical modeling. Trancossi et al. developed an analytical model of a turning jet using an exterior circular surface to entrain and turn the flow [22]. Subhash et al. analytically modeled a bifurcated nozzle capable of thrust vectoring with flow entrainment similarly to Trancossi [23]. Despite the significant efforts made to approximate solutions of Coandă flows, little progress has been found through mathematical modeling of the Coandă effect. Many researchers have turned to Computational Fluid Dynamics (CFD) to adequately model the effect of blowing on a flap. To further computational modeling, Englar et al. constructed a CFD validation case for an elliptic airfoil based on his elliptic trailing edge airfoil [19]. Swanson and Rumsey investigated the effects of different turbulence models for CC settling on $K - \omega$ SST with curvature correction [24]. Following these studies, several researchers have applied CC to investigate its optima and subsequent applications. Forster and Steijl parametrically studied a trailing edge blowing device by investigating slot geometry, Angle of Attack (AoA), and momentum blowing coefficient [25]. Montonaya and Marshall investigated the application of CC on transonic short take off and landing vehicles as part of a NASA AMES initiative [26].

Even with the significant aerodynamic improvements, especially in lift, the aviation industry has yet to adopt these new technologies. Cost of complexity and momentum generation have stymied the incorporation of trailing edge blowing in these aircraft [27]. Englar and Williams attempted to improve knowledge about the required momentum blowing coefficient by generating a class of suggested optimum geometry configurations and delineating the improvement in lift augmentation ratio [17].
2.1.2 Morphing Wing Technology

Morphing occurs where the structure of a body immersed in fluid physically changes shape during operation. This change in physical dimension can improve maneuverability, act as control surfaces, and optimize the geometry for the specific local condition during flight [28]. Surprisingly on the very first aircraft, this technology was inadvertently used. The flexible canvas used by the Wright brothers allowed the wing components to morph, but this effect was neither optimized for the flow condition or intentionally designed [29].

Psuedo-morphing technology has existed in modern commercial aircraft since the 1930s in the form of flaps and other wing augmentation methods. The main challenge and improvement is so called continuous-contour morphing wherein the shape of the airfoil remains smooth throughout deformation. The implications of continuous-contour shape morphing in aircraft are substantial. Inherently, continuous-contour shape morphing wing design is a multi-disciplinary task, requiring adequate material selection, power system implementation, control schema, and fluid-structure interactions [30]. A significant body of research has been dedicated to the material or structural design of morphing wings [3–5]. Some additional analyses have also tried to reduce the power requirements of morphing through bi-stable geometry, which remain deformed without actuation pressure [31]. While still other researchers endeavored to control the morphing wing to produce the desired geometry [32–34].

There are three main flavors of continuous-contour morphing: planform alteration, airfoil adjustment, and out-of-plane transformation [35]. The application of
these methods in flight or wind tunnel testing is quite dependant on material selection and design of the model. For applications concerning CC, airfoil modification seem to be the most promising. Recently, a compliant trailing edge was commercially introduced by Flexsys with a compliant skin to tailor the exterior geometry [6]. This application of morphing showed a 5% improvement in aerodynamic efficiency compared to a conventional wing.

Optimization plays a crucial role in the development of morphing geometry. Specifically, optimization is used to identify the shape of the geometry and the path it should take during flight. Additionally, if a system is designed to appropriately morph the geometry, a controller must be created to properly sense and then adapt the wing geometry. A significant body of work has been dedicated to the optimization of morphing wings [36–38]. Some of this body focuses on the aerodynamics of morphing wings. For example, in 2011 Kintscher et al. optimized the shape of a deforming leading edge using the SIMPLEX method to optimize the aerodynamic performance parameters for takeoff and landing [39]. While still other researchers focused on the structural mechanics of these morphing wings intended to optimize their deformative properties, power requirements, and actuation methods [40–42]. For example, Wang et al. generated a compliant structure for wingtip deformation and used a variant of a genetic algorithm to optimize the geometric constraints of an actuator and ensure the structural integrity of the wing remains uncompromised during excitation [43]. From these two components, controllers have been developed to adequately morph a given design. The combination of an optimal geometry, the structural dynamics required to generate that geometry, and the controller able to dictate the geometry
results in a currently applicable AFC method. Regardless, the costs associated with CC and the lack of understanding of the ideal constraints of CC have prohibited CCs use in modern aircraft. However, CC is poised to make efficient use of the continuous contour trailing-edge generated by morphing wings. Specifically, because the flaps are locally smooth, it is highly efficient for flow entrainment. When a flap is not locally smooth mass flow required to entrain the flow is substantially larger as explored by Carmona [44].

2.2 Parameterization and Optimization Techniques

The branch of mathematics dealing with constructing a problem space and optimizing the value within the problem space are termed parameterization and optimization, respectively. Parameterization methods aim to break down a problem space, in this instance a geometry, into a finite number of controlling variables. Optimization aims to iteratively improve an initial point within this space. This section reviews the literature in parameterization and optimization, respectively.

2.2.1 Parameterization

Parametrization methods are largely categorized into deformative and constructive methods. Deformative methods slightly adjust a baseline airfoil typically through a perturbation function applied iteratively over the domain, and constructive methods generate a new airfoil for each set of design points [45]. Deformative methods, however, are less conducive to steady state formulations as they typically
rely on flow field estimations to deform the baseline geometry [46]. Therefore, the focus of this review will solely be on constructive methods.

Popular constructive methods used to parameterize airfoil geometries are: The Parameterized Sections (PARSEC) and Class Shape Transformation (CST) methods [47]. The PARSEC method focuses on maintaining physically realizable variables: maximum thickness position, leading edge radius, and boat-tail angle [48]. The PARSEC method relies on a system of twelve linear equations to characterize the shape of an airfoil. However, the PARSEC method has two main flaws: it lacks the capability to cover the design space, and it has limited direct control over some features of the airfoil.

Several variants of the PARSEC method have emerged to solve the shortcomings the PARSEC method. Mainly these are the Sobieczky, Modified Sobieczky, and the Bézier PARSEC methods [48–50]. These methods, though, have challenges associated with the feasibility of the generated airfoils, a reduction of generality in the design space, or the inability to model conventional airfoils. On the other hand, the CST method focuses on generality and, as a result, loses the physical association with its parameters. The CST method is built on the summation of Bézier polynomials which are then transformed into an airfoil based on the Class function. Bézier polynomials have seen wide adoption in computer graphics software due to its ability to locally control curvature and ease of modification [51]. Due to the coverage of the Bézier curves over the design space, the CST method can, with enough control parameters, capture any airfoil to any degree of accuracy. Furthermore, the CST method has proven to be capable of defining a large range of airfoils with relatively few de-
sign parameters [52–54]. Though, because the CST method uniformly distributes the Bézier polynomials over a normalized chord length, trailing edge control requires a substantial increase in control parameters. A Modified CST (MCST) method was devised to improve trailing edge control simply by adding a secondary polynomial to the output airfoil of the CST method [55]. The MCST method gave more direct control over the trailing edge by modifications to secondary control parameters. Table 2.1 delineates the common methods capabilities [2].

Table 2.1: Comparison of common parameterization methods used for morphing wings [2].

<table>
<thead>
<tr>
<th>Selection Criteria</th>
<th>PARSEC</th>
<th>CST</th>
<th>Modified CST</th>
</tr>
</thead>
<tbody>
<tr>
<td>TE control</td>
<td>X</td>
<td>X</td>
<td>✓</td>
</tr>
<tr>
<td>Physical variables</td>
<td>✓</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Design space Coverage</td>
<td>X</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Reversible</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Number of variables</td>
<td>11</td>
<td>Depends on the order of the polynomial</td>
<td>CST + 6 (control on TE)</td>
</tr>
</tbody>
</table>

2.2.2 Optimization

There are two common types of optimization methods free of gradients: Population Based Optimization (PBO) and Surrogate Based Optimization (SBO). PBO methods begin with a "population" comprised of "individuals" which are randomly generated within the design space [56]. Each of these individuals represent a single point in the design space and their "fitness" corresponds to the objective functions value at that position. The population then propagates to a new position through a method which is dependant on the particular flavor of PBO. The most common,
Genetic Algorithms (GA), use a method akin to evolution, where the most fit individuals have the ability to propagate their location, encoded as genes, to the next population with some variations [45, 57, 58]. GAs are able to probabilistically converge to a global optimum over a sufficient number of populations, and have been used to optimize aerodynamic shapes [59–61]. However, GAs are notoriously data hungry, similar to most other machine learning techniques, and require a large number of individual fitness calculations in order to converge. If the optimization system takes on the order of minutes to calculate an individual’s fitness, it becomes infeasible to optimize using GAs. SBOs have a more data-limited approach to optimization [62]. SBOs operate by fitting data to a surrogate model and then operating on the surrogate model to produce a likely location for the optimum. As the surrogate model acquires new data, the location for the optimum becomes more and more accurate resulting in the optimization of the system. BO is a popular SBO method and fits the data to a Gaussian process regression model [63]. BO was identified by Blanchard in 2022 for use in AFC because of its computational efficiency, with regard to data generation, compared to other optimization techniques [64]. The use of SBO methods, like BO, produce as a by-product, a model of the cost function used which may provide additional insights into the design space.

2.3 Summary

The literature study reviewed previous work into the fields of CC, morphing wings, parameterization methods, and optimization methods. While studies into the application of individual AFC techniques have been thoroughly explored, there is a
gap in the published research pertaining to aircraft design with CC and morphing wings. This is especially true for lift-to-drag optimization. This thesis builds on previous research in both AFC technologies and incorporates both methods into one planform. Furthermore, research into optimization and parameterization techniques have paved the way for efficient optimization of aerodynamic shape and improved methods for accurately modeling airfoil geometry. This work utilizes the optimization methods described and builds additional functionality into the parameterization method for the optimization of a multi-AFC airfoil.
CHAPTER 3

FRAMEWORK DESIGN AND METHODOLOGY

An optimization framework defines a collection of interlinked components which exchange information to improve a desired characteristic of a system. In the case of aerodynamic optimization, the candidate system for optimization is a CFD simulation package. Some commercial simulation packages, for instance Ansys Fluent which is used in this work, provide their own proprietary, integrated optimization framework. Such implementations of a commercial framework necessarily restrict the access of the user to predefined, validated, and robust optimization methods. While this is usually adequate for most large scale applications, the selection of an optimization method significantly impacts the quality, usability, and speed of the optimization process. Furthermore, complex objective functions, which define the perceived quality of a given design, require significant innovation on behalf of the user to correctly define in proprietary frameworks. As a result, progress is significantly slowed. Generating an external, general optimization framework, which requires minor adjustments to interact with any simulation environment, provides the user significantly more control over the generated data, optimization process, and the objective function. Improved control, at the cost of ease-of-use, allows the user to utilize cutting-edge optimization
methods coupled with complex objective functions. The proposed framework consists of the following components: i) a communication method capable of interconnecting multiple systems into a single framework; ii) a parametrization to distill a complex geometry to a finite number of control parameters; iii) an optimization algorithm capable of iterative improving upon the design; iv) an objective function(s) capable of discerning the quality of a design, and; v) a computational solver to evaluate the key design parameters. This chapter identifies the mechanisms of interactions and the chosen framework components.

3.1 Communication Method

The Communication Method is the connective tissue between the disparate components of the framework. It is responsible for interlinking the geometry, simulation, objective function, and optimization method, and executing each of these components in turn. Figure 3.1 depicts the framework communication methods as well as the procedure for execution within the design. The communication method is broken into two main scripts. The first script, "Assembler", allows the user supplied parameterization method (geometry creation), objective function, and optimization method to communicate and utilize the secondary script. This secondary script creates an environment around the CFD simulation, which reads the results from a parameters file, creates a new design point, applies these new parameters, execute the simulations, reads the exported data from Ansys Fluent, and returns the result. Incidentally, the second, "Ansys Wrapper", script is the only component in the frame-
work which is simulation package specific. For instance, to use this framework with OpenFOAM only the second script must be adjusted.

![Diagram]

**Figure 3.1:** Communication method of the designed framework for shape optimization.

Due to the interconnection between the parameterization, objective function, and optimization methods a few assumptions have been made as to the structure of these methods. First, the methods are constructed as separate classes named Geometry, Objective, and Optimization, respectfully. Next, the executed method within each user supplied class must have the following names: Geo, Objective, and Opt. Finally, none of the supplied classes can have methods of the same name. The Assembler assembles the disparate classes into a single class, which allows communication between the supplied classes. Additionally, the ANSYS-wrapping script is included
in the Assembler to allow the execution of ANSYS within each of the scripts; though, this is typically done within the optimization class.

A GUI is constructed to select the location of the user supplied scripts. The GUI represents the main mode of interaction between the user and the framework. The Ansys-Wraper is a static class file which handles the interaction with ANSYS and is accompanied by a Workbench journal file. The first part of this ANSYS-wraper script accesses a parameters file, which is generated by the initialization of the optimization script. The ANSYS-wraper reads this file and passes this information to a workbench journal file. The ANSYS-wraper script then preforms a commandline interaction with the user-supplied simulation file, by calling the workbench journal file. After the simulation is completed, the ANSYS-wraper script collects the export file from ANSYS, parses the latest design point, and returns the result. The entire interaction is contained within the "func" method, which can be called within any user supplied script. For instance, in optimization with genetic algorithms, the "func" method can be supplied to a package, similar to pyGAD, as an anonymous function, which can then call the "func" method for each of the tested design points.

The remainder of this chapter describes the component selections for the user defined scripts and the solver conditions for CFD simulation. First, a standard parametrization method is evaluated for efficacy on a continuous-contour morphing flap. Several modifications have been made to the common CST method to better adapt the parametrization technique to discontinuous geometry generation, segmented deformation, and feasibility. Second, two optimization methods are com-
pared and implemented into the framework. Third, the objective function is identified. Finally, a computational model is initiated and the mesh evaluated for simulation.

The devised software framework is developed to be flexible and accept any parametrization, optimization, objective function, and simulation tool. To investigate the flexibility of the proposed framework a validation case is devised. The validation case used a Genetic Algorithm to optimize the lift coefficient of a NACA 0012 with a deforming trailing edge with upper surface trailing edge blowing.

3.2 Geometric Parameterization

Optimization speed is correlated with the number of variables, or parameters, which the optimization method can control. Unfortunately, for aircraft shape optimization, the controlling variables of the design space are theoretically infinite as the contours of a geometry intersect infinite points. The purpose of a parameterization method is to reduce the number of design points to a more reasonable value. One such parameterization method is the commonly used NACA four-digit series which reduces the infinite points required to define the airfoil into three parameters: the maximum camber, the position of the maximum camber, and the maximum thickness. However, the NACA n-digit series parameterization method does not cover the design space. That is, there can exist an airfoil which cannot be accurately described by these three parameters. Another common parameterization method, CST, does provide total design space coverage with relatively few parameters.
3.2.1 Class Shape Transformation Method

The CST method relies on two functions, appropriately named Class (C) and Shape (S) [55]. The class function determines the general contour of the generated airfoil depending on the class parameters $n_1$ and $n_2$ in Equation (3.1). The selection of these parameters corresponds to, for the most part, in which aerospace domain the shape is being optimized. For instance, a hypersonic airfoil, which is more angular, would have parameters $n_1 = 1$ and $n_2 = 1$ whereas common subsonic airfoils, and the parameters used in this work, would be $n_1 = 0.5$ and $n_2 = 1$. Figure 3.2 describes a number of different class functions on a symmetric airfoil.

$$C^n(x) = x^a(1 - x)^b, \forall x[0, 1]$$ (3.1)

The class function, however, only describes an airfoil broadly; the shape function describes the precise geometry of any given airfoil. The shape function is defined as a Bezier curve of order $n$ [51–55]. Bezier curves are widely used throughout most design software for their ability to be infinitely controlled and continuously span any design space. A Bezier curve is simply the weighted summation of Bernstein polynomials, $B_i$, where the weights are typically considered control points of the Bezier curve $c_i$. Since the Bezier curve, and by extension the shape function, is defined as a summation of Bernstein polynomials, as given in Equation (3.2), any change to one weight will affect the overall airfoil. It is useful, in some sense, to consider the locations of effective control of a Bezier weight within the shape polynomial. Figure 3.3 depicts the individual Bezier curves effect on the total shape function. Essentially,
Figure 3.2: Effect of CST’s class function hyper parameters, $n_1$ and $n_2$, on a baseline, symmetric airfoil.

The multiplicative terms within the Bézier curves have pronounced effects towards 0, for term $x^i$, and pronounced effects towards 1, for term $(1 - x)^{n-i}$. The overall shape of a subsonic airfoil with 5 controlling Bézier curves is given by Equation (3.3).

$$S^n(x) = \sum_{i=0}^{n} c_i \binom{n}{i} x^i (1 - x)^{n-i} = \sum_{i=0}^{n} c_i B_i(x), \forall x[0, 1]$$ (3.2)

$$y = C_1^{0.5}(x) S^4(x)$$ (3.3)

3.2.1.1 Reverse Class Shape Transformation Method

CST is deterministic and design space filling, and it is possible to generate any airfoil with any level of fidelity within a given class. This reverse CST method
**Figure 3.3:** CST’s shape function with regions of Bézier function maximum effectiveness.

converts a set of reference airfoil geometry given by \((x_{\text{ref}}, y_{\text{ref}})\) into a finite set of coefficients \(c_{\text{ref}}\). The shape function can be represented as multiplication of Bézier polynomials \((B_0, B_1, \ldots)\) and coefficient column vector \(\vec{c}\), Equation (3.4), thus the CST representation of a reference airfoil is shown in Equation (3.5). Another column vector \(\vec{Y}\) can be constructed, which corresponds to the effective shape function distribution of \(y_{\text{ref}}\) whose elements are given by Equation (3.6), resulting in Equation (3.7).

\[
S(x) = \begin{bmatrix}
B_0(0) & B_1(0) & B_2(0) & \cdots \\
B_0(x_1) & B_1(x_1) & B_2(x_1) & \cdots \\
\vdots & \vdots & \vdots & \ddots \\
\end{bmatrix}
\begin{bmatrix}
c_0 \\
c_1 \\
\vdots \\
\end{bmatrix} = \mathbf{B}(x)\vec{c} \quad (3.4)
\]
\[ y_{ref} = C(x_{ref})B(x_{ref})\bar{c}_{ref} \quad (3.5) \]

\[ Y_i = \begin{cases} 
0 & C_i = 0 \\
\frac{y_i^{ref}}{C_i} & C_i \neq 0 
\end{cases} \quad (3.6) \]

\[ \bar{Y}(x_{ref}) = B(x_{ref})\bar{c}_{ref} \quad (3.7) \]

The coefficients of an airfoil given by Cartesian coordinates \((x_{ref}, y_{ref})\) can be converted to CST parameters \((\bar{c}_{ref})\) using Equation (3.8), where \(B^+\) represents the Moore-Penrose inverse, or pseudo-inverse.

\[ \bar{c}_{ref} = B^+(x_{ref})\bar{Y}(x_{ref}) \quad (3.8) \]

### 3.2.1.2 Limitations of Class Shape Transformation Method

While CST has many beneficial attributes, primarily the design space coverage, there are some drawbacks. These drawbacks are caused primarily by simplifications in the model which allow for beneficial properties. However, the drawbacks associated with the simplifications can be overcome through increasing the complexity of the model. However, first the limitations in the CST model must be identified. CST has the following limitations: i) CST is single-valued; ii) \(x \in [0, 1]\); and iii) \(y(0) = y(1) = 0\). CST is a single-valued function and thus requires two sets of coefficients to describe the upper and lower surfaces of the airfoil. In the case of symmetric airfoils, the
reference Bézier weights for the lower surface is the additive-inverse of the Bézier weights for the upper surface. On the other hand, in cambered airfoils, two distinct sets of Bézier weights must be generated and CST method needs to be applied on both the upper and lower surface, Equation (3.9).

\[
y_{ref} = \begin{cases} 
\tilde{C}(x_{ref})B(x_{ref})\bar{c}_- & \text{Upper} \\
-\tilde{C}(x_{ref})B(x_{ref})\bar{c}_+ & \text{Lower}
\end{cases}
\]

(3.9)

Typically the \(x\) and \(y\) coordinate output of the CST method is depicted as the chord-normalized positions of a given airfoil. This rationalization works well in the case of baseline airfoils, however conventionally flapped geometries would require, at the very least, an infinite number of controlling parameters and adjustments to the AoA. This requirement, for flapped geometries, negates any advantage of a parameterization method and using the airfoil’s coordinates would likely be more efficient. Conventionally, wings with flaps rotate about a pivot point to some deflection angle, this necessarily reduces the \(x\) coordinate of the geometry as the chord-length remains the same. However, CST is only "well-behaved" when \(x\) ranges from 0 to 1 requiring a generated flap to extend through the full domain directly. Additionally, the third limitation of CST further restricts generation of flap geometries because CST requires the leading and trailing edge points of the airfoil to lie on the \(y = 0\) axis, thus no deflection in the \(y\) component is permitted. The combination of these two items does, however, guarantee that the airfoil is a closed loop with a sharp trailing
edge. In order to generate flapped geometries, some modifications must be made to the CST method.

3.2.2 Modified Class Shape Transformation

Generation of a flap with CST is largely intractable. First, the deflection of the flap would, necessarily, affect the AoA of the airfoil to retain the upper surface within the first quadrant of Cartesian space. Secondly, if the desire was to generate a discontinuity, as is required with a slotted geometry, the CST method would need a large number of controlling parameters. In this case, it may be more appropriate to use the coordinates of the airfoil as the parameters instead of the CST method. However, because of the high order of parameters, optimization in this space would be inefficient. As a result, additional complexity must be added into the CST method to deal with morphing and deflection. In literature, the Modified CST (MCST) method provides a method to address the deflection drawback where the AoA changes with deflection.

Modified CST (MCST) aims to generate a deflection in a CST generated airfoil by subtracting a polynomial from the CST method Equation (3.10). Each additional term in the polynomial \( M(x) \) provides increased control over the total deformation/morphism of the airfoil. If the morphism should be confined to the trailing edge of the airfoil then the powers \( (P_{oi}) \) must be limited values of greater than 5 and the coefficients \( (C_{oi}) \) must be limited to values below 1.5. Figures 3.4 and 3.5 depict the effect of the coefficients and powers on the designed airfoil, respectively. Unfortunately, the effect of morphism does not remain consistent, and both the coefficients and powers
have a significant effect on the deformation. Additionally, to restrict trailing edge deflection to after the slot location (80%), the morphing must only demonstrably effect the end of the trailing edge (greater than 0.96% of the chord) and leave much of the flap geometry unchanged. This effect significantly reduces control over the flap curvature limiting the effective generated design space.

\[ y = C_0^n(x)S^{nc}(x) - \sum_{i=0}^{m} C_{0_i}(x)^{P_{0_i}} = C(x)S(x) - M(x) \]  \hspace{1cm} (3.10)

**Figure 3.4:** Effect of the TE control parameters $C_{0_i}$ on the morphed airfoil geometry. [1]
3.2.3 Adjustments to the Modified Class Shape Transformation Method

The MCST method is effective at generating continuous deformation with a closed trailing edge and geometrically realizable airfoils. However, some additional modifications must be made to improve the methods generated airfoils. First, the MCST method cannot generate non-differentiable airfoils which are necessary for trailing edge blowing. The method, discontinuous Modified CST (dModCST), described by Patel was used, and improved upon, to generate an airfoil with a jump discontinuity [2]. The jump discontinuity can then be closed by a vertical line which serves as the momentum injection slot needed for CC. Second, to address the challenges with MCST, namely that morphism is not limited to the trailing edge of the
airfoil, the discontinuity generation tool used in dModCST is additionally applied

to the morphing function. As a result, the morphing is applied non-uniformly over
the airfoil and may only affect the trailing edge. This new method, discontinuous
Morphing CST (dMCST), still limitations from the CST method. Namely, limitation
ii ($x \in [0, 1]$) which requires the end of the trailing edge to occur exactly on $x = 1$.

To solve this issue a chord-limiting algorithm, which adjusts the x location of
the endpoint of the trailing edge to conserve chord-length, is applied to the dMCST
method. The combination of the chord-limiting algorithm and the dMCST method is

termed Chord-Limited discontinuous Morphing Class Shape Transformation (CLdM-
CST). CLdMCST addresses most of the limitations of the CST and MCST method
and provides both physical parameters and a method of generating conventional de-

3.2.3.1 Discontinuous Modified Class Shape Transformation Method

CC, through trailing edge blowing, has proven to be the most effective tech-

nique for lift enhancement. To actuate the flow field using trailing edge blowing,
momentum must be injected into the flow field through a slot on the upper surface
near the trailing edge of the airfoil. Neither CST or MCST have the capability, with a
reasonable number of parameters, to generate this geometry. To improve the MCST
method, the discontinuous Modified Class Shape Transformation (dModCST) was
constructed. This method constructs a replacement trailing edge at the location of
the slot discontinuity (0.8c). Then, dModCST enforces geometric constraints on the
new trailing edge to enforce slot height (h) and similarity, namely the derivatives of
the original airfoil. A vector containing the shape functions and their derivatives to be constrained is given in Equation (3.4), where \( \phi \) is the starting point of the deformable trialing edge. Equation (3.12) provides the constraints for dModCST, where the first row requires the position of the new trailing edge to be \( \frac{L}{e} \) units away from the baseline airfoil at \( \phi \). The second row requires the derivative of the new flap to be identical to the baseline airfoil at \( \phi \). By substituting Equation (3.4) into Equation (3.11) and then into Equation (3.12), it yields Equation (3.13). The vector \( c_{te} \) describes the Bézier weight coefficients for the baseline airfoil and \( c_{te} \) is the generated similar airfoil for the trailing edge for a given slot height. This method is also general and can be extended to include the second, third, etc. derivative for increased similarity to the baseline airfoil. Since, the trailing edge of the airfoil is intended to be deformed, the first derivative was deemed sufficient for similarity. Additionally, since this result does not depend on the class of the airfoil, any class function may be used.

A similar methodology was used by Patel to generate discontinuous airfoils, but only adjusted the first and second Bézier coefficients to gain similarity [2]. The proposed method improves the generality of Patel’s method and enforces a greater degree of similarity, though only the first derivative is used.

\[
\begin{bmatrix}
\tilde{S}(\phi) \\
\dot{\tilde{S}}(\phi)
\end{bmatrix}
= \begin{bmatrix}
S(\phi) \\
\dot{S}(\phi)
\end{bmatrix}
\tag{3.11}
\]

\[
\tilde{S}(\phi)_{te} = \tilde{S}(\phi)_{te} + \begin{bmatrix} h \\ 0 \end{bmatrix}
\tag{3.12}
\]
\[
\tilde{c}_e = \tilde{c}_e - \left[ \begin{array}{c}
\mathbf{B}(\phi) \\
\dot{\mathbf{B}}(\phi)
\end{array} \right]^{-1} \left[ \begin{array}{c}
h \\
0
\end{array} \right]
\] (3.13)

3.2.3.2 Discontinuous Morphing Class Shape Transformation Method

dModCST was originally intended to introduce a slot into a continuous, domain-expansive parametrization method; but, it inherits the drawbacks of the MCST technique. However, a similar discretization scheme can be used to limit morphing to just the trailing edge of the airfoil. The morphing (M) term from Equation (3.10) can be re-characterized as a piecewise polynomial Equation (3.14). Additionally, due to the normalization process on the coordinate \( x \) (i.e. \( \frac{x - \phi}{1 - \phi} \)) the derivative of the morphing function will be zero at the slot location. This means that the local curvature near the slot will be identical to the baseline airfoil, and will not be changed throughout the morphing process. Additionally, the values of \( C_0 \) take on physical meaning. At \( x = 1 \) CST requires \( y = 0 \) and dMCST yields \( M = \sum C_0 \), thus the actual deflection in the \( y \)-coordinate is \( \sum C_0 \). In addition, when \( P_0 = 1 \) a conventional flap can be generated, thus limits on the expected \( C_0 \) values can be extrapolated from the conventional case. The final description of the dMCST method is given by Equation 3.15. As with the CST method, this method must be applied to the upper and lower surfaces individually:

\[
M = \begin{cases} 
0 & x < \phi \\
\sum_{i=0}^{n_m} C_0 \left( \frac{x - \phi}{1 - \phi} \right)^{P_0} & x \geq \phi
\end{cases}
\] (3.14)
\[ y = C(x) \cdot \begin{cases} 
\sum_{i=0}^{n} B_i(x)c_i & 0 \leq x \leq \phi \\
\sum_{i=0}^{n} B_i(x)c_i - \sum_{i=0}^{n} C_{oi}(\frac{x-\phi}{1-\phi})^{P_{oi}} & \phi < x \leq 1 
\end{cases} \] (3.15)

Figure 3.6 depicts the effect of the coefficients in the morphing function of dMCST on a baseline airfoil. Due to the nature of dMCST when all powers are unity, the effect of \( C_{o3} \) is identical to the effect of any other coefficient. That is, the same geometry would be generated if \( C_{o1} \) was augmented instead. The end result is an increase in the total deflection of the trailing edge of the airfoil. The application of the dMCST method still requires the \( x \) coordinate is still fixed at 100\%c. As such, it still retains limitation ii of the CST method: \( x \in [1, 0] \). This limitation increased the overall chord-length by more than 22\%. This increase in chord-length is generally non-physical for morphing systems and does not lend well to comparisons with other similar methods of deformation.

3.2.3.3 Chord Limited Discontinuous Morphing Class Shape Transformation

The dMCST method was developed to enable the generation of a discontinuity in an airfoil and combat the limitations of the MCST method. While it generates the slot geometry necessary for CC and solves the limitations of the MCST method, namely that deformation was not confined to the trailing edge geometry, there is still one limitation inherited from the CST method. Specifically, none of the methods provided allow the maximum value of \( x \) to be less than unity. That is limitation ii, \( x \in [0, 1] \) still exists. This requirement, when paired with deflection methods, results
Figure 3.6: Effect of the morphing coefficient \( C_{0_3} \) on a baseline airfoil for the dMCST method.

in an nonphysical increase in the chord-length. The Chord-Limited dMCST algorithm was constructed to alleviate this issue and require the chord-length to remain unity, while retaining the physical effects of the \( C_{0_i} \) parameters. This algorithm uses the bisection method to calculate the scaling factor \( \beta \) of Equation (3.16) subject to the arc-length conservation condition of Equation (3.17). This process is applied discretely and iteratively.

The chord line is calculated by the average of the upper and lower surfaces of the airfoil, and the derivative is approximated in the first order sense. The integral is calculated using the trapezoid method. The Chord Limited dMCST (CLdMCST) method is thus able to create conventional and morphed flaps changes in less than 1% of the chord while maintaining all of the beneficial properties of CST and dMCST.
Figure 3.7: Effect of the morphing power \( P_{01} \) on a baseline airfoil for the dMCST method.

This method addresses limitation ii of the CST method. Specifically, it allows \( x \in [0, \zeta] \) where \( \zeta \leq 1 \).

\[
\bar{x}_{\text{new}} = \phi + \beta(\bar{x} - \phi)
\]  

\[
1 - \phi = \int_0^{\max(\bar{x}_{\text{new}})} \sqrt{1 + \left( \frac{dy}{x_{\text{new}}} \right)^2} \, dx_{\text{new}}
\]  

3.3 Optimization

Optimization time, guarantees of global optima, and solution space requirements are a few of the selection criteria for deciding an optimization method. Two methods are considered and their capabilities analyzed in each of these criteria.
3.3.1 Comparison of Optimization methods

Genetic Algorithms (GA) have gained wild popularity across all sectors which rely on optimization [65,66]. Primarily this is due to the simplicity of the algorithm and its versatility. GA, can be applied to any design space with any amount of discontinuous evaluations or parameters [67]. Additionally, modified GA methods can provide a probabilistic guarantee of global optimization if a solution is evaluate over a large number of individuals. However, while the internal calculation of the next evaluation point is very quick, GA is exceptionally data hungry, typically requiring millions of function evaluations. If the solution domain takes on the order of minutes to provide a fitness criteria, the optimization time can inflate exponentially. However, the precise optimization time depends primarily on the objective functions curvature and the number of controlling parameters.

Surrogate based optimization methods provide alternatives to PBO methods, like GA. These methods provide a more data driven optimization technique where the results of a simulation are fitted to a surrogate surface, and optimization is carried out on this reduced order model and the surrogate optimum is then tested with the simulation. The surrogate optimum is then used to update the surrogate model thus improving the models accuracy. There are, however, concessions which must be made to use these methods. First, the design space, or at the very least the region near the optimum, must be smooth. Second, SBO techniques have no probabilistic guarantees of global optima. Finally, the optimization speed of SBO methods depends heavily on the design space in question; whereas, population based methods optimize
without intrinsic knowledge of the design space and can "guess" a subsequently better position [68]. The optimization time within the design space is irrespective of the design space. The selected SBO for this analysis is the Gaussian process or BO for its superior optimization in relatively small number of evaluated design points as discussed by Patel [2].

3.3.2 Genetic Algorithms

For the validation of the framework, the selected variant of genetic algorithms is the GA with dynamic crossover and mutation rates. The dynamic portion of dynamic GA corresponds to a method of increasing convergence time, where the hyper parameters of the GA, mutation (exploration) and crossover (exploitation), are adjusted as the optimization progresses [69, 70]. Essentially, during the beginning phase of optimization exploration, or identification of regions of optima, is more important. However as the exploration progresses it becomes important to begin the optimization process to reach the local optima within the identified regions. As a result, the exploration factor (mutation rate) is initially large but decays over the generations. Conversely, the exploitation factor (crossover rate) starts initially low and then increases over time to the theoretical maximum of 1.
Figure 3.8: Genetic algorithm optimization flow chart.

Figure 3.9: Dynamic mutation and crossover values over generations in the genetic algorithm.
3.3.3 Bayesian Optimization

Bayesian optimization is a SBO method wherein data are fitted to a surrogate model and optimization is performed on this model to generate an optima candidate which is used to improve the baseline surrogate [71]. For BO, the surrogate function is the Gaussian process. The Gaussian process is a stochastic process comprising a joint normal distribution over a set of input parameters. After each data acquisition the Gaussian process’s hyper parameters are changed to fit the new data set. However, the acquisition of the new optima candidate is somewhat more sophisticated than the simple optimization of the surrogate function. Instead, an "acquisition function" is used. This acquisition function takes, as an input, the output of the Gaussian process and transforms it into a new design space. The purpose of this acquisition function is to improve the explorative capabilities of the optimization process. In doing so, the acquisition function enables the Gaussian process to explore near to the optima and avoid being trapped in the strict optima of the Gaussian process. The most popular of the acquisition functions is the Expected Improvement function (EI) Equation (3.18). Where \( x^+ \) represents the current best data point and \( x \) is a candidate optima. The argument maximum of EI is then evaluated on the desired simulation.

\[
\text{EI}(x) = \mathbb{E} \max \left( f(x) - f(x^+) , 0 \right) \tag{3.18}
\]
3.4 Objective Function

CC has already proven to be the most effective AFC method for lift enhancement; however, the cost associated with generating the mass flow needed to actuate the flow has hindered CC’s adoption into modern aircraft. To reduce the cost of incorporating CC the momentum coefficient of blowing ($C'_{\mu}$) must be reduced. This can be done by reducing the load on the aircraft’s power plant where momentum is typically ducted from to supply CC actuation. To accomplish this, while maintaining the lifting force necessary to keep the aircraft aloft, an improvement in aerodynamic efficiency ($C_l/C_d$) is required. The objective function of this study is, then, the maximization of $C_l/C_d$ at a constant $C_{\mu}$ as described in Equation (3.19).

$$Objective = \max \left( \frac{C_l}{C_d} \right) \vert_{C_{\mu}}$$ (3.19)
3.5 Computational Solver

The computational solver is responsible for generating the components necessary for the evaluation of the objective function and the continuation of the optimization process. In the present work, which considers the fluid dynamics of a deformable trailing edge and CC on an airfoil, the computational solver constitutes a CFD package, in this case ANSYS Fluent. The Ansys simulation workbench, an environment which contains many simulation and supporting packages (i.e. FEA, Electrodynamics, Computer Aided Design (CAD) software, etc.), is used for the generation of the geometry, meshing, CFD simulation, and postprocessing.

3.5.1 Geometry Generation

The computational geometry is generated using ANSYS SpaceClaim. While the framework could be easily augmented to use a more standard geometry generation tool such as Solid Works, SpaceClaim is integrated in ANSYS, has python scripting capabilities, and can pass naming schemes to the integrated mesh generators. For these reasons, SpaceClaim was selected for geometry generation.

To complete the geometry generation script, SpaceClaim journaling, using index based line naming, was used to develop the SpaceClaim-specific generation of the computational domain. First, parameters are created for the Coefficients and Powers of the morphing function in the geometry script. Then, the geometry script is loaded and the Coefficients and Powers are transferred, as inputs, to the geometry script. The output of this script defines five lines of the airfoil geometry: upper-
surface leading edge, slot height, upper-surface trailing edge, lower-surface trailing edge, and lower-surface leading edge. The Cartesian coordinates of these curves are then added to SpaceClaim and drawn. Then, the far-field inlet-outlet geometry is generated. Finally, the computation domain is split into eight blocks and each of the relevant curves are named for mesh generation. Figure 3.11 depicts a representative geometry generated in SpaceClaim.

![Figure 3.11: Representative geometry generation of computational domain in SpaceClaim.](image)

3.5.2 Meshing

Due to the large number of simulations required for optimization, the speed of both mesh generation and computation time is extremely important. Structured domains take less time to generate and typically have a smaller cell count than unstructured domains. However, structured domains do require substantially more time

44
to design. The design time of a structured mesh is relatively short compared to the mesh generation time during optimization. For this reason a structured 2D mesh was generated with a C-H topology. C-type meshes, which are an improved H-type mesh, better resolve the curvature of a wall condition, particularly the curvature experienced at the leading edge of an airfoil. The C-type topology constructs nearly orthogonal cells around the curvature and propagates the cells outward to other boundary conditions. The resulting structure of the mesh resembles the letter "C". H-type meshes are the simplest type of structured mesh block and generate highly orthogonal meshes between parallel boundary conditions. Figure 3.12 depicts the naming scheme and topology type of the disparate domains for discussion.

Figure 3.12: Truncated depictions of the structured blocks used for mesh generation in Ansys Mechanical. Blue) Inflow domain, C-type mesh topology. Green) Midfoil domain, H-type mesh topology. Yellow) Trailing edge domain, H-type mesh topology. Red) Outflow domain, H-type mesh topology. Teal) Injection domain, C-type mesh topology near the airfoil H-type mesh topology near the Outflow domain.
The inflow domain was generated by a half-circle centered on the point \((0.3c, 0)\) with a radius of \(5c\) and had a radial distribution of 60 (Vertical Nodes) cells with a symmetric type bias of 5000. While it is conventional to use an asymmetric bias with finer cells near the wall boundary condition for turbulence resolution, the orientation of the radial distribution segmentation lines, as controlled by SpaceClaim, is largely dependent on the geometry. That is, for a different airfoil the orientations of these lines may change and, thus, the asymmetric bias would refine the inlet condition leaving the near-wall region extremely coarse. For robustness, the symmetric type bias was chosen.

The circumferential distribution contained 90 cells (Horizontal Cells: Leading Edge) which were evenly distributed on the airfoils leading edge. The mid-foil domain utilized the same vertical distribution as the radial distribution of the inflow domain and utilized 90 (Horizontal Cells: Midfoil) evenly spaced cells along the airfoil. The trailing edge domain is constructed similarly to the midfoil domain except that it interfaces to the injection domain instead of the airfoil directly. The trailing edge domain is horizontally discretized by 60 evenly spaced cells (Horizontal Cells: Trailing Edge). The injection domain is the region of the geometry which spans from the slot entrance to the outflow of the geometry. It is bounded, near the trailing edge domain, by copy of the upper-surface trailing edge shifted vertically by the slot height. This slot discretization is a symmetric distribution of 25 (Vertical Cells: Injection Domain) cells with a bias factor of 100. Near the outflow domain, it is simply a rectangular region with a matching vertical distribution and a horizontal distribution identical to the outflow domain. The outflow domain used the same vertical spacing as the mid-
Table 3.1: Mesh discretization spacings and elements used for the mesh convergence study. Spacing 1 corresponds to the vertical spacing of the domain

<table>
<thead>
<tr>
<th>Mesh Elements</th>
<th>Vertical Nodes</th>
<th>Leading Edge</th>
<th>Horizontal nodes</th>
<th>Midfoil</th>
<th>Trailing Edge</th>
<th>Outflow</th>
</tr>
</thead>
<tbody>
<tr>
<td>11775</td>
<td>30</td>
<td>45</td>
<td>45</td>
<td>30</td>
<td>45</td>
<td></td>
</tr>
<tr>
<td>43350</td>
<td>60</td>
<td>90</td>
<td>90</td>
<td>60</td>
<td>90</td>
<td></td>
</tr>
<tr>
<td>94725</td>
<td>90</td>
<td>135</td>
<td>135</td>
<td>90</td>
<td>135</td>
<td></td>
</tr>
<tr>
<td>256875</td>
<td>150</td>
<td>225</td>
<td>225</td>
<td>150</td>
<td>225</td>
<td></td>
</tr>
<tr>
<td>498225</td>
<td>210</td>
<td>315</td>
<td>315</td>
<td>210</td>
<td>315</td>
<td></td>
</tr>
<tr>
<td>1008750</td>
<td>300</td>
<td>450</td>
<td>450</td>
<td>300</td>
<td>450</td>
<td></td>
</tr>
<tr>
<td>1218525</td>
<td>330</td>
<td>495</td>
<td>495</td>
<td>330</td>
<td>495</td>
<td></td>
</tr>
</tbody>
</table>

foil domain, by necessity, but used a symmetric bias on the horizontal. The horizontal distribution contained 90 (Horizontal Cells: Outflow) cells with a bias factor of 500 to better refine the wake of the airfoil. The symmetric-type distribution was chosen for the same reasons as the inflow domain.

A mesh convergence study was conducted in two parts. First, the bulk of the mesh was refined and the convergence of $C_l$ and $C_d$ was monitored using the same slot discretization. Then, the slot discretization factor was investigated for convergence of $C_l$ and $C_d$. The horizontal nodes, vertical nodes and resultant mesh sizes are delineated in Table 3.1 for the majority of the mesh and Table 3.2 for the mesh in the injection domain.

The convergence study for the lift and drag coefficients is depicted in Figure 3.13 for the majority of the mesh and Figure 3.14 for the slot spacing. The selected mesh has a total of 285,000 cells.

The resulting mesh is depicted in Figures 3.15-3.17 and contains 285,000 cells, 571,674 Faces, and 286,675 nodes. At the maximum trailing edge deflection it has a
Table 3.2: Mesh discretization of the slot geometry and corresponding mesh elements used for a mesh study concerning the Injection domain.

<table>
<thead>
<tr>
<th>Mesh Elements</th>
<th>Vertical Cells Injection Domain</th>
</tr>
</thead>
<tbody>
<tr>
<td>249375</td>
<td>5</td>
</tr>
<tr>
<td>251250</td>
<td>10</td>
</tr>
<tr>
<td>253125</td>
<td>15</td>
</tr>
<tr>
<td>255000</td>
<td>20</td>
</tr>
<tr>
<td>256875</td>
<td>25</td>
</tr>
<tr>
<td>258750</td>
<td>30</td>
</tr>
<tr>
<td>266250</td>
<td>50</td>
</tr>
<tr>
<td>275625</td>
<td>75</td>
</tr>
<tr>
<td>285000</td>
<td>100</td>
</tr>
<tr>
<td>322500</td>
<td>200</td>
</tr>
</tbody>
</table>

Figure 3.13: Mesh convergence $C_l$ and $C_d$ values for the majority mesh.

minimum orthogonal quality of 0.3319; however, less than 20% of the mesh has an orthogonal quality less than 0.90.
Figure 3.14: Mesh convergence $C_i$ and $C_d$ values for the mesh of the Injection domain.

Figure 3.15: Overview of representative chosen mesh.
3.5.3 Solver Conditions

The computational solver is tasked with simulating fluid-flow over a candidate geometry and calculating the aerodynamic forces on the geometry. The goal of this analysis is the improvement of the steady lift-to-drag ratio on an airfoil as a result of multi-AFC application. The solver considers the steady-state solution. The Coandă
effect and flow separation are inherently viscous effects; thus, the solver considers viscous flow. Instead of direct numerical simulation or higher-order methods (large eddy simulations), which are extremely computationally costly, $k - \omega$ SST model, with curvature correction, is utilized.

The $k - \omega$ SST turbulence method blends together two of the most prominent turbulence methods: $k - \omega$ and $k - \epsilon$. Blending these robust models accentuates the benefit of both Reynolds Averaged Navier Stokes (RANS) turbulence models. Specifically the $k - \omega$ SST turbulence model is relatively insensitive to the freestream/initial turbulence conditions, a vestige of the $k - \epsilon$ model, and can be solved well into the viscous sub-layer without damping which is important for shear stress prediction, the prominent feature of the $k - \omega$ model. Furthermore, the $k - \omega$ SST model is "well-behaved" in the presence of strong adverse pressure gradients like such experienced in separating flow. Since the present application deals heavily with flow separation and adverse pressure gradients, the $k - \omega$ SST RANS closure model is selected for turbulence modeling. There are, however, corrections to the model which enable improved prediction of desired quantities on CC-enabled airfoils as shown by Swanson and Rumsey [24]. The curvature correction term was added to the $k - \omega$ SST in fluent.

While the $k - \omega$ SST model is relatively insensitive to the freestream turbulence values, the solution of key force coefficients may still be affected by the selection of the freestream quantities. To quantify the effect of turbulence boundary conditions an examination using maximum deflection and $C_\mu = 0$ of the effect of the freestream turbulence intensity and turbulent viscosity ratio is conducted and compared to the
default values in Fluent. Figures 3.18 and 3.19 delineates this effect. The resulting
lift-to-drag ratio coefficients differ by less than 0.2% compared to the default values
is depicted for a range of turbulence intensities and viscosity ratios. The resulting
difference between the lift-to-drag ratios was deemed to be small and, as a result,
the default values for turbulence intensity and viscosity ratios were left to the default
values in Fluent, 5% and 10 respectively.

![Graph showing the effect of turbulence intensity on lift-to-drag ratio](image)

**Figure 3.18:** Effect of the freestream turbulence intensity on the lift-to-drag ratio
compared to the default settings in Ansys Fluent with constant turbulent viscosity
ratio (10).

To accurately characterize turbulence near the airfoil wall using the $k - \omega$
turbulence model the first cell should optimally be placed within the viscous sublayer
which the viscous effects dominate. As a result of the $k - \omega$ SSTs implementation,
within this region the $k-\omega$ turbulence model is used to close the RANS equations. The
optimal selection of $y+$ within the $k - \omega$ model is $y+ \leq 1$. However, valid selections
of the $y+$ value in the $k - \omega$ SST turbulence model ranges from $0 < y+ \leq 300$. 

52
Figure 3.19: Effect of the freestream turbulence viscosity on the lift-to-drag ratio compared to the default settings in Ansys Fluent with constant turbulence intensity (5%).

Though, at the upper portion of this range $30 < y^+$ the $k - \epsilon$ model dominates and, as a result, the viscous sublayer is not well defined which impacts the prediction of the drag coefficient. To place at least one cell within the viscous sublayer $y^+ < 5$. The $y^+$ values from the resulting mesh are depicted in Figure 3.20. The resulting $y^+$ values are less than 1.5 and as a result they exist within the viscous sublayer of the boundary layer. Additionally, the trailing edge geometry, where additional deformation and blowing actuation occurs, has $y^+ < 0.5$ to compensate for the additional velocity near the trailing-edge wall.

The solution of the $k - \omega$ SST model in Fluent, much like the other solved quantities, utilizes an Algebraic Multigrid (AMG) solver. AMG methods accelerate the convergence of a solution by using a coarsened grid to effectively damp out high-frequency spurious oscillations in the solution which is then used to correct the finer,
baseline grid. The selection of the coarseness of grid in sub-iterations is nominally dubbed "cycle" and the shape of this cycle is denoted by a letter corresponding to its shape (e.g. V-cycle). The W-cycle, while slower than the simpler V-cycle, results in faster convergence but is more computationally expensive. Due to the computational intensity, Fluent does not offer the W-cycle for use with the Coupled solver type. Instead, the F-cycle is used which acts as an intermediary point between the W and V cycles, specifically it is computationally cheap compared to the W-cycle, and results in expedited convergence when compared to the V-cycle. The transport equations relating to the $k-\omega$ SST models utilized an F-cycle with the incomplete lower upper smoother for this reason.

For the remaining freestream conditions, the Reynolds number (300,000) and a constant chord (1 meter) is used to calculate a freestream velocity of $4.381 \frac{m}{s}$. As such, the flow is entirely incompressible and the default settings for the material
properties of air are introduced: the density of air remains constant at $1.225\,\text{[kg\,m}^{-3}]$ and the viscosity is constant at $1.789 \times 10^{-5}\,\text{[kg\,m}^{-1}\text{\,s}^{-1}]$.

The boundary conditions of the computational domains are velocity inlet, wall, and pressure outlet. The types and locations of the boundary conditions are depicted in Figure 3.21. The main velocity inlet conditions, responsible for freestream external flow and depicted in Figure 3.21a, is modeled as a constant velocity with an orientation determined by the AoA of the flow. The velocity inlet corresponding to the injection slot, depicted in Figure 3.21b, spans the entire length of the slot and is determined by Equation (3.20). The pressure outlet condition uses the default conditions provided within Fluent with a normal boundary back-flow specification.

![Graphs](image)

(a) Boundary condition specification of the majority of the computational domain. (b) Boundary condition specification near the trailing edge of the airfoil.

**Figure 3.21**: Boundary conditions of the simulated domain. Red) Velocity inlet. Blue) Pressure outlet. Black) no-slip Wall.
\[ V_{jet} = \sqrt{C_\mu \frac{V_\infty^2 c}{2h}} \]  

The spatial and turbulence discretization methods are all second order within the coupled solver. Additionally, to improve convergence time, the pseudo-time option is enabled with a conservative scaling factor of 0.1. The resulting simulation is allowed to continue for 1500 solver-iterations or until the convergence criteria is reached. The number of iterations was carefully selected to allow adequate time for convergence while maintaining reasonable simulation times which is important for optimization speed. This allows for the potential of un-converged simulations, however during investigations on the results (Chapter 4) the number of simulations which were un-converged was less than 0.5%.

The convergence criteria of the solver was originally the specification of minimum force change. However, this produced results which, when compared to results simulated to residuals less than \(1 \times 10^{-6}\), had a difference of up to 15%. As such, while the solver time is greatly increased, it is required to only consider a solution converged if the residuals are less than \(1 \times 10^{-6}\). The simulation time to complete each geometry tends to increase with increased blowing.

The simulation reports the vertical and horizontal force, corresponding to lift and drag at an AoA of 0°. Additionally, the solver reports the summation of the continuity residual which is the residual which typically satisfies the convergence criteria last. This is intended to discard not converged data, monitor convergence issues, and ensure the data is valid. Typically, the sum of the continuity residual is
two orders larger than the residual monitor indicates (i.e. it is less than $1 \times 10^{-4}$). Furthermore, the simulation reports the minimum coefficient of friction ($C_f$) achieved on the flaps upper surface as well as the horizontal location of this minimum $C_f$. The combination of these two reports results in an approximate location of flow separation. While, theoretically, the $C_f$ at separation should be zero, the numerical dissipation and mesh discretization may not allow this to occur. To estimate the location of flow separation, the minimum value and location of $C_f$ is utilized instead. The flow is considered separated if it occurs before 98% of the airfoils $x/c$ coordinate and if the minimum $C_f \leq 0.005$. Both of the numerical conditions of flow separation are somewhat arbitrary. However, several cases were inspected visually to identify suitable maximum $C_f$ and separation position values.

3.5.4 Enumeration of Optimization Cases

The framing of the maximization of the lift-to-drag ratio, with respect to $C_\mu$, as a set of optimizations which occur at a fixed $C_\mu$, necessarily requires the specification of the $C_\mu$ values tested. Additionally, to compare just the effects of the multi-AFC airfoil on the lift-to-drag ratio, several parameters must be held constant. Specifically, the freestream flow velocity, the chord-length, and slot height remain fixed across all tested simulation cases. However, during preliminary investigations and validation of the framework, the chord changes according to the geometric parameters, which affects the Reynolds number as well. On the other hand, the AoA, $C_\mu$, and the parameters of the trailing edge geometry vary for each optimization case. Table 3.3 delineates the constant parameters while Table 3.4 delineates the optimization cases
Table 3.3: Simulation parameters which are held constant throughout the optimization process.

<table>
<thead>
<tr>
<th>Reynolds Number</th>
<th>Chord [m]</th>
<th>Freestream Velocity [m/s]</th>
<th>Slot Height [h/c]</th>
</tr>
</thead>
<tbody>
<tr>
<td>300,000</td>
<td>1</td>
<td>4.381</td>
<td>0.4%</td>
</tr>
</tbody>
</table>

Table 3.4: Description of the optimization cases and their variables.

<table>
<thead>
<tr>
<th>Case #</th>
<th>$C_\mu$</th>
<th>AoA [°]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>0.05</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0.05</td>
<td>7</td>
</tr>
<tr>
<td>5</td>
<td>0.1</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>0.1</td>
<td>7</td>
</tr>
<tr>
<td>7</td>
<td>0.15</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>0.15</td>
<td>7</td>
</tr>
<tr>
<td>9</td>
<td>0.2</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>0.2</td>
<td>7</td>
</tr>
</tbody>
</table>

for this analysis. Within each case the solver computes the lift, drag, and separation conditions for 200 airfoils during the optimization, this process takes approximately 2 days to complete.
CHAPTER 4

RESULTS AND DISCUSSION

This chapter discusses the validation of the framework, the results of the geometric optimization, the efficiency of single blowing optimized wings at multiple blowing coefficients, and estimates the improvement in the effectiveness of blowing caused by deformation. First, a validation of the framework and initial selections of the parameterization and optimization methods is undertaken through the use of a test case. Second, an analysis of each optimization for two angles of attack at five blowing conditions is considered. Specific attention is given to the lower blowing coefficients (0 and 0.05); while, the higher blowing coefficients will not be as thoroughly discussed as the trailing edge geometries do not vary significantly. Third, an analysis is conducted on the sweep of optimized airfoils in blowing coefficients other than their optimal. Finally, an analysis of the relative derivatives between the continuously deforming trailing edge and a fixed convention-like wing is investigated.

4.1 Framework Validation using Genetic Algorithms and dModCST

A test case was devised to evaluate the initial framework component selection GA optimization and dModCST parameterization as well as the communication
method used in the framework. This test case aimed to optimize the lift coefficient of a baseline NACA 0012 with a deforming trialing edge. This section reviews the initialization of GA through the use of a framework modified for Xfoil optimization. Then, it discusses the results of the optimization and improvements needed to the dModCST and GA algorithms which are implemented in the analysis for optimization of the lift-to-drag ratio. A full encapsulation of the method and results is given by Schweikert et al. in [1].

4.1.1 Genetic Algorithm Initialization-Xfoil

Since GAs take a significant number of simulation executions to converge, a method was developed to utilize a lower-order aerodynamic model, Xfoil, to obtain an approximate region of the global optima. This region was then used to set the initial population of the GA for higher-fidelity, CFD optimization. To accomplish this optimization, the framework’s communication method was altered to execute Xfoil simulation cases and collect data regarding the resultant aerodynamic coefficients. This framework, Python2Xfoil, utilized a variant of the GA method, namely Multi-Island Genetic Algorithm (MIGA) [68], for optimization.

MIGA uses a method similar to GA for optimization [72, 73]. It differs, however, in how the randomized design points are selected. Specifically, individuals are grouped into domain spaces, islands, which are analogous to disconnected island chains in evolution. Within each island the evolutionary algorithm is applied to generate the local optima within each island. The islands are then intermingled and then re-distributed into new clusters of islands and the process repeats. The inter-
change of individuals within islands improves the diversity, which is linked to global
optima finding, of the optimization algorithm. MIGA, as a result, performs signifi-
cantly better at identifying global optima within the design space than the traditional
GA. Due to the low cost of simulation in Xfoil, MIGA was used to approximate the
region of interest (i.e. the approximate location of the global optima). Xfoil does
have some limitations as to the geometry which it can simulate, unlike CFD. The
geometry used for optimization in Xfoil was parameterized using the MCST method.
The MCST method does have some significant drawbacks, as discussed in Section
3.2.2, but the effect of these drawbacks had not been realized until the end of the
verification analysis presented here.

4.1.2 Xfoil Optimizationcoefficient

The Python2Xfoil framework simulated approximately 4,300 disparate airfoils.
The best five performing airfoils, in terms of lift coefficient, from each population
were tabulated and provided in Figure 4.1. Each of the parameters depict bands of
multiple individuals being investigated at approximately the same design parameters
but with increasing fitness. These bands are a product of the MIGA methods high
crossover rate. The first coefficient ($C_1$) is quite large indicating that initial deflection
of the trailing edge is profitable to improve the lift coefficient. The second and third
coefficients are approximately equivalent and indicate, due to their lower value, that
a less aggressive deflection is more profitable. The bands in the power parameters
($P_1$) are much more sporadic. For $P_1$, the band with the best fitness value exists at
$P_1 \approx 5.5$ which indicates a relatively low curvature, or early deflection, is beneficial.
Figure 4.1: Analysis of the coefficients and powers of the deformation polynomial in Xfoil optimization [1].

Similar bands exist for $P_2$ and $P_3$ but the relatively low value of these powers indicate that extreme control over the trailing edge is not beneficial. Of all of the generated results, the 200 best geometries are used to generate the initial population in Ansys to reduce the search area of the GA method. Assuming a normal distribution, the mean and standard deviation of these geometries is taken and used to produce the initial population.

4.1.3 CFD Optimization

Optimization in Ansys Fluent, with Xfoil generated initial population distribution, simulated 200 airfoil cases. The resulting $C_l$ optimized airfoil trailing-edge geometries are provided in Figure 4.2. The optimized trailing edge deflections and
deformations are reasonable. Unfortunately, due to the limitations of the MCST method as discussed in Section 3.2.2, the baseline airfoil between $0.6 < x/c < 0.8$ is deformed. Additionally, the fitness values are still increasing after the GA iteration limit was met suggesting the provided airfoils are not local optima.

![Graphs showing airfoil deformations](image)

(a) $C_\mu = 0$

(b) $C_\mu = 0.10$

**Figure 4.2**: Trailing edge geometries with the largest $C_I$ produced by the optimization at $AoA = 5^\circ$ and $Re = 300,000$. a) Optimized with $C_\mu = 0$. b) Optimized with $C_\mu = 0.10$.

### 4.1.4 Review of Validation Case

An investigation of the framework components and communication method was undertaken through the use of a validation case. The GA optimization method was found not to efficiently use solver iterations and requires significant computation time for optimization. That is, the number of simulations required to optimize using GA is too large; thus, feasible optimization of multiple cases is not recommended. Bayesian optimization, which approaches optimization in a more data-driven way, will be used in the rest of this thesis. Furthermore, the parameterization method (dMod-
CST) was found to have significant drawbacks regarding baseline airfoil deformation. An adjustment to this method, specifically dMCST and the more refined CLdMCST, solve these issues. As such, CLdMCST will be used for the rest of this thesis. Finally, the communication method performed as expected in both Ansys and Xfoil simulations, validating its use and the use of the framework for Ansys optimization.

4.2 Geometric Optimization at Fixed Blowing Coefficients

Bayesian optimization was applied to a ClMCST parameterized, deformable NACA 0015 airfoil at five blowing coefficients and two angles of attack. The optimization protocol augmented the ClMCST morphing parameters utilizing the framework to simulate, in Ansys Fluent, 200 disparate airfoils for each case. The three best performing airfoil geometries from each case are provided to ensure good convergence between the cases. The generated optimal parameters are then tabulated along with the drag coefficient and lift to drag ratio. For comparison purposes, the deflection of the airfoil is compared to a conventional trailing edge where the deflection is defined by a trailing edge deflection angle ($\delta$). The resulting deflection angle is calculated by Equation (4.1) and compares the result to a conventional airfoil deflected to the same $y/c$ coordinate.

$$\delta = \sin^{-1}\left(\frac{\sum C_{o_i}}{0.2}\right)$$  \hspace{1cm} (4.1)

64
4.2.1 No Blowing Case \((C_\mu = 0)\)

For optimization which occurred with a blowing coefficient of zero, the three best geometries are depicted in Figure 4.3. Figure 4.3a outlines the optimized geometry at \(\text{AoA} = 0^\circ\). The three best performing airfoil deflections differ less than 0.4% of the airfoils chord. Additionally, the curvature components are visually similar on the airfoils. The combination of the similarity in deflection indicates good convergence in \(\sum C_{oi}\), and the visual similarity in the airfoil trailing edge indicates good convergence within the \(P_{oi}\) and their association with \(C_{oi}\). A similar argument can be made for the convergence of the optimization shown in Figure 4.3b. However, the relative deflections between the two geometries are dissimilar. The optimized geometry for the lower \(\text{AoA} (\text{AoA})\) has a deflection of approximately 5% of the chord which corresponds to a conventional deflection angle of 13°. Meanwhile, the optimized geometry for \(\text{AoA} = 7^\circ\) has a deflection of 2% of the chord corresponding to a conventional deflection of 6.5°. This is expected as the flow around the \(\text{AoA} = 0^\circ\) trailing edge has significantly less drag penalties associated with deflection than the \(\text{AoA} = 7^\circ\) case. Additionally, for the \(\text{AoA} = 7^\circ\) case, the airfoil is near the optimal condition for lift over drag, as predicted by Xfoil. As such, the small deflection in the trailing edge corresponds to a small increase in camber and effective \(\text{AoA}\) potentially indicating the actual optimal \(\text{AoA}\) is slightly above 7°. Furthermore, the slight deflection in the trailing edge, likely does not impose significant pressure drag as the increase in pressure drag is not substantially increased. For lift, however, the significant increase in curvature near the trailing edge slightly increases circulation generating a larger
lift coefficient. The combination of these factors result in a larger lift-to-drag ratio than the baseline airfoil.

(a) Case 1 \((\text{AoA} = 0^\circ, C_\mu = 0)\).  
(b) Case 2 \((\text{AoA} = 7^\circ, C_\mu = 0)\).

**Figure 4.3:** Representation of the trailing edge deformation of the three best performing airfoils for the optimized geometry of cases 1 and 2.

Table 4.1 defines the precise CLdMCST parameters generated from the optimization. Since the \(Po\) parameters define the relative position of the deflection, which is defined by \(Co\), the order of the parameters is unimportant. Instead, to retain the same geometry, the relationship between \(Co_i\) and \(Po_i\) must be preserved. For the low AoA case, \(Co_1 = 0\) which eliminates the effect of \(Po_1\). Meanwhile, \(Po_2\) is saturated to its maximum value (5) which forces the deformation to largely be relegated to the trailing edge. This high \(Po\) associated with the relatively large \(Co\) indicates a significant occurs more aft. \(Po_3\) is not saturated and thus generates a deflection farther forward on the trailing edge. However, for the \(\text{AoA} = 7^\circ\) case, the \(Po_2\) and \(Po_3\) parameters are identical; so, the contributions of each \(Po\) can be neglected. The result indicates that deflection of the trailing edge need only be defined by one set
of parameters with values $C_{o1} = 0.0227$ and $Po_1 = 5$. Since $Po_1$ value is large the
deflection mainly occurs along the trailing edge, as depicted by Figure 4.3b. The
$C_d$ expectedly increases with the increase in AoA and the performance of the NACA
0015 at AoA = 7° results in a larger $\frac{C_l}{C_d}$ at AoA = 7°.

Table 4.1: Resulting parameters and aerodynamic coefficients of the optimization
for Case 1 and 2 ($C_\mu = 0$).

<table>
<thead>
<tr>
<th>AoA[°]</th>
<th>$C_{o1}$</th>
<th>$C_{o2}$</th>
<th>$C_{o3}$</th>
<th>$Po_1$</th>
<th>$Po_2$</th>
<th>$Po_3$</th>
<th>$C_d$</th>
<th>$\frac{C_l}{C_d}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0.0267</td>
<td>0.0206</td>
<td>2.1542</td>
<td>5.0000</td>
<td>1.3373</td>
<td>0.0206</td>
<td>34.0399</td>
</tr>
<tr>
<td>7</td>
<td>0</td>
<td>0.0131</td>
<td>0.0096</td>
<td>3.5442</td>
<td>5.0000</td>
<td>5.0000</td>
<td>0.0246</td>
<td>44.2258</td>
</tr>
</tbody>
</table>

4.2.2 Low Blowing Case ($C_\mu = 0.05$)

The optimization with a blowing coefficient $C_\mu = 0.05$ the resulting deflection
is substantially larger than the $C_\mu = 0$ case. Figures 4.4a and 4.4b depict the three
best performing trailing edges for AoA = 0° and AoA = 7°, respectively. Similar to
the $C_\mu = 0$ case, the curvature and deflection of the airfoils are very similar, with
the deflection varying less than 0.05% of the chord. The conventional deflection of
the AoA = 0° airfoil is 23°. Furthermore, the far trailing edge ($0.90 < x/c < 1$)
is largely symmetrical suggesting that early deformation is more profitable than the
delayed trailing edge deformation generated in the $C_\mu = 0$ case. This is likely a
result of the increased circulation provided by the blowing as well as the increase in
momentum reattaching the flow and delaying separation. This delay in separation
further decreases pressure drag acting on the trailing edge resulting both in an increase
in lift from reattachment and improved circulation, and a decrease in drag. The
increase in lift and decrease in drag improves the lift-to-drag ratio allowing the $AoA = 0^\circ$ to perform better than the $AoA = 7^\circ$ case in the lift-to-drag ratio. That is, the lift to drag ratio generated by the $AoA = 0^\circ$ case is 7% greater than the $AoA = 7^\circ$ case. The $AoA = 7^\circ$ case has significantly less deflection than the low $AoA$ case resulting in a conventional deflection of $12.5^\circ$. Additionally, it seems that most of the curvature is more aft on the trailing edge than the $AoA = 0^\circ$ case.

![Graphs showing deflection](image)

(a) Case 3 ($AoA = 0^\circ$, $C_{\mu} = 0.05$). (b) Case 4 ($AoA = 7^\circ$, $C_{\mu} = 0.05$).

**Figure 4.4:** Representation of the trailing edge deformation of the three best performing airfoils for the optimized geometry of Cases 3 and 4.

Table 4.2 provides the exact deflection parameters for the $C_{\mu} = 0.05$ case at both angles of attack. For the row corresponding to $AoA = 0^\circ$, $Po_1$ is approximately equivalent to $Po_2$ indicating that the resultant deflection may only need to be defined by two sets of parameters. Perhaps, since $Po_2$ is close to the other power parameters, it may be possible to adequately represent the entire deflection as one set of deflection with $Po_1 = 2.114$ and $Co_1 = 0.0777$. This set of parameters positions the deflection far forward on the trailing edge leaving the far trailing edge ($0.95 < x/c < 1$) largely
Table 4.2: Resulting parameters and aerodynamic coefficients of the optimization for Case 3 and 4 ($C_\mu = 0.05$).

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$C_{02}$</th>
<th>$C_{03}$</th>
<th>$P_{01}$</th>
<th>$P_{02}$</th>
<th>$P_{03}$</th>
<th>$C_d$</th>
<th>$C_l/C_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.0388</td>
<td>0.0337</td>
<td>0.0052</td>
<td>2.0894</td>
<td>2.1514</td>
<td>2.0617</td>
<td>0.0333</td>
</tr>
<tr>
<td>7</td>
<td>0</td>
<td>0.0096</td>
<td>0.0338</td>
<td>5.0000</td>
<td>2.5010</td>
<td>2.4621</td>
<td>0.0387</td>
</tr>
</tbody>
</table>

untouched by the deformative effect. Similarly, the deflection of the $\alpha_0 = 7^\circ$ case may also be represented by one set of parameters as $P_{02} \approx P_{03}$ and $C_{03} = 0$ negating the effect of $P_{01}$. The sum of the coefficient parameters indicates a substantially smaller deformation than the $\alpha_0 = 0^\circ$ case and the $P_o$ component is larger indicating the deflection takes place farther down the trailing edge. The $\frac{C_l}{C_d}$ of the $\alpha_0 = 0^\circ$ case is substantially larger than the $\alpha_0 = 7^\circ$ case.

4.2.3 High Blowing Cases ($C_\mu = 0.1, 0.15, 0.2$)

The six cases with high blowing ($C_\mu = 0.1, 0.15, 0.2$) are largely similar to the $C_\mu = 0.05$ cases among all AoA. All six cases presented in Figure 4.5 show good convergence with minimal variation in the final deflection value and good agreement in curvature. The comparative conventional deflection angles for the $\alpha_0 = 0^\circ$ cases are 25.3°, 25.8°, and 26.6° for blowing values of 0.1, 0.15, and 0.2, respectively. For the $\alpha_0 = 7^\circ$ cases, the deflection angles are 13.9°, 15.8°, and 17.2° for blowing values of 0.1, 0.15, and 0.2, respectively. The deflection is substantially larger for the $\alpha_0 = 0^\circ$ set of cases rather than the $\alpha_0 = 7^\circ$ set for reasons discussed in Section 4.2.2.
Figure 4.5: Trailing edge deformation of the best performing airfoils with higher blowing coefficients.
Table 4.3: Resulting parameters and aerodynamic coefficients of the optimization for cases 5-10.

<table>
<thead>
<tr>
<th>$C_\mu$</th>
<th>$\alpha[\degree]$</th>
<th>$C_{O1}$</th>
<th>$C_{O2}$</th>
<th>$C_{O3}$</th>
<th>$P_{O1}$</th>
<th>$P_{O2}$</th>
<th>$P_{O3}$</th>
<th>$C_d$</th>
<th>$C_l/C_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.10</td>
<td>0</td>
<td>0.0335</td>
<td>0.0254</td>
<td>0.0234</td>
<td>1.8449</td>
<td>1.9094</td>
<td>2.4252</td>
<td>0.0432</td>
<td>50.5453</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>0</td>
<td>0.0424</td>
<td>0.0058</td>
<td>3.5851</td>
<td>2.2775</td>
<td>2.1775</td>
<td>0.0485</td>
<td>46.7323</td>
</tr>
<tr>
<td>0.15</td>
<td>0</td>
<td>0.043</td>
<td>0.0441</td>
<td>0</td>
<td>1.8624</td>
<td>1.9259</td>
<td>2.1979</td>
<td>0.0520</td>
<td>47.8070</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>0</td>
<td>0.0342</td>
<td>0.0202</td>
<td>1</td>
<td>1.9659</td>
<td>2.0246</td>
<td>0.0572</td>
<td>44.5596</td>
</tr>
<tr>
<td>0.20</td>
<td>0</td>
<td>0.045</td>
<td>0.0445</td>
<td>0</td>
<td>1.72</td>
<td>1.7889</td>
<td>3.0734</td>
<td>0.0586</td>
<td>45.1828</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>0</td>
<td>0.0466</td>
<td>0.0126</td>
<td>1.3933</td>
<td>1.76</td>
<td>1.8763</td>
<td>0.0647</td>
<td>42.4948</td>
</tr>
</tbody>
</table>

Table 4.3 delineates the trailing edge deformative parameters for the higher blowing coefficients as well as their respective AoA and aerodynamic coefficients. Focusing first on the $\alpha = 7\degree$ cases, there is a notable aversion for forward trailing edge deflection as all of the corresponding $C_{O1}$ parameters are near zero. Furthermore, where $C_{O2}$ and $C_{O3}$ are nonzero, the resulting powers are also quite similar indicating that the deformation can be constructed from one set of values. However, the relative position of the deflection, that is the magnitude of $P_O$, moves farther forward on the trailing edge. The decrease in $P_O$ also provides a seeming linearity; that is, it begins to resemble a conventional trailing edge. This potentially suggests that a nearly conventional trailing edge is optimal at very large blowing coefficients. However, the drag substantially increases with the increase in blowing and thus decreases the lift-to-drag ratio. Therefore, the resulting nearly conventional trailing edge may not be worthwhile to pursue for lift-to-drag optimization. For the cases with $\alpha = 0\degree$ a similar relationship can be seen. The optimized deformation parameters are reducible to a single set with the exception of the $C_\mu = 0.1$ case.
4.3 Comparison of Optimized Airfoils

Figure 4.6 depicts all of the optimized geometries as well as the location of minimum $C_f$. Figure 4.6a delineated the optimized geometries for cases 1, 3, 5, 7, and 9, the odd cases; whereas, Figure 4.6b depicts the optimized geometry for cases 2, 4, 6, 8, and 10, the even cases. For the odd cases, there is a surprising similarity between the trailing edges optimized with blowing, it seems the optimization simply increased the deflection of each case, but did not substantially change the shape of the trailing edge. Furthermore, assuming there is a continuous shape deformation which reaches all of the optimized geometries, there is a significant disparity between the shapes optimized with blowing and without blowing. The significant disparity in the shapes indicates critically missing phenomena and perhaps a more optimal lift-to-drag ratio. The separation location of the trailing edge exhibits a similar incongruency. The flow separates early from the non-blown trailing edge at approximately 92.6% of the chord, but remains attached due to the momentum injection from the lowest blowing coefficient.

For the even cases, Figure 4.6b, the result is somewhat more varied. First, the trailing edge deflections are much more similar throughout the cases which is likely due to an increase in drag caused by the AoA limiting the amount of profitable deflection. Second, the separation location now exists at 94.3% of the chord as the low blowing was insufficient to reattach the flow due to a more significant pressure gradient needed to be overcome. Though, there is still a geometric disparity between the non-blowing
and blowing cases suggesting that, like the odd cases, critical phenomena may be missing.

![Graphs showing geometric comparison of trailing edge deformation and approximated separation points for all optimized cases at two angles of attack.](image)

(a) $\alpha = 0^\circ$.  
(b) $\alpha = 7^\circ$.

Figure 4.6: Geometric comparison of trailing edge deformation and approximated separation points for all optimized cases at two angles of attack.

The lift-to-drag ratio of each of the cases is depicted in Figure 4.7. While there are not enough cases at differing angles of attack to provide a trend, the even cases show a marked decrease in lift-to-drag ratio when compared to the odd cases with blowing. The difference between the lift-to-drag ratio curves is explained by the difference in the flow field. First, the lower surface of the trailing edge experiences a more significant fraction of velocity head-on; that is, the frontal area contribution is increased. The pressure of the trailing edge is more significant in the even cases than the odd cases. Concurrently, the upper surface of the trailing edge has significantly less pressure due to flow separation caused by the discontinuity which increases drag. The combinations of these effects limit the amount of profitable deflection particularly in the non-blowing case where the flow has no method for reattachment. In the
blowing cases, the actuation must do much more work to reattach the flow; thus, enabling the earlier separation with deflection. However, because the baseline NACA 0015 performs much better at the higher AoA, the lift-to-drag ratio of the non-blowing AoA = 7° case performs better than the AoA = 0° case. The resulting curve shows an initially larger lift-to-drag ratio for the AoA = 7° cases, but the effect of blowing is substantially reduced compared to the AoA = 0° cases.

![Lift-to-Drag Ratio Comparison](image)

**Figure 4.7:** Lift-to-Drag ratio comparison of the optimized airfoils for AoA = 0° and AoA = 7°.
4.4 Comparison of Performance of Optimized Trailing Edges over Various Blowing Conditions

Now that the optimal geometry for each blowing coefficient has been identified, a comparison can be made regarding the benefit of a wing capable of dynamically deforming its trailing edge to generate the optimal geometry. First, a non-deforming wing which is optimized to a specific blowing condition is investigated. That is, each of the optimized case geometries are evaluated over all of the blowing coefficients with no change to the geometry. Finally, the geometry is compared to a conventional-like wing with and without deflection. Figures 4.8-4.10 depict the lift, drag, and lift-to-drag ratios, respectively, of each case over the tested $C_\mu$ values.

The lift coefficients for the cases pertaining to $AoA = 0^\circ$ are given in Figure 4.8a and the coefficients pertaining to the $AoA = 7^\circ$ are given in Figure 4.8b. Primarily, the lift coefficient serves to further validate the results obtained from the optimization. For the cases with $AoA = 0^\circ$, the lift coefficient increases with blowing and deformation. The lift coefficient of the two highest blowing optimized cases, 0.15 and 0.2, are nearly identical over all $C_\mu$ values. Furthermore, the lift coefficient of all blowing optimized cases are nearly identical at no blowing; though, this is likely due to the similarity in the trailing edge profiles. The differences in trailing edge profiles are only pronounced through blowing. For cases pertaining to the $AoA = 7^\circ$ (Figure 4.8b), the non-blowing coefficient is substantially higher than the $AoA = 0^\circ$ cases. Notably, the lift coefficients of case 9 and 10 at the high $C_\mu$ are nearly identical. The effect of blowing, however, is much more pronounced in the lower AoA with the
geometry optimized for the lower blowing coefficients (i.e. the ΔC_l between cases 1 and 3 is higher than between cases 2 and 4). This pronounced effect in C_l indicates a higher efficacy in blowing on the lower AoA than at the higher AoA.

Figure 4.8: Coefficient of lift of the optimized fixed deformed airfoils over various blowing coefficients with different angles of attack.

Figure 4.9a depicts the drag coefficients for the cases pertaining to AoA = 0°. There is a stark decrease in the drag coefficient for the AoA = 0° to flow reattachment. However, the curvature of the case 1 geometry is too intense at $\frac{x}{c} = 0.95$ to enable full reattachment; so, there is still a wake region behind the trailing edge which increases pressure drag. The starkest decrease in drag occurs with the most deflected trailing edge which has a similar enough curvature to allow most of the flow to reattach which significantly decreases drag. The cases with a AoA = 7°, depicted in Figure 4.9b are significantly less interesting. The drag coefficient increases linearly with the blowing coefficient. However, the flow reattachment caused by blowing reduces the $\frac{\Delta C_l}{C_l_{max}}$ for the low blowing cases.

76
Figure 4.9. Coefficient of drag of the optimized fixed deformed airfoils over various blowing coefficients with different angles of attack.

Figure 4.9a depicts the lift-to-drag ratio for the cases with \( \text{AoA} = 0^\circ \). The lift-to-drag of each of the optimized cases at their optimized \( C_\mu \) is, in fact, the highest ratio observed at that \( C_\mu \). The difference in lift-to-drag ratio of the optimized cases is relatively small with high blowing. If a deforming trailing edge was generated to deform to each of the optimized geometries, most of the benefit would be derived from a two mode deflection: a mode for the non-blowing case, and a mode for the low blowing case. These two modes would generate most of the benefit for a fully deformable trailing edge. Constraining a fully deformable trailing edge in this way would significantly reduce complexity.

The cases with \( \text{AoA} = 7^\circ \) are depicted in Figure 4.10b. The span of the lift-to-drag ratio curves is significantly smaller in the high AoA cases than in the low AoA cases. This would be expected if the span was just limited by the no-blowing lift-to-drag ratio, but the maximum lift-to-drag ratio in the \( \text{AoA} = 7^\circ \) is lower than
in the $AoA = 0^\circ$. The reason for this disparity was discussed in Section 4.2.2. The two modes proposed in the discussion of the $AoA = 0^\circ$ case is still applicable to the $AoA = 7^\circ$ as the no blowing and low blowing optimized geometries provide most of the benefit throughout each of the blowing regimes.

![Graph](image)

(a) $AoA = 0^\circ$.  
(b) $AoA = 7^\circ$.

**Figure 4.10:** Lift-to-drag ratio of the optimized fixed deformed airfoils over various blowing coefficients with different angles of attack.

### 4.5 Performance Comparison

To aid in the comparison of the total deforming geometry, which can smoothly change to the optimal geometry for the applied blowing, two other geometries were constructed. The first geometry was a conventional trailing edge deflected at $26.9^\circ$. The second geometry was a continuously deflecting, conventional trailing edge, which deflected to the representative angles of the optimized geometry. This second geometry was utilized to compare the effect of curvature on the total deforming geometry.
Figure 4.11 depicts the analysis of these three geometries with $\text{AoA} = 0^\circ$ (Figure 4.11a) and $\text{AoA} = 7^\circ$ (Figure 4.11b), respectively.

For $\text{AoA} = 0^\circ$ (Figure 4.11a) the deflecting trailing edge performed worse than the fixed deflection trailing edge over all, except at $C_\mu = 0$ where the lift-to-drag ratio was slightly better. While the fixed trailing edge performed better than the deflecting trailing edge, it was still significantly outperformed by the deforming trailing edge geometry. The deformation increased the lift-to-drag ratio by between 30.5\%-96.9\% when compared to the fixed geometry. For $\text{AoA} = 7^\circ$, Figure 4.11b, the results are less impressive. The deflecting geometry outperformed the fixed geometry in all blowing cases, but most significantly at $C_\mu = 0$ where it improved the lift-to-drag ratio by 10.0\%. However, similarly to $\text{AoA} = 0^\circ$, the deforming trailing edge outperformed the deflecting trailing edge. In this case the performance increase was less substantial. The increase in lift-to-drag by continuous contour deformation was between 6.3\%-22.6\% when compared to the deflecting trailing edge.

The surface of lift-to-drag for a particular AoA, in this analysis E, is defined by the morphing ($C_\delta$) and blowing ($C_\mu$) parameters. If E was known analytically, small perturbations in $C_\delta$ and $C_\mu$ would define the effect on E by Equation (4.2). Unfortunately, E is not know analytically. Instead, data have been collected on the surface of E. Specifically, each of the depicted lines in Figures 4.9a-4.11a, must lie on the surface of E because they evaluate the lift-to-drag ratio of the airfoil at a specific $C_\delta$ and $C_\mu$. For instance, the fixed case in Figure 4.11a, must lie on a line with constant $C_\delta = C_\delta_1$ projected onto the surface of E. Define the projected line as $E = h(C_\mu, C_\delta)$. Then, because there is no change in $C_\delta$, the difference in E must only
Figure 4.11: Comparison of the optimal deforming trailing edge over all blowing coefficients compared to a conventional-like fixed deflection of 26.8° and a deflecting trailing edge with a deflection similar to the deforming trailing edge with various angles of attack.

be due to the effect of $C_\mu$. Similarly, case 2 has constant $C_\delta = C_\delta_2$ then difference in $E$ must only be due to the effect of $C_\mu$. Define this projected line as $E = g(C_\mu, C_\delta)$. However, case 2 intersects the optimum geometry at $C_\mu = 0.05$, while the fixed case must not. Define the contour which describes the morphing airfoil as $E = f(C_\mu, C_\delta)$.

Using this information, an approximation of the derivatives between the fixed and deforming geometries in $C_\delta$ and $C_\mu$ can be obtained. To aid in this approximation, the values in Equation (4.2) will be discretized according to Equation (4.3) where $\partial E$ was replaced by $\Delta E$ between the deforming and fixed geometries, respectively. Specifically, this $\Delta E$ is defined as the difference between the optimal, deformed geometry, and the fixed geometry at $C_\mu = 0.05$ at which the morphing geometry was optimized. Since case 2 has the same morphing parameter as the deforming geometry at the critical $C_\mu$, the effect of deformation can be calculated between case 2 and fixed
geometries at $C_{\mu} = 0$ and is given by Equation (4.4). Similarly, the effect of blowing can be calculated by Equation (4.5).

The evaluation of these difference parameters ($\Delta E_{C_{\delta}}$ and $\Delta E_{C_{\mu}}$) does not, however, provide much information about the relative performance of blowing or deformation on a trailing edge which is capable of both. To investigate this, it is important to define an efficiency parameter for both blowing and deformation relative to the conventional-like deflection. Specifically, the efficiency parameters, as defined in Equations (4.6) and (4.7), approximate the ratio of the derivatives of $E$ with respect to $C_{\delta}$ and $C_{\mu}$ between the deformative case and the conventional-like fixed case. Using a similar approximation as Equation (4.3), the efficiency parameters of morphing and blowing can be approximated by Equations (4.8) and (4.9).

$$\partial E = \frac{\partial E}{\partial C_{\delta}} dC_{\delta} + \frac{\partial E}{\partial C_{\mu}} dC_{\mu} \quad (4.2)$$

$$\Delta E_{g(0)\rightarrow f(0.05, C_{\delta 2})} = f(0.05, C_{\delta 2}) - g(0) = \Delta E_{C_{\delta}} + \Delta E_{C_{\mu}} \quad (4.3)$$

$$\Delta E_{C_{\delta}} = h(0) - g(0) \quad (4.4)$$

$$\Delta E_{C_{\mu}} = f(0.05, C_{\delta 2}) - h(0) \quad (4.5)$$
\[ \epsilon_{C_{t_2}} = \frac{\frac{\partial E}{\partial C_t} dC_t}{\frac{\partial E}{\partial C_\delta} dC_\delta} g(0) \rightarrow h(0) \] (4.6)

\[ \epsilon_{C_\mu} = \frac{\frac{\partial E}{\partial C_\mu} h(0) \rightarrow f(0.05, C_\delta)}{\frac{\partial E}{\partial C_\mu} C_\mu, g(0) \rightarrow g(0.05)} \] (4.7)

\[ \epsilon_{C_{b_2}} = \frac{h(0) - g(0)}{f(0, C_{b_2}) - g(0)} \] (4.8)

\[ \epsilon_{C_{\mu=0.05}} = \frac{f(0.05, C_{b_2}) - h(0)}{g(0.05) - g(0)} \] (4.9)

The resultant efficiency parameters, after substitution, are \( \epsilon_{C_{\mu=0.05}} \approx 1.2288 \) and \( \epsilon_{C_{b_2}} \approx 0.4739 \). This means that the change in E with respect to blowing on the deformed geometry is approximately 23% greater than the change on a conventional fixed deflection. Additionally, the change in E with respect to deformation is 53% less effective with blowing than without blowing. Both of these efficiency parameters are within expectation.
CHAPTER 5

CONCLUSION

This thesis presents research that investigates the impact of a deformable, CC airfoil on the lift-to-drag ratio, with an aim of investigating the use of morphing to reduce the momentum blowing coefficient ($C_\mu$). A framework was developed to carry out the optimization. This framework consisted of five components: a parameterization method, an optimization method, an objective function, a communication method, and a CFD solver.

The deformable airfoil was parameterized using the developed Chord Limited discontinuous Morphing Class Shape Transformation (CLdMCST) method. CLdMCST improves over other parameterization techniques due to its chord limiting capabilities which ensure a more feasible deformation. Furthermore, the CLdMCST method enabled the generation of conventional trailing edge geometries for comparative purposes. Additionally, an improvement was made to the dModCST method which enables further constraints in the development of a discontinuity.

The optimization method was largely chosen from literature as Bayesian Optimization due to its ability to efficiently utilize the time-costly data generated by CFD. The objective function used within the framework was simply the lift-to-drag
ratio, which the optimization maximized whilst controlling the deformative parameters. The optimization took place over two AoA ($\text{AoA} = 0^\circ, 7^\circ$) and five blowing coefficients ($C_\mu = 0, 0.05, 0.1, 0.15, 0.2$) to better understand the lift-to-drag design space.

A communication method, which enabled the connection between the user-supplied parameterization, optimization, and objective functions to the CFD solver was developed. The communication method was able to adequately control the Ansys Fluent CFD simulation through the use of Design Parameters. Additionally, this communication method is also capable of controlling analysis in any Ansys Simulation package (i.e. Mechanical for use in fluid-structure interactions). The selected Ansys Fluent CFD solver utilized Ansys’s meshing tool to develop a fully structured C-H type grid around the airfoil and completed a steady state simulation for over 2000 individual airfoils. From these data, the drag force, lift force, and separation location were calculated and used for analysis.

The resulting optimization showed good convergence in optimum trailing edge geometries in both shape and deflection. For cases with $\text{AoA} = 0^\circ$ the resulting deflection was significantly larger than the cases with $\text{AoA} = 7^\circ$ due larger drag increases with deflection on the $\text{AoA} = 7^\circ$ cases. Furthermore, while the applied CLdMCST method arbitrarily employed three sets of deformative parameters ($Co$ and $Po$) the optimized geometry with blowing actuation could largely be described with two sets of parameters, in the case of $\text{AoA} = 0^\circ$, or one set directly, in the cases with $\text{AoA} = 7^\circ$. However, the cases with no blowing actuation exhibited much more complex curvature and needed to be described by all three implemented sets.
The lift-to-drag ratio of the optimized geometries was significantly higher in the AoA = 0° with blowing actuation than in the cases with AoA = 7°. This improvement was due to a smaller adverse pressure gradient in the AoA = 0° cases enabling more complete flow reattachment in the blowing cases. However, in the non-blowing cases AoA = 0° was significantly outperformed by AoA = 7° due to the baseline, NACA 0015s, superior aerodynamic performance at this AoA.

A comparison was made between each of the optimized geometries over each of the tested blowing conditions. The analysis found that most of the benefit from morphing could be derived from morphing between the $C_\mu = 0$ and $C_\mu = 0.05$ geometries because the performance for the geometries optimized for higher blowing coefficients was not significantly different than the performance of the geometries optimized at $C_\mu = 0.05$. As expected, the blowing-optimized geometries performed significantly worse without blowing actuation compared to the geometries optimized for no blowing.

An estimation of the lift-to-drag effect of blowing on an optimally deformed geometry has also been discussed. This analysis used a forward-difference approximation for the derivative of the lift-to-drag ratio with respect to the blowing coefficient at 0. The analysis then compared this derivative to that of a conventional deflection. This analysis culminated in an approximation of the efficiency of blowing on a morphed geometry compared to a conventional deflection of 23%.

Future work on the implementation of morphing technology as a method of $C_\mu$ reduction should aim to produce a candidate flight system (i.e. a small scale UAV). To accomplish this task the CFD analyses must be validated by experimental
observations and adjusted accordingly. Subsequently, a morphing wing should be designed and the CLdMCST method should be adjusted to include the strain effects of deformation. Both the morphing wing design and the optimization of the lift-to-drag ratio should be conducted iteratively to produce the structural components which are able to obtain the optimal geometry with minimal energy expenditure. That is, the optimization should be undertaken again, with $0 \leq C_\mu \leq 0.1$ while including a notional morphing structure. Then the morphing structure should be augmented to produce the optimal geometry with minimal actuation pressure. This process should be repeated until the minimized actuation pressure is obtained. The resulting optimization would produce a surrogate model for the lift, drag, and pitching moment coefficients which can then be implemented in the control law. However, it is first necessary to characterize the flow field to capture the unsteady dynamics of morphing and possible transient separation during blowing. Additionally, the relationship between aerodynamic performance and the optimal-deformed trailing edge and blowing parameter can be investigated by conducting a flow-field analysis of the unsteady simulations. Armed with the surrogate model and unsteady effects a suitable control-law for morphing and blowing actuation can be constructed and implemented on a small scale UAV. The small scale UAV may optionally utilize a conventional trailing edge deflective surface to evaluate the use of morphing as a $C_\mu$ reduction technique characterized by a single metric: the energy expenditure of the UAV.
REFERENCES


[29] Ed Pendleton. How active aeroelastic wings are a return to aviation’s beginnings and a small step to future bird-like wings, 2010.


