Coronal magnetic field extrapolation and topological analysis of multi-scale magnetic structures for solar eruptions

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Solar eruptions are manifestations of strong solar activities which occur in multiple scales and vary significantly in observations. It has also been increasingly realized that some scenarios initially proposed for relatively large-scale eruptions may persist down to much smaller/finer scales, largely owing to increasingly high-resolution observations of the Sun. However, confirming the coherent magnetic structures on the Sun remains challenging due to the lack of direct coronal magnetic field measurement. This dissertation mainly focuses on the topological analysis of multi-scale magnetic structures embodied in different solar eruptions with available observations. It is achieved by reconstructing the 3D coronal magnetic field through one unique nonlinear force-free field (NLFFF) extrapolation method, the CESE-MHD-NLFFF code, modified and tested for nonuniform embedded magnetograms. The performance of the modified code is evaluated extensively through a series of test runs based on different input magnetograms and grid constructions. Two selected solar flare eruptions are investigated to make a connection between the magnetic flux ropes (MFRs) on the Sun and their interplanetary counterparts quantitatively. For event 1, a coherent
MFR before the flare eruption is identified combining the multi-wavelength observations and the NLFFF extrapolation results. The total magnetic reconnection flux during the eruption amounts to $\sim 10^{21}$ Mx, which is measured by analyzing the associated flare ribbons via remote-sensing observations. It is significantly larger than the flux in the identified pre-eruptive MFR ($10^{19} \sim 10^{20}$ Mx). For event 2, there is no pre-eruptive MFR found with the same criteria as event 1. In both events, the total magnetic reconnection flux (in the order of $\sim 10^{21}$ Mx) agrees with the corresponding magnetic flux contents of the MFRs after the eruptions from the in situ modeling results. To study the fine-scale magnetic structures, the modified CESE-MHD-NLFFF code is applied to a flare precursor event with nonuniform embedded magnetograms from the Goode Solar Telescope (GST) and the Solar Dynamics Observatory (SDO). By comparing the extrapolation results with the simultaneous SDO and high-resolution GST observations, the magnetic field lines originating from the precursor brightening regions show a more consistent configuration with observations. The resolved fine-scale magnetic structures exhibit low-lying sheared arcades characteristic of a plausible configuration for precursor magnetic reconnection before the main flare onset. In addition, a recurrent solar jet event is also studied by employing two sets of embedded magnetograms from GST and SDO. We find small closed loops surrounded by open field lines in two runs, which indicates a scenario of interchange reconnection leading to the occurrence of the jets. These quantitative investigations contribute to understanding the important role of magnetic reconnection in shaping the topological features seen in observations and shed light on the underlying physical mechanisms driving the solar eruptions.
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# List of Acronyms

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<th>ACRONYM</th>
<th>DEFINITION</th>
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<tbody>
<tr>
<td>ACE</td>
<td>Advanced Composition Explorer</td>
</tr>
<tr>
<td>AIA</td>
<td>Atmospheric Imaging Assembly</td>
</tr>
<tr>
<td>AMR</td>
<td>Adaptive Mesh Refinement</td>
</tr>
<tr>
<td>AR</td>
<td>Active Region</td>
</tr>
<tr>
<td>au</td>
<td>astronomical unit, 1 au = 149,597,870,700 meters</td>
</tr>
<tr>
<td>Å</td>
<td>Angstrom, 1Å = 10^{-10}m</td>
</tr>
<tr>
<td>BBSO</td>
<td>Big Bear Solar Observatory</td>
</tr>
<tr>
<td>CESE</td>
<td>Conservation-Element/Solution-Element</td>
</tr>
<tr>
<td>CME</td>
<td>Coronal Mass Ejection</td>
</tr>
<tr>
<td>DKIST</td>
<td>Daniel K. Inouye Solar Telescope</td>
</tr>
<tr>
<td>EUV</td>
<td>Extreme Ultraviolet</td>
</tr>
<tr>
<td>FOV</td>
<td>Field of View</td>
</tr>
<tr>
<td>GST</td>
<td>Goode Solar Telescope</td>
</tr>
<tr>
<td>HMI</td>
<td>Helioseismic and Magnetic Imager</td>
</tr>
<tr>
<td>Hα</td>
<td>H-alpha spectral line with a wavelength of ( \sim 6563 , \text{Å} )</td>
</tr>
<tr>
<td>ICME</td>
<td>Interplanetary Coronal Mass Ejection</td>
</tr>
<tr>
<td>LASCO</td>
<td>Large Angle and Spectrometric COronagraph</td>
</tr>
<tr>
<td>LFFF</td>
<td>Linear Force Free Field</td>
</tr>
<tr>
<td>MFR</td>
<td>Magnetic Flux Rope</td>
</tr>
<tr>
<td>MHD</td>
<td>Magnetohydrodynamics</td>
</tr>
<tr>
<td>Mx</td>
<td>Maxwell, CGS unit of magnetic flux, 1 Mx = 1 G-cm²</td>
</tr>
<tr>
<td>NLFFF</td>
<td>NonLinear Force-Free Field</td>
</tr>
<tr>
<td>PIL</td>
<td>Polarity Inversion Line</td>
</tr>
<tr>
<td>Acronym</td>
<td>Description</td>
</tr>
<tr>
<td>---------</td>
<td>----------------------------------</td>
</tr>
<tr>
<td>PSP</td>
<td>Parker Solar Probe</td>
</tr>
<tr>
<td>SDO</td>
<td>Solar Dynamics Observatory</td>
</tr>
<tr>
<td>SHARP</td>
<td>Space-weather HMI Active Region Patches</td>
</tr>
<tr>
<td>SOHO</td>
<td>SOlar and Heliospheric Observatory</td>
</tr>
<tr>
<td>STEREO</td>
<td>Solar TErrestrial RElations Observatory</td>
</tr>
<tr>
<td>UV</td>
<td>Ultraviolet</td>
</tr>
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</table>
Chapter 1. Introduction

“If the Sun had no magnetic field, it would be as uninteresting as most astronomers think it is.” – R. B. Leighton: unpublished remark (Moore and Rabin, 1985)

1.1 Structure of Solar Atmosphere and Measurement of Solar Magnetic Field

To humankind, living with the Sun, our stellar neighbor, there are many distinct features that make this star unique. It is the closest star around us and is currently the only star with well-resolved atmosphere and interior structure. And the prosperity of life on the Earth depends on the Sun. In particular, solar activities have a close relationship with the space weather, which could bring a huge impact to the modern society. In general, the intensity of the solar activity varies in a solar cycle around 11 years. Over the past few decades, an abundant variety of solar activities in multiple scales are observed and investigated. The connection between the Sun and the Earth is also further established and characterized with the aid of the accumulation of solar observations.

The Sun is a typical main-sequence star with the spectral type G2V (Zombeck, 1990). Its internal structure can be divided into three parts: the
core, the radiation zone, and the convection zone. Beyond the internal structure of the Sun and above several thin surface layers, the solar atmosphere extends to a long range into the interplanetary space. The solar atmosphere covers a broad range of temperatures and plasma densities. Near the solar surface and beyond, there are mainly two thin layers, the photosphere and the chromosphere, and the extended solar corona. As the innermost layer, the photosphere is the layer below which the Sun becomes opaque to visible light. This layer is only about several hundreds kilometers thick (far less than the solar radius of about $6.96 \times 10^5$ km), while most visible light originates from the photosphere, and its temperature varies between about 4000 K to 6500 K. Most of the photosphere appears to be covered by the granulation motion while some features stand out in the active regions corresponding to sunspots. Sunspots, characterized by dark regions on the solar surface with strong magnetic field, vary in the number of occurrence over an approximate 11-year solar cycle. During a solar cycle, more sunspots appear when the Sun becomes more active which leads to stronger solar activities. The chromosphere spans from the top of the photosphere to a height about 2000–3000 km without a regular upper boundary. The temperature increases gradually to about 20,000 K in the chromosphere at first and suddenly experiences a rapid increase in a thin layer (about 100 km in thickness) called the transition region, then ultimately reaches millions of K in the lower corona. The corona gas becomes very tenuous with a low particle number density of around $10^{15} - 10^{16} \ m^{-3}$ and it is thoroughly ionized given such a high temperature. In addition, the corona expands to the interplanetary space permeating the solar system far beyond the
Earth’s orbit, maintaining a supersonic flow velocity. Although the exact heating mechanism for the hot corona is still under intense debate and investigation, it is generally believed that the major contribution lies in the solar magnetic field.

Table 1.1: Major Instruments and Progress for Solar Magnetic Field Measurement.

* : no specific instrument corresponds to the associated progress.

<table>
<thead>
<tr>
<th>Year</th>
<th>Instrument</th>
<th>Major progress</th>
</tr>
</thead>
<tbody>
<tr>
<td>1950s</td>
<td>*</td>
<td>point measurement of vector magnetic field</td>
</tr>
<tr>
<td>1970s</td>
<td>Marshall Vector Magnetograph</td>
<td>vector magnetogram</td>
</tr>
<tr>
<td>1995</td>
<td>SOHO/MDI</td>
<td>routine photospheric LOS magnetogram, full disk, 0.6”/pixel</td>
</tr>
<tr>
<td>2006</td>
<td>Hinode</td>
<td>photospheric vector magnetogram, local FOV, 0.32”/pixel</td>
</tr>
<tr>
<td>2008</td>
<td>1.6 meter Goode Solar Telescope</td>
<td>photospheric vector magnetogram, local FOV</td>
</tr>
<tr>
<td></td>
<td>(GST)</td>
<td></td>
</tr>
<tr>
<td>2010</td>
<td>SDO/HMI</td>
<td>routine photospheric vector magnetogram, full disk, 0.5”/pixel</td>
</tr>
<tr>
<td>2015</td>
<td>Chromospheric LAyer</td>
<td>five-minute flight for the measurement of magnetic field in the chromosphere</td>
</tr>
<tr>
<td></td>
<td>SpectroPolarimeter (CLASP)</td>
<td></td>
</tr>
<tr>
<td>2020</td>
<td>4 meter Daniel K. Inouye Solar</td>
<td>routine photospheric/chromospheric vector</td>
</tr>
<tr>
<td></td>
<td>Telescope (DKIST)</td>
<td></td>
</tr>
<tr>
<td>2020</td>
<td>Solar Orbiter/PHI</td>
<td>routine photospheric vector magnetogram, full disk</td>
</tr>
</tbody>
</table>

Accurate measurement of the solar magnetic field is fundamental to understanding the diverse solar activities. While nowadays it’s commonly accepted that solar activities are mainly driven by complicate magnetic environment, i.e., the accumulation and release of magnetic free energy in solar eruption events, the development of the solar magnetic field measurement has gone through a long history. The measurement of the solar magnetic field only becomes feasible after the Zeeman effect was discovered by Pieter Zeeman (Zeeman, 1897). Later on, George Ellery Hale found the Zeeman splitting in sunspots for the first time (Hale, 1908). After that, a lot of progress has been made with consistent efforts.
Table 1.1 provides a list of selected major instruments for solar magnetic field measurement in the last few decades from both ground-based and space-based facilities. Generally, the Zeeman effect is an effect which could result in the splitting of a spectral line into two or three components with the presence of the magnetic field as shown in Figure 1.1 by one example. The simplest case is that a spectral line splits into three parts after interacting with the magnetic field. The splitting distance for each sub-component is a function of the magnetic field magnitude and the wavelength of the passing light. However, in practice, the Zeeman splitting distance (line displacement) is often too small to be measured directly due to a finite spectral line width. Given that the Zeeman effect also introduces unique polarization signatures, current solar magnetic field measurements are mostly based on observations of polarization signals (Solanki et al., 2006). The state of such polarizations is described by four Stokes parameters I, Q, U, V, which are the sums or differences of measurable intensities in different directions. These parameters can be measured directly. After relating them to the vector magnetic field \( \mathbf{B} \) based on a radiation transfer theory, the vector \( \mathbf{B} \) can be derived through a procedure solving the inversion problem. In addition to the Zeeman effect, there are also some other physical effects which are still under development for utility to measure the solar magnetic field, like the Hanle effect, bremsstrahlung radiation based on radio observations, etc. In any case, it is always essential to pursue an accurate measurement of solar magnetic field as well as higher resolutions to further explore the underlying mechanisms driving different phenomena on the Sun. Currently, the accurate measurement of solar
vector magnetic field is only routinely available on the photosphere, in the form of a vector magnetogram, or 2D discrete data arrays in digital format. In this dissertation, we will make use of available photospheric vector magnetograms from different telescopes to explore multi-scale magnetic structures in different solar eruptions.

1.2 Observations for Solar Eruptions and Associated Theoretical Models

Solar eruptions are manifestations of strong solar activities, which include spectacular events in different scales through multi-wavelength imaging observations like jets, erupted filaments, flares, coronal mass ejections (CMEs), and so
on. It is conventional to study those phenomena on the Sun by using remote-sensing observations from both space-based telescopes, like the Solar Dynamics Observatory (SDO), and ground-based telescopes like the 1.6 meter Goode Solar Telescope (GST) at the Big Bear Solar Observatory (BBSO), as shown in Figure 1.2. Along with the development of advanced techniques and instruments, multi-wavelength observations with higher spatial, temporal and spectral resolutions become available containing substantial information from versatile solar structures. General estimations of physical properties like the composition, temperature, plasma velocity, and radiation distribution (radio, X-ray) of the solar atmosphere can be extracted from different observations. Meanwhile, polarimetry observation offers diverse and important diagnostics of the Sun, especially the
vector magnetic field at certain heights (mostly on the photosphere only for the
time being). Figure 1.3 illustrates examples of the remote-sensing observations
on the Sun for various structures/processes during solar eruptions from different
telescopes. Those transient phenomena generally differ significantly in shapes,
sizes and duration.

Figure 1.3: Various remote-sensing images at one instant time (snapshots) of different
structures in solar eruptions. (a) Fine-scale sunspot structure observed by DKIST (im-
age credit: NSO). (b) Two-ribbon flare structure observed by GST in Hα channel (Jing
et al., 2016). (c) A partial coronal mass ejection taken by the white-light coronagraph
onboard STEREO; the inner white circle represents the boundary of the Sun (image
courtesy of NASA). (d-f) Obtained from SDO/AIA observations with corresponding
wavelengths denoted (images credit: SDO/NASA): the observation of a solar jet (Ster-
ling et al., 2015), the observation of post-flare loops, and a filament during the eruption
presented as a twisted loop, respectively.

To explain the underlying physical mechanisms within different observa-
tional phenomena on the Sun, there exists many theoretical models for solar
eruptions, especially for solar flares, in the past few decades. As a basic model mostly based on remote-sensing observations developed over years, the standard two-dimensional (2D) flare and CME model, so-called the CSHKP model (developed by Carmichael (1964), Sturrock (1966), Hirayama (1974), Kopp and Pneuman (1976)), has stimulated significant progress in the study of solar flares. This

![Figure 1.4](image)

**Figure 1.4:** (a) Schematic plot of the 2D standard CSHKP model which describes the existence of an MFR, the reconnection process beneath the flux rope (see the label ‘X’) and the post-flare loops (marked by ‘C’). (b) A quasi-3D version of the CSHKP model. Reconnection takes place between adjacent loops consecutively and therefore produces the closed twisted field lines (marked ‘FR’) with two feet attached to the bottom boundary where the reconnected field line (post-flare loop) footpoints form the two ribbons (marked by ‘R’) on the two sides of the polarity inversion line (‘PIL’), respectively (adapted from Longcope *et al.* (2007)).

model introduces a conceptual scenario that a magnetic flux rope (MFR) in the corona (defined more generally as a bundle of coherent twisted magnetic field lines, thus appearing closed in 2D) is ejected into the interplanetary space and produces a disconnected plasmoid in a strict 2D geometry, as illustrated in Figure 1.4. However, it cannot be ignored that multi-wavelength observations of solar eruptions also show various 3D features including the sigmoid structure, erupting
filaments (flux ropes), shear transition in post-flare loops and morphology change of flare ribbons (corresponding to the poloidal and toroidal fluxes of the MFR; Qiu et al. (2007); Hu et al. (2014)). Therefore, although different descriptions of the 2D standard flare-CME model have been proposed to include various observational features in flare-CME processes, the three-dimensional (3D) aspect beyond an idealized 2D model should be considered for complying with much broader ranges of measurements with 3D features intrinsic to realistic solar eruptions. Figure 1.4(a) gives a schematic illustration of the CSHKP model and an extension to a quasi-3D or 2.5D configuration is given in Figure 1.4(b), where an MFR (“FR”) structure is presented as a result of flare related reconnection and forming the core structure of the subsequent CME eruption (Longcope et al., 2007; Kazachenko et al., 2017).

Some 3D models have been proposed in the form of cartoons (Shibata et al., 1995; Moore et al., 2001; Priest and Forbes, 2002). Moore et al. (2001) give a 3D magnetic field configuration sketch of the standard model to describe the development for both confined and eruptive magnetic explosions for solar flares and CMEs. As illustrated in Figure 1.5, before the eruption, the initially strong sheared and twisted arcades near the neutral line could undergo relatively small-scale (or precursor) reconnection. For eruptive flares, the overlying magnetic field lines are stretched by the rising plasma and the reconnection process continues. Thus the flare ribbons will form during such a process and the plasmoid will be released in the end. Later on, some attempts have also been made to provide extensions of the standard flare model in 3D by utilizing numerical simulations.
For example, Aulanier et al. (2012) conducted a generic 3D MHD simulation for the formation of the post-flare loops to understand the 3D nature of magnetic reconnection during flares. A comprehensive review on the 3D models of solar flares can be found in Janvier et al. (2015). During flare eruptions, the magnetic reconnection process could add significant amount of magnetic flux to an ensuing magnetic flux rope and may help it get away from the Sun. Thus, despite the variable levels of complexity among different models, an MFR structure always constitutes a common and important element. Therefore, there are always strong interests in the community focusing on the formation and characterization of MFRs before and during eruptions.

So far we have seen the continuous refinement and improvement of both theoretical models and observations for the origin and evolution of solar eruptions on the Sun. It has also been increasingly realized that some scenarios initially proposed for relatively large-scale eruptions may persist down to much smaller scales. In order to characterize those multi-scale magnetic structures manifested in solar eruptions as displayed in Figure 1.3, especially their magnetic field topology in the solar source region, often in the so-called active regions (ARs), it becomes imperative to obtain 3D magnetic field distribution in the solar atmosphere.

1.3 Magnetic Field Modeling in the Solar Atmosphere

Given the lack of direct magnetic field measurements in the corona, the 3D coronal magnetic fields are usually studied through numerical modeling methods. Depending on the specific purpose of studying the solar eruptions, there are a
variety of numerical simulations to model the solar magnetic field with different assumptions. Traditionally, it is common to use idealized data as the boundary condition or the initial condition to derive the 3D magnetic field and drive its evolution. These models are mainly designed to focus on the key features in solar eruptions (Amari et al., 2003; Aulanier et al., 2012; Jiang et al., 2021b). Figure 1.6 indicates the 3D geometry from one study by an MHD simulations based on initial conditions built with an asymmetric current-free bipolar magnetic field. Al-

Figure 1.5: The 3D standard model for both eruptive and confined magnetic explo-
sions (flares and CMEs) illustrated by Moore et al. (2001).
though the idealized models are more advantageous to show fundamental physical mechanisms with more free and adjustable conditions, those models are simplified considering the complicated and dynamic realistic observational features for a majority of solar eruption events. Therefore, the realistic models have been developed naturally which utilize realistic observations as a part of constraints. The history of realistic numerical simulations for the 3D coronal magnetic field adopting the photospheric magnetogram as the boundary conditions can date
back over a half century ago (Altschuler and Newkirk, 1969), which makes use of the line-of-sight photospheric magnetogram to derive the potential field in the solar corona. As time goes on, dynamic models driven by time-varying observational magnetograms gradually develop to be a powerful tool to determine the 3D magnetic field and to study the initiation, evolution as well as eruption mechanisms of solar eruptions with simultaneous solar observations. For example, as illustrated in Figure 1.7, Jiang et al. (2016) carried out a time-dependent MHD simulation driven directly by a series of photospheric magnetograms to study the whole evolution process of a solar eruption event from the pre-eruptive state to the eruptive state. The simulated coronal magnetic field lines were shown to demonstrate a good consistency with the corresponding EUV observations from SDO/AIA.

Alternatively, another numerical simulation approach is often adopted to reconstruct a realistic 3D coronal magnetic field under some assumptions. According to the estimate of plasma $\beta$, the ratio between the plasma pressure and the magnetic pressure, by Gary (2001), the plasma $\beta$ is relatively low over a certain range of heights in the solar atmosphere, especially in the low corona immediately above the photosphere/chromosphere. It means the magnetic field is dominant in the low corona. Therefore, the force-free assumption is put forward such that other non-magnetic forces can be ignored in the momentum equation and the Lorentz force should be self-balanced, $J \times B = 0$. From Ampere’s law, $\nabla \times B = \mu_0 J$ for a static state. So the magnetic field $B$ satisfies the equation $\nabla \times B = \alpha B$ with $\alpha$ an unknown parameter. There are several special cases
Figure 1.7: The comparison between (a) a time sequence of the SDO/AIA observations and (b-c) the corresponding simulated coronal magnetic field lines from the MHD simulation based on realistic photospheric vector magnetograms (Jiang et al., 2016). Panels (b) and (c) present the top view and the side view, respectively.

under the force-free assumption. The simplest one is the potential field when $\alpha = 0$. If $\alpha$ becomes a non-zero constant, it is a linear force-free field. The nonlinear force-free field (NLFFF) corresponding to $\alpha$ being a function of space is more complex and could provide a more accurate description of the magnetic field for realistic ARs, complying with the realistic magnetogram on the bottom boundary. Since our interest lies in the magnetic structures in the source region for solar eruption events on a local scale with high spatial resolutions, the most common and practical way to reconstruct the coronal magnetic field is the NLFFF extrapolation method. Different numerical methods to reconstruct the NLFFF will be introduced in more details in Chapter 2.
1.4 Characterization of Multi-scale Magnetic Structures in Solar Eruptions

As we can see, starting from diverse solar observations, theoretical models and numerical simulations are developed jointly to explain the underlying physics in solar eruptions on the Sun. On the one hand, study of the evolution and propagation of large-scale magnetic structures embedded in strong solar eruptions has been greatly advanced given the availability of multi-point in situ measurement by multiple spacecraft. Therefore, it becomes feasible to make a physical connection for the origin of those magnetic structures on the Sun with their interplanetary counterparts. On the other hand, accompanied by the improvement of the remote-sensing observations, more frequent and widespread small-scale solar eruptions (like solar jets) or small-scale phenomena preceding the flare eruptions (so-called precursors) with fine-scale observational features are well identified. Some observations show that jet eruptions are initialized by a mini-filament eruption which consists of a small-scale MFR (Sterling et al., 2015). In other words, characterization of magnetic structures in multiple scales becomes necessary to achieve a more comprehensive physical understanding for different realistic solar eruptions.

1.4.1 Core Magnetic Structures in Solar Eruptions: MFRs

MFRs are generally believed to be the core magnetic structure of CMEs and ICMEs (Qiu et al., 2007; Hu et al., 2014). So characterization of MFRs re-
lated to their formation and evolution remains an important topic to be explored. Erupted MFRs that propagate into the interplanetary space are commonly detected in-situ as the large-scale magnetic clouds among ICMEs (Burlaga, 1992; Hu et al., 2014). On the Sun, there is various evidence in remote-sensing observations for MFRs, including filaments, coronal cavities, sigmoids, and hot channels (Cheng et al., 2017). These phenomena can be unified into one framework for the underlying magnetic structure as distinct manifestations of MFRs, depending on different observational wavelengths and perspectives (Liu, 2020). Among them, most of the latest recognized observational features are attributed to the observations from the SDO. It is worth noting that the development of large ground-based solar telescopes also provides a growing prospect to capture the dynamic fine-scale structures related to the MFR formation in the low corona within a small field of view (FOV). These ground-based facilities contribute to the study of MFR physics with very high spatio-temporal resolutions, including the Swedish 1 m Solar Telescope at La Palma, the 1.6 m GST at Big Bear Solar Observatory (BBSO/GST, Goode et al. (2010)), the 1 m New Vacuum Solar Telescope at Yunnan Observatory (NVST, Liu et al. (2014)) and also the 4 m Daniel K. Inouye Solar Telescope (DKIST, Rimele et al. (2020)). Signatures of MFRs can be collected nearly continuously in different heights of the solar atmosphere. For example, in addition to the aforementioned observational signatures of MFRs, a pre-existing twisted MFR was witnessed clearly in the chromosphere via the BBSO/GST Hα observations by Wang et al. (2015) in high spatial resolu-
tion. More detailed properties of MFRs, especially their magnetic properties, are expected to be revealed by multiple spectral lines with the newly built DKIST.

Given the important role of CME/ICMEs in space weather and the advancement of multi-point measurements, the connection between a solar MFR and its interplanetary counterpart in the same solar eruption event inevitably gains more attention, which motivates the subsequent quantitative characterization and topological analysis of MFRs. Such characterizations require the development of numerical simulations to produce more realistic 3D magnetic field configurations, which is commonly obtained based on vector photospheric magnetograms. For instance, a number of static extrapolation studies, which utilize the NLFFF method by adopting photospheric vector magnetograms, have been carried out and the results are compared to multi-wavelength observations on the Sun to investigate the MFRs in the complicated magnetic environment (Wiegelmann and Sakurai, 2021). The formation and eruption processes of MFRs in the low corona are further analyzed in different event studies, by dynamic MHD simulations, which typically demands much more computational resources than static extrapolations for a same event (Jiang et al., 2014a; Inoue et al., 2014). Observationally, Zhu et al. (2020) reported a statistical study of 60 CME-flare events to investigate the properties of CMEs and associated magnetic reconnection in the low corona during the early phase of the eruption. This type of study further establishes the close relationship between CME-MFR structure and flare-associated magnetic reconnection processes.
1.4.2 Small-scale Magnetic Structures in Solar Eruptions

While a lot of efforts have been made to understand large-scale coherent magnetic structures like MFRs in the main phase of solar eruptions, there are many nontrivial small-scale structures which are also found from different solar eruptions, including flare precursors and jet-like eruptions. These phenomena show much smaller size and less energy release compared to observations in the main-phase of flares and CMEs. Nowadays, instruments with higher spatial and spectral resolutions improve the ability to resolve these features in a finer scale rather than under-resolved signals with a few pixels. More importantly, ground-based telescopes also show the ability to regularly measure magnetic field as well as local plasma properties in ultra-high resolutions with a designated pointing FOV near specific solar source regions (Rast et al., 2021). Those observations including imaging and spectrum, especially the high-level data products with the vector magnetic field measurements, provide us a unique opportunity to extend our understanding for small-scale solar eruptions and their implication for other solar activities through studies of fine-scale structures in these eruptions.

Before the availability of high-resolution observations of solar eruptions, there are already a wealth of studies focusing on small-size solar eruptions or small-scale structures in solar eruptions, like coronal jets, mini-filament eruptions and flare precursors (Shibata et al., 2007; Sterling et al., 2015; Tappin, 1991). Given the observations of various small-scale structures in different wavelengths, plenty of studies concentrate on the morphological characteristics and thermal
properties during the evolution. It’s found that jet-like eruptions often display morphological changes which are closely related to magnetic reconnection (Sterling et al., 2015; Tiwari et al., 2018; Tian et al., 2018; Hou et al., 2017). To understand the physical processes of solar eruptions behind these features, different cartoons and theories are put forth to illustrate the evolution process for solar jets (Sterling et al., 2015; Panesar et al., 2016b). Meanwhile, some efforts are also made to understand the jet-like eruptions through numerical simulation. Based on idealized initial conditions, Wyper et al. (2017) introduced a universal model for solar eruptions based on simulations for jet-like eruptions and large-scale CME eruptions based on a scenario of breakout reconnection. Recently, some numerical simulation and extrapolation results based on the realistic observations are also analyzed to understand the initiation or triggering mechanisms of solar jets (Guo et al., 2013; Zhu et al., 2017; Nayak et al., 2019).

Why do we care small-scale structures? In general, resolving small-scale magnetic structures is not only important to understanding small-size solar eruption events, but may be also essential for understanding multi-scale magnetic structures in flares/CMEs, which will in turn help us improve the modeling technique of large-scale solar eruptions. It can be expected that there are potential contributions from small-size events to the associated large-scale events. For example, Yan et al. (2022) combined both high-resolution NVST imaging and spectral observations as well as data-constrained MHD modeling results of a confined flare to study the process of fast plasmoid-mediated reconnection from the current-sheet to the plasmoid scale and even larger macroscopic scales. With the
combination of more regularly available ultra-high-resolution observations with a small appointed FOV and the routine full-disk magnetic field measurement, new understanding towards the characteristics of those small-scale structures could be expected, like the magnetic topology, the initiation and formation mechanisms, the related energy built-up and release processes and so on. Among different focuses for small-scale magnetic structures or small-size solar eruptive events, below are some related questions we aim to explore in this dissertation:

(a) What do the 3D magnetic field topology and field line connectivity look like for the fine-scale structures? (b) How do we identify the magnetic reconnection site specifically? (c) How do we resolve and characterize those small-scale structures consistently in both observations and simulations? (d) By comparing small and large magnetic structures in extrapolation results, are there any similarities between CME eruption and solar jet? (e) How is the redistribution of magnetic flux related to the topological change and what are the implications for flux cancellation and flux emergence?

1.5 Motivation, Objectives and Dissertation Outline

During different phases of solar eruptions, including periods before, during, and after the eruption, the magnetic structures show specific topology in multiple spatial scales. Some structures propagate into the interplanetary space and maintain traceable features throughout the evolution reaching distances as far as the outer heliosphere. For the formation and evolution of CME-related MFRs, magnetic reconnection plays an important role that converts magnetic free en-
ergy to kinematic and thermal energy, and contribute to enhance radiation and energetic particle acceleration (Priest and Forbes, 2002; Qiu et al., 2002, 2004).

In order to study the nature of CME-related MFR, finding signatures associated with magnetic reconnection is helpful for characterizing MFRs on the Sun. To understand the physical process of solar eruptions consistently and further improve the accuracy of space weather prediction, it is important to characterize those multi-scale magnetic structures in the early stage. In this dissertation, we aim to characterize the topological properties of magnetic structures embodied in solar eruptions by reconstructing the 3D coronal magnetic field with the NLFFF extrapolation method. To optimize the analysis method for magnetic structures consistently across different scales, the NLFFF method is also modified for nonuniform embedded magnetograms from realistic magnetograms with different spatial resolutions and FOVs. Specifically, Chapter 2 and Chapter 3 focus on the quantitative characterization of MFRs with the NLFFF method for uniform photospheric magnetograms, while Chapters 4, 5, and 6 concentrate on the tests and application of the updated NLFFF method for different nonuniform embedded magnetograms. In what follows, we describe the main content for each chapter.

Chapter 2 describes the routine procedures for the preparation of input magnetograms, calculation and examination performed to get a converged extrapolation result before any further analysis. The basic equations and numerical setup for the CESE-MHD-NLFFF extrapolation method are introduced in detail. Specifically, two sets of photospheric vector magnetograms are selected as
examples to demonstrate the overall quality of the preprocessing results and the corresponding performance of the convergence through an iteration process. The quality of the extrapolation results (mainly the force-freeness and divergence-freeness of the 3D magnetic field $B$) is also evaluated based on a series of NLFFF metrics.

Chapter 3 presents the effort to investigate the properties of MFRs quantitatively and to establish the connection between MFRs on the Sun and their interplanetary counterparts in the same solar eruption events. Two events are selected with confirmed flare-CME and CME-ICME associations. Before the CME eruption, the coherent MFR structure can be identified based on the observations and extrapolation results via the NLFFF method. The magnetic reconnection flux during the eruption is measured based on remote-sensing solar observations. In addition, the magnetic properties of MFRs are characterized quantitatively and compared with the corresponding in situ modeling results.

Chapter 4 introduces an updated version of the CESE-MHD-NLFFF code for the nonuniform embedded magnetogram and the associated results from test runs. Major differences lie in the interfaces between the different pieces of the magnetograms with different resolutions when inputting for a nonuniform magnetogram rather than a uniform one as the bottom boundary condition for the NLFFF extrapolation. The associated grid construction is described. Firstly, the preliminary test runs for nonuniform SDO/HMI magnetograms are conducted and compared to extrapolation results from the corresponding uniform HMI magnetograms. Then we apply the updated method to the embedded photospheric
magnetograms from the high-resolution GST/NIRIS observation in a small FOV and the SDO/HMI observation in a larger FOV. Then the quality of different test runs from different settings is presented.

Chapter 5 presents the investigation of the fine-scale structures of the precursors before a main flare eruption by employing the CESE-MHD-NLFFF method for a nonuniform embedded magnetogram tested in Chapter 4. Two additional extrapolation runs utilizing a uniform SDO/HMI magnetogram and a uniform GST/NIRIS magnetogram, respectively, as the bottom boundary conditions, are also performed for comparison. These three extrapolation results are compared in terms of their magnetic field line connectivity around the observed corresponding precursor brightenings. Further analysis of the fine-scale magnetic topological features near the precursor brightenings helps to resolve the potential configuration for the magnetic reconnection during the flare precursors before the main flare eruption for this one particular event.

Chapter 6 presents another attempt to study the small-scale magnetic structures in a recurrent solar jet event, by applying the CESE-MHD-NLFFF method to the available nonuniform embedded magnetograms from the GST/NIRIS and the SDO/HMI. For this jet event, we adopt two sets of magnetograms at two different times to conduct the NLFFF extrapolations and compare the extrapolation results with the corresponding remote-sensing observations. Similar topological analysis is performed to resolve the fine-scale magnetic topological features as well as the relevant field line connectivity. We also aim to compare the footpoints
identified from observations with our extrapolation results and further explore the associated triggering mechanism for the recurrent solar jets.

In Chapter 7, we conclude our findings and discuss our future work.
Chapter 2. Extrapolation Method by a CESE-MHD-NLFFF Code

While the photospheric magnetograms are routinely obtained, the measurements of the magnetic field in the corona are only available for a few individual cases. In order to model the coronal magnetic field based on limited measurements, it is necessary to make some assumptions, most commonly, the force-free approximation. The plasma $\beta$, the ratio between the plasma pressure and the magnetic pressure, is generally assumed to be much less than 1 in the middle corona according to the estimation by Gary (2001), which indicates that the magnetic field plays a dominant role. Therefore, the non-magnetic forces including the gravitational, the pressure gradient and the inertial forces can be ignored for a static and time stationary equilibrium in the ideal MHD momentum equation. Under this circumstance, the Lorentz force should be self-balanced, i.e., it should satisfy the equation, $\mathbf{J} \times \mathbf{B} = 0$, which means that the electric current density $\mathbf{J}$ is parallel to the magnetic field $\mathbf{B}$, with $\mathbf{J} = \alpha \mathbf{B}$ (the so-called force-free parameter $\alpha$ may vary in space). The simplest case is the potential field when $\alpha \equiv 0$, which indicates that no current exists for a potential field. If the force-free parameter $\alpha$ is not zero, then there are two situations, depending on whether $\alpha$ is constant or varying in space. One is the linear force-free field
(LFFF) for \( \alpha \equiv \text{const} \) and the other one is the nonlinear force-free field (NLFFF). Considering the magnetic structures in the ARs on a local scale, especially the fine-scale structures, it is a common and practical way to reconstruct the static three-dimensional coronal magnetic field in a volume with the NLFFF extrapolation method by using the photospheric magnetograms as the bottom boundary conditions (Wiegelmann and Sakurai, 2021).

2.1 NLFFF Extrapolation Method

Compared to a potential field solution, NLFFF solution contains free magnetic energy and electric current which could drive solar eruptions. The force-free parameter \( \alpha \) is usually dependent on spatial dimensions based on the vector magnetograms and the assumption of an NLFFF describing a static equilibrium in an AR, which is more general than an LFFF solution. Furthermore, the NLFFF extrapolation becomes a better candidate for initiating further MHD simulations than the potential field and LFFF (Jiang et al., 2022). Therefore, the NLFFF extrapolation method is often used to derive quantitatively the magnetic topology and other magnetic properties of ARs. Besides, the lack of quantitative observation of plasma parameters and limitation of computational resources for the numerical simulation capability of a more realistic full MHD model also lead to the popularity of NLFFF extrapolation.

There are a variety of numerical methods proposed to apply the NLFFF models for the given boundary conditions and sometimes pseudo-initial conditions, \textit{e.g.}, via upwind integration, Grad-Rubin iteration, MHD relaxation, and
optimization approach, etc (see a review by Wiegelmann and Sakurai (2021)). In addition to the fundamental differences in methods, specific realizations differ significantly in many aspects ranging from software to hardware implementation, e.g., the grid configuration, the numerical scheme, the specific boundary conditions, the degree of parallelization and also the hardware architecture. Therefore the computation speed and accuracy of different numerical methods can vary significantly. A detailed comparison of several representative codes was carried out by Schrijver et al. (2006), Metcalf et al. (2008) and De Rosa et al. (2009) (see also the review by Wiegelmann and Sakurai (2021)). The NLFFF model has been widely evaluated and proved to be a practical tool to study the realistic solar eruptions. Considering its computational efficiency and performance, we mainly study the magnetic field configurations on the Sun with the NLFFF extrapolation method.

In this dissertation, we apply an MHD-relaxation based method with a conservation-element/solution-element (CESE) solver (Jiang et al., 2011) for the NLFFF extrapolation. The so-called CESE-MHD-NLFFF code (Jiang and Feng, 2013) has been tested by different benchmark cases including a standard benchmark from a semi-analytic force-free solution (Low and Lou, 1990) and a more complex van Ballegooijen reference model which contains a weakly twisted flux rope (van Ballegooijen and Mackay, 2007; Metcalf et al., 2008). And it has also been widely applied to the extrapolation of realistic solar magnetograph data from SDO/HMI. The extrapolation results have shown good agreements with ob-
servational features like sigmoids, coronal loops and even an elongated quiescent filament (Jiang and Feng, 2013; Jiang et al., 2014a; Duan et al., 2019).

2.2 MHD Relaxation Method and CESE-MHD Scheme

The MHD relaxation method solves the MHD momentum equation and the magnetic induction equation iteratively until a stationary magnetic field solution is reached. In particular, the magnetofrictional method can be regarded as a special case of an MHD relaxation method. In this method, an artificial dissipative term $D(v)$ is included to balance the momentum equation with flow velocity $v$. The term $D(v)$ can either be in a frictional form like $D(v) = -\nu v$ or in a viscous form such that $D(v) = \nabla \cdot (\nu \rho \nabla v)$ with mass density $\rho$. Here we adopt the frictional form where $\nu$ is the frictional coefficient used to control the dissipative speed (Jiang and Feng, 2012), together with the continuity equation,

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho v) = 0,$$

$$\rho \frac{dv}{dt} = \rho (\frac{\partial v}{\partial t} + v \cdot \nabla v) + [\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho v)]v$$

$$= \frac{\partial (\rho v)}{\partial t} + \nabla \cdot (\rho vv)$$

$$= -\nabla p + \rho g + J \times B - \nu v. \tag{2.1}$$
Here the current density is written as \( J = \frac{1}{\mu_0} \nabla \times B \). Based on the force-free field model, the pressure, gravity and inertial forces can be eliminated, which leads to,

\[
\rho \frac{d \mathbf{v}}{dt} = J \times B - \nu \mathbf{v}.
\]  

(2.2)

If the inertial term in equation (2.1) is neglected, which is usually adopted in the magnetofrictional method (Valori et al., 2007), the time-independent momentum equation is further reduced to

\[
J \times B = \nu \mathbf{v}.
\]  

(2.3)

Considering the magnetic induction equation,

\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}),
\]  

(2.4)

the velocity from equation (2.3) can be inserted into equation (2.4), and then the only equation to be solved is the magnetic induction equation. Proper construction of the frictional coefficient \( \nu \) can make it possible to control potential numerical instabilities, and to obtain a converged solution reaching a quasi-static equilibrium state \( \mathbf{v} \approx 0 \) and \( \frac{\partial}{\partial t} \approx 0 \).

In order to utilize the CESE-MHD solver, the inertial term is partially reserved in the CESE-MHD-NLFFF code. Specifically, the modified time-dependent
momentum equation is written as (Jiang and Feng, 2012, 2013),

\[
\frac{\partial (\rho \mathbf{v})}{\partial t} = (\nabla \times \mathbf{B}) \times \mathbf{B} - \nu \rho \mathbf{v},
\]

\(\rho = |\mathbf{B}|^2 + \rho_0.\) \hfill (2.5)

Here the \(\nabla \cdot (\rho \mathbf{v} \mathbf{v})\) term in equation (2.1) is ignored to speed up the convergence. Since the velocity \(\mathbf{v}\) is approaching zero for the final quasi-static state when the solution converges, omission of this term has little effect on the final converged solution. Here the (pseudo) mass density \(\rho\) also contains the permeability constant \(\mu_0\) for simplicity and is assumed to be related and largely proportional to \(|\mathbf{B}|^2\) in order to roughly equalize the speed of the evolution of the entire field with nearly uniform Alfvén speed \(v_A = \frac{|\mathbf{B}|}{\sqrt{\rho}} \approx \text{const}\). To enhance the ability of handling noisy data in realistic solar magnetograms, a small value \(\rho_0, \text{ e.g., } \rho_0 = 0.1\) (in the same unit as \(|\mathbf{B}|^2\)) is added, to the original pseudo density \(\rho\). Equation (2.5) takes such a form mostly for numerical implementations based on prior experiences (Jiang and Feng, 2012, 2013), and the fact that for a converged solution with \(\mathbf{v} \approx 0\), an approximate force-free state is obtained, \(\mathbf{J} \times \mathbf{B} \approx 0\).

For the induction equation, it is written,

\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) - \mathbf{v} \nabla \cdot \mathbf{B} + \nabla (\mu \nabla \cdot \mathbf{B}).
\]

\(\) \hfill (2.6)

Two extra terms are added to control the divergence cleaning of the magnetic field. The first term, \(-\mathbf{v} \nabla \cdot \mathbf{B}\), is from Powell’s method (Powell \textit{et al.}, 1999) and the second term is for additional control of \(\nabla \cdot \mathbf{B}\) with a diffusive coefficient \(\mu\).
(Marder, 1987; Dedner et al., 2002). These terms ensure that the numerically
generated magnetic charge cannot accumulate locally once produced, which is
one of the so-called standard divergence cleaning procedures.

For computational speed and accuracy, equations (2.5) and (2.6) are solved
by the CESE-MHD scheme. This is a modern advanced scheme with proven suc-
cess in prior applications to magnetic field extrapolations of solar ARs (Feng et al.,
2012; Jiang and Feng, 2012). The above system of equations in Cartesian coordi-
nates can be written in a general conservation form with dependent variables ($B$,
$\rho v$) for the CESE code. The CESE scheme has many advantages. Unlike tradi-
tional numerical methods including the finite-difference or finite-volume schemes,
the CESE code “treats both space and time as one entity”. More details of the
CESE method for MHD simulations can be found in Zhang et al. (2006), Feng
et al. (2007), Jiang et al. (2010), and also Jiang et al. (2012).

2.3 Initial Grid Setup and Boundary Conditions

Considering the speed and accuracy of the computation in terms of high-
resolution and large field-of-view solar magnetograms, a nonuniform grid struc-
ture within a block-structured parallel computation framework is adopted with
the help of the PARAMESH software package (MacNeice et al., 2000) for the
CESE-MHD-NLFFF code. PARAMESH offers structured grid blocks and sup-
ports for the equations written as conservation laws. Then the whole computa-
tional domain is divided into blocks with different spatial resolutions, and all
blocks have identical logical structures (each block has $N_x \times N_y \times N_z$ interior
grids and several guard grids near the boundaries of each block) which are evenly distributed among processors. The smallest grid is the smallest computational unit for the spatial domain, which usually corresponds to the spatial resolution of the input magnetogram. Within such a parallel computation framework, we have more freedom to configure a domain of nonuniform grids and save computational resources, as compared with an ordinary set of uniform grids.

![Figure 2.1: Configuration of a block structure: the whole computational domain is divided into blocks with different spatial resolutions and each block has $8 \times 8 \times 8$ grids. The vertical slice of the computation domain shows the arrangement of different blocks and the bottom map shows $B_z$ distribution and a uniform block (grid) structure on the photosphere. The images to the right show the $B_z$ distribution on the bottom boundary ($z = 0$, photosphere) and at $z = 10$ Mm, respectively. Credit: Jiang and Feng (2012).](image)

For the grid initialization, the whole computation domain includes the preset core region and the surrounding buffer region to reduce the influence of the side boundaries (Jiang and Feng, 2013). A rectangular main computation region, the core region, on the bottom layer will be set up first. The length and width
of the core region are the same as the input magnetogram while the height is pre-set and fixed during the relaxation process. Figure 2.1 gives an example of the initial nonuniform block structure (mainly in the vertical dimension $z$ corresponding to the height into the corona). Since the magnetic field strength mainly concentrates near the photosphere and the field generally expands quickly into the higher corona, it becomes smoother and weaker with increasing heights (Gary, 2001). As illustrated in Figure 2.1, we generally apply a grid structure with decreasing resolutions (increasing grid/block sizes) when the height increases: the grid resolution matches the resolution of the magnetogram at the bottom boundary and decreases by about four times at the top of the computational domain.

After the grid initialization, the initial solution for the whole computation domain is assigned with the potential field derived from the bottom boundary condition by using the Green’s function method (Chiu and Hilton, 1977; Metcalf et al., 2008). The bottom boundary condition is consistent with the preprocessed observed vector magnetogram which corresponds to the core region. All the other side boundaries far away from the core region are also fixed during the computation with the values obtained from the initial potential field. Subsequently in Chapter 4, we show the implementation of the nonuniform grid structures in all three dimensions, i.e., in both the x-y plane and the vertical height.
2.4 Data and Preprocessing Method

2.4.1 Photospheric Vector Magnetogram Data Products

Although it is difficult to measure the magnetic field in situ in the solar atmosphere near the Sun, it has been common to obtain the solar vector magnetograms from the remote-sensing observations. The vector magnetic field in different locations at variable heights albeit mostly on the photosphere can be retrieved from inversion techniques by measuring a set of Stokes polarization parameters across magnetically sensitive spectral lines based on the Zeeman and Hanle effects (Wiegelmann and Sakurai, 2021; LaBonte et al., 1999). Nowadays, vector magnetograms are available in different resolutions and FOVs via various space-based telescopes, e.g., Hinode, SDO, Advanced Space-based Solar Observatory (ASO-S) and Solar Orbiter, and also the large-aperture ground-based telescopes like BBSO/GST, GREGOR solar telescope, and DKIST, etc.

The Helioseismic and Magnetic Imager onboard the SDO (SDO/HMI, Schou et al. (2012)) can map the full-disk photospheric vector magnetic field routinely with high cadence and spatial resolution. It observes the full Sun at six shifted wavelength points across the Fe I 6173 Å spectral line and measures six polarization states that are combined to derive the physical parameters (Schou et al., 2012). Raw filtergrams recorded by the camera are collected and converted to observable quantities with a rapid time cadence. It takes 135 s to complete each set of filtergrams for the vector magnetic field data. To obtain the vector magnetograms, the Stokes parameters are first derived from combined filtergrams.
over 720 s interval and then inverted through the Milne-Eddington based algorithm by the Very Fast Inversion of the Stokes Vector code (VFISV, Borrero et al. (2011)) optimized for the HMI pipeline. The remaining 180° azimuthal ambiguity is resolved with the Minimum Energy algorithm (ME0, Leka et al. (2009)). A detailed description on the vector magnetogram production can be found on the website, http://jsoc.stanford.edu/HMI/Vector_products.html.

We will use the Space-weather HMI Active Region Patches (SHARPs, Bobra et al. (2014)) vector magnetogram data product, hmi.sharp_cea_720s, as the input for the extrapolation. The SHARP data series provide maps following each patch of significant solar magnetic field concentration for its entire lifetime. Quantities like the photospheric vector magnetic field and its uncertainty are included. The active region patches are automatically identified and assigned with an HMI Active Region Patch (HARP) number, similar to the concept of an NOAA Active Region number. The HARP database generally captures more patches of solar magnetic activity than the NOAA active region database (Bobra et al., 2014). With a cadence of 720 s and a spatial resolution of 0.5″, the SHARP data is de-rotated to the disk center and remapped using the cylindrical equal area (CEA) Cartesian coordinates to generate data arrays corresponding to the Cartesian components of the vector magnetic field over the patch.

Compared with the unique detection abilities of space-based telescopes in frequencies and wavelength coverage across the entire electromagnetic spectrum, the ground-based telescopes also have distinct advantages. For example, they can be built much larger in aperture with less money, and they are easier to be
upgraded and maintained. Therefore, the ground-based telescopes can offer observations with ultra high-resolutions in relatively smaller FOVs nowadays. We also take advantage of the vector magnetograms from the 1.6 meter BBSO/GST, which offers ultra high-resolution magnetograms from the Near-InfraRed Imaging Spectropolarimeters (NIRIS; Cao et al., 2012). The GST/NIRIS is equipped with the infrared detector and the dual Fabry-Perot interferometers system, and provides the spectropolarimetric data (at Fe i 1565 nm doublet, 0.2 Å bandpass). The spectropolarimetric data are processed with the NIRIS data processing pipeline including dark and flat field corrections, instrument crosstalk calibration and Milne-Eddington Stokes inversion, from which the vector magnetic fields could be extracted. The pixel size and temporal cadence of the resulting vector magnetograms are $\sim 0.08^\prime\prime$/pixel and 87 s, respectively. In general, the space-borne instrumentation provides a larger FOV and more continuous observations while the ground-based counterpart has a relatively smaller FOV and more sporadic temporal coverage, but has a much higher spatial resolution. Therefore, it is desirable to combine these two sets of observations to make the best use of their data products. In Chapters 5 and 6, we present the unique studies of NLFFF extrapolations by embedding the higher resolution GST/NIRIS magnetogram into the corresponding part of the lower resolution, larger FOV SDO/HMI magnetogram.

2.4.2 Preprocessing of the Photospheric Vector Magnetograms

NLFFF extrapolation is an initial and boundary-value problem, which needs boundary conditions like the vector magnetogram for the bottom boundary.
But considering the force-free assumption, there are still some limitations that cannot be ignored. Passing through the inhomogeneous plasma environment, the plasma $\beta$ could vary from $\beta > 1$ in the photosphere to $\beta \ll 1$ in the low and middle corona, and to $\beta > 1$ again in the upper corona (Gary, 1989, 2001). Metcalf et al. (1995) found that the magnetic field is not force-free especially in the photosphere and it becomes force-free roughly at a height 400 km above the photosphere from observations of the chromospheric vector magnetic field in a chosen active region. And the non-force-freeness of the solar magnetic field in the photosphere has also been investigated and supported by many studies (Moon et al., 2002; Tiwari, 2012; Liu et al., 2013). Using the vector magnetic field measurement of the upper chromosphere is one choice to meet the force-free assumption but such accurate magnetic field measurement in the chromosphere is generally difficult. There are a few individual cases which have applied available chromospheric magnetograms as the input of the extrapolation (Yelles Chaouche et al., 2012) or compare the NLFFF extrapolation results with the chromospheric magnetic field (Kawabata et al., 2020). However, the application of the chromospheric magnetogram as the bottom boundary condition is still under exploration and needs further validation combined with more upcoming observations. Another practical way to get a more consistent boundary condition for the force-free assumption is to modify the raw photospheric magnetogram to mimic a force-free chromospheric one, which is called preprocessing and was first proposed by Wiegelmann et al. (2006).

Before the evaluation of the force-free assumption, we assume the flux balance on the bottom boundary (the x-y plane) at the photosphere ($z = 0$)
within a finite region $S$, 

$$
\int_S B_z(x, y, 0) \, dx \, dy = 0. \tag{2.7}
$$

According to the derivations of Molodensky (1974) and Aly (1989), there are several necessary constraints which must be fulfilled for the magnetogram input for NLFFF extrapolations. They are listed as follows.

(1) The following set of quantities proportional to the total force on the boundary has to vanish:

$$
F_x = \int_S B_x B_z \, dx \, dy,
F_y = \int_S B_y B_z \, dx \, dy = 0, \tag{2.8}
F_z = \int_S (B_x^2 + B_y^2 - B_z^2) \, dx \, dy = 0.
$$

(2) The following set of quantities proportional to the total magnetic torque on the boundary also has to vanish:

$$
T_x = \int_S y (B_x^2 + B_y^2 - B_z^2) \, dx \, dy = 0,
T_y = \int_S x (B_x^2 + B_y^2 - B_z^2) \, dx \, dy = 0, \tag{2.9}
T_z = \int_S (y B_x B_z - x B_y B_z) \, dx \, dy = 0.
$$

These equations are derived from volume integrals of the magnetic force, torque and magnetic divergence over the whole computational domain with the volume
where $\mathcal{T}$ refers to the magnetic tensor $\mathcal{T} = -\frac{B^2}{2} \mathcal{I} + BB$. Moreover, to apply the preprocessing method, the magnetic field on the bottom boundary should be isolated and close to the disk center. Here “isolated” means that most magnetic flux is concentrated within the field of view of an AR and is relatively well balanced (Wiegelmann et al., 2006).

To evaluate the quality of force-freeness of a real magnetogram, three metrics (all normalized) are usually computed (Wiegelmann et al., 2006),

$$
\begin{align*}
\epsilon_{\text{flux}} &= \frac{\int_S B_z \, dx \, dy}{\int_S |B_z| \, dx \, dy}, \\
\epsilon_{\text{force}} &= \frac{|F_x| + |F_y| + |F_z|}{\int_S P_B \, dx \, dy}, \\
\epsilon_{\text{torque}} &= \frac{|T_x| + |T_y| + |T_z|}{\int_S \sqrt{x^2 + y^2} P_B \, dx \, dy},
\end{align*}
$$

(2.11)

where $P_B = B_x^2 + B_y^2 + B_z^2$. The smaller these quantities are, especially, for $\epsilon_{\text{flux}}, \epsilon_{\text{force}}$ and $\epsilon_{\text{torque}} \ll 1$, the better the input magnetogram is for the NLFFF extrapolation.

For the purpose of obtaining an approximate force-free chromospheric magnetogram from preprocessing, the main task is to improve the quality of the raw
magnetogram to make it closer to being force-free as best as it can. Another more routine task is to smooth the raw data. Smoothing is generally necessary to help reduce the measurement uncertainties and the numerical error from the computation (Jiang et al., 2014a).

In this dissertation, we use the preprocessing code developed by Jiang et al. (2014a) which is consistent with the CESE-MHD-NLFFF extrapolation code by adopting an optimal magnetic field splitting form. The raw magnetogram $B$ is split into two parts, $B = B_0 + B_1$, a sum of the potential field part $B_0$ and the non-potential field part $B_1$, which will be processed separately. The initial $B_0$, calculated from the observed vertical component $B_z$ by using a potential field solver, will be replaced ("smoothed") simply by taking the values at a plane that is usually one pixel size (i.e., about 360 km for SDO/HMI data, at $z = 0.5''$) in height above the photosphere for the purpose of smoothing, from the numerical 3D potential-field solution. The processing of the non-potential part $B_1$ is much more complicated. It is done by utilizing the improved preprocessing method developed by Jiang et al. (2014a), which adopts the basic principles similar to Wiegelmann et al. (2006), to reach a force-free state. As a general rule of thumb, values of the force-freeness and smoothness metrics calculated from the smoothed field $B_0$ are used as a reference to evaluate the quality of the preprocessing results as we will demonstrate for a few selected examples.
2.4.3 Quality of Preprocessed Magnetograms: Examples

The magnetograms from the ARs for at least 10 minutes before the flare onset times (estimated from the corresponding soft X-ray measurements of the GOES satellite) of the two events, AR 11719 and AR 12158, respectively, are preprocessed to get necessary boundary conditions and derive the initial conditions from a potential field solver for the NLFFF extrapolations. The original spatial resolution of the magnetograms is about 0.5″ per pixel. We rebin them to 1″ per pixel for the preprocessing and the subsequent extrapolations. The set of parameters indicating the quality of the raw and preprocessed magnetograms is given in Table 2.1 along with the results from their corresponding numerically calculated potential field. The total magnetic flux of the whole magnetogram of AR 11719 chosen at 06:36 UT on 2013 April 11 is not so well balanced and its distribution has a complex and diffusive pattern (see the magnetograms in Figures 3.1 and 3.2). In both cases, the parameter $\epsilon_{\text{flux}}$ doesn’t change much while preprocessing helps reduce $\epsilon_{\text{force}}$ and $\epsilon_{\text{torque}}$ by more than one order of magnitude. They become comparable with the values for the corresponding potential field, and are of the same orders of magnitudes.

Another set of parameters, $S_x$, $S_y$ and $S_z$, is calculated to indicate the degree of smoothness of each magnetic field component $B_m$ ($m = x, y, z$), which is defined as follows (see Jiang et al. (2014a)),

$$S_m = \frac{\sum_p [(\Delta B_m)^2]}{\sum_p [(\nabla B_m)^2]}, \quad (2.12)$$

41
Here the sum $\sum_p$ refers to summation over all pixels of the magnetogram, and the operator $\Delta$ is a typical finite-difference approximation of a Laplace operator in two dimensions using a five-point stencil, i.e., for pixel $(i, j)$,

$$\Delta B_{i,j} \equiv B_{i+1,j} + B_{i-1,j} + B_{i,j+1} + B_{i,j-1} - 4B_{i,j}, \quad (2.13)$$

and correspondingly,

$$\tilde{\Delta} B_{i,j} \equiv B_{i+1,j} + B_{i-1,j} + B_{i,j+1} + B_{i,j-1} + 4B_{i,j}. \quad (2.14)$$

<table>
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<th>Data</th>
<th>$\epsilon_{flux}$</th>
<th>$\epsilon_{force}$</th>
<th>$\epsilon_{torque}$</th>
<th>$S_x$</th>
<th>$S_y$</th>
<th>$S_z$</th>
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<td>0.319</td>
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<td>0.0835</td>
<td>6.90E-03</td>
<td>2.33E-03</td>
<td>1.88E-04</td>
<td>2.86E-04</td>
<td>2.17E-04</td>
</tr>
<tr>
<td>Potential field (AR 12158)</td>
<td>0.0835</td>
<td>2.52E-03</td>
<td>4.14E-03</td>
<td>1.78E-04</td>
<td>2.48E-04</td>
<td>2.17E-04</td>
</tr>
</tbody>
</table>

From Table 2.1, the smoothness levels of the preprocessed data among three components are relatively equal. And in each run, both preprocessed maps have the same level of force-freeness and smoothness as their corresponding poten-
tial field. As judged from the set of quality control parameters, the preprocessed magnetogram is deemed to be more suitable for the NLFFF extrapolation than the raw data.

Figure 2.2 (similar as Figure 3.1) shows the overall smoothing effect as a result of preprocessing for AR 11719 by comparing the raw and preprocessed maps of the magnetograms and the current density $J_z$ distributions. Random noise is suppressed obviously in the $J_z$ map.

**Figure 2.2:** First row: (left) the $B_z$ component of the raw magnetogram and (right) the derived vertical current density distribution $J_z$ for AR11719, at 06:36 UT on 2013 April 11. Second row: similar maps as the first row from the preprocessed magnetogram. The whole field of view of the map is $540'' \times 344''$. The blue box shows the subregion for Run 2 with a size of $300'' \times 250''$. And an additional subregion (the red box) is also chosen in Run 1 for the calculation of the convergence metrics.
Figure 2.3: Convergence metrics of two extrapolation runs for AR11719 in every 200 iteration steps. Top four panels show (clockwise from top left) the metrics of CWsin, $C^2$Wsin, $\langle |f_i| \rangle$, and the total magnetic energy (arbitrary unit), respectively. The bottom two panels show the residuals for Run 1 (left) and Run 2 (right), respectively. Three sets of results for Run 1 and Run 2 are presented in the top four panels (see the legend). See the text for details.
2.5 Convergence Study and NLFFF Extrapolation Metrics

The extrapolation result is obtained through a relaxation process to achieve convergence as judged by additional metrics. In this section, we study the convergence of the computation by checking the iterative evolution of several metrics (see details in Jiang and Feng (2012, 2013)). Also see detailed descriptions of those metrics in Appendix A.

As an example, for AR 11719, we carry out two extrapolation runs based on different field of views in the magnetogram and then calculate the convergence metrics for every 200 iteration steps as summarized in Figure 2.3. Before the extrapolation, the original vector magnetograms with about 0.5′′ per pixel resolution are rebinned to 1′′ per pixel and then preprocessed. One extrapolation (Run 1) utilizes the full size of the SHARP vector magnetogram (540″ × 344″) as part of the bottom boundary condition for the core region. Another one (Run 2) is performed by using a subregion (the blue box in Figure 2.2) of the rebinned magnetogram with a size of 300″ × 250″. In Run 1, we also calculated the convergence metrics of a sub-volume with the bottom side corresponding to a subregion (the red box in Figure 2.2) for comparison. As shown in Figure 2.3, the residual of Run 1 goes through a gradual increase before ∼ 6500 iteration steps because the bottom boundary condition drives the system away from the initial potential field (Jiang and Feng, 2012). Even though obvious fluctuations appear after the initial driving process, the overall trend of the residual toward the end is decreasing, accompanied by small oscillations. After ∼ 30,000 iteration steps, the residual is
reduced to \( \sim 10^{-5} \) and still maintains a declining trend. The other metrics of Run 1 also remain stable afterwards. In contrast, the performance of the residual for Run 2 is constantly fluctuating at a relatively high value. And the evolution of metrics for Run 2 also shows an unstable state compared to the evolution for Run 1, which may result from the different domains chosen for the two runs. Thus the extrapolation result of Run 1 can be considered a converged solution when terminated after 40,000 iteration steps. It is noticed that there are some oscillations in the convergence process of Run 1, which may be caused by the broad distribution of the weak field and random noise from the input magnetogram. The total computation time for Run 1 to converge (until 40,000 iterations) took about 95 hours with 19 processors on a 24-processor desktop with 48 GB memory. For AR 12158, we carry out one run with the smallest grid size 1'' and use the full magnetogram. Similar analysis has been done for the convergence study as shown in Chapter 3. And these extrapolation results will also be analyzed in Chapter 3.
Chapter 3. Quantitative Characterization of MFR Properties for Two Solar Eruption Events

In order to bridge the gap between heliospheric and solar observations of coronal mass ejections (CMEs), one of the key steps is to improve the understanding of their corresponding magnetic structures like the magnetic flux ropes (MFRs). But it remains a challenge to confirm the existence of a coherent MFR before or upon the CME eruption on the Sun and to quantitatively characterize the CME-MFR due to the lack of direct magnetic field measurement in the corona. In this study, we investigate the MFR structures, originating from two active regions (ARs), AR 11719 and AR 12158, and estimate their magnetic properties quantitatively. We perform the nonlinear force-free field extrapolations with preprocessed photospheric vector magnetograms. In addition, remote-sensing observations are employed to find indirect evidence of MFRs on the Sun and to analyze the time evolution of magnetic reconnection flux associated with the flare ribbons during the eruption. A coherent “pre-existing” MFR structure prior to the flare eruption is identified quantitatively for one event from the combined analysis of the extrapolation and observation. Then the characteristics of MFRs for two events on the Sun before and during the eruption, forming the CME-MFR, including the axial magnetic flux, field-line twist, and reconnection flux,
are estimated and compared with the corresponding in situ modeling results. We find that the magnetic reconnection associated with the accompanying flares for both events injects significant amount of flux into the erupted CME-MFRs. The following content has been published in the Astrophysical Journal (He et al., 2022). And the appendix of the paper has been moved to Appendix A of this dissertation.

3.1 Introduction

Coronal mass ejections (CMEs) are spectacular eruptions of plasma often accompanied with rapid release of magnetic energy from the solar atmosphere. When CMEs propagate away from the Sun and into interplanetary space, they are often called interplanetary CMEs (ICMEs) which exhibit a distinct set of observational signatures from in situ measurements. They are recognized as drivers of major space weather events which could severely impact human activities in modern society. Erupted flares have a close relationship with CMEs and strong flares are often observed along with CMEs (Chen, 2011). In the past few decades, a lot of efforts have been made on the development of the flare-CME model in order to explain the underlying physical mechanism(s). Among them, the standard two-dimensional (2D) flare and CME model, so-called CSHKP model (developed by Carmichael, 1964; Sturrock, 1966; Hirayama, 1974; Kopp and Pneuman, 1976), has successfully explained the morphological evolution of eruptive two-ribbon flares. One of the essential components in the model is a CME magnetic flux rope (MFR). Upon its ejection, field lines below the erupting rope reconnect.
and form an arcade of flare loops observed in the X-ray and extreme ultraviolet (EUV) wavelengths, and two ribbons observed in optical and ultraviolet (UV) wavelengths demarcating the feet of the arcade. In the 2D model, the same amount of magnetic flux encompassed by the flare loops is also turned into the poloidal flux of the erupting MFR, and this amount of flux can be measured from flare ribbons sweeping through photospheric magnetic field (Forbes and Priest, 1984; Qiu et al., 2004, 2007). Since MFRs are generally believed to be the core magnetic structure of CMEs and ICMEs (e.g., Gibson et al., 2006; Hu et al., 2014; Gopalswamy et al., 2017, 2018; Liu, 2020), studies of MFR properties related to their formation and evolution remain an important topic to be explored. We note that hereafter we use “MFR” for a more generic reference to a magnetic flux rope at different stages of evolution, and “CME-MFR” for the specific reference of a magnetic flux rope at the final stage of a CME eruption (i.e., when the MFR has well formed after the flare reconnection).

In recent years, the study of evolution and propagation of CMEs/ICMEs between the Sun and the Earth is greatly advanced with the help of multiple measurements from multi-spacecraft missions. The large-scale magnetic clouds (MCs) in ICMEs which are commonly detected in situ provide direct evidence for the existence of erupted CME-MFRs that come from the Sun (Burlaga, 1992; Qiu et al., 2007; Hu et al., 2014). In addition, there are also remote-sensing observations available throughout interplanetary space. For example, the twin spacecraft, Solar TErrestrial RElations Observatory (STEREO, Kaiser et al. 2008), can trace CMEs from the high corona to the inner heliosphere via coronagraphic observa-
tions. The STEREO mission provides two more viewpoints towards the Sun, in addition to the viewpoint from Earth provided by the Solar and Heliospheric Observatory (SOHO), in the past decades. There are also various signatures for MFRs on the Sun from remote-sensing observations, including filaments, coronal cavities, sigmoids, and hot channels (Cheng et al., 2017). These solar phenomena can be unified into one framework as distinct manifestations of MFRs (Liu, 2020). Most of the latest recognized observational features are attributed to the observations from the Solar Dynamics Observatory (SDO, Pesnell et al., 2012). Recent development of large ground-based solar telescopes also becomes an indispensable way to reveal the fine-scale structures and dynamics of MFR formation in the low corona (e.g., Wang et al., 2015).

Compared to various studies based on in situ measurements of MFR structures after the eruption, the origin of CME-MFRs before and during eruptions still remains elusive due to the complex environment in the solar source region and limited observations. At the present time, there are certain hypotheses on the formation process of MFRs. Some studies indicate that MFRs could exist prior to the eruption. For example, both Fan (2001) and Magara (2004) reported magnetohydrodynamic (MHD) simulation results that a twisted MFR initially formed below the photosphere can partially emerge into the low corona by magnetic buoyancy. While some studies suggest that the presence of pre-eruptive MFRs is not necessary and MFRs could be built up in the corona via magnetic reconnection processes associated with flares (Amari et al., 2003; Moore et al., 2001; Antiochos et al., 1999; Jiang et al., 2021a,b). To understand the physical
processes more precisely for the flare-CME events, extensions of the standard 2D flare model have been proposed to account for much broader ranges of quantitative measurements with three-dimensional (3D) features intrinsic to realistic solar eruptions (Longcope et al., 2007; Aulanier et al., 2012; Priest and Longcope, 2017; Aulanier and Dudík, 2019). For example, quasi-3D models have been developed with a non-vanishing magnetic field component along the axis of the MFR and to illustrate a scenario that sequential reconnection along magnetic polarity inversion line (PIL) forms the MFR in the first place (van Ballegooijen and Martens, 1989; Longcope et al., 2007; Schmieder et al., 2015). This scenario has been widely applied to infer and interpret the magnetic reconnection properties based on the observed flare ribbon morphology (Qiu et al., 2002, 2004, 2010; Hu et al., 2014; Kazachenko et al., 2017; Zhu et al., 2020). From such analyses, Qiu et al. (2004) illustrated that there is a temporal correlation between the magnetic reconnection rate and the acceleration of the CME (considered as the eruptive MFR) in the low corona. Such a correlation has been further established by Zhu et al. (2020) based on a statistical study of ∼60 events. In addition, Qiu et al. (2007) and Hu et al. (2014) showed the correlation between the magnetic reconnection flux and the flux contents of the corresponding ICME/MC flux ropes based on modeling results employing in situ spacecraft measurements. These results support the hypothesis, especially concerning CME-MFRs, that the CME-MFR can be formed by magnetic reconnection during the corresponding flare process. And recent simulation results also indicate clearly that the reconnection flux contributes to
the axial (toroidal) flux of the CME-MFR in the early stage (Jiang et al., 2021a; Inoue et al., 2018).

For the quantitative MFR identification in the solar source region, numerical models can be applied to find MFRs in addition to observations. For the topological analysis of a solar MFR, the 3D magnetic field configuration is commonly obtained through coronal magnetic field extrapolation based on photospheric magnetograms. A number of high resolution extrapolation results employing different numerical schemes has been compared to observations to study the properties of MFRs in the magnetically dominant environment on the Sun (Schrijver et al., 2008; Thalmann et al., 2008; Wheatland and Régnier, 2009; De Rosa et al., 2009; Wiegelmann et al., 2012; Sun et al., 2012; Jiang et al., 2014b; Guo et al., 2016). Among many extrapolation studies, the force-free approximation is commonly adopted for the case of low plasma \( \beta \) (the ratio between the plasma pressure and magnetic pressure) over certain heights in the solar atmosphere above the photosphere (Gary, 2001). Under this assumption, the non-magnetic forces including the inertial force can be ignored for a static and time stationary equilibrium. Therefore, the Lorentz force should be self-balanced, and it should satisfy the equation, \( \mathbf{J} \times \mathbf{B} = 0 \), which means that the electric current density \( \mathbf{J} \) is parallel to the magnetic field \( \mathbf{B} \), with \( \mathbf{J} = \alpha \mathbf{B} \) (\( \alpha \) is the so-called force-free parameter). The simplest case is the potential field when \( \alpha \equiv 0 \). If \( \alpha \) is not zero, there are two situations depending on whether \( \alpha \) is constant or varying in space. One is the linear force-free field (LFFF) for \( \alpha \equiv \text{const} \) and the other is the nonlinear force-free field (NLFFF). Since our interest lies in the
magnetic structure in the ARs on a local scale with high spatial resolution, the most common and practical way to reconstruct the coronal magnetic field is the NLFFF extrapolation method.

There are a variety of numerical methods proposed to reconstruct the NLFFF for an AR from boundary conditions and sometimes pseudo-initial conditions, including the upward integration, Grad-Rubin iteration, MHD relaxation, optimization approach, and so on (see a review by Wiegelmann and Sakurai, 2021). The computation speed and accuracy of different numerical methods can vary significantly given the differences in algorithms and their specific realizations in many aspects. We apply a kind of MHD-relaxation method with a conservation-element/solution-element (CESE) solver (Jiang et al., 2011). The so-called CESE-MHD-NLFFF code (Jiang and Feng, 2013) has been tested by different benchmark cases (Low and Lou, 1990; van Ballegooijen and Mackay, 2007; Metcalf et al., 2008). And it has also been widely applied to the magnetic field extrapolation of realistic solar magnetic field data (Jiang and Feng, 2013; Jiang et al., 2014b; Duan et al., 2017, 2019). As these numerical extrapolation results have gone through a series of quality check by different kinds of metrics and have shown a good agreement between the extrapolated field lines and the observational features like sigmoids, coronal loops and even an elongated quiescent filament, it is practical and promising to use this method to contribute to a quantitative study of CME-MFRs. In addition, we will also combine with the in-situ spacecraft measurements of the interplanetary counterpart to help with the interpretation and validation of our analysis results.
The understanding towards the formation and evolution process of the CME-MFRs will ultimately help us make a definitive and physical connection between the origin of solar MFRs (including the MFR before and after the eruption) and their interplanetary counterparts. Such a connection can be pursued through a quantitative study of MFR's physical characteristics (e.g., magnetic flux, field-line twist, and electric current). Specifically, one critical step is the detailed analysis of available solar observations in order to determine whether an MFR exists or how one can form prior to and during the eruption. Characterization of MFR properties not only plays a major role in understanding the physical mechanisms underlying solar eruption and the subsequent evolution, but also contributes to the improvement of the forecast ability in space weather.

In this chapter, we carry out the coronal magnetic field extrapolation based on the method developed by Jiang and Feng (2013) for two events to obtain 3D magnetic field topology of the AR prior to eruption. In addition to the magnetic field extrapolation, we estimate the possible locations of the MFR’s footpoints prior to eruption and measure a number of magnetic field parameters (including the magnetic reconnection flux derived from flare ribbons) in the corresponding AR during the eruption process from different observations. Then the results from extrapolation and observation are combined to identify whether there is a coherent pre-existing MFR before the eruption and to interpret how such a structure may evolve during and following the corresponding flare and CME eruption process. The magnetic properties of the CME-MFR will be further analyzed and compared with the in situ ICME/MC modeling results which are obtained separately.
This chapter is organized as follows. The two selected events and extrapolation method are introduced in Section 3.2. Then we describe the identification methods and analyze results associated with the MFRs on the Sun in Section 3.3. The magnetic properties of MFRs are estimated quantitatively and presented in Section 3.4 based on results from both the solar source region and in situ modeling. The conclusion is given in Section 3.5.

### 3.2 Events Selection and Extrapolation Method

#### 3.2.1 Events Overview

For the purpose of performing a quantitative study of the CME-MFR, we search for appropriate event candidates from a list of reconstructed MFRs based on photospheric magnetograms before the eruption by Duan et al. (2019). They extrapolated the 3D magnetic field in the active regions for 45 major flare eruption events employing the NLFFF extrapolation code by Jiang and Feng (2013). With a set of criteria similar to those by Jing et al. (2018), all major flares that are above GOES-class M5 and occurred within 45° of the solar disk center from 2011 January to 2017 December are selected. Moreover, we also examine the associations between flare and CME, and between CME and ICME to ensure that there exists a well-established one-to-one connection among a flare, an associated CME and an ICME based on the work by Zhu et al. (2020). Two events are selected for this study as shown in Table 3.1. Both ARs related to the two events located near the disk center. The CMEs associated with the corresponding flares
in these two events have been both observed by SOHO and STEREO spacecraft close to the peak times of the corresponding flares (Vemareddy and Zhang, 2014; Cheng et al., 2015; Cheng and Ding, 2016; Joshi et al., 2017). And the associated ICME/MC events were also observed by the WIND and ACE spacecraft at 1 au, which have been reported by Hu et al. (2021) and Kilpua et al. (2021), respectively.

Table 3.1: Timelines of two Flare-CME-ICME events observed by multiple spacecraft (all times are in UT).

<table>
<thead>
<tr>
<th>Flare peak time</th>
<th>Flare peak time</th>
<th>Magnitude</th>
<th>Location</th>
<th>Location</th>
<th>CME time</th>
<th>CME time</th>
<th>Arrival time</th>
</tr>
</thead>
<tbody>
<tr>
<td>2013-04-11T07:16</td>
<td>2014-09-10T17:45</td>
<td>M6.5</td>
<td>N07E13</td>
<td>N11E05</td>
<td>07:39</td>
<td>07:24</td>
<td>04-14T17:00</td>
</tr>
<tr>
<td></td>
<td></td>
<td>X1.6</td>
<td></td>
<td></td>
<td>17:54</td>
<td>18:00</td>
<td>09-12T22:00</td>
</tr>
</tbody>
</table>

In event 1, an M6.5 flare was produced at ∼ 6:55 UT on 2013 April 11 (N07E13). Then a halo CME appeared in the field of view of SOHO/LASCO after 07:24 UT, and the same CME was also observed simultaneously by both STEREO A and B spacecraft after ∼ 07:39 UT. The corresponding ICME/MC passing Earth was detected about three days later (Hu et al., 2021). Similar examination is conducted for the second event, which started with an X1.6 flare peaking at ∼ 17:45 UT on 2014 September 10 (N11E05). There was also a halo CME following the flare based on the observations from SOHO/LASCO and the coronagraph of STEREO B (data from STEREO A was unavailable for event 2). After two days, the WIND spacecraft encountered the subsequent ICME/MC structure at Earth (Kilpua et al., 2021). Therefore, the connections of CME-
MFRs from the Sun to Earth are well established for these two events. We will mainly present the quantitative study of the CME-MFR before the eruption hereafter.

3.2.2 CESE-MHD-NLFFF Extrapolation Method

The CESE-MHD-NLFFF code solves the MHD momentum equation and the magnetic induction equation iteratively until a stationary magnetic field solution is reached, similar to a magnetofrictional approach. As a special case of the MHD relaxation method, the magnetofrictional method includes an artificial dissipative term \( D(\mathbf{v}) \) to balance the momentum equation with flow velocity \( \mathbf{v} \). Specifically, in the CESE-MHD-NLFFF code, \( D(\mathbf{v}) \) is written in a frictional form \( \nu \rho \mathbf{v} \) (see below, and Jiang and Feng, 2012) along with some modifications in order to utilize the existing CESE-MHD solver. The modified momentum equation and the induction equation are written as (Jiang and Feng, 2012, 2013),

\[
\frac{\partial (\rho \mathbf{v})}{\partial t} = (\nabla \times \mathbf{B}) \times \mathbf{B} - \nu \rho \mathbf{v}, \quad \rho = |\mathbf{B}|^2 + \rho_0. \tag{3.1}
\]

\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) - \mathbf{v} \nabla \cdot \mathbf{B} + \nabla (\mu \nabla \cdot \mathbf{B}). \tag{3.2}
\]

Here the (pseudo) mass density \( \rho \) also contains the constant \( \rho_0 \) for simplicity and is assumed to be largely proportional to \( |\mathbf{B}|^2 \) in order to roughly equalize the speed of the evolution of the entire field with nearly uniform Alfvén speed \( (v_A = \frac{|\mathbf{B}|}{\sqrt{\rho}} \approx \text{const}) \). To enhance the ability of handling noisy data in realistic solar magnetograms, a small value \( \rho_0 \) is added, e.g., \( \rho_0 = 0.1 \) (in the same unit as
$|\mathbf{B}|^2$, to the original pseudo density $\rho$. Two extra terms are added to equation 3.2 to control the divergence of the magnetic field in the induction equation. More details can be found in Jiang and Feng (2012, 2013).

![Figure 3.1](image_url)

**Figure 3.1:** First row (from the left to right panels): the three components $B_x$, $B_y$ and $B_z$ of the raw magnetogram and the derived vertical current density $J_z$ distribution for event 1, AR11719, at 06:36 UT on 2013 April 11. The red box marks the bottom side of a sub-volume. Second row: the corresponding maps from the preprocessed magnetogram. The size of each map is $540'' \times 344''$.

### 3.2.3 Data Preprocessing and Grid Initialization

Passing across the inhomogeneous plasma environment, the plasma $\beta$ could vary from $\beta > 1$ in the photosphere to $\beta \ll 1$ in the low and middle corona, and to $\beta > 1$ again in the upper corona (Gary, 1989, 2001). So the force-free condition may not be always satisfied especially at the photosphere (Metcalf et al., 1995). One way to get a more consistent boundary condition for NLFFF extrapolation is to modify the raw photospheric magnetogram to mimic a force-free chromo-
Figure 3.2: The raw and preprocessed maps for event 2, AR 12158, at 17:00 UT on 2014 September 10. Format is the same as Figure 3.1. The size of each map is 282″ × 266″.

The high-resolution vector magnetograms are routinely observed by Helioseismic and Magnetic Imager (HMI; Schou et al. 2012) onboard SDO. Specifically, the Space-weather HMI Active Region Patches (SHARPs, Bobra et al.
(2014)) vector magnetogram data product, hmi.sharp_cea_720s, provided by SDO/HMI, is used as the input for the extrapolation. The SHARP data series provide maps following each patch of significant solar magnetic field for its entire lifetime and the data are also de-rotated to the disk center and remapped using the cylindrical equal area (CEA) Cartesian coordinates. Photospheric vector magnetograms are included with a cadence of 720 s and a spatial resolution of 0.5′′ (∼0.36 Mm). For the two selected events, vector magnetograms from the ARs at least 10 minutes before the flare onset times (estimated from the soft X-ray measurement of the GOES satellite) are preprocessed to get necessary boundary conditions and derive the initial conditions from a potential field solver for the NLFFF extrapolations. The original magnetograms are rebinned from 0.5′′ per pixel to 1′′ per pixel for the preprocessing procedure and the subsequent extrapolations. The overall smoothing effect is presented in Figures 3.1 and 3.2 as a result of preprocessing for the two events by comparing the raw and preprocessed maps of magnetograms and current density $J_z$ distributions. Random noise is obviously suppressed in the $J_z$ maps. The unsigned vertical flux in $z$ direction of AR11719 in Figure 3.1 is about $1.556 \times 10^{22} Mx$, and about $2.386 \times 10^{22} Mx$ for AR12158 in Figure 3.2. The ratios of the total net magnetic flux to the total unsigned flux are about 0.0747, and 0.0752 for the two events, respectively. The CESE-MHD-NLFFF code does not require exact flux balance on the bottom boundary.

For the consideration of the speed and accuracy of the computation in terms of high-resolution and large field-of-view solar magnetograms, a non-uniform
grid structure within a block-structured parallel computation framework is adopted with the help of PARAMESH software package (MacNeice et al., 2000) for the CESE-MHD-NLFFF code. For the grid initialization, the whole computation domain includes the pre-set main computation region and the surrounding buffer zones to reduce the influence of the side boundaries (Jiang and Feng, 2013) since the magnetic field at these numerical boundaries are simply fixed as their initial values (i.e., those of the potential field). Then the whole computational domain is divided into blocks with different spatial resolutions and all blocks have identical logical structures which are evenly distributed among processors. As we vary the grid size only in height (the $z$ dimension) for this study, the grid resolution matches the resolution of the magnetogram at the bottom boundary and decreases by four times at the top of the computational domain. After the grid initialization, the initial condition of the whole computation domain is assigned by a potential field solution derived from the input magnetogram by using the Green’s function method (Chiu and Hilton, 1977; Metcalf et al., 2008).

### 3.2.4 Convergence Study and Extrapolation Metrics

Before we start our analysis for the two events, we also examine the relaxation process by several metrics to obtain converged extrapolation results. These include the residual of the field between two successive iteration steps, the force-freeness of the numerical result $CWsin$, the divergence-free condition $⟨|f_i|⟩$ (Wheatland et al., 2000; Metcalf et al., 2008) and the total magnetic energy $E_{tot}$ (see their definitions in the Appendix).
Figure 3.3: Two sets of convergence metrics of two extrapolation runs for AR11719 (event 1, left column) and AR12158 (event 2, right column). The top three rows show the metrics of $CWsin$, the total magnetic energy $E_{tot}$ (arbitrary unit), and $\langle |f_i| \rangle$, respectively. Panels in the bottom row show the evolution of residuals for the two events.
For event 1, we carry out the extrapolation based on the whole SHARP vector magnetogram (540″ × 344″) and then calculate the convergence metrics for every 200 iteration steps as shown in the first column of Figure 3.3 with the finest grid size 1″. As shown in Figure 3.3, the residual goes through a gradual increase before ∼ 6500 iteration steps because the bottom boundary condition drives the system away from the initial potential field (Jiang and Feng, 2012). Even though obvious fluctuations appear after the initial driving process, the overall trend of the residual toward the end is decreasing, accompanied by small oscillations. After ∼ 30,000 iteration steps, the residual is reduced to ∼ 10^{-5} and still maintains a declining trend. Other metrics also display a trend with little variation and both CWsin and ⟨|f|⟩ reach relatively small values. Thus this extrapolation result can be considered as a converged solution. It is noticed that there are some oscillations in the convergence process, which may be caused by the broad distribution of the weak field and random noise from the input magnetogram. The total computation time (to converge until 40,000 iteration steps) took about 95 hours with 19 cores on a 24-core local desktop with 48 GB memory.

For event 2, the size of the SHARP magnetogram is 282″ × 266″. One run is carried out with the smallest grid size 1″ and the full size magnetogram. The second column of Figure 3.3 shows a smooth convergence process. The residual converges very fast after an initial rise exceeding 10^{-4} to an order of magnitude smaller, < 10^{-5}, within 11,000 iterations. All the other metrics show clear monotonic decreases and stabilize after ∼ 11,000 iterations, which is consistent with an
optimal convergence pattern in the previous tests of this code (Jiang and Feng, 2012; Jiang et al., 2012; Jiang and Feng, 2013). This convergence process is relatively smooth without any spurious oscillations, so a final solution with good indication of convergence is readily obtained for subsequent analysis. It took about 23 hours with 19 cores on the same local desktop for the extrapolation result to converge (after 20,000 iteration steps).

Table 3.2: NLFFF Extrapolation Metrics of Force-freeness and Divergence-freeness for Two Active Regions.

| AR           | CWsin  | ⟨|f_i|⟩   | Ev×B     | Ev·B     |
|--------------|--------|----------|----------|----------|
| AR11719 Full Volume | 0.449  | 2.59 × 10^{-4} | 0.195    | 3.38 × 10^{-2} |
| AR11719 Sub Volume   | 0.384  | 4.56 × 10^{-4} | 0.177    | 2.61 × 10^{-2} |
| AR12158 Full Volume  | 0.368  | 3.81 × 10^{-4} | 0.161    | 4.05 × 10^{-2} |

To further check the quality of force-freeness and divergence-freeness of the converged extrapolation results, calculations of metrics like CWsin and ⟨|f_i|⟩ for two events are also shown in Table 3.2. The CWsin values for two ARs are around 0.3 ∼ 0.5, which are much larger than the values obtained from previous tests for ideal benchmark solutions (which are less than 0.1, Jiang and Feng 2013) but are similar to many other reported NLFFF extrapolations results for realistic magnetograms. The typical range for CWsin is from 0.1 to 0.5 (see, e.g., De Rosa et al. 2009; DeRosa et al. 2015; Jiang and Feng 2013; Kawabata et al. 2020). It should also be noted that although CWsin ≪ 1 indicates a force-free field, large CWsin value does not mean the opposite (Jiang et al. 2012; Malanushenko et al. 2014), considering that this is one weighted average over the whole computational domain. In the current-free region, J = ∇× B may be non-
zero due to the numerical finite differences. In addition, small-scale structures in the magnetograms (where the magnetic field strength is usually small) may also increase the $CWsin$ value as the derived currents might not be small and have random orientations. Here we add two additional metrics $E \nabla \times B$ and $E \nabla \cdot B$ to evaluate the force-freeness and divergence-freeness by analyzing the residual force for a chosen volume (see descriptions in the Appendix). Considering the broad distribution of the weak field in event 1, the metrics for a sub-volume with strong magnetic field as marked by the red box in Figure 3.1 are also derived. All the metrics are very close to the previously reported results for the CESE-MHD-NLFFF extrapolation of different ARs (Jiang and Feng, 2013; Duan et al., 2017).

3.3 Characterization of MFRs on the Sun

3.3.1 MFR Identification Method

Both extrapolation and observation results are critical for the MFR identification on the Sun. As for the observational analysis of MFRs, we analyze the data from Atmospheric Imaging Assembly (AIA, Lemen et al. 2012) and HMI on board the SDO spacecraft to study the evolution of the corresponding flares for the two events. SDO/AIA provides full-disk images in 7 extreme ultraviolet (EUV) and 2 ultraviolet (UV) wavelength channels with a high spatial resolution (0.6" per pixel and a total of $4096 \times 4096$ pixels per image) and a moderate time cadence (12 s in EUV channels and 24 s in UV channels).
To provide additional support for the MFR identification and characterization of the corresponding CME-MFRs at a different stage besides the extrapolation, we also analyze the evolution of flare ribbons and the corresponding magnetic reconnection properties. Flare ribbons map the footpoints of reconnected field lines. Magnetic reconnection beneath the erupting MFR forms flare loops, and the same amount of reconnected magnetic flux is injected into the MFR. Reconnection may also take place between the erupting MFR and the ambient magnetic field, although this is not the main focus of this study. Therefore, magnetic reconnection flux associated with flare ribbons is useful to establish a quantitative connection between MFRs on the Sun (both before and after the flare eruption) and their interplanetary counterparts. The amount of accumulative magnetic reconnection flux can be measured by summing up the magnetic flux in newly brightened UV pixels within flare ribbons. In this study, we employ 1600 Å data from SDO/AIA and vector magnetograms from SDO/HMI to measure the magnetic reconnection flux and magnetic reconnection rate from the brightening pixels, following the automated approach developed by Qiu et al. (2002, 2004). The brightening ribbon pixels are chosen when the intensity of a pixel is greater than 4, 5 or 6 times the median intensity which is fixed and determined from the average of a 6-minute time period (for a region of interest before the eruption). These threshold values are used to distinguish flare brightening from the plage emission. A flare ribbon pixel also should stay bright for at least 4 minutes, which helps minimize effects due to saturation or projection of bright coronal ejecta. The reconnection flux quoted in the table is the average of
the measurements using these three thresholds and in the positive and negative
magnetic fields, and the standard deviation of these measurements is quoted as
the measurement uncertainty. The discussion of the measurement method and
uncertainties has been given in Qiu et al. (2007, 2010) and more recently in Naus
et al. (2022).

While an MFR is generally considered as a group of coherent winding field
lines with both ends rooted on the photosphere before eruption, it has not been
quantitatively defined in a universal way. Identifying a coherent MFR based on
the reconstructed coronal magnetic field derived from the real magnetogram can
be difficult, given the complex magnetic topology. Liu et al. (2016) suggested
that the magnetic twist number $T_w$ can serve as a good proxy for finding the axis
of an MFR. The twist number $T_w$ measures how many turns two infinitesimally
close field lines wind about each other (see Berger and Prior 2006), and is defined
by

$$T_w = \int \frac{\mu_0 J_{\parallel}}{4\pi B} dl = \frac{1}{4\pi} \int_L \left( \nabla \times B \right) \cdot B dl,$$

$$= \frac{1}{4\pi} \int_L \alpha dl, \text{ if } \nabla \times B = \alpha B. \tag{3.3}$$

Here $\alpha$ is the force-free parameter and the integral is taken along one magnetic
field line with path length $L$, starting from one end point of the field line on
the boundary to the other. For both events, extrapolation results are generated
utilizing magnetograms that are chosen at least 10 minutes before the flare onset
times. We calculate the twist number $T_w$ at each grid point in the whole volume
with the same grid size as the resolution of the input magnetogram, i.e., $1''$. Then
we start the topology analysis with the definition of Liu et al. (2016) that an MFR has a bundle of field lines spiraling around the same axis or each other by more than one turn (|T_w| ≥ 1, see also Duan et al. 2019). Combined with the field-line topology, one may also require that such constrained MFR volume be a single tube without multiple bifurcations. However in reality, such bifurcations are common, which often indicate that the pair of identified footpoint regions with positive and negative polarities may not contain the same amount of magnetic flux. In other words, the field lines originating from the positive polarity region may not all end in the corresponding conjugate negative polarity region. In addition, the footpoints of MFRs should be restricted within or close to main flare ribbon areas identified from AIA observations, given the general relation between the magnetic reconnection process during flares and the formation of erupting CME-MFRs (Moore et al., 2001; Qiu et al., 2004; Qiu, 2009; Zhu et al., 2020).

3.3.2 Results for AR 11719

For event 1 in AR 11719, simultaneous observations of the flare’s time evolution in SDO/AIA 94, 131 and 1600 Å wavelength channels before and during the flare eruption are given in Figure 3.4. From the EUV observations in 94 Å and 131 Å, some curved sigmoidal structures are present near the center before the flare eruption, which were recognized as hot channels in Cheng and Ding (2016). Such an S-shaped sigmoidal structure before and during the flare eruptions has been widely found and discussed (Rust and Kumar, 1996; Sterling and Hudson,
Figure 3.4: Observations from SDO/AIA in 94 Å, 131 Å and 1600 Å (from top to bottom row) wavelength channels at three different times as marked in each panel (from left to right) of AR11719 for event 1. Contours of flare ribbons in red as observed in 1600 Å are also overlaid on 94 Å and 131 Å plots which are observed at the same times.

But a sigmoid-like structure based on emission-line images does not necessarily yield a similarly continuous magnetic field-line configuration (Titov and Démoulin, 1999; Schmieder et al., 2015; Cheng and Ding, 2016; Duan et al., 2017), i.e., that of an MFR. Instead, these brightened features may correspond to groups of short sheared arcades which are discontinuous, based
on the extrapolation result as to be demonstrated below. In the bottom panels of Figure 3.4 for the 1600 Å UV observation, there is a typical flare morphology with two brightening ribbons lying nearly in parallel with each other, expanding and then drifting away from each other during the time evolution. The contours of flare ribbons coincide with the curved brightening structures in 131 Å observation at the central region, especially towards the “hooked” ends, which gives us a rough estimation of possible positions for the MFR footpoints for this event.

The time evolution of the flare ribbons in the corresponding SDO/HMI magnetograms which are remapped to the sub areas in SDO/AIA’s field of view is shown in Figure 3.5, the left column. Besides, we add the X-ray flux measurement of the whole Sun provided by the GOES satellite for the wavelengths of soft X-ray (1 - 8 Å) during the same time period in the right column, together with the concurrent measurements of accumulative magnetic reconnection flux and magnetic reconnection rate by the approach of Qiu et al. (2002). This M6.5 flare eruption starts at ∼06:55 UT according to the rapid change of the soft X-ray flux curve, which is consistent with the onset of the magnetic reconnection flux increase shown in the second panel in the right column. Based on the average of the total unsigned magnetic flux in each enclosed ribbon area with one dominant polarity (either positive or negative, Kazachenko et al. 2017), the final accumulative magnetic reconnection flux reaches the magnitude of $17 \pm 2.8 \times 10^{20}$ Mx after the eruption. Given the association between the magnetic reconnection flux and the flux content of the corresponding ICME/MC flux ropes (e.g., Qiu et al., 2007; Hu et al., 2014), such flux measurement from flare ribbons can be helpful for
Figure 3.5: Left column: observation of flare ribbons in 1600 Å passband (top panel) and the time evolution of flare ribbons overplotted on the co-aligned HMI magnetogram for event 1 (bottom panel), where the areas swept by flare ribbons are colored by the elapsed time in minutes as denoted by the colorbar. The thick red curve lying in the middle of the lower panel marks the PIL. Right column: GOES soft X-ray (1-8 Å) flux measurement for the flare in event 1 (top panel), the magnetic reconnection flux measured from 1600 Å observation (middle panel) for both positive (red) and negative (blue) flux measurements with uncertainty limits based on different background removal criteria, while the unsigned mean flux is shown in black with the standard deviation represented by the errorbars, and the corresponding magnetic reconnection rates for event 1 (bottom panel). The dashed lines in all three panels indicate the flare onset time of event 1.

making further connections of MFRs on the Sun and their in situ counterparts, as to be laid out in Section 3.4.
Figure 3.6: Identification of an MFR for event 1: (a) the overall field line configuration superimposed on an AIA 94 Å image which is observed at 06:36 UT, (b) field line groups identified from the extrapolation result based on the criterion of $|T_w| > 1$ overplotted on an AIA 1600 Å image observed at 06:59 UT, (c) the field lines of the identified MFR, and the underlying PIL in red drawn over the corresponding line-of-sight HMI magnetogram, and (d) the same as (c) except for the additional superimposed flare ribbons, which are color coded by elapsed time in the same way as Figure 3.5.

In Figure 3.6(a), some sample field lines are drawn over the corresponding AIA 94 Å image to give a qualitative comparison between the extrapolation result and the observation. Several loop structures are recovered overlapping with selected field lines, and a set of twisted field lines lying around the PIL takes the shape resembling the middle of the inverse “S” sigmoid as seen in 94 Å channel (see also Figure 3.6(c)). Among the comparisons with the 1600 Å observation in Figure 3.6(b), footpoints of the twisted field lines locating close to the flare...
ribsns are associated with grid points with negative $T_w$ values. On a plane near the bottom layer (at $z = 2''$ above the photosphere), we pick all the points with $T_w \leq -1$ around the central sigmoidal structure, and plot field lines passing through this set of seed points. We eliminate open field lines which only have one end point attached to the bottom boundary thus not “closed”, and also ill-defined $T_w$ values. As a result shown in Figure 3.6(b), four groups of field lines are distinguished starting with the selected seed points. Compared to the locations of the flare ribbons, three groups of field lines are excluded since a part of their footpoints extends out of the ribbon sites. Therefore the remaining bundle of field lines shown in Figure 3.6(c) is identified to be the most likely candidate MFR for the 2013 April 11 event before the flare eruption. After determining the MFR, we can find the axial field line with the maximum $|T_w|$ of the MFR. The axis of the identified MFR in event 1 possesses $T_w = -1.5$ which lies close to the bottom boundary and reaches a maximum height at $z \sim 19''$. The time sequence of flare ribbons after 6:42 UT is then co-aligned with the bottom boundary magnetogram and overplotted in the usual way, color coded by elapsed time in Figure 3.6(d). It shows that two groups of identified MFR footpoints locate on the opposite sides of the flare ribbons near the far ends, consistent with the scenario proposed by Moore et al. (2001). If the MFR is strictly confined to be the field lines shown here with two and only two opposite-polarity ends all rooted on the photosphere, then its footpoints are just part of the regions identified based on the $|T_w|$ threshold conditions as we described earlier in Section 3.1.
Figure 3.7: The enlarged and side views of the identified MFR in Figure 3.6(b) for the criteria of (a) $|T_w| > 1$, and (b) $|T_w| > 0.8$, respectively, and similarly for Figure 3.6(c): the distribution of $|\mathbf{J}|/|\mathbf{B}|$ as indicated by the colorbar on a vertical slice across the identified MFR for $|T_w| > 1$ with the MFR field-line intersection points colored by the corresponding values according to the colorbar, (d) same as (c) but overplotted with the MFR field lines, and (e-f) the distributions of the squashing degree $Q$ on the same slices as (c) and (d).

To further confirm the existence of the MFR, we also check different topological properties from a side view. In Figure 3.7, the coherent MFR structure is still maintained with a different $|T_w|$ threshold as seen in Figure 3.7(a) and 3.7(b). The distribution of the quantity $|\mathbf{J}|/|\mathbf{B}|$ as a proxy to current density is displayed in Figure 3.7(c) and 3.7(d) on a cross section plane nearly perpendicular to the MFR. The current density $|\mathbf{J}|$ itself also shows a similar distribution. Based on the current density distribution at the intersections between the identified MFR field lines and the vertical slice in Figure 3.7(c), the flux rope goes through a region with relatively high current density. The geometric boundary of an MFR can also be estimated by the location of a quasi-separatrix layer (QSL), a very thin layer
where there is a strong gradient of the field line connectivity (Demoulin et al., 1996). Such a feature is typically defined mathematically by the high squashing factor $Q$ (Titov et al., 2002; Vemareddy, 2021). As shown in Figure 3.7(e) and 3.7(f), the identified group of field lines with small $Q$ values is surrounded indeed by a clear boundary with high squashing degree $Q$.

**Figure 3.8:** Observations from SDO/AIA in 94 Å, 131 Å and 1600 Å (from top to bottom row) wavelength channels of AR12158 for event 2. The format is the same as Figure 3.4.
3.3.3 Results for AR 12158

Observations of the flare ribbon evolution before and during the flare eruption for event 2 in AR 12158 are shown in Figure 3.8. Event 2 also exhibits a two-ribbon flare morphology, with two ribbon areas co-located near the two ends of an inverse “S” shape sigmoidal structure. The southward ribbon has a more dominant swept area in size than the other. Similarly, we use the overlapping regions between the flare ribbon areas and the curved brightening sigmoidal structures in 131 Å observation to approximate the possible locations of MFR footpoints in this event.

In Figure 3.9, we show the same set of panels for event 2, as in Figure 3.5. We find that the initial enhancement of the X-ray flux is earlier than the significant increase of the magnetic reconnection flux. After a slow rise phase with small reconnection rate, a strong flare is produced quickly after $\sim 17:20$ UT. The final accumulative magnetic reconnection flux reaches the magnitude of $47 \pm 7.5 \times 10^{20}$ Mx for event 2. The flare ribbon morphology still exhibits general features for a “two ribbon” flare, albeit asymmetry of the spatial distributions is more pronounced, indicating perhaps more significant deviation from a “standard” 2D geometry.

For event 2, the configuration of magnetic field lines from the extrapolation result has a good visual correspondence with the AIA observations as shown in Figure 3.10(a) and 3.10(b). There is a clear inverse “S” sigmoid structure near the center imaged by AIA 94 Å passband. However, the core field in our extrapolation
result mainly consists of several groups of sheared arcades structure over-arched by higher coronal loops, rather than one continuous inverse “S” structure (see also Duan et al. 2017). Based on the long-term evolution before the eruption, Cheng et al. (2015) found that there was a central sigmoid structure initially appearing in the AIA 94 Å passband at \( \sim 15:10 \) UT and then it had gone through repetitive disappearance and re-appearance processes. So they suggested that a nascent MFR was under formation prior to the major eruption by tether-cutting
reconnection. After \( \sim16:55 \) UT, the sigmoid develops quickly and produces an X1.6 flare and a CME. In order to find a possible MFR structure prior to the flare, we take a look at the \( T_w \) distribution and find that the majority of the core field region has a negative and relatively small twist number such that \(|T_w| < 1\). This indicates the absence of a twisted coherent MFR according to the criterion we are using (Liu et al., 2016; Duan et al., 2019). In Figure 3.10(c) and 3.10(d), we show the isosurfaces of \( T_w = -1 \) and \( T_w = -0.8 \) in the central volume. There is no coherent structure under the \( T_w = -1 \) criterion, though several coherent structures appear for a lower threshold in magnitude \( T_w = -0.8 \). Comparing these field line bundles with the locations of the flare ribbons, there are two coherent weakly twisted field line groups as presented in Figure 3.10(e). The left group has a maximum \(|T_w| \sim 0.82\) but extends to a relatively far away location from the ribbon (also to a height of \( z \sim 50'' \)). It also appears to be nearly perpendicular to the local PIL. Another group of field lines lies close to the bottom boundary and has a maximum \(|T_w| \sim 0.97\), which still, strictly speaking, fails to satisfy the MFR criterion. In addition, the current density distribution in Figure 3.10(f) at the intersections of a vertical slice with two field line bundles also shows less clearly-defined concentrations in those field-line regions.

The NLFFF extrapolation result by Zhao et al. (2016) demonstrated the existence of a strongly twisted MFR, which used magnetograms at different times from ours as the input and an alternative method to process the boundary conditions. Kilpua et al. (2021) used a data-driven magneto-frictional method that was driven by several days of time-varying magnetograms, and thus the underlying
Figure 3.10: Magnetic field topology analysis for event 2: (a-b) selected field lines superimposed on the $B_z$ map, and an AIA 131 Å image at 17:00 UT, respectively, the isosurfaces of (c) $T_w = -1$ and (d) $T_w = -0.8$ over the background of the AIA 1600 Å observation at 17:23 UT, (e) selected field line bundles based on the threshold condition $|T_w| > 0.8$ with a composite background of $B_z$ map and color-coded flare ribbons (the same as the bottom left panel in Figure 3.9), and (f) the distribution of $|J|/|B|$ on a vertical slice intersecting two field line bundles in (e).

Model and approach are different from ours. To obtain the MFR topology, they also needed to adjust the free parameters related to the injection of twist at the bottom boundary. On the other hand, Liu et al. (2018) and Shen et al. (2022) also found that there is no twisted MFR but two J-shaped sheared arcades and overlaying arcades based on the line-of-sight photospheric magnetogram before the flare using the flux rope insertion method, which is consistent with our result. So it is generally understood that different studies can yield different results owing to different numerical methods and boundary conditions employed. One unique
aspect in our study is that we intend to compare with the additional analysis of the interplanetary counterparts of the CME-MFRs based on in-situ measurements, which provides support for the physical interpretations we present and the reliability of our results.

3.4 Estimation of magnetic properties of MFRs

After the identification of MFRs on the Sun for the two events, we look further into the magnetic properties of MFRs at different stages or locations and try to find a potential correlation among them. For example, the total magnetic flux (generally considered conserved) is one of the most important quantitative properties of MFRs that can be measured or derived to make a connection between CME-MFR and the corresponding ICME/MC (Qiu et al., 2007; Hu et al., 2014).

Specifically, for the identified pre-existing MFR in event 1, we give below a quantitative description of its magnetic properties. The axial magnetic flux enclosed by the pre-existing MFR’s footpoints is calculated for further analysis. Regions of footpoints are obtained by extracting the intersection points between field lines from the MFR and a slice parallel to the bottom boundary (photosphere). We choose a slice at a height of 1.0″ where two well-separated groups of footpoints are obtained. In general, more grid points are included under a smaller $|T_w|$ threshold value for the MFR criterion, i.e., more points with $|T_w|$ exceeding such a value. Here we denote the region dominated by the positive magnetic field in the MFR footpoints as ‘$FP_+$’ and the region taken up mainly by the negative magnetic field in the MFR footpoints as ‘$FP_-$’. The total axial (or toroidal) mag-
Table 3.3: Magnetic properties of the footpoint (FP) regions associated with an MFR in AR11719 for event 1 with different criteria at $z = 1.0''$: $\Phi_z$, the sum of normal magnetic flux over all associated grids for each group of identified footpoints; the fourth column shows the number of grids on the chosen slice containing the identified footpoints; $\langle B_z \rangle$, the average vertical magnetic field for each group of footpoints; $\langle J_z \rangle$, the average current density in the $z$ direction for each group of footpoints, $J_z = \frac{\langle \nabla \times B \rangle_z}{\mu_0}$, and $I_z = \sum (J_z dS)$, the total current in the $z$ direction for each group. (+): positive flux; (−): negative flux.

| $|T_w|$ | $\Phi_z$ (10$^{20}$ Mx) | # of Grids | $\langle B_z \rangle$ (G) | $\langle J_z \rangle$ (10$^{-3}$ A·m$^{-2}$) | $I_z$ (10$^{10}$ A) |
|-------|-----------------------------|-------------|---------------------------|---------------------------------|------------------|
| $|T_w| > 0.8$ | FP$_+$ (+) 0.510 | 165 | 58.2 | -0.79 | -6.9 |
|       | FP$_+$ (−) 0 | | | | |
|       | FP$_-$ (+) 0 | 64 | -394 | 5.8 | 20 |
|       | FP$_-$ (−) -1.34 | | | | |
| $|T_w| > 0.9$ | FP$_+$ (+) 0.385 | 114 | 63.6 | -0.60 | -3.6 |
|       | FP$_+$ (−) 0 | | | | |
|       | FP$_-$ (+) 0 | 48 | -367 | 5.8 | 15 |
|       | FP$_-$ (−) -0.934 | | | | |
| $|T_w| > 1.0$ | FP$_+$ (+) 0.283 | 68 | 78.5 | -0.19 | -0.68 |
|       | FP$_+$ (−) 0 | | | | |
|       | FP$_-$ (+) 0 | 35 | -343 | 5.8 | 11 |
|       | FP$_-$ (−) -0.638 | | | | |

Magnetic flux for both regions is calculated based on $\Phi_z = \iint B_z dS$, where $B_z$ is the vertical magnetic field component. After two well-separated groups of footpoints are obtained, the integration is estimated by summing up the magnetic flux from all grid points (pixels) within the identified footpoints regions on the slice.

Table 3.3 shows the result of flux calculations of the identified MFR. The differences of $\Phi_z$ between $FP_+$ and $FP_-$ are within one order of magnitude for different $T_w$ criteria, though they get smaller for a larger $|T_w|$ threshold. Given...
Table 3.4: Summary of magnetic properties for the MFRs in two events.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Source Region Results</th>
<th>in situ Modeling Results$^a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(all flux in $10^{20}$ Mx)</td>
<td>2D</td>
<td>3D</td>
</tr>
<tr>
<td><strong>Event 1: 2013-04-11</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Axial flux $\Phi_z$</td>
<td>0.3 - 1.3 $^b$</td>
<td>5.7</td>
</tr>
<tr>
<td>Twist $\tau$</td>
<td>$\sim 1.5^b$(axis)</td>
<td>1.6 /au</td>
</tr>
<tr>
<td>Reconnection flux$^c$</td>
<td>$17 \pm 2.8$</td>
<td>...</td>
</tr>
<tr>
<td>Poloidal flux</td>
<td>...</td>
<td>9.2 /au</td>
</tr>
<tr>
<td><strong>Event 2: 2014-09-10</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Axial flux $\Phi_z$</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>Twist $\tau$</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>Reconnection flux$^c$</td>
<td>$47 \pm 7.5$</td>
<td>...</td>
</tr>
<tr>
<td>Poloidal flux</td>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>

$^a$ In situ modeling results of magnetic clouds cited from Hu et al. (2021). For the 3D model, the poloidal flux is approximated by $\tau \Phi_z$.

$^b$ Parameters for identified “pre-existing” MFR only.

$^c$ The reconnection flux is estimated based on Figures 3.5 and 3.9.

that the largest $\Phi_z$ in magnitude is still significantly smaller than the reconnection flux measured from the flare ribbons after the eruption, especially for the typical criterion $|T_w| > 1.0$, we believe that the MFR found from the extrapolation is likely a seed MFR before the eruption. It should be noted that the difference in fluxes for the positive and negative footpoint regions results exists because the field lines originating from the footpoints in one region as we identified mainly based on the $|T_w|$ threshold condition do not all end in the other polarity region. In this case, the flux is larger in the negative footpoint region than the positive region since one group of footpoints ($FP_-$) locates closer to one main negative polarity of the magnetogram than the other group of footpoints ($FP_+$) to any main positive polarity. And $FP_-$ takes up a rather smaller area compared to
\(FP_+\), but the latter has much smaller average magnetic field \(\langle B_z \rangle\), current density \(\langle J_z \rangle\) and total current \(I_z\).

A summary of quantitative results for MFRs in two events is given in Table 3.4. The corresponding in situ modeling results for the two events are provided by Hu et al. (2021) by applying two magnetic cloud reconstruction methods. One of the modeling methods is the Grad-Shafranov reconstruction technique yielding a 2D configuration of the MFR (Hu and Sonnerup, 2002; Hu, 2017). The other method is the optimization approach based on a more general linear force free formulation to obtain a more complex quasi-3D structure (Hu et al., 2021; Hu et al., 2021). For event 1, the twist of the MFR identified in the source region is relatively consistent with the in situ modeling results of the MFR structure, considering the uncertainty of the total twist numbers. The axial magnetic flux calculated from the in situ modeling results is significantly larger than the seed MFR identified in the source region before the eruption, while the reconnection flux measured from the source region after the eruption is generally larger than the axial flux from the in situ modeling results. The poloidal magnetic flux, obtained from the in situ modeling results, appears to agree better with the reconnection flux, subject to the uncertainty in the axial length. Specifically if one assumes an axial length \(\in [1, 2]\) au typically (see Hu et al. 2015) for an MC flux rope, this amounts to the total poloidal flux in the range \(9.2 - 18 \times 10^{20}\) Mx for event 1, based on the 2D MC modeling result.

For event 2, the axial (toroidal) flux content from the in situ 3D model agrees with the reconnection flux within their respective uncertainty ranges.
Other parameters in the source region are not available (marked by “...”) since we did not find a pre-existing MFR structure before the eruption. The 2D MC model also failed to yield an acceptable solution. The twist of the MFR from the in situ modeling is generally larger than the twist we found in the groups of field lines in Figure 3.10 (the maximum $|T_w| \sim 0.97$). The CME-MFR containing significant amount of flux was likely formed during the eruption through dynamic evolution process in the solar atmosphere. Recently it was demonstrated by unique observational analysis and data-inspired numerical simulation (Xing et al., 2020; Jiang et al., 2021a) for the “increase-to-decrease” behavior in the toroidal flux of the CME-MFR. However the applicability of such analysis to our events is beyond the scope of the current study. To study such a process usually requires discerning multiple flux systems with complex and constantly evolving topologies. And it remains a challenge to separate the toroidal and poloidal flux contents from the reconnection flux, although one pioneering approach developed by Qiu (2009) for detailed analysis of the reconnection sequence can help in future studies.

3.5 Conclusions

In this paper, we have identified the MFR structures in the solar source regions and established the connection between MFRs on the Sun and their in situ counterparts quantitatively for two selected events. One event began on 2013 April 11 (event 1, AR 11719) and the other on 2014 September 10 (event 2, AR 12158), respectively. Each event exhibits a sequence of flare, CME and the corresponding ICME observed by multiple space-borne instruments. We perform
coronal magnetic field extrapolations for each AR by the CESE-NLFFF-MHD method, which utilizes the preprocessed photospheric magnetograms and the results are also examined through a set of convergence metrics. Remote-sensing observations from SDO are analyzed to find evidence of MFRs and trace the evolution of the associated flares. Specifically we measure the amount of magnetic reconnection flux by analyzing the temporal and spatial evolution of flare ribbons. We combine the extrapolation results with observations to identify MFRs on the Sun before the eruption and estimate their magnetic properties. The main results of our study are summarized as follows.

1. Observational evidence of MFR footpoints and associated magnetic reconnection flux during the flare eruption are inferred from multi-wavelength observations. From the comparison of EUV observations, there are signs of MFRs for the two events. Based on the flare ribbon measurements, the total magnetic reconnection flux reaches $17 \pm 2.8 \times 10^{20}$ Mx for event 1, and $47 \pm 7.5 \times 10^{20}$ Mx for event 2, respectively, which corresponds to the amount of available flux to be injected into the final CME-MFRs.

2. From the combination of extrapolation and observation results, a coherent MFR structure before the flare eruption is identified for event 1. However, there is no pre-existing MFR found for event 2, based on the same set of MFR criteria, including the requirement for the field-line twist number $|T_w| > 1.0$ and also both regions of the MFR field-line footpoints close to the main flare ribbons. For event 1, a coherent pre-eruption MFR is determined,
which carries a maximum $T_w = -1.5$ and its two ends are located near the opposite ends of the respective flare ribbons across the PIL.

3. The magnetic properties of MFRs on the Sun are summarized and compared with their corresponding in situ modeling results from Hu et al. (2021) in Table 3.4. For event 1, the axial magnetic flux from in situ modeling results is in the order of $10^{20} \sim 10^{21}$ Mx, while the total magnetic reconnection flux after the eruption from the source region is in the order of $\sim 10^{21}$ Mx. Both are significantly larger than the flux in the identified pre-existing MFR’s footpoints area which is in the order of $10^{19} \sim 10^{20}$ Mx.

4. For event 2, there is no pre-existing MFR identified. The amount of the magnetic reconnection flux, $47 \pm 7.5 \times 10^{20}$ Mx, agrees with the corresponding ICME MFR toroidal flux, $\sim 16 - 81 \times 10^{20}$ Mx, within the limits of the uncertainty ranges.

These results for the two events indicate the dynamic and complex nature of the MFR formation during its evolution process while some properties (like magnetic flux and twist) are useful for making connections between the formation of MFRs on the Sun and their in situ characteristics in a quantitative manner. Based on these quantitative results, we conclude that the magnetic reconnection process, manifested during solar flares, injects significant amount of magnetic flux into the ensuing CME-MFR. For event 1, the identified pre-existing (or pre-eruption) MFR from the NLFFF extrapolation is likely a seed MFR before the eruption and additional flux is injected through the magnetic reconnection
process associated with the flare. Furthermore, based on the comparison among various inter-related magnetic flux contents and the corresponding flare ribbon morphology for each event, we conclude that for event 1, a quasi-2D configuration of the MFR is largely valid for which the poloidal flux is more meaningfully defined and compared more favorably with the corresponding reconnection flux than the axial flux. For event 2, however, we believe that the MFR topology deviates more from a 2D configuration, but is better described by a quasi-3D model for which the axial flux agrees with the reconnection flux. In this case, the poloidal flux is not readily defined geometrically because there does not exist a straight field line representing the central axis of a flux rope (see Hu et al., 2021). Therefore, for the 3D model, we choose to approximate the poloidal flux by the product of the average field-line twist and the axial flux (see, e.g., Hu et al., 2014).

This study represents an effort to make a physical connection between a solar MFR (including the MFR before and after the eruption) and the corresponding ICME/MC by the quantitative comparison of the magnetic properties under different scenarios through extrapolations and observations. It is usually not easy to envisage the existence of a coherent pre-existing MFR, reconstruct it before the eruption in the solar source region for a CME event and make one-to-one connection with its interplanetary counterpart. Efforts have been made continuously on the quantitative description of the MFR configuration with more advanced observations and improved numerical simulation techniques, which is helpful for further understanding the formation and evolution processes of the CME-MFRs. Future studies including more events (see the identified candidate event list in Appendix
B) will be carried out for a deeper understanding of CME-MFRs, where improvements to the extrapolation method and use of high-resolution ground-based data can be implemented.
Chapter 4. Updated CESE-MHD-NLFFF Code for Nonuniform Magnetograms

With the development of the new techniques and instruments, observations from multiple telescopes in different resolutions and fields of view (FOVs) available for the same solar eruption event gradually become more common. Therefore, it is important to accommodate multi-instrument observations to the existing method, i.e., the photospheric magnetograms from both space-based and ground-based telescopes. In order to take advantage of both the ultra high-resolution magnetograms with a small FOV from large-aperture ground-based telescopes (like GST) and the magnetograms in a larger FOV with a relatively lower resolution from another telescope (like SDO), we propose an updated version of the CESE-MHD-NLFFF code to be described below.

4.1 Grid Construction for the Updated CESE-MHD-NLFFF Code

Compared to the original CESE-MHD-NLFFF code for a set of uniform vector magnetograms as described in Chapter 2, this updated code is mainly updated to adjust the associated interfaces for the nonuniform embedded magnetograms, which will serve as the bottom boundary condition (BC) for the NLFFF extrapolation. To adopt such a nonuniform bottom BC, we redesign the grid
structure for the whole computational domain, divide the whole computational
domain into multiple regions, and modify the associated initialization process
based on the embedded magnetograms.

The construction of nonuniform spatial grids is mainly achieved by the re-
finement process in the PARAMESH package (MacNeice et al., 2000) for all three
dimensions. The block structures distributed in the computational domain can be
partly refined or de-refined. When refining a block structure, the original parent
block will be divided into eight child blocks in 3D space with the grid size cut by
a half. Such structured blocks thus provide flexibility for embedding nonuniform
magnetograms as bottom BCs in the CESE-MHD-NLFFF code. Specifically, it
should be noted that there is a routine process to check the refinement levels
between the refined block and its neighboring blocks to ensure that the difference
in the refinement level is no more than one level. For example, in a uniform grid
structure with grid size $dx = 8''$, the blocks cannot jump separately from $dx = 8''$
to $dx = 1''$, while it can be refined once to $dx = 4''$. To reach the finest grid
size $dx = 1''$, there should be an additional layer with the corresponding grid
size $dx = 2''$. Then one more level of refinement will match the finest grid size
$dx = 1''$. The major processes for the updated code are shown in Figure 4.1(a).

As illustrated in the flowchart, the first step is to build up the initial
grid structure properly before the computation. To employ an embedded mag-
etogram with nonuniform resolutions that differ by more than a factor of 2,
like Figure 4.1(b), as the bottom BC, we need additional intermediate regions
between the central core region and the buffer region due to the constraint from
Figure 4.1: (a) The flowchart for the CESE-MHD-NLFFF extrapolation code, as applied to both uniform and nonuniform (embedded) input magnetograms. (b) The $B_z$ map for a nonuniform embedded magnetogram and (c) the corresponding block distribution on the bottom layer. Here each block contains $8 \times 8 \times 8$ grids. The ratio between the side lengths of the block/grids in the buffer region and those in the core region is 8.

the PARAMESH package that no blocks can vary separately by more than one refinement level in space. So different from the grid distribution prepared for a uniform magnetogram which mainly consists of a core region and the surrounding buffer region, the bottom boundary for the embedded magnetogram will mainly be divided into three parts as illustrated in Figure 4.1(c): the inner core region for inserting the ultra high-resolution magnetogram (corresponding to a smaller FOV within the red box), the intermediate regions from the similar but rebinned ultra high-resolution magnetogram, the surrounding buffer region populated by
the lower-resolution magnetogram (with a larger FOV). For numerical reasons, sometimes possibly additional surrounding grids beyond the FOV of the nonuniform embedded magnetogram can be added. One important principle for our embedding is to keep the ultra high-resolution magnetogram in its entirety as much as possible, so the intermediate regions between the core region and the buffer region are generally kept as narrow as possible, although their sizes can be adjusted. Once the relative positions for the two magnetograms are properly aligned, the size of the computational domain and the position of the core region will be determined. Then the grid structure will be set up through several refinement processes. To begin with, all grids in the computational domain will be refined uniformly to the same pixel size of the larger lower-resolution magnetogram, e.g., 1″/pixel. Next, the grids located within the boundaries of the defined core region will be refined until they reach the highest resolution of the inner magnetogram (for the GST data which is rebinned to 0.125″/pixel, the grids will be refined for three times). Due to the constraint for the grid refinement from PARAMESH as mentioned before, some grids within the core region which are next to the boundaries between the core region and the buffer region (grids in 1″/pixel) will not go through the whole refinement process. Those border grids then make up the narrow intermediate regions with the corresponding intermediate sizes. For the vertical dimension z in height, a similar grid structure is obtained as to be shown later.

After the grid initialization, the initial solutions for all blocks in the whole computation domain will be assigned by a potential field solution derived from
the innermost higher-resolution magnetogram via the Green’s function method (Chiu and Hilton, 1977). To apply the embedded magnetogram for the bottom BC, we assign values from different magnetograms for different regions. According to the grid setting, values from the higher-resolution map will be assigned to both the innermost core region and the intermediate regions (with proper rebinning). In contrast, the buffer region will adopt values as many as possible from the (available) lower-resolution magnetogram. The bottom BC is usually applied gradually reaching the assigned values during the initial iteration steps. Then the bottom BC will be fixed during the remainder of the computation.

4.2 Test Runs for Nonuniform SDO/HMI Magnetograms

![Figure 4.2:](image)

**Figure 4.2:** The $B_z$ map of the bottom boundary for the test run based on a nonuniform magnetogram (left) and the corresponding block structure (right) for AR 11719. The total FOV of the magnetogram is 512″ × 320″. Each block contains 8 × 8 × 8 grids.

To demonstrate the feasibility of the updated CESE-MHD-NLFFF code for nonuniform magnetograms, we perform a series of test runs utilizing this code for AR 11719 (SOL2013-04-11) at first. Instead of using the uniform HMI magnetogram which has been commonly employed as BCs for the NLFFF extrapolations
in various studies, we adopt a nonuniform HMI magnetogram as the bottom BC for test runs. This nonuniform magnetogram consists of two regions, the inner region and outer region. The inner rectangular region uses the original resolution of 0.5″/pixel (∼365 km) which is obtained directly from the HMI SHARP magnetogram) with a FOV of 225″ × 175″. And the outer surrounding region has a relatively lower resolution of 1″/pixel (∼730 km) that is rebinned from the original HMI SHARP magnetogram in a FOV of 512″ × 320″. To make use of such a nonuniform magnetogram in the bottom BC, it is natural to reconstruct a nonuniform grid structure correspondingly. Following steps presented in Section 2.3, the grid structure is built up before any calculations. In our tests, the FOV of the core region is very close to the map size of the inner finer magnetogram, while the FOV of the whole computational domain is slightly smaller than the size of the nonuniform map. Accordingly, the grid sizes on the bottom boundary vary from 0.5″/pixel (inner core region) to 1″/pixel (outer buffer region) which are consistent with the input nonuniform magnetogram. Figure 4.2 shows the $B_z$ component of the bottom BC from a preprocessed nonuniform HMI magnetogram and the associated block structure.

From this initial setting, we experiment with different nonuniform grid configurations assigned to the nonuniform bottom BC for the NLFFF extrapolations to check potential influences on the extrapolations results, e.g., the dimensions of the inner core region and the outer buffer region in the x, y and z directions. To get a general estimation of the convergence for different test runs, we mainly focus on the evolution of residual to compare the relaxation process of the NLFFF
Figure 4.3: Different grid structures from test runs on the bottom plane for updated CESE-MHD-NLFFF method for nonuniform magnetogram: (a-b) for the nonsymmetric and symmetric grid structures; (c) for the embedded magnetogram; (d) for the uniform magnetogram. The BCs for the innermost FOV is almost the same for all test runs while the test run in (c) uses a rebinned magnetogram as a part of the bottom BC for the outer buffer region.

extrapolation outputs from various grid configurations. In particular, we vary the height of the core region, the symmetry of the buffer region, and the resolution variation for the intermediate regions between the inner core region and the outer buffer region. Some examples of the grid configurations are illustrated in Figure 4.3. The results are shown in Table 4.1. Six extrapolation test runs (1-6) are chosen with the variation of grid structures in the grid symmetry, grid resolution
Table 4.1: Grid structures for the tests of updated CESE-MHD-NLFFF code (AR11719). Resolution Variation: indicate the variation of the grid resolutions from the inner core region, through the optional intermediate regions, to the outer buffer region in the whole computational domain.

<table>
<thead>
<tr>
<th>Test</th>
<th>Grid Structure</th>
<th>Resolution Variation</th>
<th>Height of the Core Region</th>
<th>Convergence (when the residual ≤ 10⁻⁵)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>non-symmetric</td>
<td>1'' ⇒ 2'' ⇒ 4'' ⇒ 8''</td>
<td>16''</td>
<td>After ∼ 8930 steps</td>
</tr>
<tr>
<td>2</td>
<td>non-symmetric</td>
<td>1'' ⇒ 2'' ⇒ 4'' ⇒ 8''</td>
<td>32''</td>
<td>Not converged</td>
</tr>
<tr>
<td>3</td>
<td>symmetric</td>
<td>1'' ⇒ 2'' ⇒ 4'' ⇒ 8''</td>
<td>16''</td>
<td>After ∼ 8790 steps</td>
</tr>
<tr>
<td>4</td>
<td>symmetric</td>
<td>1'' ⇒ 2'' ⇒ 4'' ⇒ 8''</td>
<td>32''</td>
<td>Not converged</td>
</tr>
<tr>
<td>5</td>
<td>embedded</td>
<td>0.5'' ⇒ 1''</td>
<td>16''</td>
<td>After ∼ 8430 steps</td>
</tr>
<tr>
<td>6</td>
<td>embedded</td>
<td>0.5'' ⇒ 1''</td>
<td>32''</td>
<td>After ∼ 18500 steps</td>
</tr>
<tr>
<td>7</td>
<td>uniform (300'' × 250'')</td>
<td>1'' ⇒ 2'' ⇒ 4''</td>
<td>20''</td>
<td>∼Not converged</td>
</tr>
<tr>
<td>8</td>
<td>uniform (540'' × 344'')</td>
<td>1'' ⇒ 2'' ⇒ 4''</td>
<td>20''</td>
<td>After ∼ 30000 steps</td>
</tr>
</tbody>
</table>

as well as the core region height. As a reference, we also include two test runs (7-8) based on the uniform magnetograms with different FOVs, which have already been investigated in Section 2.5.

The residual evolution of all test runs for the whole computational domain is also summarized in Figure 4.4. From the residual evolution, the symmetry of grids in the buffer region does not obviously influence the convergence process. It is shown that Tests 1 and 3 (black and blue solid lines), which have the same heights of the core region, have a similar decreasing trend toward the end. According to Table 4.1, Tests 1 and 3 also reached converged solutions with approximately the same numbers of iteration steps. Similarly, Tests 2 and 4 (black and blue dash lines) both present a non-converging behavior with continuous fluc-
Figure 4.4: Residual evolution of all test runs in Table 4.1 for different grid structures. 

During the evolution, most test runs failed to yield a converged result with a small residual when the height of the core region is larger than $z = 16''$, 
\textit{e.g.}, Tests 2, 4 and 7. However, there are two exceptions that reached a state with a small residual. One is Test 8 based on a uniform SHARP magnetogram with a larger FOV. Another one is Test 6 employing a nonuniform magnetogram, which also shows a faster convergence behavior than Test 8 in Figure 4.4. In general, the test runs with the nonuniform magnetogram converge faster than other test runs with the same grid configuration of the core region. To further evaluate the convergence process and the quality for these extrapolation results, we also calculate several NLFFF quality metrics for force-freeness and divergence-freeness.
Table 4.2: NLFFF metrics for different CESE-MHD-NLFFF results

| Test No. | Calculation step | CWsin | $\langle |f_i| \rangle$ | $E_{\nabla \cdot B}$ | $E_{\nabla \times B}$ |
|----------|-----------------|-------|------------------------|----------------------|----------------------|
| 1        | 22000           | 0.477 | 9.6310$^{-4}$           | 3.4310$^{-2}$        | 0.320                |
| 6        | 22000           | 0.466 | 5.9610$^{-4}$           | 3.7210$^{-2}$        | 0.319                |
| 8        | 32000           | 0.377 | 6.0610$^{-4}$           | 4.5210$^{-2}$        | 0.290                |

specifically for Tests 1, 6 and 8 as given in Table 4.2. All metrics in Table 4.2 are calculated in a domain size of $225'' \times 175'' \times 150''$ at the calculation step after convergence. In addition to the parameters $CWsin$ and $\langle |f_i| \rangle$, we also calculate two additional metrics, $E_{\nabla \cdot B}$ and $E_{\nabla \times B}$, to evaluate the divergence-freeness and force-freeness (see Appendix A for the definitions). From Table 4.2, the values of those quality metrics from Test 6 based on the embedded magnetogram are generally smaller than or comparable with Test 1 which uses a uniform magnetogram as the bottom BC. For comparison, Test 8 is more force-free with a smaller $CWsin$ and $E_{\nabla \times B}$ than Tests 1 and 6. While the order of magnitude in all force-freeness and divergence-freeness metrics is the same, Test 6 takes much less iteration steps to converge than Test 8.

In conclusion, these results give us a general estimation of the influences from different grid configurations for the inner core region and the buffer region. More importantly, these results indicate the feasibility of the implementation of the embedded magnetograms to comply with the consistency requirement of the code and also the potential to improve the quality of the NLFFF extrapolation result.
4.3 Test Runs for the Embedded GST-SDO Magnetogram

After the preliminary tests for the updated CESE-MHD-NLFFF code with the nonuniform HMI vector magnetograms, we apply this updated code to the embedded photospheric vector magnetograms from the GST/NIRIS and SDO/HMI instruments. Unlike the nonuniform SDO/HMI magnetogram that we constructed for tests in Section 4.2, the GST magnetogram has a much higher spatial resolution of $\sim 0.083''$/pixel ($\sim$54 km) than the SDO/HMI magnetogram, which has a pixel size of $0.5''$ ($\sim$325 km). On Jun 22, 2015, the GST telescope captured the whole eruption process of a solar flare, which is rare in the observations for the small-FOV, ground-based telescopes. In the small FOV of GST, an M6.5 flare reached a peak GOES soft X-ray flux at $\sim$ 18:23 UT on 2015 June 22 (N13W06). And the evolution of a corresponding two-ribbon structure for the flare was also captured in the AIA observations with a much larger FOV (Jing et al., 2016).

The availability of the photospheric magnetograms from both GST and SDO observations for this solar eruption event thus offers a unique chance to make an embedded magnetogram as the bottom BC for the updated CESE-MHD-NLFFF extrapolation code.

To prepare the nonuniform embedded magnetograms, the high-resolution GST magnetogram in a small FOV is embedded into the interpolated SDO magnetogram to form an embedded magnetogram with a uniform spatial resolution of $0.083''$/pixel. Then the embedded vector magnetograms are rebinned from $0.083''$/pixel to $0.125''$/pixel and preprocessed uniformly. The preprocessed em-
Figure 4.5: Top row: rebinned preprocessed embedded vector magnetogram in 1″/pixel (left) and the nonuniform embedded magnetogram similar as the left one while the inner core region maintains a high resolution (0.125″/pixel) on the right. Bottom: the top and the side views of the grid structures for the extrapolation based on the nonuniform magnetogram.

bedded magnetograms are converted into two different resolutions to generate a nonuniform BC for the updated CESE-MHD-NLFFF method, which is similar to the test runs in Section 4.2. A smaller map in a rectangular FOV (54.24″ × 51.84″), which corresponds to the FOV of the original GST/NIRIS magnetogram, is cho-
sen as the inner core region of the nonuniform bottom BC. Meanwhile, the whole preprocessed map is rebinned to 1″/pixel and provides the input for the outer region of the nonuniform bottom BC. The $B_z$ component of the whole preprocessed embedded magnetogram and the nonuniform BC generated from the preprocessed magnetogram are illustrated in the first row of Figure 4.5, respectively.

Given one constraint from the PARAMESH package that the difference of the refinement levels between the refined blocks and their neighboring blocks should not exceed one, additional intermediate regions are added between the inner core region and the outer region as the resolutions between the two regions differ by eight times. So we need to refine the grids three times to change the grid size from 1″/pixel to 0.125″/pixel. For the purpose of maintaining the high-resolution GST magnetograms as much as possible and avoiding interpolating the low-resolution HMI data, the intermediate region is designed to be as narrow as possible. The grid structure and the underlying embedded magnetogram for the bottom boundary, including the core region, the intermediate region and a part of the outer buffer region, are shown in the second row of Figure 4.5. The bottom boundary will be updated with values from the nonuniform embedded magnetograms consistently. In particular, the values for the grids in the intermediate region will be assigned with the rebinned GST magnetogram correspondingly.

Besides the grid configuration for the whole computational domain as considered in the previous section, the pre-processing method for the embedded magnetogram may also contribute to the quality of the extrapolation results as the embedded magnetogram from GST and SDO contains significant difference in
Table 4.3: List of quality control parameters (see definitions in equations 2.11 and 2.12) for the three sets of magnetograms for June 22, 2015 eruption event.

<table>
<thead>
<tr>
<th>Data</th>
<th>(\epsilon_{flux})</th>
<th>(\epsilon_{force})</th>
<th>(\epsilon_{torque})</th>
<th>(S_x)</th>
<th>(S_y)</th>
<th>(S_z)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HMI magnetogram (204′′ × 204′′, resolution: 1′′)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Raw map</td>
<td>0.301</td>
<td>3.64E-02</td>
<td>4.20E-02</td>
<td>4.39E-03</td>
<td>2.93E-03</td>
<td>1.76E-04</td>
</tr>
<tr>
<td>Preprocessed map</td>
<td>0.342</td>
<td>1.02E-03</td>
<td>1.28E-02</td>
<td>3.85E-04</td>
<td>2.48E-04</td>
<td>3.22E-04</td>
</tr>
<tr>
<td>Embedded GST-SDO magnetogram (204′′ × 204′′, resolution: 0.125′′)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Raw map</td>
<td>0.304</td>
<td>3.66E-02</td>
<td>4.48E-02</td>
<td>3.92E-03</td>
<td>2.74E-03</td>
<td>5.82E-04</td>
</tr>
<tr>
<td>Preprocessed map (z=0.125′′)</td>
<td>0.318</td>
<td>8.53E-03</td>
<td>9.84E-03</td>
<td>4.67E-05</td>
<td>3.14E-05</td>
<td>2.42E-04</td>
</tr>
<tr>
<td>Preprocessed map (z=1′′)</td>
<td>0.347</td>
<td>1.04E-02</td>
<td>1.19E-02</td>
<td>3.43E-05</td>
<td>2.25E-05</td>
<td>1.14E-07</td>
</tr>
</tbody>
</table>

the spatial resolutions between the inner and outer regions. Table 4.3 shows the quality of the raw and preprocessed input magnetograms from the SDO/HMI magnetogram alone and the embedded magnetogram based on some parameters which have also been used in Chapter 2. Specifically, given that the spatial resolution of the embedded magnetogram is high with 0.125′′/pixel, it is preprocessed with two different heights, 0.125′′ and 1′′, respectively. The quality of two raw magnetograms is comparable as the values of those six parameters are close for the two maps. The preprocessed embedded map does yield two parameters that are about one order of magnitude smaller than the corresponding parameters for the preprocessed HMI map. And the preprocessing height does not cause an obvious difference for the quality of the preprocessed embedded magnetogram.

To see the actual influence to the quality of the extrapolation results, we perform four test runs based on different preprocessed maps as well as different grid structures for the updated CESE-MHD-NLFFF method. For a better comparison, all test runs are performed with one same height of the core region
and the same finest grid resolution in 0.125″/pixel. Run 1 and Run 2 adopt the bottom BCs from the embedded GST-SDO magnetograms (204″ × 204″) preprocessed at heights of \( z = 0.125″ \) and \( z = 1″ \), respectively. Run 3 applies the same bottom BC as Run 1 while an additional buffer region is added with the purpose to eliminate the influence of the boundary effect. And we also perform Run 4 with a differently preprocessed magnetogram, which is obtained from a preprocessed SDO/HMI magnetogram with a size of 204″ × 204″ (1″/pixel resolution) and a separately preprocessed GST/NIRIS magnetogram (0.125″/pixel resolution) replacing the inner smaller FOV of 54.24″ × 51.84″. The analysis on the convergence and quality of these test runs is discussed as follows.

In Figure 4.6, the evolution of the residual and different NLFFF quality metrics during the relaxation process is presented for each run. The evolution of the residual and the magnitudes of the magnetic energy \( E_{\text{tot}} \) in four runs are very similar. The major difference lies in the evolution of the force-freeness parameter \( CW_{\sin} \) and the divergence-freeness parameter \( \langle |f_i| \rangle \). By utilizing the preprocessed maps obtained at different heights, Run 2 shows smaller \( CW_{\sin} \) values than Run 1 while Run 1 yields a smaller \( \langle |f_i| \rangle \) value than Run 2 during the iteration process. As for Run 4, which employs a nonuniform BC from two separately preprocessed magnetograms, it shows a slightly smaller residual, smaller \( CW_{\sin} \) and \( \langle |f_i| \rangle \) metrics during the evolution. But in general, the difference is small within one order of magnitude and the evolution trend of different quality metrics for these three runs are very similar. In particular, the evolution of the \( CW_{\sin} \) metric in Run 3 decreases faster than the other three test runs after \( \sim 28,000 \) steps and
Figure 4.6: Evolution of different metrics for June 22, 2015 event with different bottom BCs (see the legend). Run 1 and Run 2 use bottom BCs from the embedded magnetogram preprocessed at heights of $z = 0.125''$ and $z = 1''$, respectively. Run 3 applies the same bottom BC as Run 1 and an additional buffer region. Run 4 employs a nonuniform BC from a combination of separately preprocessed magnetograms.

drops to a smaller CWsin value. Run 3 with an additional buffer region also shows a smoother trend and smaller value for the evolution of the divergence-freeness metric $\langle |f_i| \rangle$. Such an implementation could be considered if there is enough computational resource. Beyond the examination of the NLFFF quality metrics, some sampled fieldlines from the same seed points are also drawn to illustrate the differences from these four extrapolation runs more straightforwardly. Figure 4.7 displays the uniformly distributed fieldlines in a FOV of $65'' \times 112.5''$ for all four runs. In the central region near the two major magnetic polarities, the fieldlines
from Run 1 and Run 2 are a bit more twisted than the other runs. Generally the fieldlines connectivity does not vary much in a large scale. Further examination of the fieldline distribution in a finer scale will be performed in the next chapter.

Figure 4.7: Comparison of magnetic field lines of different BCs and grid structures for four test runs within the height of 15 ″ (~10,800 km) over the $B_z$ map. Field lines for all plots are chosen from the same set of seed points. From left to right: Run 1 test with the preprocessed map at $z=0.125''$; Run 2 test with the preprocessed map at $z=1''$; Run 3 test with the preprocessed map at $z=0.125''$ with a larger buffer region; Run 4 test with separately preprocessed outer region and inner map at $z=0.125''$.

4.4 Summary

In this chapter, we mainly propose an updated CESE-MHD-NLFFF code for the nonuniform embedded magnetogram and test it with the nonuniform HMI magnetograms and the embedded GST-SDO magnetograms. For the latter, the ratio between the native resolutions of the magnetograms from the two different instruments can be as large as 8. Different grid structures are constructed and
evaluated for a few test extrapolation runs. Then an overall estimation of the quality for the extrapolation results is discussed based on the calculations of the NLFFF metrics during the relaxation process.

Based on the test runs for the nonuniform HMI magnetograms and the realistic nonuniform embedded GST-SDO magnetogram, we demonstrate the feasibility of the updated CESE-MHD-NLFFF code for a nonuniform BC and analyze the improvement of the updated code compared to the original code for the uniform BC. The results from Section 4.2 indicate that the consistency in performance still maintains comparable with the extrapolation results for the uniform magnetograms and the convergence can be improved. For example, the computation can be accelerated with less computational resources based on a nonuniform grid setting for nonuniform magnetograms, especially when we aim to study the fine-scale structure in the lower corona with a higher core region height. For example, in Chapter 5, the detailed information about the computational cost and efficiency will be provided for one specific event study.

Another important lesson we learn from these test runs is the criteria for quality evaluation of the NLFFF extrapolation. Given the inconsistency between the realistic photospheric magnetograms and the force-free assumption, it is necessary to preprocess the photospheric magnetograms before the NLFFF extrapolation. So it is always a standard step to check the associated NLFFF metrics throughout the iteration process and to compare the reconstructed magnetic field lines with the corresponding multiple coronal observations to study realistic solar eruption events. Such comparisons will be explicitly presented in Chapter 5.
Chapter 5. Coronal Magnetic Field Extrapolation and Topological Analysis of Fine-Scale Structures during Solar Flare Precursors

Magnetic field plays an important role in various solar eruption phenomena like flares, coronal mass ejections, etc. The formation and evolution of characteristic magnetic field topology in solar eruptions are critical problems that will ultimately help us understand the origination of these eruptions in the solar source regions. With the development of advanced techniques and instruments, observations with higher resolutions in different wavelengths and fields of view have provided more quantitative information for finer structures. So it is essential to improve our method to study the magnetic field topology in the solar source regions by taking advantage of high-resolution observations. In this study, we employ a nonlinear force-free field (NLFFF) extrapolation method based on a nonuniform grid setting for an M-class flare eruption event (SOL2015-06-22T17:39) with embedded vector magnetograms from the Solar Dynamics Observatory (SDO) and the Goode Solar Telescope (GST). The extrapolation results employing the nonuniform embedded magnetogram for the bottom boundary are obtained by maintaining the native resolutions of the corresponding GST and SDO magnetograms. We compare the field line connectivity with the simultane-
ous GST/Hα and SDO/AIA observations for these fine-scale structures associated with precursor brightenings. Then we perform a topological analysis of the field line connectivity corresponding to fine-scale magnetic field structures based on the extrapolation results. The analysis results indicate that by combining the high-resolution GST magnetogram with a larger HMI magnetogram, the derived magnetic field topology is consistent with a scenario of magnetic reconnection among sheared field lines across the main polarity inversion line during solar flare precursors. The following content is adapted from the manuscript published in the Astrophysical Journal (He et al., 2023).

5.1 Introduction

There are different kinds of spectacular eruptions in the solar atmosphere, such as flares, coronal mass ejections (CMEs) and jets, which release energy in various spatial and temporal scales. In particular, solar flare eruption attracts a lot of attention among these eruption phenomena as an explosive, energetic phenomenon with enhanced emission throughout the electromagnetic spectrum in a dynamic and complicated process. In multi-wavelength observations, a flare usually goes through three major phases, namely the preflare, the impulsive and the gradual phases. And the life of a flare spans from tens of seconds to several hours (see a review by Benz, 2017). During the flare eruption, the energy release could be as large as $10^{32}$ ergs, while the major contribution comes from the magnetic energy comparing to other sources. To figure out the underlying physical mechanisms, i.e., the source of energy for release, a lot of efforts have been made
on the different perspectives of the main phase (impulsive and gradual phases) of solar flares. For example, the standard two-dimensional (2D) flare model (so-called CSHKP model, Carmichael, 1964; Sturrock, 1966; Hirayama, 1974; Kopp and Pneuman, 1976) proposes that magnetic reconnection plays a major role in the energy release during the evolution of a flare.

In addition to the main phase of the flare eruption, it is noteworthy that there are also some interesting small-scale localized energy release phenomena in the precursor phase (before the flare main phase or before the time of the peak X-ray flux emission), e.g., the so-called flare precursor brightenings. Bumba and Křivský (1959) introduced the concept of flare precursors which were observed as a short-term and small brightening before the main flare onset. Later it has been observed in many flares through multiple wavelengths including X-ray, optical, ultraviolet/extreme ultraviolet (UV/EUV) and microwave observations (Awasthi et al., 2014; Bamba et al., 2014, 2017). Tappin (1991) performed a statistical study based on X-ray observations to investigate the correlation between flare precursors and flare onsets and they summarized that most flares as measured by X-ray emissions are preceded by one or more soft X-ray precursors with 10 to 60 minutes prior to the flare onset (see also a more recent statistical study by Gyenge et al., 2016). Later on, Chifor et al. (2007) reported that the precursors locate near or on the polarity inversion line (PIL) and hard X-ray precursor brightenings move rapidly along a PIL before the flare main phase based on the analysis of a list of preflare events by combining multi-wavelength observations with the evolution of photospheric magnetic fields. Their study also provides evidence of
the spatial and temporal correlation between the preflare activities and the fila-
ment eruption onsets. Therefore, the investigation of the flare precursors is an
important subject not only for the initiation mechanism of flares but also for the
associated eruption phenomena. Given the relatively smaller-scale energy release
of flare precursors with respect to the main phase of the flare evolution, observa-
tions at higher spatial and temporal resolutions are required. In the meantime, it
is also essential to validate different eruption initiation mechanisms with a better
understanding of the magnetic field topology for flare precursors. However, the
inherent fine-scale three-dimensional (3D) magnetic field topology change is still
unclear owing to a lack of quantitative study by using high-resolution vector mag-
etograms. Here we intend to perform an analysis of the fine-scale magnetic field
structures associated with flare precursor brightenings through nonlinear force-
free field (NLFFF) extrapolations by employing recently available high-resolution
vector magnetograms from multiple sources.

With the recent development of observational techniques and instruments,
more advanced high-resolution solar observational data become available, includ-
ing space-based telescopes like the Solar Dynamics Observatory (SDO), the Hin-
odde satellite, and the Solar Orbiter, and also the ground-based telescopes like the
1.6-meter Goode Solar Telescope (GST), the 4-meter Daniel K. Inouye Solar Tele-
scope (DKIST) and so on. More advanced observations will be definitely crucial
to improving our understanding of small-scale energy release processes like flare
precursors and their connections to the following flare main phase. Therefore,
there is a growing demand for taking full advantage of data from multiple instru-
ments with necessary improvements to the existing methods. For instance, the ultra high-resolution observation in a smaller field of view (FOV) can contribute to the analysis of the fine-scale structure of small-scale events (Jing et al., 2016; Wang et al., 2017; Zhao et al., 2022). Alternatively, a relatively lower-resolution observation in a larger FOV has more advantages to extend the spatial coverage and to describe the magnetic connections to surrounding structures. As for the analysis of coronal magnetic structures in solar eruptions, vector magnetograms may be obtained from multiple instruments with different spatio-temporal resolutions and FOVs (mostly on the photosphere). One desirable approach is to be able to combine these vector magnetograms for the numerical extrapolation of the coronal magnetic field, while preserving their respective advantages.

Numerical simulations can be a viable tool to derive unavailable data like the 3D coronal magnetic fields with reasonable assumptions (Jiang et al., 2022). For example, the photospheric vector magnetograms are often employed as bottom boundary conditions (BCs) in different numerical simulation methods to reconstruct the 3D coronal magnetic field in the solar source region. However, the limitations of computational resources for the numerical simulation capability of a more realistic full magnetohydrodynamics (MHD) model also lead to the use of the NLFFF extrapolation method to reconstruct static 3D coronal magnetic field based on a force-free assumption (Wiegelmann and Sakurai, 2021; Jiang and Feng, 2012). Different kinds of numerical methods have been proposed to reconstruct the NLFFF for the coronal magnetic field from specific BCs and sometimes pseudo-initial conditions, including the upward integration, Grad-Rubin iteration,
MHD relaxation, optimization approach, and so on (see a review by Wiegelmann and Sakurai, 2021). The computation speed and quality of different numerical modeling results may vary significantly for the realistic solar magnetograms not only due to the differences in algorithms and their specific realizations, but also the quality of the input magnetograms. For instance, the spatial resolution of input vector magnetograms as the bottom BC has been proven to have an essential effect on NLFFF extrapolation results, including the magnetic energy and associated magnetic field topology, as reported by Thalmann et al. (2013) and DeRosa et al. (2015). In the meanwhile, more solar observations become available and can provide vector magnetograms in different spatial resolutions and FOVs for the same solar source region. Therefore, in order to improve the computational efficiency and maximize the advantage of available observations, it becomes natural to incorporate the available higher-resolution magnetograms (often in a small FOV) into the bottom BC along with the lower-resolution magnetograms (with a larger FOV) to conduct the extrapolation, thus maintaining the native resolution of the higher-resolution magnetogram and a larger FOV at the same time, especially for the study of fine-scale structure in flare precursors.

An M6.5 class flare erupted close to the solar disk center (8°W 12°N) on June 22, 2015 in active region NOAA 12371. The impulsive phase of the flare starts at \(\sim 17:51\) UT. Two short-duration small-scale brightenings were observed in unprecedented spatio-temporal resolution by the 1.6-m GST along with photospheric magnetic field dynamics and reported as flare precursors by Wang et al. (2017). That study focused on the two short episodes of flare precursors by utiliz-
ing high-resolution Hα and photospheric magnetic field from GST observations, complemented by X-ray and microwave observations. And these observations indicate the evidence of successive reconnection process during the evolution of the precursor periods, which may contribute to the onset of the main flare. Many studies have been performed for this event in terms of different physical processes. The fine-scale structure of this flare and associated large-scale dynamic motion of flare ribbons have been discussed by Jing et al. (2016, 2017). Liu et al. (2018) and Xu et al. (2018) looked into the relationship between the flaring signatures and the evolution of photospheric vector magnetic fields by taking advantage of the GST observations. For the flare onset process, some authors (Awasthi et al., 2018; Kang et al., 2019; Liu et al., 2022b) have studied the pre-eruptive magnetic configuration with reconstructed 3D magnetic field by the NLFFF extrapolation method based on the SDO/Helioseismic and Magnetic Imager (HMI, Schou et al., 2012) magnetograms. A multi-instrument comparative study was also conducted by Liu et al. (2022a), which offers a quantitative description of the thermal behaviors for flare precursor over a large temperature range. In addition, Jing et al. (2023) analyzed the 3D magnetic properties of two light bridges in both small and large scales before the flare by utilizing the photospheric vector magnetograms from GST and SDO separately for the NLFFF extrapolation. For all these studies, it is always essential to compare the derived magnetic field configuration with the corresponding multi-wavelength imaging observations to help validate and interpret the extrapolation results when applicable.
In this study, we apply a type of MHD relaxation method with a conservation-element/solution-element (CESE) solver, so-called the CESE-MHD-NLFFF method (see details in Jiang et al., 2011; Jiang and Feng, 2013), to obtain the 3D coronal magnetic field in an approximate force-free state. It has been widely applied to the analysis of magnetic field topology with realistic solar magnetic field data (Jiang and Feng, 2013; Duan et al., 2017, 2019; He et al., 2022). For example, in Chapter 3, this method was applied to characterize the properties of magnetic flux ropes (MFRs) on the Sun and the results are then compared to the properties of the corresponding interplanetary counterparts quantitatively (He et al., 2022). The results indicated the importance of flare associated magnetic reconnection process in that the magnetic reconnection flux estimated from the analysis corresponds well to the magnetic flux content found in the MFR formed during the main phase of solar flares. A subsequent study (Hu et al., 2022) further implied the variability in the magnetic field topology changes of an MFR as manifested in the analysis of multiple observations of the associated flare/CME eruption process. For the present study, with the available high-resolution GST observations for the aforementioned M6.5 flare, we develop an updated version of the existing CESE-MHD-NLFFF code for embedded magnetograms by incorporating the high-resolution GST magnetogram and the larger-FOV SDO/HMI magnetogram as the bottom BC with non-uniform grid spacing. The results will be compared to the extrapolations with single-set uniform magnetograms and the associated observations, mainly the high-resolution GST/Hα images during the flare precursors.
The chapter is organized as follows. First, the instrumentation and data used in this paper are described in Section 5.2. Then we present the modified CESE-MHD-NLFFF method and the associated convergence study in Section 5.3. In Section 5.4, the magnetic field topology from the extrapolations with different bottom BCs is presented and investigated in detail. The major results are summarized and conclusions are given in Section 5.5.

5.2 Instrumentation and Data

For this event, we make use of the observational data from both SDO and GST. SDO can provide full disk observations of the Sun routinely. Specifically, the Space-weather HMI Active Region Patch (SHARP; Bobra et al., 2014) vector magnetograms are used as the input bottom BCs of the NLFFF extrapolations. The SHARP data product offers photospheric vector magnetograms with a pixel size of 0.5” (∼365 km) in a cadence of 720 seconds. On the other hand, the corresponding remote sensing observations in UV and EUV wavelength channels are provided by the Atmospheric Imaging Assembly (AIA, Lemen et al., 2012) onboard SDO with a spatial sampling of around 0.6”/pixel (∼438 km) and a moderate time cadence (12 s for EUV channels and 24 s for UV channels). For the June 22, 2015 flare event, the GST at the Big Bear Solar Observatory (BBSO) also obtained high-resolution observations during ∼16:50 – 23:00 UT (Jing et al., 2016; Wang et al., 2017). The \( H\alpha \) images at the line center and off-bands (±0.6Å and ±1.0 Å) are taken by the Visible Imaging Spectrometer (VIS; Cao et al., 2010) with a FOV of ∼57” × 64” (42 × 47 \( Mm^2 \)). The GST/VIS observations
have a pixel size as small as $\sim 0.03''$ ($\sim 22$ km) and a time cadence of 28 s. The Near InfraRed Imaging Spectropolarimeter (NIRIS; Cao et al., 2012) of GST, equipped with the infrared detector and the dual Fabry-Perot interferometers system, provides the spectropolarimetric data (at Fe $\mathrm{I}$ 1565 nm doublet, 0.2 Å bandpass). The spectropolarimetric data are processed with the NIRIS data processing pipeline including dark and flat field corrections, instrument crosstalk calibration and Milne-Eddington Stokes inversion, from which the vector magnetic fields could be extracted. The pixel size and temporal cadence of the resulting vector magnetograms are $\sim 0.08''$ ($\sim 59$ km) and 87 s, respectively. As for this event, the GST/NIRIS vector magnetograms and the SDO/HMI products have been properly aligned and compared at about the same time by Liu et al. (2018) and both the horizontal and vertical magnetic fields from NIRIS and HMI measurements show a high correlation (with correlation coefficients greater than 0.85) for the flare core region.

In general, the space-borne instrumentation provides a larger FOV and more continuous observations while the ground-based counterpart has a relatively smaller FOV and more sporadic temporal coverage, but has a much higher spatial resolution. Therefore it is desirable to combine these two sets of observations in order to make the best use of their data products, by embedding the higher resolution GST/NIRIS magnetogram into the corresponding part of the lower resolution, larger FOV SDO/HMI magnetogram.

To generate such a comprehensive NIRIS-HMI magnetogram, the first and most important step is the time-consuming alignment between the NIRIS and
HMI magnetograms. Using the data here as an example, the FOV of the NIRIS magnetogram is $52'' \times 52''$ ($38 \times 38$ $Mm^2$), consisting of $650 \times 650$ pixels with a spatial sampling of $0.08''$/pixel, while the FOV of HMI is about $200'' \times 200''$ ($146 \times 146$ $Mm^2$), consisting of $400 \times 400$ pixels, $0.5''$/pixel. By manual alignment, we find the exact position, with subpixel precision, of the NIRIS magnetogram of a small FOV on the HMI magnetogram of a large FOV. Since the spatial sampling rates of the two magnetograms are different, we interpolate the data array of the HMI magnetogram to $2500 \times 2500$ pixels with the same FOV, so that each of its pixels has the same spatial scale as the NIRIS magnetogram, i.e., $0.08''$/pixel. Then, we embed the GST/NIRIS magnetogram in the middle of such an HMI magnetogram. After these steps, an embedded magnetogram with a FOV of $200'' \times 200''$ ($146 \times 146$ $Mm^2$) and a pixel size of $0.08''$/pixel ($\sim 60$ km/pixel) is derived containing data from both HMI and NIRIS. However, it is very costly to perform computations for such a magnetogram with a uniform ultra-high resolution. We will construct a nonuniform grid structure for the NLFFF extrapolations and utilize the embedded magnetograms with nonuniform resolutions as our bottom BC.

5.3 NLFFF Extrapolation Code for Embedded Magnetograms

5.3.1 Extrapolation Method by the CESE-MHD-NLFFF Code

The CESE-MHD-NLFFF code is similar to a magnetofrictional method, which can be regarded as a special case of the MHD relaxation method. And it
is mainly designed to solve the modified momentum equation and the magnetic induction equation (Jiang and Feng, 2012, 2013),

\[
\frac{\partial (\rho \mathbf{v})}{\partial t} = (\nabla \times \mathbf{B}) \times \mathbf{B} - \nu \rho \mathbf{v}, \quad \rho = |\mathbf{B}|^2 + \rho_0, \quad (5.1)
\]

\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) - \mathbf{v} \nabla \cdot \mathbf{B} + \nabla (\mu \nabla \cdot \mathbf{B}). \quad (5.2)
\]

These equations are solved as a kind of Dirichlet-type boundary value problem. Based on the force-free field assumption (plasma \(\beta \ll 1\)), the magnetic force plays a major role. So other forces including the plasma pressure, gravity and inertial forces can be ignored. In order to balance the Lorentz force, an artificial term \(\nu \rho \mathbf{v}\) in a frictional force form involving velocity \(\mathbf{v}\) is added in the momentum equation (5.1). In addition, a pseudo mass density \(\rho\) is assumed to take the form given. And \(\rho\) is modified with a small value \(\rho_0\), e.g., \(\rho_0 = 0.1\) (in the same unit as \(|\mathbf{B}|^2\)), to deal with the case of very weak magnetic field. For the magnetic induction equation, two extra terms are added to control the divergence of the magnetic field. The equations (5.1) and (5.2) are solved through the iteration process until a converged solution of a quasi-static equilibrium state is approached.

The computation proceeds by iterations until a converged solution is reached as judged by a series of metrics. For the NLFFF extrapolation, a well-known problem is that the force-free condition may not always be satisfied in the inhomogeneous solar atmosphere, especially in the photosphere (Gary, 2001). Wiegelmann \textit{et al.} (2006) proposed that a more consistent bottom BC for an NLFFF extrapolation can be obtained by modifying the original photospheric magnetogram.
to mimic a force-free chromospheric magnetogram. Such a practice commonly adopted for NLFFF extrapolations is called preprocessing. The basic method for preprocessing proposed by Wiegelmann et al. (2006) is to obtain a target magnetogram by minimizing a function, which includes several additional constraints, through an optimization method. The individual constraints include the surface integrals of the total force, total torque, deviation between the updated magnetogram and the observed magnetogram, and the smoothness of the updated magnetogram (see details in Chapter 2). The optimization and smoothing algorithms could vary in different preprocessing codes. Here we follow the basic approach of Wiegelmann et al. (2006), but use the specific preprocessing code developed by Jiang et al. (2014a) to get the bottom BC for the CESE-MHD-NLFFF extrapolation code before the computation. This preprocessing code works by splitting the magnetogram into the potential field part and the non-potential field part. Then the non-potential part will be optimized and smoothed following the general approach of Wiegelmann et al. (2006) to get close to a force-free state. The preprocessed magnetogram can usually achieve the same level of force-freeness as the potential field part as evaluated by the standard set of metrics (Jiang et al., 2014a).

5.3.2 Grid Construction and Modified CESE-MHD-NLFFF Code for Embedded Magnetograms

Considering the speed and accuracy of the computation for the realistic solar magnetograms, a nonuniform grid structure within a block-structured (one
Table 5.1: Boundary Conditions (BCs) for Different NLFFF Extrapolation Runs: all computations are performed with 19 cores on a 24-core local desktop with 48 GB memory. And \(1'' \sim 730 \text{ km}\).

<table>
<thead>
<tr>
<th>Runs</th>
<th>Bottom BCs</th>
<th>Resolution</th>
<th>FOV of the magnetogram</th>
<th>Count of blocks</th>
<th>Computation Time (to 40,000 steps)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Run 1</td>
<td>Nonuniform embedded magnetogram</td>
<td>Outer region: (1'') Core region: (0.125'')</td>
<td>(204'' \times 204'') (149 \times 149 Mm(^2))</td>
<td>41040</td>
<td>157 hrs</td>
</tr>
<tr>
<td>Run 2</td>
<td>Uniform SDO magnetogram at 17:36 UT</td>
<td>(1'')</td>
<td>(204'' \times 204'') (149 \times 149 Mm(^2))</td>
<td>3080</td>
<td>12.5 hrs</td>
</tr>
<tr>
<td>Run 3</td>
<td>Uniform GST magnetogram at 17:32 UT</td>
<td>(0.125'')</td>
<td>(50'' \times 46'') (37 \times 34 Mm(^2))</td>
<td>13850</td>
<td>52.5 hrs</td>
</tr>
</tbody>
</table>

block contains a group of cells) parallel computation framework has been adopted for the CESE-MHD-NLFFF code with the help of the PARAMESH software package (MacNeice et al., 2000). For the grid initialization of CESE-MHD-NLFFF code, the whole computational domain includes the pre-set central core region and the surrounding buffer region to reduce the influence of the side boundaries (Jiang and Feng, 2013). Then the whole computational domain is divided into blocks with different spatial resolutions, and all blocks have identical logical structures. The blocks are evenly distributed among processors. The block structures can be refined (increase the grid resolution) or de-refined (decrease the grid resolution), which provides the flexibility for embedding nonuniform magnetograms as bottom BCs.

To apply the embedded magnetograms with nonuniform spatial resolutions for the NLFFF extrapolation, we develop an updated version of the CESE-MHD-NLFFF code to utilize the embedded map as the bottom BC. Figure 5.1(a) and (b) show the difference of the bottom boundary layers between the nonuniform embedded and uniform magnetograms. Specifically, we redesign the grid struc-
Figure 5.1: Bottom boundary layers of $B_z$ component for (a) the embedded magnetogram, (b) the uniform HMI magnetogram, and (c) a zoomed-in portion of (a) as outlined by the orange box. Panel (d) shows one associated nonuniform grid structure on the bottom boundary for (c). The whole domain is divided into blocks with equal sides as illustrated in (d) for the bottom boundary by the solid lines, and each block contains $8 \times 8 \times 8$ cells. The side length of the cell for the innermost block is $0.125''$ (91 km) corresponding to the side length $1''$ (730 km) of the blocks that form the core region as shown. It doubles three times to reach the cell size $1''$ for the outermost region which is the gt region. In-between is the intermediate region.

Figure 5.1: Bottom boundary layers of $B_z$ component for (a) the embedded magnetogram, (b) the uniform HMI magnetogram, and (c) a zoomed-in portion of (a) as outlined by the orange box. Panel (d) shows one associated nonuniform grid structure on the bottom boundary for (c). The whole domain is divided into blocks with equal sides as illustrated in (d) for the bottom boundary by the solid lines, and each block contains $8 \times 8 \times 8$ cells. The side length of the cell for the innermost block is $0.125''$ (91 km) corresponding to the side length $1''$ (730 km) of the blocks that form the core region as shown. It doubles three times to reach the cell size $1''$ for the outermost region which is the gt region. In-between is the intermediate region.

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the refinement level between the refined block and its neighboring blocks which should be no more than one level. For example, in a uniform grid structure with grid size \( dx = 8 \), the grid size in the core region cannot be refined once to \( dx = 1 \) instantly, but it can only be refined once to \( dx = 4 \). To reach the finest grid size \( dx = 1 \), two additional intermediate regions are required with the grid sizes, \( dx = 2 \) and \( dx = 4 \) (see an illustration of the intermediate regions in Figure 5.1(d)), respectively. Therefore, for a nonuniform embedded magnetogram (the resolution ratio should be integral powers of two), we need additional intermediate regions between the central core region and the buffer region due to the constraint from the PARAMESH package. So different from the grid structure in a uniform magnetogram which mainly consists of a core region and the surrounding buffer region, the bottom boundary for an embedded magnetogram will mainly be divided into three parts: the inner core region for the higher-resolution magnetogram (in a smaller FOV), the intermediate regions from the rebinned higher-resolution magnetogram, and the surrounding buffer region populated by the lower-resolution magnetogram (with a larger FOV). One important principle for our embedding is to keep the higher-resolution magnetogram in its entirety as much as possible, so the intermediate regions between the core region and the buffer region are generally kept as narrow as possible. Once the relative positions for the two aligned magnetograms are obtained, the grid structure for the whole computational domain can be set up.

After the grid initialization, the initial solutions for all blocks in the whole computation domain will be assigned by a potential field solution derived from the
higher-resolution magnetogram in the core region via Green’s function method (Chiu and Hilton, 1977). On the bottom boundary, values from the higher-resolution magnetogram will be assigned to the innermost core region and the intermediate regions with proper rebinning. In contrast, the buffer region will adopt values from the lower-resolution magnetogram. The bottom BC is usually applied gradually, reaching the assigned values during the initial iteration steps, and then it will be fixed during the remainder of the computation.

5.3.3 Convergence Study and NLFFF Quality Metrics

To verify the quality and accuracy of the NLFFF extrapolation results, routine check and evaluation of the extrapolated coronal magnetic field in a volume, including the force-freeness and divergence-freeness metrics, and comparison with coronal observations are usually required according to various validation studies of NLFFF modeling results (Schrijver et al., 2006; Metcalf et al., 2008; DeRosa et al., 2015). In this study, we calculate several NLFFF quality metrics to examine the trend of the extrapolation results along the convergence process. The quality metrics include the residual of the field between two successive iteration steps; the $\text{CWsin}$, a current-weighted sine metric defined by the mean sine of the angle $\theta$ between the electric current density $\mathbf{J}$ and the magnetic field $\mathbf{B}$, weighted by $|\mathbf{J}|$; the $\langle |f_i| \rangle$ metric, a volume-averaged divergence of the magnetic field (see a more complete description in He et al., 2022). In addition, the total magnetic energy $E_{tot}$ is also obtained.
For the June 22, 2015 event, we perform three extrapolation runs with different bottom BC inputs and grid structures. Associated descriptions about the bottom BC inputs are listed in Table 5.1. The extrapolation Run 1 employs the nonuniform embedded magnetogram with a FOV of 204″ × 204″ (149 × 149 Mm$^2$). The higher-resolution GST/NIRIS magnetogram with a FOV of 50″ × 46″ (37 × 34 Mm$^2$) has been embedded into a larger SDO/HMI map with 1″/pixel resolution. The inner core region is thus constructed with a volume size of 50″ × 46″ × 16″ (37 × 34 × 12 Mm$^3$) with a grid size of 0.125″ (91 km). Two additional runs are conducted for comparison. Run 2 is carried out based on the uniform SDO/HMI vector magnetogram in a FOV of 204″ × 204″, with a rebinned spatial resolution at 1″/pixel, which forms a core region with a height of 16″ (12 Mm, the same physical height as Run 1). Run 3 employs the uniform higher-resolution vector magnetogram from GST/NIRIS in a smaller FOV of 50″ × 46″ with a pixel size of 0.125″. The volume size of the core region for Run 3 is 50″ × 46″ × 2″ (16 pixels in height). The corresponding NLFFF quality metrics are calculated in a larger volume of 192″ × 192″ × 96″ (140 × 140 × 70 Mm$^3$) for Run 1 and Run 2, and in a smaller domain of 62.5″ × 53.75″ × 37.5″ (46 × 39 × 27 Mm$^3$) for Run 3 only.

From Figure 5.2, the residuals from three runs all decrease to a small magnitude of the order 10$^{-6}$ toward the end of the iteration. As for divergence-freeness, all three runs become stable after ~20,000 steps. Figure 5.2(b) shows that the convergence of the current-weighted sine metric in Run 1 is more complex than the other runs. In a larger domain of 192″ × 192″, the CWsin value for Run 1 keeps decreasing but it is relatively higher (~0.6, ⟨θ⟩ ~ 37°) than the result from
Run 2 within 40,000 steps. Considering the nonuniform BC we applied for Run 1, such a high CWsin value in a large domain may be due to the difference between the part of the updated outer bottom boundary from the HMI magnetogram and the potential field solution based on the inner GST magnetogram in Run 1. Therefore, we also calculate the CWsin values in a smaller domain for Run 1 with a size of 62.5'' × 118.75'' on the bottom boundary (orange curves in Figure 5.2) for consistency check, which reduces to \(\sim 0.34\) \((\langle \theta \rangle \sim 20^\circ)\) after \(\sim 40,000\) steps. This corresponds to the main part of the volume in which the subsequent topological analysis will be performed (see Section 5.4). For reference, the CWsin value for Run 3 becomes almost stable and equal to Run 2 after 25,000 steps.

For the magnetic energy \(E_{\text{tot}}\) in a larger domain, Run 2 becomes stabilized while Run 1 shows a gradual increasing trend. For a smaller domain, the evolution of \(E_{\text{tot}}\) for Run 3 and Run 1 (smaller) becomes stabilized with a similar trend. And \(E_{\text{tot}}\) is also larger than the corresponding potential field energy \(E_{\text{pot}}\) for all three runs. The computational time for three extrapolation runs varies and is generally proportional to the count of blocks as shown in Table 5.1. To account for the low-lying magnetic structures for the nonuniform grid configurations and the available computational resources, the core regions with the same and modest physical heights are assigned for Run 1 and Run 2. So significantly more blocks are created in Run 1 with a smaller grid size which takes more time to converge.

In general, the computational time can be shortened significantly with a smaller core region. For example, another run with a similar setting as Run 1 for a core
region at a height of 16 pixels (2′′, 1.46 Mm) took about ∼19 hrs to converge (40,000 steps) with the same computational resource.

**Figure 5.2:** Evolution of the convergence metrics for the three extrapolation runs: (a) the residual of the magnetic field, (b) the current-weighted metric CWsin, (c) the total magnetic energy $E_{tot}$, and (d) the divergence-freeness parameter $\langle |f_i| \rangle$. In practice, these metrics in (b)-(d) are calculated for different domains. The domains size for metrics calculation are 192′′ × 192′′ × 96′′ (140 × 140 × 70 Mm$^3$) for Run 1 and Run 2, and 62.5′′ × 53.75′′ × 37.5′′ (46 × 39 × 27 Mm$^3$) for Run 3, respectively. Another smaller domain with a size of 62.5′′ × 118.75′′ × 37.5′′ (46 × 87 × 27 Mm$^3$), the same as the FOV of Figures 5.5 and 5.6, is applied additionally for Run 1.

To get a converged and consistent result for later analysis, we further check the quality of these extrapolation results with additional quality metrics for force-freeness and divergence-freeness. Here we show the comparison of NLFFF quality metrics at 40,000 iteration steps from three extrapolation runs in Table 5.2. For Run 1, the additional metrics in a smaller volume (62.5′′ × 118.75′′ × 37.5′′)
are derived. Similarly, the additional metrics for Run 2 and Run 3 are also calculated, with volume sizes of $192'' \times 192'' \times 96''$ and $62.5'' \times 53.75'' \times 37.5''$, respectively. The $CWsin$ values for all three runs are around $0.24$–$0.34$ ($\theta \sim 14^\circ$-$20^\circ$), which are consistent with other NLFFF extrapolation results for realistic solar magnetograms (Jiang and Feng, 2013; De Rosa et al., 2009). Given that the small-scale structures in the magnetograms with weak magnetic field may increase the $CWsin$ value due to weak currents, we also evaluate the force-freeness and divergence-freeness with two additional metrics, $E_{\nabla \times \mathbf{B}}$ and $E_{\nabla \cdot \mathbf{B}}$, which estimate the residual force in the extrapolation results. The residual force comes from two parts: one is the Lorentz force $(\nabla \times \mathbf{B}) \times \mathbf{B}$, and the other one is due to the non-vanishing divergence of the magnetic field $\mathbf{B} \nabla \cdot \mathbf{B}$ from the numerical errors (see the detailed descriptions in Duan et al., 2017). The results of the two additional metrics for all extrapolation runs are small and of the same orders of magnitude and consistent with the previous reports (Jiang and Feng, 2013; Duan et al., 2017; He et al., 2022). Thus these extrapolation results extracted for the aforementioned smaller volume can be considered as converged solutions and are to be further analyzed and compared with observations.

**Table 5.2: NLFFF Quality Metrics for Force-freeness and Divergence-freeness**

| Runs          | CWsin | $\langle |\mathbf{j}| \rangle$ | $E_{\nabla \times \mathbf{B}}$ | $E_{\nabla \cdot \mathbf{B}}$ |
|---------------|-------|-------------------------------|-------------------------------|-------------------------------|
| Run 1 (smaller) | 0.338 | $4.50 \times 10^{-4}$        | 0.219                         | $2.10 \times 10^{-2}$       |
| Run 2         | 0.255 | $8.90 \times 10^{-4}$        | 0.161                         | $3.76 \times 10^{-2}$       |
| Run 3         | 0.243 | $2.02 \times 10^{-4}$        | 0.157                         | $1.97 \times 10^{-2}$       |

As a routine and qualitative validation of the extrapolation results, Figure 5.3 shows a general comparison between an AIA 131Å image and each extrapo-
Figure 5.3: Magnetic field lines from three extrapolation runs within a height range of [0'', 60''] ([0, 44 Mm]) overplotted on the coaligned AIA 131 Å image at 17:36 UT. (a) and (b) show field lines from Run 1 and Run 2 rooted in the same set of uniformly distributed seed points in a FOV of 190'' × 190'' (139 × 139 Mm²) which is similar to Figure 5.1(a). Panel (c) shows field lines from Run 3 in a smaller FOV due to a limited computational domain of Run 3. And the white dashed box indicates the FOV of Figure 5.7.

lation run with arbitrarily selected magnetic field lines rooted on the same set of seed points. In general, the distribution of field lines from Run 1 is similar to Run 2 and both results match well with the large-scale coronal structures in the coaligned AIA 131Å observation while field lines in Figure 5.3(a) show a more compact shape near the flare core region than Figure 5.3(b). In contrast, Figure 5.3(c) exhibits a finer field line connectivity pattern around the center, which is confined in a smaller FOV given the limited computational domain of Run 3.

5.4 Results from Extrapolations and Observations

Before the main flare eruption, the two small-scale precursor brightenings were identified as P1 and P2 near the PIL at ∼17:24 UT (P1, see Figure 5.4(b)) and ∼17:42 UT (P2, see Figure 5.4(c-d)) from the study utilizing the high-resolution GST observations by Wang et al. (2017). Part of the results from
that analysis is reproduced in Figure 5.4(b)–(g). Figure 5.4(a) shows the GOES X-ray flux during the flare precursors. There are two small peaks appearing before the main flare eruption. These impulsive emission times of the GOES X-ray flux also coincide with the corresponding Hα brightenings as marked. Figure 5.4(b)–(g) are regenerated from Wang et al. (2017) to show the structural evolution of the flare precursors from the high-resolution GST/Hα observations. As shown in (b) and (d), the brightening P1b and P2a (“a” and “b” for each precursor period are named by Wang et al. 2017 based on the chronological order of their occurrence times) are almost co-spatial while the P1a lies southward in a relatively different area from the brightening P2b. The corresponding GST/NIRIS magnetogram of $B_z$ component is presented in (g) and overplotted with the PIL (colored by the yellow contour). During the precursor P1, a brightening point was also observed in the coaligned SDO/AIA 193 Å image in (f), which located close to the P1a brightening region.

5.4.1 Field Line Connectivity and GST Observations

To have a better understanding of the fine-scale structures in the 3D volume for the precursor brightenings, we compare the available GST/Hα observations with the 3D coronal magnetic field topology from the static extrapolations Run 1 to Run 3, respectively. Figure 5.5 illustrates the selected magnetic field line connectivity near precursor P1. Firstly, two areas of interest for regions P1a and P1b are identified from the GST/Hα image at 17:24:18 UT in light blue scales (the same as Figure 5.4(b)) when the brightening intensity of a pixel is greater
Figure 5.4: Precursor brightenings and associated observations. Panel (a) shows the GOES X-ray flux during the precursors. Two dashed lines mark the emission times for the two precursors which occurred at $\sim$17:25 UT and $\sim$17:43 UT, respectively. (b-e) Structural evolution of the H\(\alpha\) brightenings P1a/P1b and P2a/P2b from the GST observations as identified by Wang et al. (2017) before the peak of the main flare (reproduced from Figure 1 in Wang et al. (2017)), and (f) the corresponding image observed in the SDO/AIA 193 Å wavelength. Panel (g) shows the corresponding GST/NIRIS magnetogram of the $B_z$ component. The yellow contour marks the polarity inversion line (PIL).

than a certain threshold in Figure 5.4(b). Two groups of such brightening pixels from P1a and P1b are selected and colored by red (cyan) corresponding to the positive (negative) magnetic field polarity. The majority of the pixels are in red
with positive magnetic polarity, and the corresponding conjugate footpoints in cyan are marked across the PIL to the south based on the Run 1 result. Then the field lines that originate from the red points for all three NLFFF extrapolation runs are drawn for comparison and are colored by the vertical height along each field line in (b)–(d).

**Figure 5.5:** Analysis of selected magnetic field line connectivity near the precursor brightening P1. (a) Two groups of brightening pixels from P1a and P1b are selected over the GST/Hα+0.6Å image (in light blue shades) at 17:24:18 UT together with their conjugate field-line footpoints based on the extrapolation result from Run 1. They are color-coded by the magnetic polarity of the corresponding field-line footpoints on the bottom boundary: positive in red and negative in cyan. Panels (b-d) show the field lines that originate from the red points in (a) for all three NLFFF extrapolation runs (Run 1, 3, and 2, respectively) which are colored by the height. In each panel, the $B_z$ map for Run 1 on the bottom boundary is drawn in gray scales with the PIL indicated by the white contours. In (a) and (b), the two white lines S1 and S2 indicate the positions of two vertical slices to be displayed in Figures 5.9 and 5.10.

Most field lines from Run 3 in Figure 5.5(c) are short-sheared arcades appearing within the FOV of the GST observation. And the conjugate footpoints for the field lines originating from P1a mainly attach to a major negative polarity
Figure 5.6: Similar analysis for the selected field lines corresponding to the precursor brightening P2, based on the GST/Hα+0.6Å image at 17:42:19 UT and the extrapolation results from all three runs. The format is the same as Figure 5.5.

region of the background GST magnetogram, while field lines from P1b are generally open exiting the limited computational domain. The field lines from Run 2 are less sheared and extend longer than those results from the other two runs. Those long field lines in (d) come from P1a and P1b extending to an area beyond the FOV of the GST magnetogram and further southward of the cyan dots in (a). It is shown that some field lines from P1a in (d) are not closed within the selected domain as shown. In contrast, the corresponding field lines from Run 1 in (b) show conjugate negative polarity footpoints (cyan dots in (a)) extending beyond the FOV of the GST magnetogram, but well within the FOV of the HMI magnetogram and across the main PIL between the two major positive and negative polarities. The group of selected field lines from P1b in (b) stays closed on the bottom boundary while another group of field lines from P1a appears as
a shorter sheared arcade and lies above the group that originates from P1b. We can see that the field line bundles in Run 1 and Run 3 reach a similar maximum height of \( \sim 11 \) Mm (15”). From these comparisons, Run 1 shows a reasonable consistency for the sheared arcade structures shown across the main PIL where the two strong polarity regions are separated based on the GST and HMI magnetograms. The results for the other two runs are clearly affected by the sizes of their computational domains with each maintaining a uniform grid setting.

For precursor P2, we also draw the field lines originating from chosen brightening pixels based on similar criteria for the GST/H\( \alpha \) observation at 17:42:19 UT, as presented in Figure 5.6. Similar to Figure 5.5, the field lines from Run 1 in Figure 5.6(b) show the magnetic structures corresponding to the precursor brightenings with both a fine scale and a spatial extent along the main PIL confined within the strong field regions albeit at lower heights. The H\( \alpha \) observations show that the brightening region P2a is nearly co-spatial with P1b, whereas another region P2b lies in a different area from P1a. In terms of the height distribution, the field line bundle originating from P2b situates at a lower height compared to the field lines from P1a while the field lines from P1b and P2a have similar heights. As a general feature, the two groups of field lines from Run 1 lie almost parallel to the PIL. What’s more, the positive polarity footpoints of the sheared field line bundle from P2b are close to a part of the conjugate negative footpoints from P2a, as indicated by the red and cyan points in Figure 5.6(a), which is a potential configuration favorable for magnetic reconnection. Further topological analysis results will be presented in Section 5.4.3.
Figure 5.7: Comparison between the field line connectivity and AIA observations in different wavelengths. (a) All selected field lines from Run 1 in Figures 5.5 and 5.6 are superimposed on the AIA 131 Å observation. (b) Contours of flare ribbons colored by the elapsed time since 16:55 UT (see the color bar) are overplotted with the set of field lines over an AIA 1600 Å image. (c-d) The footpoints with positive (red) and negative (cyan) magnetic polarity for the set of field lines are drawn over the corresponding AIA 1600 and 1700 Å images.

5.4.2 Field Line Connectivity and AIA Observations

Due to the constraint of limited FOV, the fine-scale GST observation is not available in a larger FOV encompassing the pairs of conjugate field line footpoints in the negative polarity regions. To further verify the identified conjugate negative footpoints from the extrapolation Run 1, we also compare the field line connectivity with the corresponding SDO/AIA observations in a larger FOV (the white box marked in Figure 5.3(c)) to find its connection to the precursor brightenings. In Figure 5.7, a series of AIA observations during the precursor P1 are used as the background images along with the extrapolated field line bundles from Run 1. In (a), the field lines that originate from the selected Hα brightening pixels during precursors P1 and P2 are overplotted and they have a good consistency with the central hot loops in AIA 131 Å at 17:24 UT. In addition, in (b), the flare ribbon brightenings are also marked and color-coded by the elapsed
time since 16:55 UT. The positive and negative footpoints of identified field lines from precursors P1 and P2 are overlaid to AIA observations in (c)-(d). During precursor P1, the identified positive footpoints (red) from Run 1 near P1a and P1b are co-spatial with the brightening patches observed simultaneously in AIA 1600 Å and 1700 Å wavelengths. And the identified negative footpoints overlap partially with the brightening patches to the south in (c) and (d). Furthermore, as the flare ribbons can be used as an estimation for footpoints of reconnected magnetic field lines (Qiu et al., 2002, 2004), the overall flare ribbon evolution is superimposed in Figure 5.7(b). It indicates the initiation of the main flare reconnection closer to the PIL at earlier times and the subsequent extension of the ribbons away from the PIL when reconnection proceeds during the flare main phase. The multiple bundles of field lines identified from precursors P1 and P2 have most footpoints located inside or near the flare ribbons at earlier times, which offers additional evidence in support of this magnetic field configuration for the precursor magnetic reconnection between low-lying arcades followed by the main phase flare reconnection (see, e.g., Moore et al., 2001).

5.4.3 Additional Topological Analysis near Precursors

From the previous comparisons between the extrapolations and observations, several groups of sheared arcades over the main PIL have been successfully reconstructed corresponding to the Hα precursor brightenings. The conjugate footpoints of precursor brightenings based on the extrapolation Run 1 also show consistency with the alternative and subsequent brightening regions in the AIA
observations. But it remains a question: how to find the potential sites for magnetic reconnection? How do precursor brightenings evolve and what is the subsequent reconnection sequence? To look into such questions, especially the first one, additional parameters like the normalized current density (equivalent to $|\mathbf{J}|/|\mathbf{B}|$, in the unit of $1/\Delta$, where a uniform grid size $\Delta$ is used), the magnetic twist number $T_w$, and the squashing degree $Q$ are calculated to analyze the magnetic topology in a specific volume. The magnetic twist number $T_w$ gives a good estimation of how many turns two infinitesimally close field lines wind about each other (Berger and Prior, 2006; Liu et al., 2016). And the squashing degree $Q$ quantifies the change of magnetic connectivities (Demoulin et al., 1996; Titov et al., 2002). For example, complex 3D magnetic structures can be distinguished near high $Q$ regions, where the gradient of the field line connectivity as measured by the $Q$ value is large.

The top views of the twist number $T_w$ and the squashing degree $Q$ distributions on the bottom boundary are shown in Figure 5.8 along with the footpoints from the identified field lines corresponding to the precursor brightenings P1 and P2. They show again the spatial distribution of the field-line footpoints for P1 and P2 along and within the extent of the main PIL which is characterized by a sharp “ridge” like feature in these parameter distributions. Figures 5.9 and 5.10 show the distributions of $|\mathbf{J}|/|\mathbf{B}|$, the twist number $T_w$ and the squashing degree $Q$ on the two vertical slices, S1 and S2, as marked in Figures 5.5 and 5.6. The extrapolation result based on Run 1 is obtained with the embedded vector magnetograms at 17:32 UT (GST) and 17:36 UT (HMI), which provides
Figure 5.8: The top views of the distributions of (a) the twist number $T_w$ and (b) the squashing degree $Q$ (in base-10 logarithmic scale) on the bottom boundary. The footpoints for the four groups of field lines illustrated in Figures 5.5 and 5.6 are overlaid in different colors: precursor P1a in magenta, P1b in yellow, P2a in green, and P2b in cyan.

a snapshot of the magnetic field topology at a time between precursors P1 and P2. For the slice S1 in Figure 5.9, the field lines from P1b and P2a (colored by yellow and green respectively) go through a region with relatively high current density. Besides, the squashing degree $Q$ around those field lines exhibits a complex pattern intermixed with high values. Such complexity also remains around the intercepting field-line points on the slice S2 in Figure 5.10(d), indicating the potential sites for magnetic reconnection between these field lines, which could result in the co-spatial brightenings at their footpoints as observed in P1b and P2a areas. Considering the magnetic topology near other brightenings (P1a and P2b), the field line bundles originating from P1a and P2b (pink and cyan) are next to the other two with modest $Q$ values in Figure 5.10(d). The twist numbers are all insignificant for these field lines. However, from the distribution of the in-
Figure 5.9: Topological analysis of the field lines corresponding to brightenings P1 and P2 near the vertical slice S1 (location marked in Figures 5.5 and 5.6). (a) The distribution of $|\mathbf{J}|/|\mathbf{B}|$ on the slice S1. The identified field lines corresponding to precursor P1a (magenta), P1b (yellow), P2a (green), and P2b (cyan) are also shown and marked. (b) The same distribution on S1 as (a) but with the corresponding magnetic field line intercepting points in the same colors as the field lines overplotted on the vertical slice S1 and the bottom layer. Those intercepting points are also drawn in (c) and (d). Panels (c) and (d) show the distributions of the twist number $T_w$ and the squashing degree $Q$ on slice S1 respectively, as indicated by the colorbars. For all panels, the $B_z$ map with the PIL highlighted by white contours is drawn on the bottom layer.

The intercepting points of different field lines on the slice S2 in Figure 5.10(b), the cyan field line bundle from P2b is lower than the pink bundle from P1a and it is close to yellow/green field line bundles from P1b/P2a. In addition, the cyan field lines from P2b and green field lines from P2a are separated by a high $|\mathbf{J}|/|\mathbf{B}|$ region in Figure 5.10(b). Such a configuration may correspond to the initial stage of the
Figure 5.10: Similar analysis as shown in Figure 5.9 with the distributions of the corresponding topological parameters on the vertical slice S2. The format is the same as Figure 5.9. The same sets of the magnetic field lines and the intersection points with corresponding colors are shown.

tether-cutting reconnection scenario described in van Ballegooijen and Martens (1989) and Moore et al. (2001). That is the magnetic reconnection among and between the sheared magnetic flux bundles as shown across the main PIL may take place, resulting in the brightenings of the associated field line footpoints without significant changes in their positions. And this could be one result to explain the spatial features consistent with the observed brightenings, but not the temporal changes. The purpose of this extrapolation is to provide a snapshot at a specific time. To examine the temporal change in the magnetic field topology
is beyond its capability. Nonetheless, we also performed an additional extrapolation run for the embedded magnetogram around 17:48 UT, at a time that is a few minutes after P2. The result shows a similar magnetic field line topology for the precursor regions as we have presented in this section. It probably implies that the reconnection associated with the precursors only involved small amounts of flux and the reconnection did not significantly change the flux distributions of the precursor regions. And the small amount of flux change distributed outside the FOV of GST may not be captured by the SDO/HMI magnetogram with its modest spatial resolution.

5.5 Summary and Conclusions

In this study, we have applied the CESE-MHD-NLFFF extrapolation method to a nonuniform embedded magnetogram for the first time to study the fine-scale structures of precursors before the main flare eruption. Three extrapolation results are obtained with different bottom BCs and grid structures, namely, Run 1 with a nonuniform embedded magnetogram, Run 2 with a uniform SDO/HMI magnetogram, and Run 3 with a uniform GST/NIRIS magnetogram. In the convergence study, the residual and the divergence-freeness parameters for all three runs become sufficiently small during the iteration, while the force-freeness parameter shows more complicated behaviors. The CW\text{sin} value in a larger computational domain for Run 1 is higher than the other results, but it reduces to \(\sim 0.34\) (\(\langle \theta \rangle \sim 20^\circ\)) for a smaller volume in which the magnetic field topology is examined in detail. The deviation from a strict force-free state in Run 1 could be
due to the nonuniform BC and grid structure designed for the embedded magnetogram. Nonetheless, the $CWsin$ values calculated in the regions of interest for all three runs after $\sim 40,000$ iteration steps are around $0.24 - 0.34$ ($\langle \theta \rangle \sim 14^\circ - 20^\circ$), which are consistent with prior NLFFF extrapolation results that are considered to be converged solutions for realistic solar magnetograms. After the converged results are obtained, we look into the reconstructed 3D magnetic field topology around the precursor brightenings, and compare the field line connectivity with the GST/H$\alpha$ and SDO/AIA observations. Additional topological features for the extrapolation Run 1 are investigated by focusing on the fine-scale structures around the precursor brightenings and across the main PIL. The main results are listed as follows:

1. For all three extrapolation runs, the field line connectivity around the precursor brightenings is compared with the GST/H$\alpha$ observations. The magnetic field lines originating from the precursor brightening regions based on Run 1 exhibit a configuration of the fine-scale magnetic structures beyond the small FOV of GST but confined within the larger FOV of HMI, more consistent with the spatial extent of the main PIL between two major magnetic polarities. Multiple sheared flux bundles are found overlying across the main PIL with groups of footpoints rooted in the positive magnetic polarity regions and coinciding with each set of the observed H$\alpha$ brightening patches, P1 and P2, respectively.

2. The selected field line bundles originating from the H$\alpha$ brightening patches from Run 1 show an overall shape consistent with the corresponding AIA
observations in different wavelengths. Those selected field lines have a good correspondence with the hot loops observed in AIA 131 Å passband. And their footpoints are attached to the inner sides of the flare ribbons with the closest distances from the PIL, which indicates a potential configuration for the magnetic reconnection during the flare precursors at earlier times.

3. With the magnetic field topological analysis near the precursor brightenings based on the extrapolation Run 1, including the distributions of the normalized current density $|\mathbf{J}|/|\mathbf{B}|$, the magnetic twist number $T_w$ and the squashing degree $Q$, a plausible configuration for magnetic reconnection is found. Such structures may correspond to the initial stage of the tether-cutting reconnection scenario, before the main “flare onset”, for instance.

These results based on Run 1 represent the application of the CESE-MHD-NLFFF extrapolation method for an embedded photospheric magnetogram from the GST/NIRIS and SDO/HMI observations. By utilizing different analyzing tools for the extrapolation results together with additional observations, the fine-scale magnetic structures around flare precursors are found to be consistent with the associated high-resolution GST/Hα observations. We conclude that the reconstructed magnetic field line topology/connectivity across the main PIL from Run 1 is more plausible for the subsequent magnetic reconnection among the sheared flux bundles, resulting in the corresponding precursor brightenings. We thus provide a viable approach to investigate the fine-scale structures associated with solar eruptions by combining the high-resolution magnetogram in a smaller FOV with another set of magnetogram in a larger FOV. By resolving the poten-
tial site for the small-scale precursors before the main flare eruption, this study demonstrates the merit of employing the ultra high-resolution magnetogram with its native resolution. The reconstructed magnetic field over the whole computation volume could also be further analyzed and the results could contribute to improving our understanding on how to make a connection between the small-scale energy release processes and the main phase of solar eruptions at larger scales, including the filaments, flares, CMEs and so on. This will be pursued in future studies.
Chapter 6. Topological Analysis for a Solar Jet Event with the Updated NLFFF Code and Observations

In this Chapter, we aim to apply the updated CESE-MHD-NLFFF extrapolation method evaluated in Chapters 4 and 5 to a recurrent solar jet event with another set of nonuniform embedded vector magnetograms from both SDO and GST to study the change of fine-scale magnetic structures related to the observed appearance of a recurring jet occurred in AR 12583. In a study by Zhao et al. (2022), the magnetic origin of the observed fan-shape jets has been investigated based on analysis of high-resolution observations. It was found that the magnetic cancellation has been observed in a small FOV along with the horizontal converging flow, which may be associated with the triggering mechanism of the recurrent jet.

6.1 Introduction

The amount of released energy in solar eruptions can vary a lot from coronal mass ejections (CMEs), flares, filament eruptions to localized solar jets. As a result, multi-scale magnetic structures could be found in different solar transients in the solar atmosphere and some of those structures may play an essential role in the formation and evolution process of solar eruptions. We have seen numerous
efforts made to understand the formation and evolution of large-scale magnetic
structures in the associated solar eruptions. For example, the magnetic flux rope
(MFR), defined as a coherent structure which consists of a group of twisted mag-
netic field lines, is generally believed to be the core magnetic structure and it
has been found in a variety of solar eruptions. The magnetic properties of MFRs
during the evolution of large-scale solar eruptions from the Sun to interplanetary
space have been an important topic to explore. In addition to focusing on the
large-scale coherent magnetic structures, it also becomes important and feasible
to improve our understanding and the analysis method towards the fine-scale
magnetic structures. They often occur during the earlier phase before the onset
of strong solar eruption like the flare precursour phase as well as relatively weak
solar activities like solar jets.

Solar jets usually represent dynamic, small and narrow ejections which
occur in different sizes and originate in different layers of the solar atmosphere.
And they are considered to be one of the important sources to supply mass and
energy to the upper solar atmosphere and solar wind (Raouafi et al., 2016; Shen,
2021). Observations of solar jets have been reported ubiquitously as features with
small spatial scales in multiple wavelengths, such as UV, EUV, X-ray, and Hα
(Shibata et al., 2007; Sterling et al., 2015; Panesar et al., 2016a; Tian et al., 2018;
Joshi et al., 2020). Many jets have been observed close to sunspots (concentrations
of strong magnetic field on the solar surface) while some have been found above
sunspot light bridges (which has a generally weaker and more inclined magnetic
field than the neighboring elements) for more complex sunspots (Tian et al., 2018;
In particular, some previous chromospheric observations in H$_\alpha$ and Ca II passbands also reveal the occurrence of the recurring surge-like (or jet-like) activity above light bridges in a long period, which are also named as H$_\alpha$ surges, plasma ejections, chromospheric jets, or light walls by different authors (Tian et al., 2018; Roy, 1973; Asai et al., 2001; Louis et al., 2014; Robustini et al., 2016). With the improvement of the technology and instruments, more coordinated observations tracking the same solar source region become available and in turn provide us unique opportunities to investigate such dynamic solar activities in different spatio-temporal resolutions, spectral lines and fields of view (FOVs). Therefore, to gain insight into solar jets, it is desirable and practical to make use of multiple observations to find associated fine-scale magnetic structures and study their roles in the formation and evolution processes of solar jets.

Although the energy release of solar jets is much smaller than the commonly observed flares and CMEs, jets share many common features with those strong phenomena, like the formation, evolution and driving mechanisms. Analysis for those features in solar jets could make a contribution to understanding the complexity in large scale solar eruptions. It is usually assumed that magnetic reconnection plays a dominant role to initiate and to drive those small solar transients. Many efforts have been made to find evidence of small-scale magnetic reconnection like high-speed motions of the outflow and strong current density (Louis et al. 2014; Toriumi et al. 2015; Robustini et al. 2016). Similar as debates in the formation mechanism of MFRs in strong solar eruption, there are basically two theoretical models for the initiation mechanisms of solar jets. One
is the flux emergence model which suggests that the jets are formed when newly emerged magnetic flux undergoes magnetic reconnection with the surrounding magnetic field (Shibata et al., 1992). Another model proposes that the on-disk jet forms through the eruption of a small-scale filament or so-called mini-filament (see Sterling et al. (2015); Adams et al. (2014)). Due to the limitation of the observations, the magnetic properties of solar jets during the evolution processes have not been fully investigated. For example, one may investigate the fine-scale magnetic structures associated with the initiation and driving mechanisms for recurrent jets. In addition, more detailed study of solar jets can be explored with the help of numerical simulations. Motivated by observations, many studies have simulated the dynamic process of jets based on idealized initial conditions and make a significant contribution to improving our understanding toward the universal/fundamental physical process in solar jets (Pariat et al., 2015; Wyper et al., 2018). To date, there are also simulation results which aim to reconstruct the magnetic topology more comparable to the corresponding detailed realistic observations for solar jet by employing the realistic vector magnetograms (see simulation results by Guo et al. (2013); Zhu et al. (2017); Nayak et al. (2019)).

In this Chapter, we apply the updated CESE-MHD-NLFFF extrapolation method for the available embedded vector magnetograms to resolve fine-scale structures in a recurrent solar jet and to compare those results with the remote-sensing observations from SDO and GST. In our previous study, the nonuniform embedded magnetogram generated by GST and SDO has been applied successfully as input to the updated CESE-MHD-NLFFF code and the fine-scale mag-
netic field topology in flare precursors before the main flare onset is identified (He et al., 2023). Therefore, it is promising to utilize the GST and SDO observations jointly for the solar jet event to study the evolution of fine-scale magnetic structures and to understand the associate triggering or formation mechanisms for solar jets.

6.2 Data and Method

6.2.1 Data and Event Overview

On September 7, 2016, a series of intermittently fan-like jets was observed in a sunspot group of NOAA AR 12583 (N07°, W25°) by the GST at BBSO and lasted for several hours. The analysis on the observational results from GST has been performed by Zhao et al. (2022) based on the data in a selected time range between 17:00 and 17:44 UT. To study the fine-scale magnetic structures in the solar jet event, we mainly reconstruct the 3D magnetic field by using the embedded GST-SDO magnetograms chosen in the same time interval and compare the results with various high-resolution multi-wavelength observations from GST. For example, the Hα images are obtained from the Visible Imaging Spectrometer (VIS, Cao et al., 2010) on GST with a scanning wavelength range shifting from -1.2 Å to +1.2 Å of the Hα line center of 6563 Å. The pixel size is 0.029” and the temporal resolution is 33 s. The TiO images are taken by the Broadband Filter Imager (BFI, Cao et al., 2010) using 7057 Å line with a passband of 10 Å. In addition to the GST data, the corresponding remote-
sensing observations in UV and EUV wavelength channels are provided by the Atmospheric Imaging Assembly (AIA, Lemen et al., 2012) onboard SDO with a spatial sampling of around 0.6″/pixel (∼438 km) and a moderate time cadence (12 s for EUV channels and 24 s for UV channels). However, due to the regular calibration activities of SDO, the guiding information was lost for the time period of interest. Therefore, we manually calibrated the AIA observations and use the coaligned data as an assistance for comparison in the following analysis.

The vector magnetograms can be derived from two telescopes. One is the Near InfraRed Imaging Spectropolarimeter (NIRIS, Cao et al., 2010) on GST, and it provides the full stokes profiles from the photospheric line Fe i 1565 nm doublet. The spectropolarimetric data are processed with the NIRIS data processing pipeline. After that, the vector magnetic field is extracted by using a Milne-Eddington Stokes inversion method (Wang et al., 2017) and the 180° ambiguity in the directions of the transverse field vectors is resolved based on a minimum-energy approach. The same methods have also been applied for deriving vector magnetograms from the Helioseismic Magnetic Imager (HMI, Schou et al., 2012) onboard the SDO. The pixel size and temporal cadence of the retrieved GST vector magnetograms are ∼0.08″ (∼59 km) and 87 s, respectively. To construct the embedded magnetogram in a large FOV, we also employ the SHARP vector magnetogram products from SDO/HMI. Given the vector magnetogram was not available after 16:20 UT, the HMI SHARP magnetogram at 16:12 UT is employed for the embedding purpose as a compromise. Then the high-resolution GST/NIRIS magnetograms at two times, 17:15 UT and 17:44 UT,
Figure 6.1: The $B_z$ map of the preprocessed embedded magnetogram from the combination of two magnetograms. The region inside the red box comes from a GST/NIRIS magnetogram observed at 17:15 UT in a pixel size of 0.125$''$/pixel. The area outside the red box mainly comes from a rebinned SDO/HMI magnetogram obtained at 16:15 UT with a pixel size of 1$''$/pixel. The yellow box shows the corresponding FOV of the panels in Figure 6.2.

respectively, are chosen to be embedded into the HMI magnetogram separately in slightly different sizes of FOV. The FOVs of the GST/NIRIS magnetograms embedded into the larger SDO/HMI magnetogram are 39$''$ × 40$''$ at 17:15 UT, and 40$''$ × 41.5$''$ at 17:44 UT, respectively. Figure 6.1 shows the preprocessed embedded magnetogram from GST (17:15 UT) and SDO/HMI (16:12 UT). Here the FOV of the outer HMI magnetogram is 320$''$ × 238$''$, which encloses the inner GST magnetogram in a FOV of 39$''$ × 40$''$. 
As for the data alignment, the TiO images are aligned with the NIRIS Stokes intensity data at first. Then the images at Hα far wings are coaligned with the TiO images and the near-line-center Hα data can be coaligned following the same transformation process as the data in far wings. The SDO/HMI continuum

![Figure 6.2](image_url)

**Figure 6.2:** Evolution of recurrent solar jets by coaligned multi-wavelength observations from GST and SDO. First column: Hα+0.4Å observations by GST/VIS. The red arrows indicate the locations of the fan-like ejection features from solar jets. Second column: TiO (705.7 nm) images from GST/BFI. Third column: GST/NIRIS continuum intensity (1564.85 nm) maps. Fourth column: SDO/AIA 1600 Å images (manually aligned based on cross correlation). The brightening spot near the jet feature is also marked by the red arrow.
images are aligned with the simultaneous TiO images. Then the subsequent AIA images from different wavelengths are coaligned with the continuum in turn. Figure 6.2 shows the overall coalignment of different data as well as the temporal evolution of the recurrent jets from the multi-wavelength observations from GST and SDO. In the first column, the corresponding Hα observations from GST/VIS show different patterns of fan-like dark ejection features at different times as their footpoints locate closely in the same place. From the coaligned AIA 1600 Å observations, there is a brightening feature clearly observed near the footpoints of the jets. The intensity of this brightening feature decreases in the following snapshots. To understand the characteristics and the underlying physical process in the periodically appearing ejections on the Sun, especially associated magnetic structures, two sets of embedded magnetograms are used to perform the NLFFF extrapolation and results will be compared with different observations to help us further explore this event.

6.2.2 NLFFF Extrapolation Method

To study the 3D fine-scale magnetic structure during the recurrent jet period, we apply the CESE-MHD-NLFFF extrapolation method for the nonuniform embedded magnetogram to reconstruct the 3D magnetic field. The CESE-MHD-NLFFF method is similar to a magnetofrictional method, which is a special case of the MHD relaxation method. The method has been described in detail in Chapters 2 and 4.
To handle the photospheric magnetograms in a high spatial resolution and a large FOV, a nonuniform grid configuration for a parallel computation framework is constructed with the help of the PARAMESH package (MacNeice et al., 2000), a toolkit to provide parallelization and adaptive mesh refinement support with structured mesh. More details for the grid initialization are described in Chapter 5. Figure 6.3 illustrates a side view of the nonuniform grid structure near the core region which contains grids in the highest spatial resolution corresponding to pixels with the finest resolution of the GST magnetogram in the input nonuniform embedded magnetogram. The grid size increases from 0.125″/pixel to 1″/pixel from the inner core region to the outer buffer region. Similar to the previous study, the whole computational domain is divided into several regions in different spatial resolutions to accommodate the nonuniform embedded magnetograms that will be adopted for the bottom boundary condition (BC). In this study, we use a core region height up to 24″ to resolve fine-scale structures.

After the grid initialization, the initial solution for all grids will be derived from the potential field solver based on the high-resolution GST magnetogram in the core region using the Green’s function method (Chiu and Hilton, 1977). After that, the bottom BC will be replaced gradually by the embedded magnetogram and associated equations will be solved through an iteration process until a converged solution is reached.
6.2.3 Convergence Study for Extrapolation Results

To prepare a converged result for further analysis, some metrics are utilized for the extrapolation results as a routine evaluation to study the convergence, including the force-freeness and divergence-freeness. Following various studies for NLFFF modeling results (Schrijver et al., 2006; Metcalf et al., 2008; DeRosa et al., 2015) and our previous studies, we calculate several NLFFF quality metrics to inspect the quality and accuracy of the extrapolation result along the relaxation process, including the residual of the magnetic field between two successive steps; the current-weighted sine metric $CWsin$; the metric $\langle |f_i| \rangle$, a volume-averaged divergence of the magnetic field (see a more complete description for these metrics...
in Appendix A). In addition, the total magnetic energy $E_{\text{tot}}$ is also acquired to oversee the overall convergence process.

**Figure 6.4:** Evolution of the NLFFF convergence metrics for three extrapolation runs with two sets of bottom BCs at 17:15 and 17:44 UT, as well as different core region heights. The size of a larger domain for metrics calculation is $288'' \times 224'' \times 43.75''$ and the size of another smaller one is $90'' \times 50'' \times 50''$.

As mentioned before, two sets of embedded magnetograms, which contain the GST magnetograms at two different times, have been obtained during the selected period (17:00–17:44 UT) of the recurrent jets. In this study, three extrapolation runs are conducted for the two different nonuniform embedded magnetograms which are used as the bottom BC inputs. Two test runs are performed
for the embedded magnetogram at 17:15 UT with two core region heights, 16″ (Run 1) and 24″ (Run 2), respectively. And we perform another run for the embedded magnetogram at 17:44 UT with a core region height of 24″ (Run 3). And similar grid structures for the inner core regions are assigned to accommodate the different GST magnetograms in two embedded magnetograms. The volume sizes of the inner core region are 39″ × 40″ × 16″ for Run 1, 39″ × 40″ × 24″ for Run 2, and 40″ × 41.5″ × 24″ for Run 3. To study the convergence of extrapolation results during the relaxation, the NLFFF metrics are calculated in two selected domains. The larger volume size is 288″ × 224″ × 43.75″ and the smaller volume size is 90″ × 50″ × 50″. As shown in Figure 6.4, the residuals for all runs drop below a small value of 10^{-5} after ∼20,000 steps and keep the decreasing trend toward the end of the iteration steps. For the evaluation of the divergence-freeness, the evolution of ⟨|f_i|⟩ and magnetic energy for all runs become stable after ∼20,000 steps. While the result from Run 1 shows a relatively higher ⟨|f_i|⟩ value than Run 2 and Run 3. The current-weighted CWsin keeps decreasing within 40,000 steps and shows different behaviors for different domain sizes. In a larger domain, CWsin falls to 0.725 for Run 2 and 0.713 for Run 3, which is relatively higher. With a lower core region height, Run 1 presents a higher CWsin value than Run 2 and Run 3. Such complex behavior for the evolution of CWsin is similar to our previous extrapolation result for another nonuniform embedded magnetograms and the high CWsin value in a large domain may be due to the difference between the part of the updated outer bottom boundary from the HMI magnetogram and the inner GST magnetogram. So the evolution of CWsin values in a smaller do-
main is also calculated for comparison. In a smaller domain (90″ × 50″ × 50″; the bottom area is close to the FOV of the inner GST map), $CW_{\text{sin}}$ parameter becomes smaller for both runs. At 40,000 iteration steps, it reduces to 0.463 in Run 2 and 0.434 in Run 3. These values are close to $CW_{\text{sin}}$ values in our previous study for the nonuniform embedded magnetogram (He et al., 2023) and the previous NLFFF extrapolation results for realistic uniform magnetograms (De Rosa et al., 2009; DeRosa et al., 2015). Therefore, the extrapolation outputs from a smaller volume show a more reasonable calculation of quality metrics and can be extracted for the subsequent analysis.

**Figure 6.5:** Similar as figure 6.4 but for the evolution of the maximum velocity and the average velocity (both normalized by the Alfvén speed) during the relaxation process.

To further evaluate the convergence and consistency of the extrapolation result for subsequent topological analysis, we also check the evolution of other parameters. In equation (5.1), the velocity $v$ would get closer and closer to zero during the relaxation process as the solution gradually approaches a static and stationary equilibrium force-free state. Therefore, we can further examine
the evolution of the average velocity $v_{\text{average}}$ and the maximum velocity $v_{\text{max}}$ in the extrapolation results, as a supplemental way to verify the convergence and consistency for the extrapolation results. Such metrics are also applied for the convergence study toward the near-force-free state in Jiang and Feng (2012). Figure 6.5 shows the evolution of the averaged velocity and the maximum velocity in the whole computational domains for two runs. In Run 2 and Run 3, the $v_{\text{average}}$ values begin decreasing and maintain the decreasing trend after about 14,000 steps, which is later than Run 1 ($v_{\text{average}}$ drops after 10,000 steps). At 40,000 steps, $v_{\text{average}}$ is close to zero, which is $3.978 \times 10^{-3}$ in Run 1, $3.646 \times 10^{-3}$ in Run 2 and is $3.416 \times 10^{-3}$ in Run 3, respectively. Another parameter $v_{\text{max}}$, becomes stable and fixed after around 20,000 steps for these three runs. The evolution of $v_{\text{average}}$ and $v_{\text{max}}$ indicate that a nearly static equilibrium state has been reached with a small average residual velocity in the order of $10^{-3}$. In general, for Run 1 and Run 2 which are conducted based on the embedded magnetogram at 17:15 UT with different core region heights, Run 2 shows smaller $v_{\text{average}}$ than Run 1. And Run 2 also gives smaller $C W\sin$ and $\langle |f_i| \rangle$ values than Run 1. Therefore, we will use the results from Run 2 and Run 3 for later analysis.

Before analyzing the results, it is also conventional to check the quality of the extrapolation qualitatively based on the consistency between the reconstructed field line distribution and coronal observations. Therefore, we also plot the field line distribution originated from a group of uniformly selected seed points and compare them with the corresponding H$\alpha$ observations. Figure 6.6 shows a general comparison between the H$\alpha$ observations by GST and the reconstructed
magnetic field lines. The open and close field lines are highlighted in yellow and

Figure 6.6: Magnetic field lines originated from evenly distributed footpoints over-plotted on the coaligned simultaneous Hα+0.4Å observations by GST/VIS at 17:15 (left column) and 17:44 (right column). The first and second rows illustrate the corresponding GST/NIRIS magnetogram of the $B_z$ component and the Hα+0.4Å images, respectively. The white contour marks the polarity inversion line (PIL). The colored boxes indicate the FOVs analyzed in Figures 6.7 (red) and 6.8 (yellow).

red separately. The appearance of dark jet features change from 17:15 to 17:44 UT. A bunch of closed field lines in the south of the jet surrounded by open magnetic field lines is identified and show consistency with some fibril structures observed in the Hα wavelength. And there are also some small closed loops and
open field lines near the jet. In general, the overall field line distributions in a large scale are very similar in two runs, and more comparison for small-scale structure associated with the jet eruption will be discussed in the next section.

### 6.3 Results from Extrapolations and Observations

According to a series of multi-wavelength observations during 17:00 – 17:44 UT analyzed by Zhao et al. (2022), this fan-like jet event appears periodically near the same area and it was indicated that the photospheric origin of these recurrent jets locates near the intergranular lanes between two sunspots based on the associated GST observations. Therefore, it would be interesting to find the associated fine-scale magnetic structures near the potential jet origin and to make potential connections for the recurrent jet evolution by analyzing the magnetic field topology via the extrapolation results from the nonuniform embedded vector magnetograms and the multi-wavelength high-resolution observations.

#### 6.3.1 Comparison between Extrapolations and Observations

Figure 6.7 shows the coaligned observations in SDO/AIA 1600 Å and GST/Hα passbands. There is a small brightening covering a few pixels found in the AIA 1600 Å image at 17:15 UT near the footpoints of the fan-like dark jet feature observed in the Hα image. Such a brightening may imply a potential connection to magnetic reconnection. To investigate the fine-scale magnetic topology near the footpoints of jets, we select a small area surrounding the brightening and the footpoints of the fan-like dark jet feature (the white box marked
Figure 6.7: Open and closed magnetic field lines from extrapolations start from a small selected region (white square) close to the origin of the recurrent jets at 17:15 (left column) and 17:44 (right column). The first and second rows show closed field lines overlaid with AIA 1600 Å images and GST/Hα+0.4 Å images. The third row is similar with the second row with additional open field lines from the selected region. The fourth row represents a side view of field lines in the third row over the bottom $B_z$ maps from extrapolations.
in Figure 6.7). The open and closed magnetic field lines originating from this small area are plotted in different colors. Results from Run 1 and Run 2 are shown in Figure 6.7(a-d) and (e-h), respectively. From Run 1, a small closed arch in red is identified with both ends of its footpoints located within the selected area and near the AIA brightening, while another larger arch originating from the selected area attaches to a negative polarity in the distance as illustrated in (d). For Run 2, there is also a small arch found within the selected FOV which lies a little southward than the small arch identified in Run 1. Figure 6.7 (b) and (f) show the same closed field lines as (a) and (e), but overlaid with the Hα + 0.4 Å images. The two small arches found in Run 1 and Run 2 are both situated near the footpoints of the fan-like dark features. The top view and side view of open and close field lines are shown in the third and fourth rows. The open (long) field lines rooted in the small selected white box enclose the small arch. The lower part of those open field lines generally orientated along the fan-like dark features while the larger closed loop in red lies beneath these open field lines.

To better understand the fieldline connectivity of these recurrent jets, we show the field line orientations by overplotting the selected field lines over the Bz maps along with the PILs in Figure 6.8. At 17:15 UT, all footpoints of the small closed arch distribute between two strong positive polarities. In 6.8(a), several small areas with the negative polarity are surrounded by large positive regions. The identified small arch has one end attaches to a small negative polarity on as shown in (c). In Figure 6.8(b), the distribution of PIL illustrates that the area of the small negative polarity expands. Figure 6.8(d) shows that the positive
Figure 6.8: Magnetic polarity distribution for the small regions as denoted by the yellow box in Figure 6.6. (a-b): $B_z$ component of the photospheric vector magnetogram overlaid with PIL (cyan). (c-d): same background as the first row overplotted with the selected close magnetic field lines from extrapolation results.

Footpoints of the small closed arch drift southward and get close to the region with stronger positive magnetic field. Since the identified small closed loops are co-spatial with the footpoints of the fan-like dark features, the evolution of those fine-scale magnetic structures may also help us to understand the formation and evolution processes of the recurrent jets, i.e., the potential sites and favorable magnetic topology for magnetic reconnection between the small closed loops and surrounding open (long) magnetic field lines.
6.3.2 Topological Analysis of Fine-scale Magnetic Structures in Recurrent Solar Jets

As suggested in the above comparison between observations and extrapolations, the field line connectivity of two separate extrapolation results shows a similar magnetic configuration consisting of a small closed loop surrounded by open magnetic field lines. The photospheric magnetogram used for the bottom boundary also exhibits the change of negative polarities associated with the closed loops. Meanwhile, there is a shift of positions for the corresponding small closed loops from two different extrapolation results. To further understand the potential role of the identified fine-scale magnetic structures in the formation and evolution mechanisms of the fan-like recurrent jet, we perform additional topological analysis for the extrapolation results. Figure 6.9 illustrates different topological features on the bottom boundary. The open and closed magnetic field lines in the selected domain are mainly distinguished based on whether two ends of each field line both attach to the bottom boundary or not. The first row show the distribution of open (red) and closed (blue) magnetic field-line footpoints on the \( XY \) plane at \( z=0 \). In (c) and (d), the footpoints of the small closed loops found in two runs generally distribute on the boundary of the squashing degree \( Q \) (where \( log_{10} Q \sim 0 \)). By comparing the distribution of large normalized current density \( (|J|/|B|) \) at \( z=0 \) layer illustrated in the third row with the distribution of open/closed magnetic field lines in the fourth row, we find that there are some areas with high \( |J|/|B| \) values spreading between the identified small closed loops.
Figure 6.9: Topological analysis of the field line connectivity on the bottom boundary for two times. For each column, the panels from the top to the bottom are: distribution of open and close field line footpoints on the bottom boundary, selected close magnetic field lines overlaid with the distribution of the squashing degree $Q$ on the bottom boundary, the distribution of the normalized current density $|J|/|B|$ over the background $B_z$ map. The field lines drawn in the fourth row are the same with field lines in Figure 6.7. The small white boxes in all panels indicate the same white FOV marked in Figure 6.7.
and open field lines in two runs. The formation of the recurrent jets could result from magnetic reconnection near the high $|\mathbf{J}|/|\mathbf{B}|$ region between open and closed magnetic field lines, a process that can be recurring as well. In general, open magnetic field lines pass through regions of high $|\mathbf{J}|/|\mathbf{B}|$ values have a good correspondence with the fan-like dark features at 17:15 UT and 17:44 UT, while the twist number $T_w$ and the squashing degree $Q$ values are variable. For these two runs, we do not find any twisted magnetic flux rope structures near the white box.

### 6.4 Summary and Discussions

In this study, we investigate the fine-scale magnetic structures in a recurrent solar jet event by employing the CESE-MHD-NLFFF extrapolation method for two sets of nonuniform embedded magnetograms from GST and SDO at two different moments. After the routine convergence study for two extrapolation runs, the extrapolation results are compared to high-resolution observations from GST and SDO. Additional topological analysis are also performed to investigate the potential role of the identified fine-scale magnetic structures near the footpoints of jet features in the formation and evolution mechanisms for this jet event.

Both of the two extrapolation results show a configuration composed of small closed loops surrounded by open field lines near the footpoints of the fan-like jet features, while the corresponding magnetograms also show the minor change of negative polarities near the closed loops. There is also a shift of positions for the corresponding small closed loops from two different extrapolation results.
Such a magnetic configuration is favorable for the magnetic reconnection of an interchange type, \textit{i.e.}, the reconnection between open and close magnetic field lines. This can be typically a recurring process, leading to little or no significant change in the distribution of the magnetic polarities on the photosphere. This scenario seems to be compliant with the magnetograms.

To better understand and interpret the fine-scale magnetic structures found in the NLFFF extrapolations, other physical properties of solar jets can be derived and be compared to our current results. For example, the motion patterns of photospheric flow from the Doppler velocity distribution by the DAVE method, the temperature distribution via the DEM analysis, and etc. A more comprehensive analysis could be done in our future work to understand the physical mechanisms behind the evolution of these fine-scale structures identified in the recurrent jets.
Chapter 7. Conclusions and Future Work

In this dissertation, we present our work and progress made to study a variety of magnetic structures at multiple spatial scales based on extrapolations and observations. Those structures can be found in different stages of solar eruptions. Their magnetic properties are analyzed and interpreted. The application of uniform photospheric vector magnetograms and multi-point measurements makes it possible to trace some large scale magnetic structures during their evolution from the Sun to the Earth. Additionally, the availability of high-resolution solar observations inspires us to modify the original NLFFF extrapolation method for uniform vector magnetograms to take input from nonuniform embedded vector magnetograms. By utilizing the embedded vector magnetogram combining the most modern space-borne and ground-based observations, the updated NLFFF method is tested, evaluated and then applied to analyze the magnetic topological properties of fine-scale magnetic structures in two selected solar eruptions. The main results are summarized as follows:

- The formation and evolution processes of CME-MFRs are complex and dynamic. A coherent MFR structure before the flare/CME eruption could be identified through the combination of observations and results from the NLFFF
extrapolations for one flare/CME event. However, the same MFR criteria could also fail to identify a pre-eruption MFR for another event.

– The connection between MFRs on the Sun and their interplanetary counterparts can be built up quantitatively based on the combination of various observations and the reconstructed magnetic field topology. Some magnetic properties (like magnetic flux and twist) are useful to bridge the gap between the MFRs developed on the Sun and their in situ characteristics quantitatively.

– Consequently, the quantitative results from two selected flare/CME events indicate that the magnetic reconnection process, manifested during solar flares, injects significant amount of magnetic flux (in the order of up to $\sim 10^{21}$ Mx) into the following CME-MFRs. In particular, the amount of the reconnection flux may correspond to either the toroidal (axial) flux or the poloidal flux content in the corresponding ICME flux rope.

– A updated version of the CESE-MHD-NLFFF extrapolation method for the nonuniform magnetograms is tested and applied for nonuniform embedded magnetograms from the SDO/HMI and GST/NIRIS for the first time. Based on three extrapolation runs with different BCs, the value of the force-free metric $CWsin$ in the updated extrapolation method is higher in a larger computational domain than the other extrapolation runs based on uniform vector magnetograms. On the other hand, the $CWsin$ values in the regions of interest are generally smaller ($\leq 0.34$) and consistent with prior NLFFF extrapolation results for realistic solar magnetograms.
From the comparison between the GST/Hα observations and the three extrapolation runs for one flare precursor event, the reconstructed magnetic field line connectivity near the main PIL from the extrapolation via the nonuniform embedded magnetograms shows a more consistent configuration within a larger FOV. Those selected field lines also have a good correspondence with the corresponding SDO/AIA observations of low-lying loops.

The fine-scale magnetic structures associated with the flare precursor brightenings are investigated. The magnetic field lines originating from the precursor brightenings via the extrapolation for the nonuniform embedded magnetograms show a plausible configuration for the subsequent magnetic reconnection among sheared flux bundles. And additional topological analysis indicates that those magnetic structures may correspond to the initial stage of the tether-cutting reconnection scenario.

With available embedded vector magnetograms from GST and SDO, the fine-scale structures in a small-scale recurrent jet event are also resolved and studied by applying the updated CESE-MHD-NLFFF extrapolation method. The comparison between the high-resolution GST observations and the extrapolation results helps to identify a configuration of small closed flux bundles surrounded by open field lines near the footpoints of fan-like jet features. And there are multiple alternating polarities near the closed field lines. Such a magnetic configuration is favorable for the interchange reconnections, which may be a common cause for the recurring behavior of this jet event.
For the current solar cycle, more vector magnetic field measurements become available via various solar telescopes including different space-based and ground-based observatories like the SDO, Solar Orbiter, GST, DKIST and etc. More coordinated observations will become available and can be used as inputs for numerical simulations to understand the dynamic natures of multi-scale magnetic structures in different solar eruptions. Further topological analysis and interpretations are planned to be done for more selected events with observations in higher spatio-temporal resolutions as well as measurements from multiple sources. There are several future topics to be pursued as listed below:

1. Conduct a statistical study on the magnetic properties for CME/ICME MFRs based on more selected events for different solar cycles. For example, see the identified flare-CME-ICME event list in Appendix B.

2. Analyze multi-scale magnetic structures for solar eruptions with a better understanding of the extrapolation results through other quantitative parameters, like the calculation of magnetic helicity as well as some thermal and kinematic properties like the temperature, velocity and density.

3. Apply the updated version of the CESE-MHD-NLFFF code for more high-resolution magnetograms, like the vector magnetograms from the DKIST and the Solar Orbiter. Along the way, we can also improve the consistency and flexibility of the CESE-MHD-NLFFF method for the embedded magnetogram, for example, by adding the adaptive mesh refinement technique for the grid construction.
4. Beyond the static extrapolation method, study the evolution process in solar eruptions with data-driven simulations and high-resolution observations. For example, the result of the CESE-MHD-NLFFF extrapolation for the embedded magnetogram can be used as the initial condition for dynamic MHD simulations.

5. Make a connection for the MFRs on the Sun to their potential interplanetary counterparts with the help of the static global coronal magnetic field reconstruction method and multi-point measurements.
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Appendix A. Convergence Metrics

The metrics for the convergence study of the computation during the iteration process are defined below. These include the residual of the field between two successive iteration steps $n$ and $n-1$ ($n > 1$),

$$\text{res}^n(B) = \sqrt{\frac{1}{3} \sum_{\delta=x,y,z} \sum_i (B^n_{i\delta} - B^{n-1}_{i\delta})^2 \sum_i (B^n_{i\delta})^2}, \quad (A.1)$$

where the subscript $i$ refers to the linear indices of grid points and runs over all grid points in the computational volume. We also estimate the force-freeness of the numerical result by a current-weighted metric, $CWsin$, which is defined by (equivalent to the weighted sine of the angle between $J$ and $B$),

$$CWsin = \frac{\sum_i |J_i| \sigma_i}{\sum_i |J_i|}; \quad \sigma_i = \frac{|J \times B|_i}{|J_i||B_i|}. \quad (A.2)$$

In addition, a current-square-weighted metric ($C^2Ws$) is similarly defined with more weight on the strong-current regions to reduce uncertainties from the weak field,

$$C^2Ws = \frac{\sum_i |J_i|^2 \sigma_i}{\sum_i |J_i|^2}. \quad (A.3)$$

Other important parameters including the total magnetic energy $E_{tot}$ and the divergence-free condition $\langle |f_i| \rangle$ (Wheatland et al., 2000; Metcalf et al., 2008)
are defined by,

\[ E_{\text{tot}} = \sum_i B_i^2 \Delta V_i, \]

\[ \langle |f_i| \rangle = \frac{1}{M} \sum_i \frac{(\nabla \cdot B)_i}{6|B_i|/(\Delta x)_i}, \]

(A.4)

where \( \Delta x \) is the grid spacing and \( M \) refers to the total number of grid points contained, and \( \Delta V_i = (\Delta x)_i \times (\Delta y)_i \times (\Delta z)_i \) representing a volume element.

Since the metric \( CWsin \) is not always reliable for evaluating the force-freeness, two additional metrics are proposed (see Jiang et al. 2012; Duan et al. 2017) by analyzing the residual force in the extrapolation result. The residual force for the numerical “force-free” field consists of two parts, the Lorentz force and a force induced by a non-zero \( \nabla \cdot B \). The non-zero divergence of the field introduces a force \( F = B \nabla \cdot B \) parallel to the field line (Dellar, 2001), which is unphysical and only results from numerical errors. To find a reference value of these two forces, the decomposition of the Lorentz force can be used,

\[ (\nabla \times B) \times B = (B \cdot \nabla)B - \nabla(B^2/2). \]

(A.5)

Here the first component is the magnetic tension force and the second component is the magnetic pressure gradient force. These two terms should be balanced for a force-free field. So another metric for the force-freeness can be defined as, for a
chosen volume $V$,

$$E_{\nabla \times B} = \frac{1}{V} \int_V \frac{|B \times (\nabla \times B)|}{|(B \cdot \nabla)B| + |\nabla (B^2/2)|} dV. \quad (A.6)$$

This equation is also the same as Equation (10) proposed by Malanushenko et al. (2014). Similarly, one can define an additional metric for the divergence-freeness by measuring the force induced by the non-zero $\nabla \cdot B$ by,

$$E_{\nabla \cdot B} = \frac{1}{V} \int_V \frac{|B(\nabla \cdot B)|}{|(B \cdot \nabla)B| + |\nabla (B^2/2)|} dV. \quad (A.7)$$
Appendix B. Identified Flare-CME-ICME Event List

(2010/08-2017/09)

<table>
<thead>
<tr>
<th>No.</th>
<th>Flare Peak</th>
<th>CME Peak Arc. Time</th>
<th>AR</th>
<th>Flare Class</th>
<th>STA/CME</th>
<th>LASCO/CME</th>
<th>ICME Start</th>
<th>ICME End</th>
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Figure B.1: Identified flare-CME-ICME candidate event list based on observations from SDO, STEREO and Wind spacecraft.

This event list shows candidate events which contain large-scale solar eruptions with confirmed observations of flares, CMEs and ICMEs. See the larger version in http://fluxrope.info/flare_cme_icme/list1.html. These candidate events are mainly selected from two published events lists. One is the flare eruption event list (2011.2–2017.9) studied by Jing et al. (2018), which is further studied combined with the NLFFF extrapolation by Duan et al. (2019). The flare-CME event list (2010–2013) is reported and investigated by Zhu et al. (2020). The erupted flare and CME observations come from GOES soft X-ray data and remote-sensing observations of SDO and STEREO, respectively. The ICME cor-
respondence is identified based on the in situ measurement by Wind spacecraft and in situ modeling results by Dr. Qiang Hu. Several confirmed CME and ICME events without the corresponding flare observations are also included.