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Uncertainty analysis of probability of kill in indirect fire simulations

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UNCERTAINTY ANALYSIS OF PROBABILITY OF KILL IN INDIRECT FIRE SIMULATIONS

by

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ABSTRACT
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The objective of this thesis is to analyze the uncertainty of indirect fire weapons against targets of various types and vulnerabilities as they appear on the battlefield. In order to analyze the effectiveness and associated uncertainty of a firing event, the appropriate analytical methods must be selected, and then weapon and target characteristics can be applied. Such characteristics include sensor error, aiming and delivery errors of weapons, and target vulnerabilities.

For this analysis, a Monte Carlo simulation was used to conduct probability of kill calculations. The uncertainties of the sensor, shooter and round were analyzed to understand the overall impact those errors have on the effectiveness of the firing event.

This study found that the type of distribution selected to represent the input data for the Monte Carlo simulation significantly influences the outcome of the engagement. Also, if the accuracy of the shooter and round is high, the sensor error dominates the resulting probability of kill, making the design for the sensor critical to the success of the engagement. When the accuracy of the shooter and round is low, the error of the shooter has a greater influence on the probability of kill than the error of the round. When this is true, the focus of the design effort should be placed on improving the design of the shooter rather than the accuracy of the round.

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TABLE OF CONTENTS

Page

LIST OF FIGURES vii
LIST OF TABLES viii
CHAPTER
1. INTRODUCTION 1
   1.1 Problem Overview and Objective 1
   1.2 Background 2
   1.3 Criteria and Parameter Restrictions 3
   1.4 Methodology 3
   1.5 Thesis Organization 5
2. LITERATURE REVIEW 7
3. PROBABILITY OF KILL FUNDAMENTALS AND INPUT DATA 10
   3.1 Circular Error Probable (CEP) 11
   3.2 Sensors 13
   3.3 Mean Point of Impact (MPI) and Precision Error (PE) 14
   3.4 Shooter and Round 14
   3.5 Mover Location Error 15
   3.6 Probability of Kill (P(k)) 16
   3.7 Targets 17
4. MODELING AND SIMULATION 19
   4.1 Model Setup 19
   4.2 Monte Carlo Simulation 20
   4.3 Sources of Error 21
   4.4 Random Number Generation 23
   4.5 Distribution Types 24
5. RESULTS 25
   5.1 Run Matrix 25
   5.2 Mortar Error Factors 28
   5.3 Cannon Error Factors 32
   5.4 Long Range Cannon Error Factors 34
   5.5 Percent Difference of Isolated Error Factors 36
   5.6 Dispersions 37
   5.7 Necessary Iterations 39
   5.8 Results Summary 40
6. CONCLUSIONS AND RECOMMENDATIONS 42
   6.1 Error Sensitivity Observations 42
   6.2 Influence of Distribution Type 43
   6.3 Impact on Design 44
APPENDIX: Excel Implementation of Random Number Generation 45
REFERENCES 50

LIST OF FIGURES

Figure Page

3.1. Probability of Kill Contributors 11
3. 2 CEP Example 11
4.1 Monte Carlo Simulation Process [6] 20
4.2 X-direction error calculation depiction 22
4.3 Y-direction error calculation depiction 23
4.4 Random Number Generation Example 24
5.1 Normal Distribution Probability of Kill for Run Matrix 27
5.2 Uniform Distribution Probability of Kill for Run Matrix 27
5.3 Probability of Kill Percent Difference Between Normal and Uniform Distributions 28
5.4 Probability of Kill for Isolated Mortar Variables using a Normal Distribution 29
5.5 Probability of Kill for Isolated Mortar Variables using a Uniform Distribution 30
5.6 Isolated Metrics for Y-direction PE Effects on Probability of Kill – Mortar, Normal 30
5.7 Isolated Metrics for X-direction MPI Effects on Probability of Kill – Mortar, Normal 31
5.8 Isolated Metrics for Y-direction PE Effects on Probability of Kill – Mortar, Uniform 31
5.9 Isolated Metrics for Y-direction MPI Effects on Probability of Kill – Mortar, Uniform 32
5.10 Probability of Kill for Isolated Cannon using a Normal Distribution 32
5.11 Probability of Kill for Isolated Cannon Variables using a Uniform Distribution 33
5.12 Isolated Metrics for X-direction PE Effects on Probability of Kill – Cannon, Normal 33
5.13 Isolated Metrics for Y-direction PE Effects on Probability of Kill – Cannon, Uniform 34
5.14 Probability of Kill for Isolated Long Range Cannon Variables using a Normal Distribution 35
5.15 Probability of Kill for Isolated Long Range Cannon Variables using a Uniform Distribution 35
5.16 Distribution Type Percent Difference Results for Isolated Variables – Mortar 37
5.17 Distribution Type Percent Difference Results for Isolated Variables – Cannon 37
5.18 Normal Distribution Dispersion of Shots for Sensor 1, Mortar, Exposed Soldier 38
5.19 Uniform Distribution Dispersion of Shots for Sensor 1, Mortar, Exposed Soldier 39
5.20 Number of Iterations Necessary for Calculating Probability of Kill 40

LIST OF TABLES

Table Page

3.1 Example Sensor Data for Range vs. CEP....13
3.2 Sensor Data for Simulation....14
3.3 Shooting system input data....15
3.4 Target Lethal Area Data....18
5.1 Run Matrix....25
5.2 Individual Error Factors Impact on Probability of Kill....41

DEDICATION

To my husband, Patrick; parents, Nan and John Gebus; and grandma, Helen Walterhouse.

CHAPTER 1

INTRODUCTION

1.1 Problem Overview and Objective

In the military, when a system fires in the vicinity of a target, several factors contribute to the success of the engagement including the sensor used to locate the target, the shooter or gun platform itself, the round fired, the target, the weather, etc. For this analysis, there are three factors which have statistical descriptions that enter into the probability of kill analysis: the sensor, shooting system, and target. This study focuses on the performance of three types of sensors. Sensor 1 is a nominal representation of a sensor with good performance. Sensor 2 represents a sensor with average performance. Sensor 3 is a relatively poor performing sensor.

Three shooting systems are considered. The shooting system is composed of two elements. The first element contributing to the aiming error is the gun platform, which is referred to as the shooter in this thesis. The second element is the round fired from the shooter. The shooting systems selected for this analysis are all indirect fire systems (meaning the aiming is done using indirect methods) with high explosive (HE) rounds. The three shooting systems represent indirect fire systems that are used at various ranges. The first is the Mortar which is used for shorter ranges. Next is the Cannon which is used for longer ranges. The third shooting system is the Long Range Cannon used for extended ranges. Each shooting system yields four sources of error: x and y error components for the shooter and x and y components for the round. The details of these errors are discussed in Chapter 3.

Targets selected for this analysis are limited to four generic types: exposed soldier, truck, light armored vehicle, and medium armored vehicle. The targets represent a range of vulnerabilities due to their sizes and armaments, with the exposed soldier being more vulnerable than the truck and armored vehicles. The targets selected affect the probability of kill by their associated lethal areas, which are a function of the shooting system and range to the target. The target details are discussed in Chapter 3.

Each of these factors has associated errors and uncertainty. A failure in execution by any of these components could cause an unsuccessful engagement. Therefore, the success of the engagement is a result of the aggregation of these factors. This aggregation makes it difficult to understand how the errors associated with each independent factor affect the success of the engagement.

It is the intent of this thesis to analyze the uncertainty associated with the individual components of an engagement. By understanding the individual uncertainties, the probability of success of the engagement can be traced back to its source. If a particular component integral to the engagement is much more sensitive to error than another, the design for that component should be held to a higher tolerance than a less sensitive component.

1.2 Background

Total system accuracy can be viewed as an "error budget" in considering each source contributing to the total error. In this case, the "error budget" is a set of systematically defined error sources (sensor, shooter, round), each contributing some portion to the total inaccuracy of the system. The idea of an error budget allows each error to be assessed to ensure that no single component exceeds a given limit, reducing the probability of kill.

In the development of new military systems, it is important to model weapon lethality, i.e., the probability of that system killing a target. In the firing of guns or missiles for ground impact, there results a two-dimensional pattern of shots or impact points which exhibits an

amount of scatter depending on the round-to-round aiming error and the ordinary "ballistic" dispersion [1]. The two-dimensional pattern of shots gives rise to various measures of dispersion which are used to summarize the "accuracy" of the pattern of shots. All of these measures require statistical analysis, in that they are really random variables from one firing group to another, especially because the individual impacts themselves are randomly scattered about the target area. The scatter pattern varies from one group to another. It is this random variation which is modeled in order to estimate the probabilities of a warhead killing a target. The probability of killing a target depends not only on the proximity of the warhead, but on the vulnerability of the target as well.

1.3 Criteria and Parameter Restrictions

On the battlefield, numerous methods of assessing the damage inflicted on a target are available. Damage to the target can be assessed based on surveillance, such as visual observation or ground sensors, human intelligence, or signal intelligence. This thesis focused specifically on damage assement methods based on modeling and simulation. When the process of damage assessment is modeled, equations have to be developed to estimate the damage sustained. Probability of kill equations are tailored to the specific scenario and exist for direct fire, indirect fire, high explosive munitions, improved conventional munitions, smart munitions, etc.

The type of round fired has a large impact on the damage incurred by the target. A target can be engaged using any number of types of rounds. This thesis was restricted to the effects of high explosive rounds fired by indirect fire systems. The range at which the shooter fires can also vary in a multitude of ways. Weapons usually have a recommended minimum and maximum range. This study focused on a selected mid range specific to each individual shooting system.

1.4 Methodology

The purpose of this study is to perform an in depth analysis on the uncertainty of each factor contributing to the probability of kill for an indirect fire system using a high explosive

round. A computer based simulation is used to investigate a matrix of error factors contributing to the success of the engagement.

The first step is to gain a general understanding of how target damage is assessed in military modeling and simulation applications. The assumptions and limitations within the models need to be understood and also which equations are used to represent damage assessment. The second step is to identify the type of weapon systems to be analyzed, i.e., direct fire, indirect fire, smart weapons, etc. The type of round must also be identified, and high explosive (HE) rounds are one of the more common types of shells used today. Equations used for damage assessment exist for specific weapon types. The present analysis focused on the equations and variables pertaining to indirect fire HE rounds. Details are discussed in Chapter 3.

Indirect fires with HE rounds use an elliptical damage function to measure probability of kill, whereas, for example, attrition of dismounted personnel by grenades uses a circular damage function. Once the proper equations have been determined, the appropriate input data is gathered. This set of inputs includes the sensor types, shooter and round data, and targets of interest. Representative data is gathered for each component necessary to evaluate the uncertainty of the overall system.

Next, the analysis method is defined. Typical uncertainty analysis methods can include the propagation and the Monte Carlo methods. Since multiple error configurations were considered, the most practical method for conducting this uncertainty analysis was the Monte Carlo method. A Monte Carlo simulation is used to approximate the probability of specific outcomes by iteratively generating random values for variables and observing the result. The method is useful for obtaining numerical solutions to problems which are quite complicated for analytical solution.

Once a method of analysis is defined, a computer simulation is designed to calculate the overall uncertainty. A Monte Carlo simulation is conducted for each factor contributing to the probability of kill. The simulated error for each factor is summed together and then input into the

data reduction equation to compute the probability of kill. For each simulation run and error source configuration, a different set of random numbers is calculated, thus providing a unique uncertainty measurement that corresponds to the input factors.

A distribution type must be chosen in order to generate random variables. Typically, weapon systems are simulated using a normal distribution. For this study, both a normal distribution and a uniform (rectangular) distribution of random numbers were analyzed.

The next step is to develop a method for displaying metrics and determining the data of interest. This is accomplished by viewing sample data and determining how best to display each configuration. Once the method for displaying data is defined and the data of interest is identified, the simulation is run to generate and record the data. The results are displayed and analyzed, and conclusions drawn.

1.5 Thesis Organization

The first chapter briefly introduces the topic of this thesis and explains the background of the study. This chapter outlines a few of the criteria and parameter restrictions associated with round selection, analysis method, etc. The chapter then explains the overall methodology followed to understand the problem and perform the uncertainty analysis.

Chapter Two provides a literature review which explores some previous works on the probability of kill uncertainty analysis. This chapter also introduces a classic reference widely used in the defense industry.

Chapter Three discusses the fundamentals needed for understanding the terminology used in performing the uncertainty analysis. Such terminology includes defining probability of kill, circular error probable, mean point of impact, and precision error. The probability of kill equation is introduced and the variables discussed. This chapter also explains the various "kill chains" selected for analysis. This includes information on selecting sensors, shooting systems, and targets and their resulting configurations and assumptions for analysis.

Chapter Four explains the modeling and simulation procedure used for generating data. Within the simulation, several key processes need to be understood. This chapter explains how the errors are summed, the process of random number generation as it pertains to the Monte Carlo simulation, and the various distribution types used.

Chapter Five presents the results generated using the simulation. The results are displayed graphically and analyzed in detail. As the initial results were generated, additional questions surfaced, requiring further investigation which is also presented here.

The final chapter discusses conclusions drawn from the data and analysis. This chapter also presents recommendations based on the outcome of the study.

CHAPTER 2

LITERATURE REVIEW

The U.S. Army has decades of experience in the analysis of weapon systems. To capture the background in this area of military operations research studies and weapon systems evaluations, the Army Weapon Systems Handbook [1] was developed. This classic reference is widely used as the recommended methodology for evaluation of Army weapon systems and material. The fundamental concepts and equations as they relate to target damage assessment, delivery error characteristics, and lethality are explained in this handbook. These fundamentals are today incorporated into data models such as Combined Arms and Support Task Force Evaluation Model (CASTFOREM), Janus, and other military supported attrition based simulations as mentioned by Davidson [2].

In an attempt to find open source literature on analyzing weapon effectiveness and to understand what work has been done, this chapter reviews several studies on probability of kill error calculations. The principles and conclusions that have been established in previous studies are identified. While many specifics about probability of kill error contributions have been studied, much of the work is not available open source and it does not specifically address many things such as indirect fire systems shooting HE rounds, using different distribution types to simulate error, and de-aggregating the factors that contribute to the probability of kill. Finally,

this chapter shows how the work of this thesis is related to previous research of probability of kill error calculations and how it is unique.

Davidson [2] analyzed the modeling of precision munitions in urban terrain. He conceded that the algorithms and data in models such as CASTFOREM and Janus simplify the complexity of the engagements, and that these simplifications lead to an overestimation of precision munitions' capabilities. The study establishes that each event in firing a munition has associated errors and that these errors are usually aggregated within a simulation such that it is difficult to identify how the failure in execution by any of these events will cause an unsuccessful engagement. Simply put, the data does not capture the independent nature of these events as represented in several combat models. His work also analyzes the specifics of precision munitions and the physics of their capabilities in urban terrain, and touches briefly on the concept of system delivery error, which is the ability of the round to arrive at a given point in space. This specific error includes drift, gun misalignment and crosswind.

Davidson's work supports the theory that errors in military simulations are aggregated together such that the individual errors are unrecognizable. However, his analysis is concentrated on precision weapons in urban terrain.

Jacques and Leblanc [3] of the Air Force Research Laboratory conducted an effectiveness analysis for wide area search munitions. They used Monte Carlo based effectiveness models to conduct much of their study. Since their effort was focused on autonomous munitions, the munitions were evaluated independently of the delivery platform. Jacques and Leblanc claim that for several like munitions (autonomous) flying over the same search path and in the same direction, a high degree of correlation would be expected in the munitions behavior when encountering a stationary target. However, if an effectiveness model performs a random draw each time a target is encountered, uncorrelated behavior is assumed because there is no change in probabilities based on previous munition behavior. Therefore, the Monte Carlo model assumes uncorrelated behavior at the target. Their paper also examines one subset of performance metrics

that can be incorporated into engagement level effectiveness models and simulations. Although autonomous wide area munitions differ greatly from indirect high explosive rounds, the modeling methodology is similar.

Thomas and Gremmill [4] developed an expression for increasing salvo kill probability through aim point patterning. They assume that the target is circular in nature, the aiming error and round-to-round error is governed by a circular normal distribution, and there is independence between the aiming error and round error. They further assume that the rounds in the salvo are aimed at a pattern of points about the target center. By restricting the pattern of aim points to equidistant points on a circle centered at the target center, they obtain an expression that increases the probability of kill. Their assumptions are similar in nature to some of those discussed in this study.

The literature reviewed in this chapter gives an overview of previous work conducted that relates to analyzing the uncertainty associated with probability of kill. Open source documentation on the topic is limited, so this thesis provides an analysis of the uncertainties as they pertain to probability of kill by using realistic, nominal values that can be discussed without restriction.

CHAPTER 3

PROBABILITY OF KILL FUNDAMENTALS AND INPUT DATA

The probability of killing a military target of interest is quite involved. Before the results of the analysis can be fully understood, it is important to cover some of the terminology and definitions as they pertain to probability of kill. To do this, the definitions for circular error probable (CEP), mean point of impact (MPI), precision error (PE), probability of kill (P(k)) and lethal radius are reviewed. These basic definitions are key to understanding the input data.

The input data for this thesis includes sensor, shooter, round, and target data. There are many combinations of these inputs. In order to draw conclusions relating to error budget without considering every possible combination, a representative selection of variables is chosen. This selection is enough to understand the interactions between the variables and their associated influence on the uncertainty of the firing event.

Each input variable affects the overall probability of kill of the target. Figure 3.1 shows a diagram of the probability of kill contributors which are explained individually in detail in this chapter. Again, these factors include the sensor, shooter, round, and target.

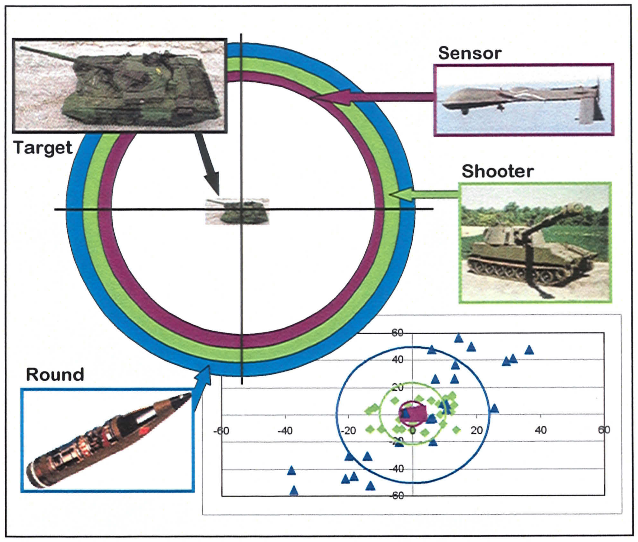


Figure 3.1 Probability of Kill Contributors

3.1 Circular Error Probable (CEP)

CEP is defined as the radius of a circle about the aim point inside of which there is a 50 percent chance that the weapon will impact and/or detonate. Using Figure 3.2 as an example, if 10 rounds were fired, 5 would land within the radius of the circle and 5 outside of the circle. If Sensor A has a CEP of 100m and Sensor B has a CEP of 20m, then the smaller radius of Sensor B would mean the shots would be more accurate. From this, it is apparent that a smaller CEP has more shots that are closer to the intended target.

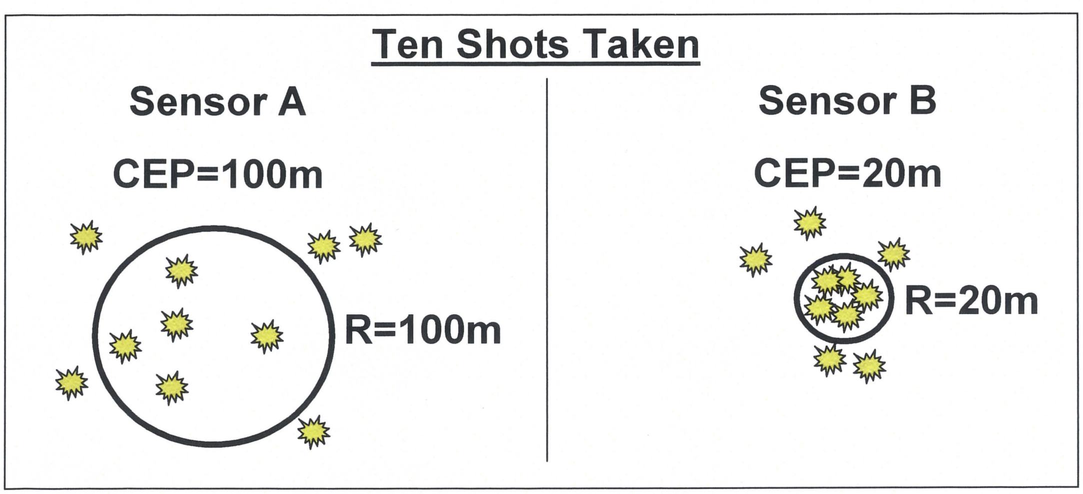


Figure 3.2 CEP Example

The accuracy of a sensor is often expressed in terms of CEP. The CEP has been developed primarily for the "circular" normal distribution [1]. The circular normal density function is described as

$$f(x,y) = \left[1/\left(2\pi\sigma^2\right)\right]\exp\left[-\left(x^2+y^2\right)/2\sigma^2\right]. \tag{3.1}$$

With the aim point at the origin, the function can be integrated over a circular target with the center at the origin and the result equal to one-half

$$\iint_{x^2+y^2 \le R_{0.50}^2} f(x,y)\,dxdy = 0.50. \tag{3.2}$$

In equation (3.2) the CEP is equal to the "fifty percent" radius $R_{0.50}$. By transferring the variables to polar coordinates where

$$x = r\cos\Theta \tag{3.3}$$

$$y = r\sin\Theta \quad 0 \le \Theta \le 2\pi, \tag{3.4}$$

equation (3.2) is now equal to

$$\left[1/\left(2\pi\sigma^2\right)\right]\int_0^{R_{0.50}}\int_0^{2\pi} d\Theta \exp\left[-r^2/\left(2\sigma^2\right)\right] r\,dr = 1-\exp\left[-R_{0.50}^2/\left(2\sigma^2\right)\right] = 0.50. \tag{3.5}$$

By expressing equation (3.5) as $\exp\left[-R_{0.50}^2/\left(2\sigma^2\right)\right] = 0.5$, the desired radius R, or CEP is described by

$$CEP = R_{0.50} = \sqrt{2\ln 2}\,\sigma = 1.1774\sigma. \tag{3.6}$$

So, the CEP is 1.1774 times the round-to-round standard deviation in one direction. Standard deviation σ is really a univariate or one-directional measure of dispersion, whereas the CEP, which depends on the coefficient of 1.1774, is often understood as a measure of bivariate or two-dimensional scatter of weapon shots, even though it depends on the same standard deviation in either direction [1].

3.2 Sensors

Sensor performance data as used in modeling and simulation is commonly quantified as CEP versus range. The data is typically presented such that the CEP for that sensor is given at a set of ranges from the shooting system to the target. Actual numerical values for specific sensors are not available through open source documentation, so Table 3.1 shows nominal sensor data presented as Range vs. CEP.

Table 3.1 Example Sensor Data for Range vs. CEP

** nominal data **	Sensor A	Sensor B
Range (km)	CEP (m)	CEP(m)
1	11	10
2	25	16
3	34	20
4	50	26

The uncertainty is determined from the CEP, which is translated into a standard deviation and used in the Monte Carlo Simulation. Several representative CEPs can be selected in place of range to target as an input for the sensor. For this analysis, one "sensor" defines a single CEP value. Early in the study, eight CEP values were selected to represent a range of performance from a high performing sensor to a low performing sensor. The original eight sensors were then narrowed to a representative three to help reduce simulation runtime. The three CEP values were selected to represent a sensor with good performance (smaller CEP), a sensor with poor performance (larger CEP), and a sensor with average performance. Table 3.2 displays sensor data for the three representative sensors.

Table 3.2 Sensor Data for Simulation

SENSOR	CEP (m)	STANDARD DEVIATION (m)
1	14.13	12
2	40.03	34
3	94.19	80

As mentioned earlier, the sensor data is given in terms of a circular error probable indicating that the error in the x- and y-directions are the same. For the Monte Carlo simulation, two sets of independent random numbers are generated to represent error in the x-direction and in the y-direction. Using CEP, the standard deviations used for each distribution are also the same in the x- and y-direction.

3.3 Mean Point of Impact (MPI) and Precision Error (PE)

The mean point of impact (MPI) is the arithmetic average point of impact of the group of rounds fired from a single gun on a particular occasion. Precision error (PE) is the uncorrelated variation or dispersion of the individual round impacts about the MPI. The pattern of shots on a target varies from group to group in a random manner. The MPI and measures of dispersion also vary randomly due to the random round-to-round ballistic dispersion, or precision error. The MPI movement and round-to-round precision error during the firing of a group of rounds affect the probability of hitting a target, which is of great importance in evaluating the effectiveness of a weapon [1]. Both the MPI and PE have a range and deflection component that is represented as a standard deviation and used in the Monte Carlo Simulation.

3.4 Shooter and Round

The shooting system data is comprised of four variables: the mean point of impact (MPI) error or bias in the range (y) and deflection (x) directions (2 variables), and the precision error (PE) or round to round error in both the x and y directions (2 variables). The MPI error is the

error associated with the shooter. The PE is the error attributed to the round. Three high explosive (HE) munitions are selected as representative rounds shot by indirect fire systems. To keep the data open source, the shooting system data used here is plausible, generic data that is a realistic representation of weapons as they appear in the battlefield. These representative shooting systems include a mortar, a cannon, and a long range cannon. Typically, the engagement range for a mortar is smaller than that of a cannon. Mortars usually have a high approach angle and plunging fire which is usually smaller in diameter than a cannon round [5]. The long range cannon is intended to be more accurate at longer ranges than the conventional cannon.

Input data for each shooter and round is represented in terms of the standard deviation as a function of several ranges. To reduce the number of combinations to be analyzed, a mid range for each shooting system was used to complete most runs. The representative input data used for this analysis is displayed in Table 3.3.

Table 3.3 Shooting system input data

Shooting System	Range (m)	MPI-x (m)	MPI-y (m)	PE-x (m)	PE-y (m)
Mortar	4500	26	120	27	44
Cannon	15000	30	97	13	57
Long Range Cannon	25000	6	6	5	5

* Numbers for MPI and PE are standard deviations

3.5 Mover Location Error

The mover location error, or the distance between the shooter's perceived location and actual location will be zero for this analysis. This error is usually an estimated distance and is treated as a uniform distribution.

3.6 Probability of Kill (P(k))

There are infinite potential outcomes to any engagement, but for most military purposes, the primary concern is that the target is removed from action. It is in this sense that the target is considered "killed." The probability of kill is a statistical measure of the likelihood that a target will be removed from action. Apart from the inputs mentioned thus far, the P(k) also depends on the nature of the target, how vulnerable it is to certain effects, and the proximity of the round to the intended target. The proximity of the round to the target takes into account the accuracy of the sensor, shooter, and round.

In many military modeling applications, the probability of kill is assessed using the elliptical Carleton damage function shown in equation (3.7) [1]. This equation specifically assesses the damage effects of indirect fire rounds, namely, conventional high explosive (HE) munitions.

$$P(k) = D_o * \exp\left[-D_o \left(\frac{\delta_x}{r * \sqrt{c}} \right)^2 + \left(\frac{\delta_y}{r / \sqrt{c}} \right)^2 \right], \qquad (3.7)$$

where

D_o = a scaling factor, determined by the type of munition

δ_x, δ_y = coordinate of target relative to local deflection and range axis centered at the impact point, takes into account error of sensor, shooter, round

r = lethal radius of round and attrition class

c = ratio of elliptical deflection axis to the range axis

The probability of kill is determined by comparing the calculated value from equation (3.7) to a random variable which is generated from a uniform 0-1 distribution. If the probability of kill is greater than or equal to the random variable, then the unit has incurred a kill.

The lethal area is used to determine the relative effectiveness of munitions and is more or less an "average" area in which incapacitations of target elements would occur if they happen to be at points located in the lethal fragment spray [1]. For the purpose and ease of analysis, lethal areas in this analysis will be considered to be a regular figure, such as a circle. The lethal radius is simply determined from the lethal area by $A=\pi r^2$.

The lethal area of a target is a function of the target range and shooting system selected. The more heavily protected the target, the smaller the lethal area, i.e., a heavily armored tank will have a much smaller lethal area than a soldier when considering a single shooting system. The lethal radius for assessment is obtained by a linear interpolation between the range band input data given for a particular shooting system and target.

3.7 Targets

Military targets exist in various sizes, shapes, and vulnerabilities. Potential targets might be an exposed soldier without any cover or protection, or a heavily armored tank with the latest armament technology. In this study, the targets are limited to four generic types: exposed soldier, truck, light armored vehicle, medium armored vehicle. These four types were chosen to represent four different threat classes based on their differing sizes, shapes, and vulnerabilities. The threats were also chosen based on their likelihood of being engaged with the set of proposed shooting systems. For example, a heavily armored vehicle was not considered because the shooting systems selected would typically not be used to fire against such a fortified vehicle.

As open source data, the target data presented is plausible, generic data that is a realistic representation of the target types selected. The target input data is expressed in terms of a lethal area for each target and is a function of range and shooting system. A round is lethal to a target if the round lands within the lethal area, which is a circular area surrounding the target. As the vulnerability of the target decreases, so does the lethal area. This means to be lethal a round must land much closer to a medium armored vehicle than an exposed soldier. The representative target

data is displayed in Table 3.4. By using the P(k) calculation together with the input and target data discussed, the performance of weapons against targets of different types and vulnerabilities can be quantified.

Table 3.4 Target Lethal Area Data

Shooting System	Range (m)	Exposed Soldier (m^2)	Truck (m^2)	Light Armored Vehicle (m^2)	Medium Armored Vehicle (m^2)
Mortar	4500	2041	987	74	37
Cannon	15000	1206	583	109	55
Long Range Cannon	25000	1908	583	109	55

CHAPTER 4

MODELING AND SIMULATION

Probability of kill (P(k)) is calculated through the use of a Monte Carlo simulation. This analysis calculates P(k) by running a Microsoft Excel based model; however, this chapter discusses simulation design and execution in general.

4.1 Model Setup

The model used to calculate P(k) is setup by generating lookup tables containing the input data. A table is constructed for each input variable, for example, the sensor table (Table 3.2) contains sensor options one through three and the corresponding standard deviations used in the Monte Carlo simulation. The analyst selects one input from each table to create one "case." For example, Sensor 1 can be selected with a Mortar shooting system and an exposed soldier target. This combination would comprise one "case." Error multiplication factors ranging between 0.25 and 1.5 are applied to either increase or decrease the standard deviation of each individual variable. Through the use of the Monte Carlo simulation, these inputs are used to generate the P(k). The appendix displays an example of an input data selection tab used in Microsoft Excel.

4.2 Monte Carlo Simulation

As mentioned earlier, a Monte Carlo simulation is used to approximate the probability of certain outcomes by randomly generating values for certain variables over and over. Figure 4.1 (from Coleman and Steele [6]) describes the Monte Carlo Simulation.

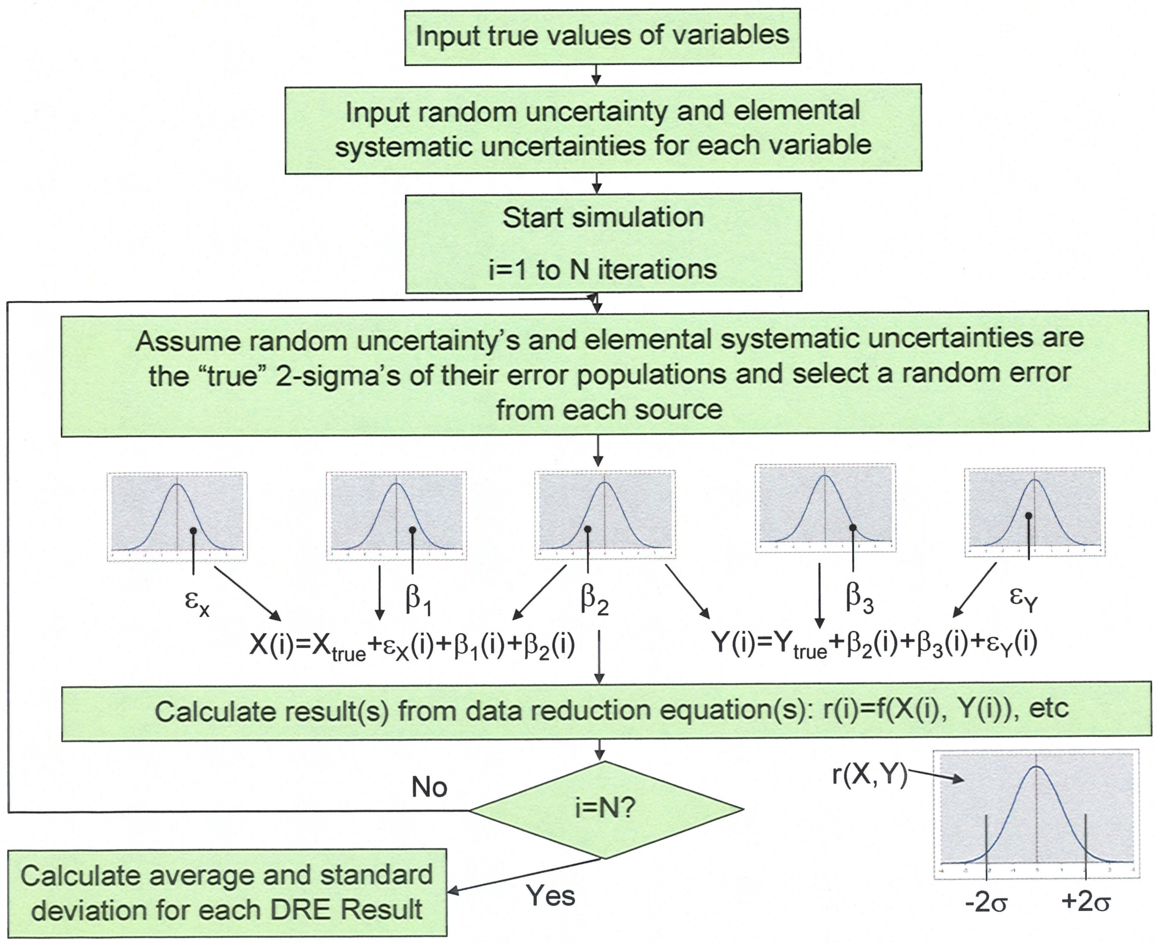


Figure 4.1 Monte Carlo Simulation Process [6]

Since the Monte Carlo simulation calculates random variables for up to 10,000 iterations in this analysis, it is convenient to automate the random number generation. This can be accomplished in several ways; however, here the process is explained through the use of Visual Basic macros as used in Microsoft Excel. Sample macros are displayed in the appendix.

Using the sensor input variable as an example, the macro computes a list of 10000 x 1 random variables for sensor error in one direction (x or y). The mean and standard deviation of the selected sensor define a distribution from which a set of random variables are generated. Two random variable lists for n = 1 to 10,000 iterations are generated for the sensor in each direction (x and y). A separate macro is written for the sensor, shooter, and round.

4.3 Sources of Error

The probability of killing a target is dependent upon several key contributing factors and the errors associated with each. The steps for a potentially successful engagement are as follows: First, the object must be seen and identified as a target of interest by a sensor. The targeting sensor has some degree of error associated with its ability to correctly locate the target. Next, the shooter or gun must be aimed at the target. There is error associated with the uncertainty of the aimpoint. Finally, the fired round must travel from the shooter to the target. There is error associated with the accuracy of the trajectory and the ability of the round to land at the intended location. All of these errors are summed together to give an overall error which directly affects the probability of kill.

Probability of kill is calculated using equation (3.7). This equation uses the miss distance error as a function of the sensor, shooter, and round error in the x- and y-direction. These errors are aggregated together in the equation as a single error for each the x- and y-directions.

Starting with the sensor error, the mean is assumed to be zero. Standard deviation is dependent on the sensor selection. A set of 10000 x 1 randomly generated numbers populates two identically sized error sets, representing sensor errors in the x and y (deflection and range).

Next, the shooter error is calculated based on sensor randomization. Random numbers for the shooter are based upon the corresponding random number from the sensor data set and the standard deviation dependent shooter selection. The result is two sets of displacement errors (10000 x 1) associated with the shooter, one for deflection and one for range.

Finally, the round error is calculated based on the shooter randomization in the same manner as previously described. This time, random numbers for the round are based upon the corresponding random number from the shooter data set and the standard deviation dependent round selection. The range and deflection error for the round now includes the error of the sensor and shooter and this value can be used in calculating the probability of kill.

Figures 4.2 and 4.3 illustrate this process for generating overall miss distance errors. A normal distribution is shown; however, the distribution was also defined as a uniform distribution using the Visual Basic macros prior to random number generation.

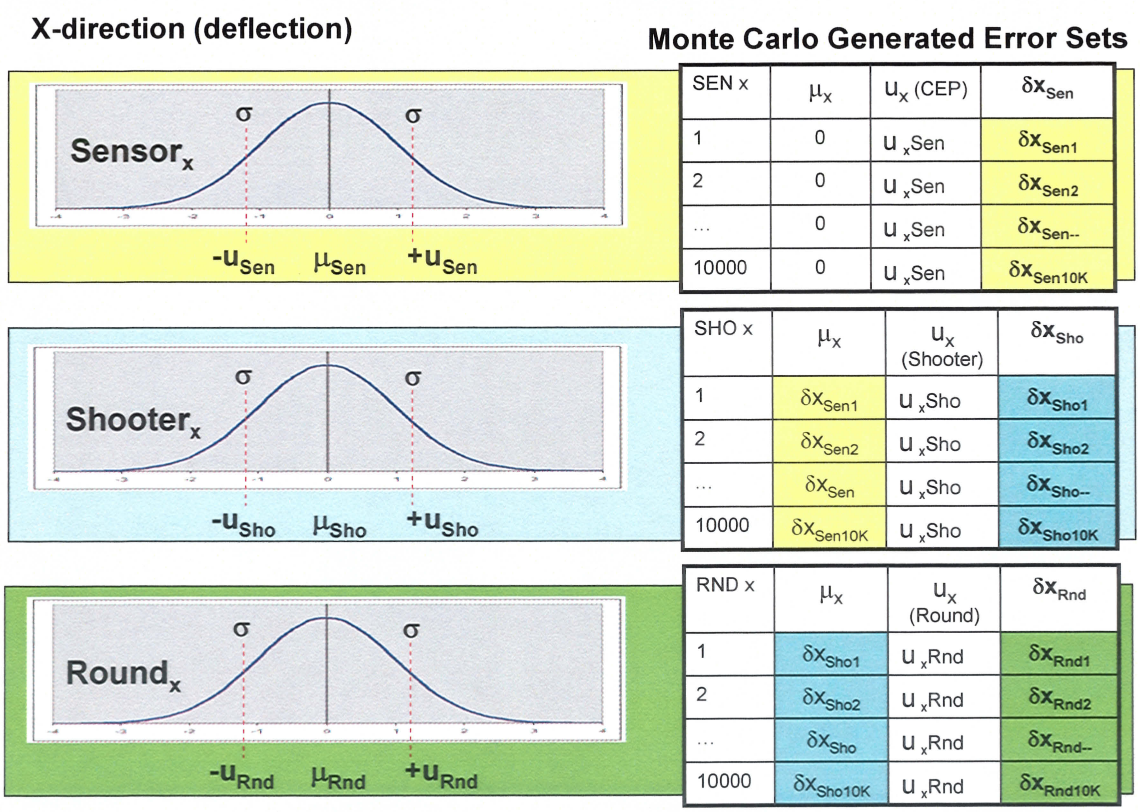


Figure 4.2 X-direction error calculation depiction

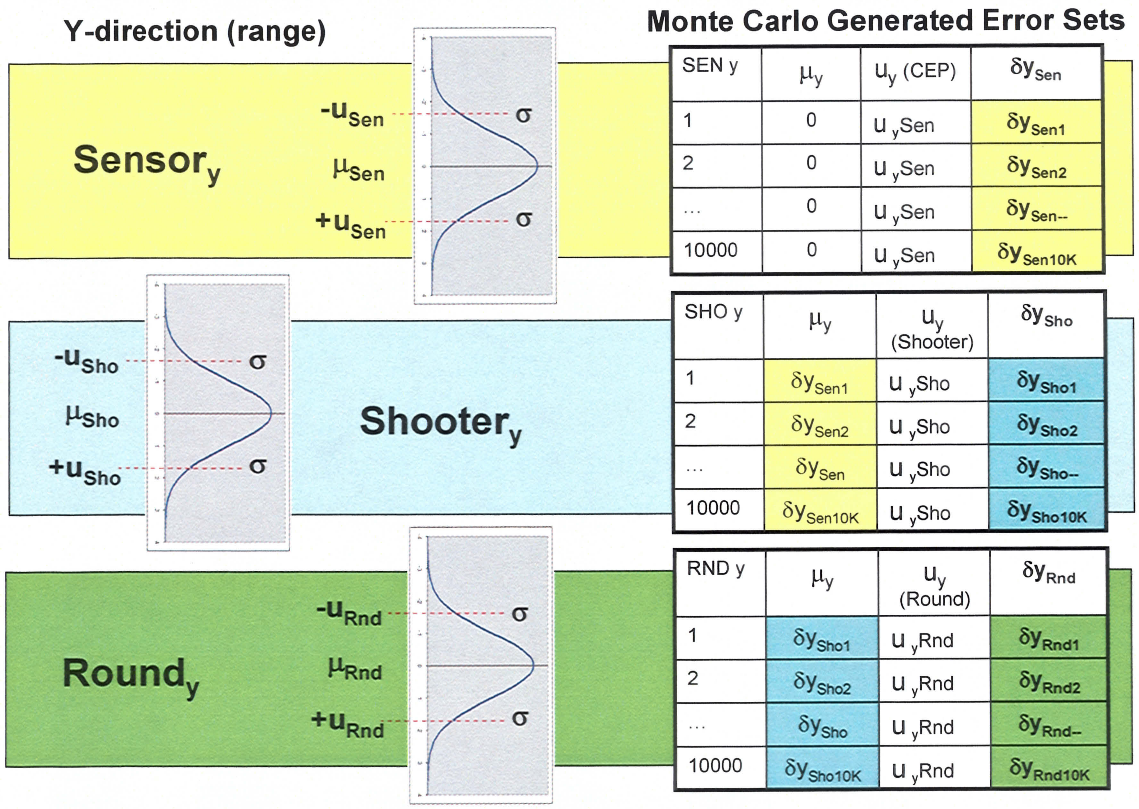


Figure 4.3 Y-direction error calculation depiction

4.4 Random Number Generation

For this analysis, random numbers are generated in Excel by using the Data Analysis Tool Pack and the Random Number Generation tool. This process is written into a Visual Basic macro and defined to specify the number of random variables to be created, the distribution type, and the output location specified by the analyst. Figure 4.4 shows an example of how random numbers are generated from the worksheet view in Microsoft Excel. The appendix displays the code for each macro that generates the random numbers for this analysis.

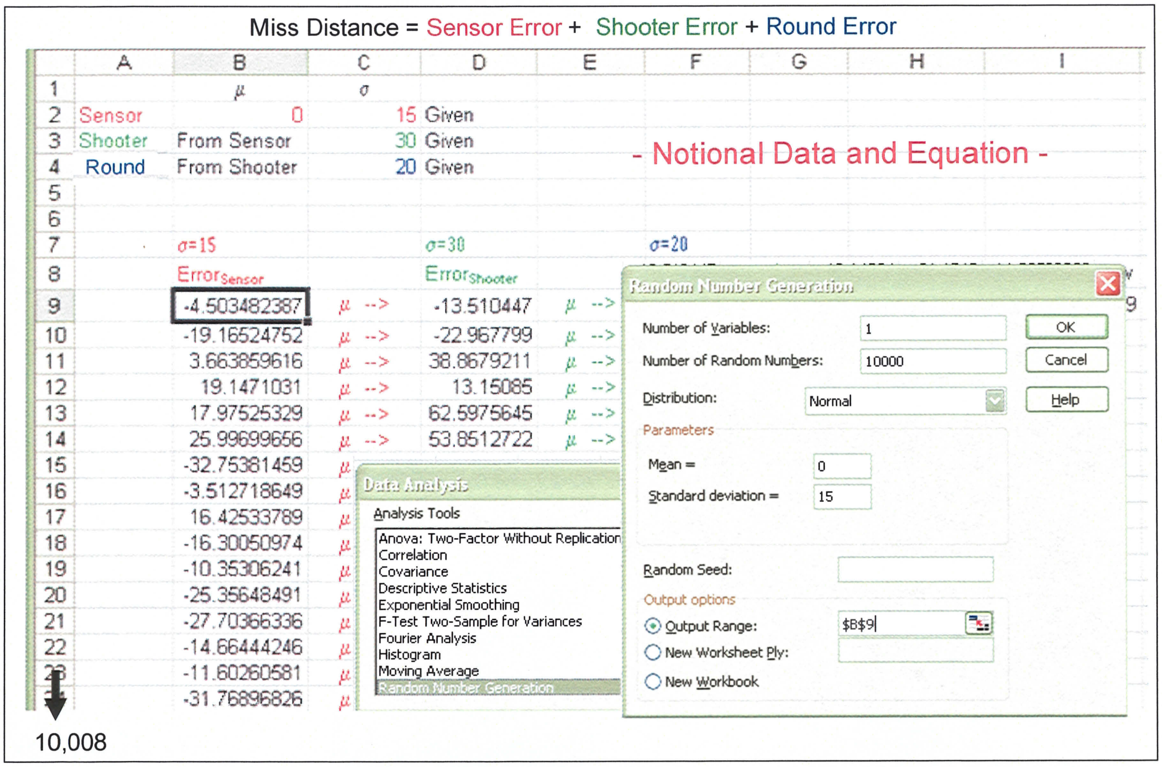


Figure 4.4 Random Number Generation Example

4.5 Distribution Types

Numerous types of distributions can be selected using the random number generator. For this analysis, two distributions are considered: a normal distribution and a uniform distribution.

The normal distribution is defined using a mean and standard deviation and resembles a symmetric curve with a single peak about the mean. The distribution is not finite; however, about 99.7% of the values lie within three standard deviations of the mean.

A uniform distribution has a finite range and all values of the set are equally probable. For a uniform distribution running from –A to +A with a mean of zero, the value of A is the square root of three times the value of the standard deviation. Since the standard deviation is given for each error factor, the values of –A and +A are calculated and used in the simulation.

CHAPTER 5

RESULTS

The method for generating data for this study is as follows: First, a run matrix is generated to define the different combinations of sensor, shooter, round, and target of interest. Next, the method of presenting the data is developed, such as a chart or graph that displays the data in a manner that is easily understood. Once the initial results are analyzed, a second run matrix is identified to collect additional data. This process is continued until enough data has been collected and analyzed to draw conclusions about the analysis.

5.1 Run Matrix

The initial run matrix includes every combination of the three sensors, three shooting systems at their respective mid ranges, and the four targets. Table 5.1 shows the initial matrix.

Table 5.1 Run Matrix

Shooting System	Target	Sensor 1	Sensor 2	Sensor 3
Mortar				
	Exposed Soldier			
	Truck			
	Light Armored Vehicle			
	Medium Armored Vehicle			
Cannon				
	Exposed Soldier			
	Truck			
	Light Armored Vehicle			
	Medium Armored Vehicle			

Table 5.1 Cont.				
Shooting System	Target	Sensor 1	Sensor 2	Sensor 3
Long Range Cannon				
	Exposed Soldier			
	Truck			
	Light Armored Vehicle			
	Medium Armored Vehicle			

The initial run matrix is executed for both a normal and uniform distribution. The results are presented by charting the output as seen in Figures 5.1 and 5.2 which show the results for the normal and uniform distributions respectively. The shooting system is defined by the color, the target is defined by the shape of the data point and the sensor is defined across the top of the plot. Figures 5.1 and 5.2 display the probability of kill as a function of sensor, shooting system and target. Each case number along the x-axis represents a different combination of inputs.

From Figures 5.1 and 5.2 it is observed that when the shooting system errors are low, the sensor error dominates the outcome of the probability of kill. The long range cannon has the lowest standard deviation of any shooting system; therefore, the sensor dominates the predicted probability of kill. Here, it is not clear whether the mean point of impact error or the precision error of the shooting system is influencing the probability of kill output. The individual error components need to be further broken down to understand their contributions.

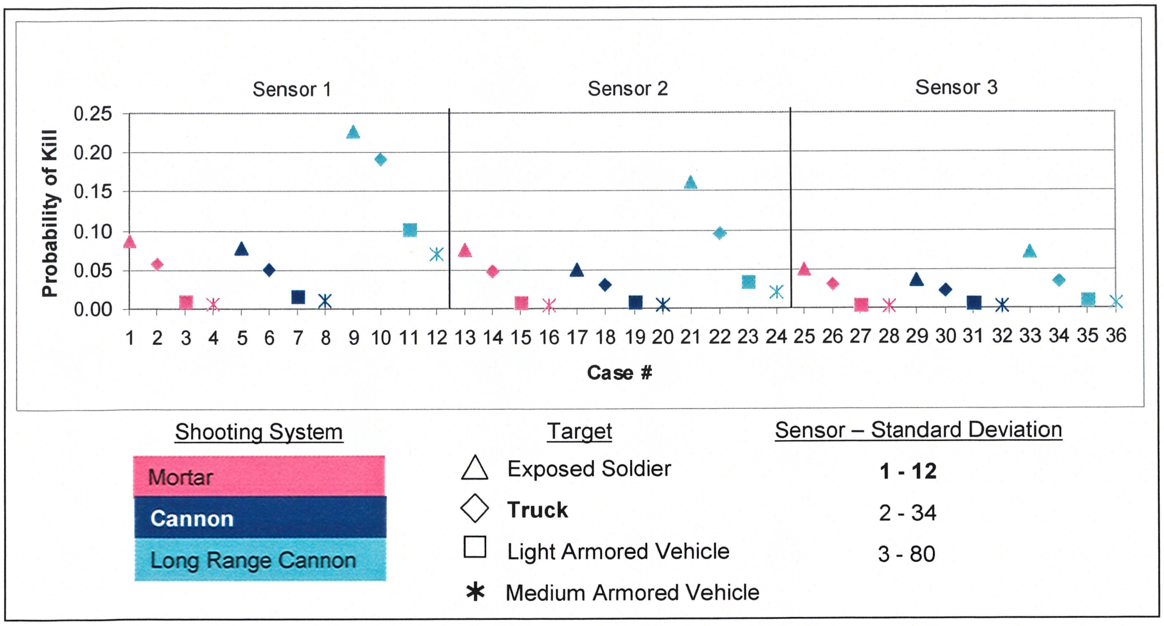


Figure 5.1 Normal Distribution Probability of Kill for Run Matrix

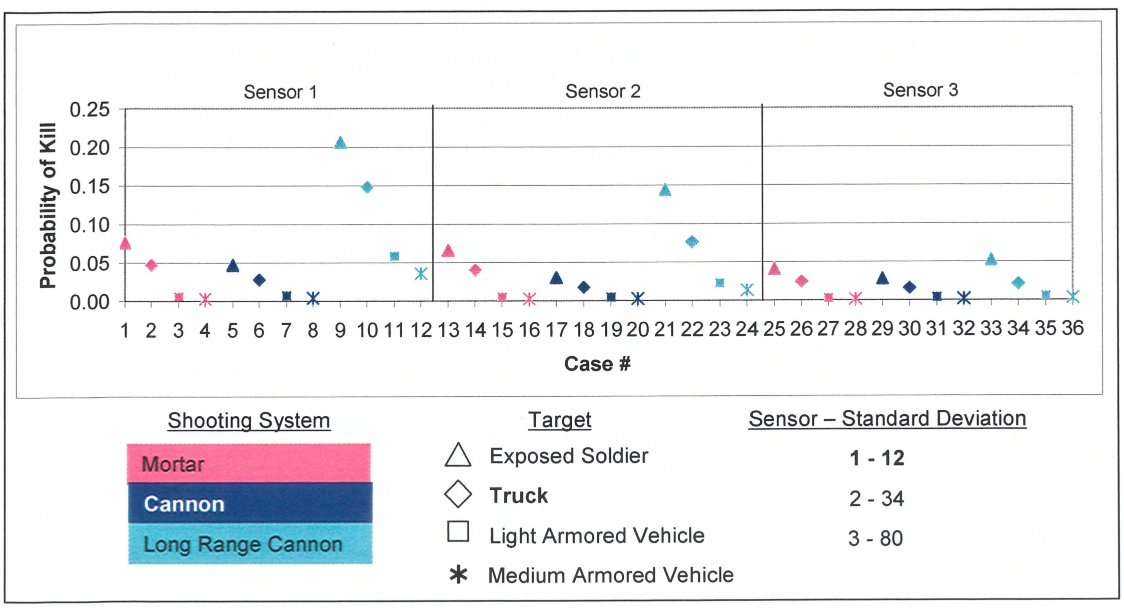


Figure 5.2 Uniform Distribution Probability of Kill for Run Matrix

To compare the effects of using a normal distribution versus a uniform distribution, the percent difference between the two distributions is plotted, where Percent Difference = (|Normal – Uniform|/Normal) * 100. Figure 5.3 displays the percent difference plotted for each case. Each

case corresponds numerically to those in the previous figures. It should be noted that taking the percent difference for very low probabilities of kill sometimes results in misleadingly high percentage values. Therefore, some values are circled in red and should be disregarded for this reason.

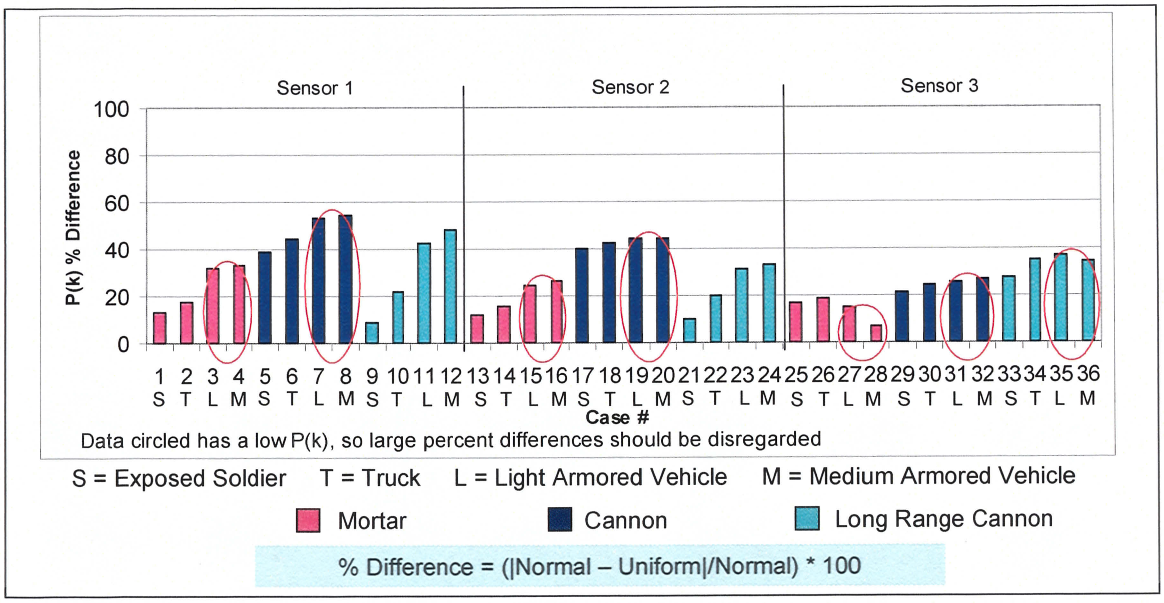


Figure 5.3 Probability of Kill Percent Difference Between Normal and Uniform Distributions

Figure 5.3 shows that the percent difference between the distributions is slightly reduced as the sensor's performance gets worse. The percent difference is greater when the target is better protected. When the target is better protected, i.e., armed vehicle versus soldier, the lethal area is smaller, which more noticeably affects the percent difference.

5.2 Mortar Error Factors

The next step is to understand the individual factors behind the results represented in Figures 5.1-5.3. Each shooting system is addressed separately. This section presents the results for the Mortar only. Sensor 1, with the smallest CEP, is selected for all runs. The exposed soldier target is also selected for each run. With the sensor and target constant, the MPI and PE errors can be modified and the resulting P(k) monitored. In turn, each of the MPI and PE errors

are increased and decreased by multiplying the error by 1.5 and 0.5 respectively to represent a 50% increase and decrease in error. This is done for both the normal distribution and the uniform distribution.

Figure 5.4 shows the results for the isolated MPI and PE errors for the normal distribution. The blue, "base" column represents the run where all errors are held constant. The green shaded columns represent the errors which are improved by reducing the error by 0.5. The red shaded columns represent the errors which are made worse by increasing the error by a multiplier of 1.5. Intuitively, when the error is reduced, the probability of kill should increase. Likewise, when the error is increased, the probability of kill should decrease. However, this is not always true and instances where error scaling produces a counterintuitive or converse effect on P(k) are indicated with an asterisk in the figure. The same follows for Figure 5.5 which displays the results for the uniform distribution.

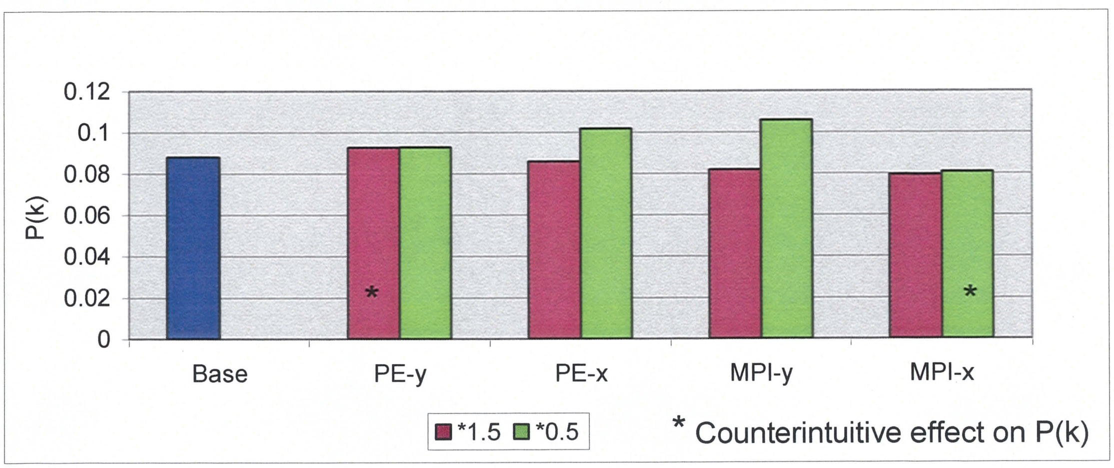


Figure 5.4 Probability of Kill for Isolated Mortar Variables using a Normal Distribution

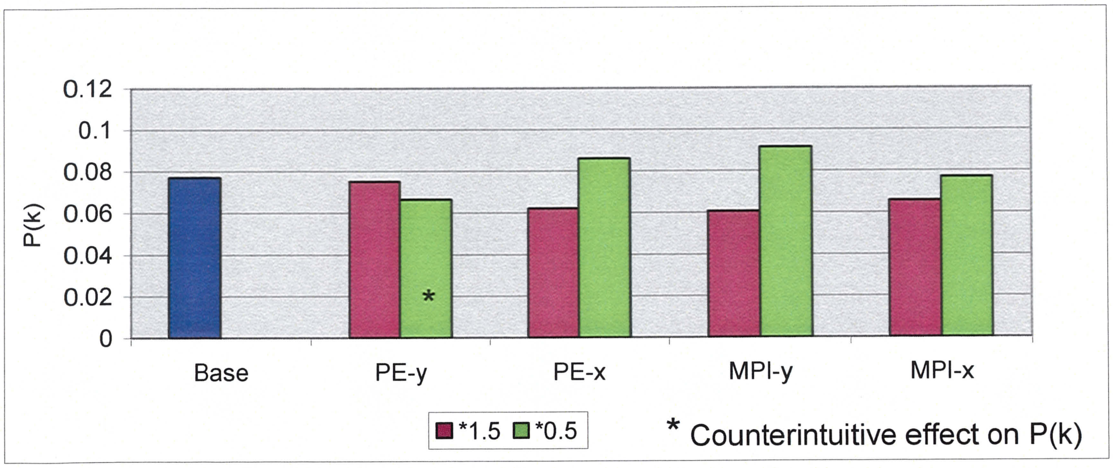


Figure 5.5 Probability of Kill for Isolated Mortar Variables using a Uniform Distribution

In order to understand why P(k) behaves in an opposite manner with respect to error scaling, some additional investigation is done. To do this, the metrics indicated with an asterisk are further analyzed such that they are multiplied by several numbers, ranging from 0.25 to 1.5. These results are displayed in Figures 5.6, 5.7, and 5.8. In these plots, the numbers less than one (which should increase P(k)) are indicated with a green marker and those greater than one (which should decrease P(k)) are indicated with a red marker.

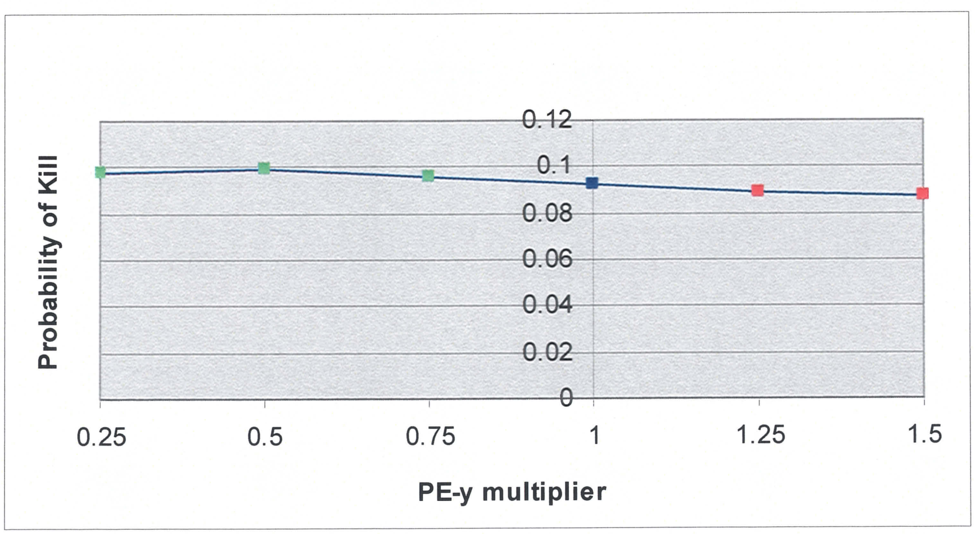


Figure 5.6 Isolated Metrics for Y-direction PE Effects on Probability of Kill – Mortar, Normal

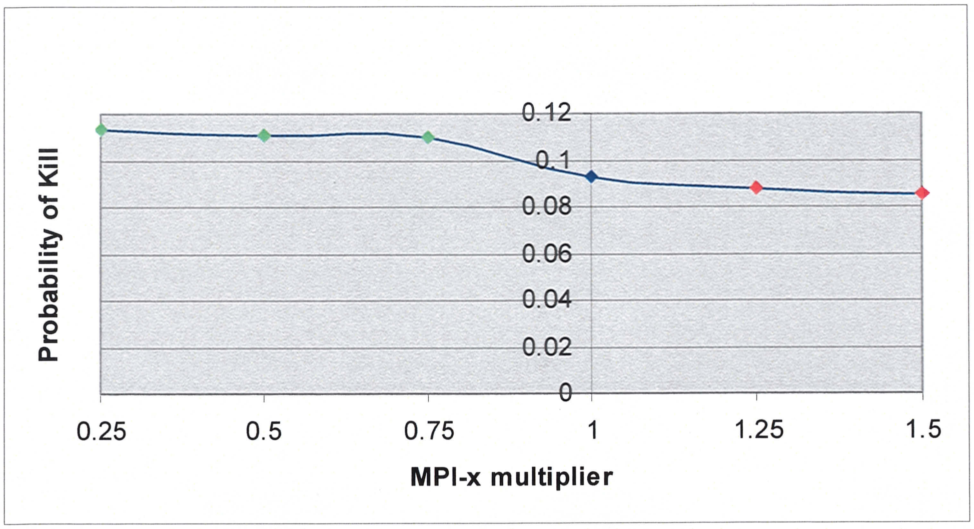


Figure 5.7 Isolated Metrics for X-direction MPI Effects on Probability of Kill – Mortar, Normal

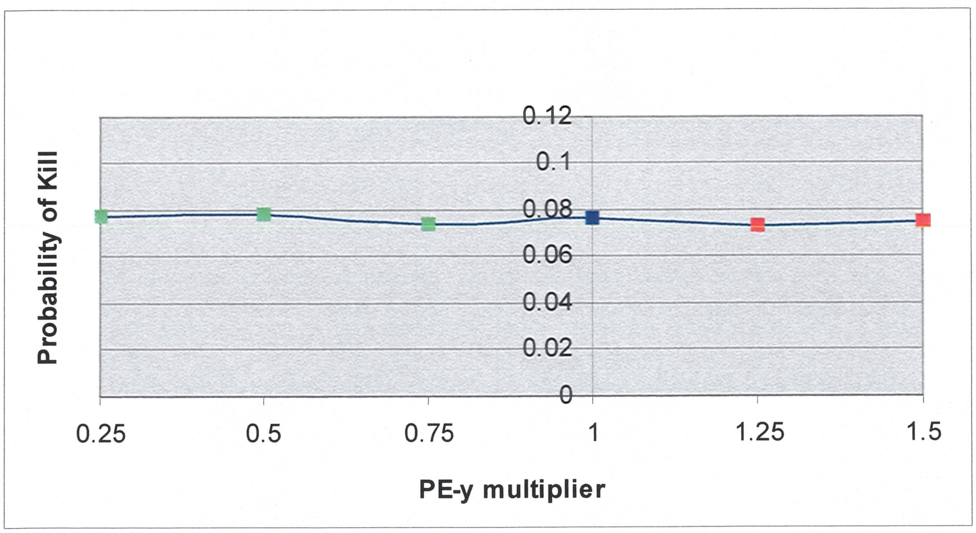


Figure 5.8 Isolated Metrics for Y-direction PE Effects on Probability of Kill – Mortar, Uniform

Despite some anomalies, the slope of the line seen in Figures 5.6-5.8 is as expected. This slope is compared to an isolated variable which follows the expected trend. For the uniform distribution, when the MPI-y error is multiplied by 0.5 and effectively reduced, the P(k) is increased. Figure 5.9 shows MPI-y error as it is modified by several multipliers. The slope of the line here is much steeper than with the cases affected conversely by scaling. The scale of the y-axis has to be increased to include all data points.

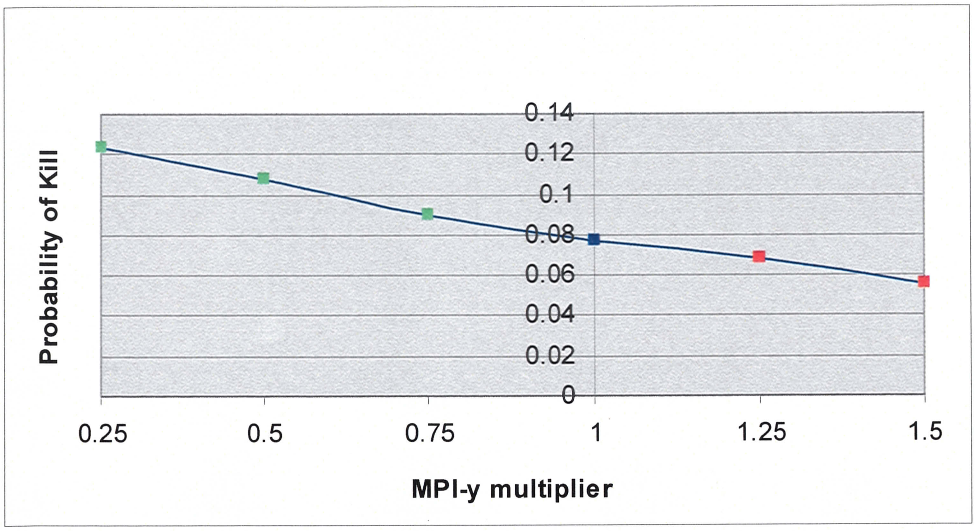


Figure 5.9 Isolated Metrics for Y-direction MPI Effects on Probability of Kill – Mortar, Uniform

5.3 Cannon Error Factors

The Cannon is analyzed in the same manner as the Mortar in the previous section. Once again, Sensor 1 and the exposed soldier are used for each run while each of the MPI and PE errors are increased and decreased via 1.5 and 0.5 scaling factors. This is done for both the normal distribution and the uniform distribution. Figures 5.10 and 5.11 show the Cannon results for the isolated MPI and PE errors for the normal distribution and uniform distribution respectively.

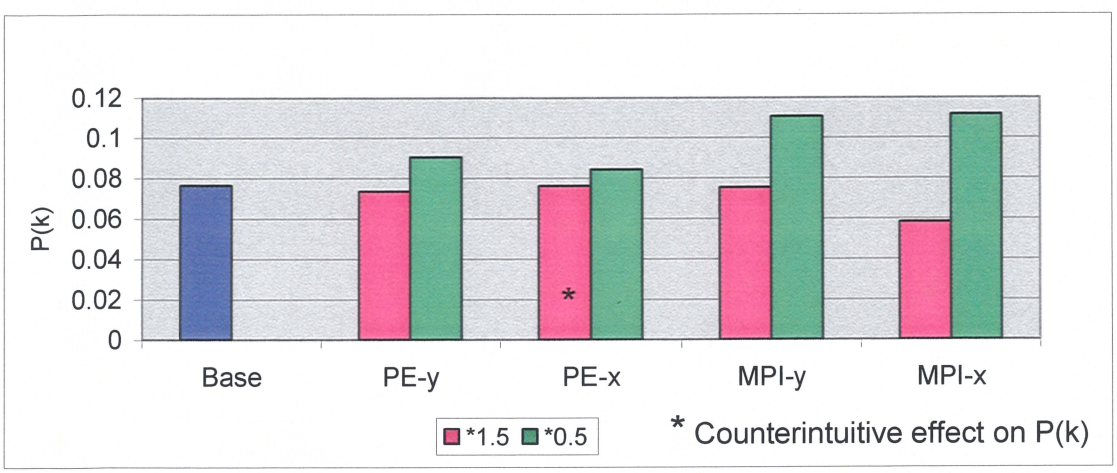


Figure 5.10 Probability of Kill for Isolated Cannon using a Normal Distribution

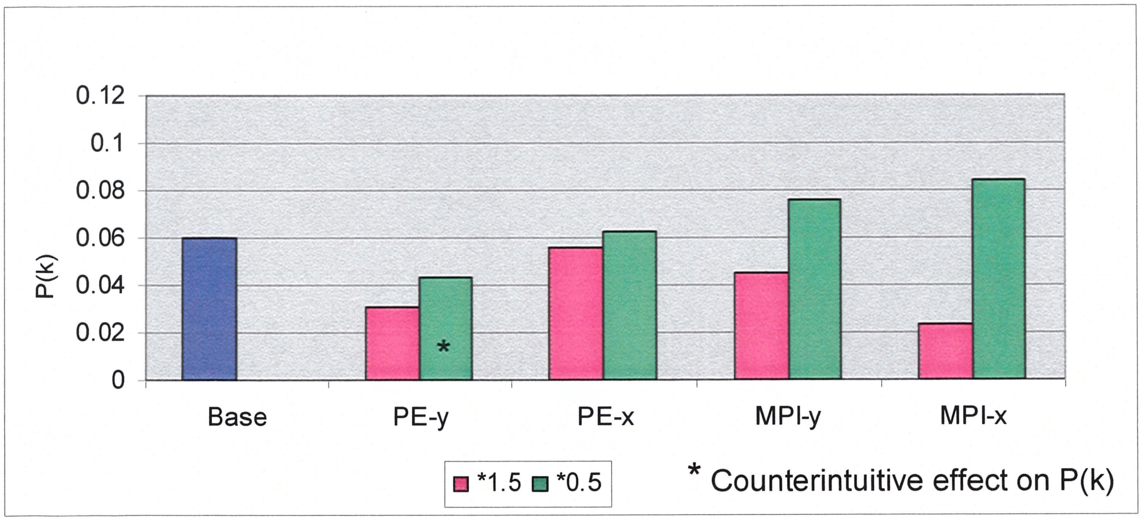


Figure 5.11 Probability of Kill for Isolated Cannon Variables using a Uniform Distribution

The instances where the resulting P(k) is conversely affected by the scaling differ from those instances observed for the Mortar. However, it is interesting to note that the MPI-y scaled results are as expected for both the Mortar and Cannon. Additional investigation is needed once again to understand the "counterintuitive" cases by scaling from 0.25 to 1.5. These results are displayed in Figures 5.12 and 5.13.

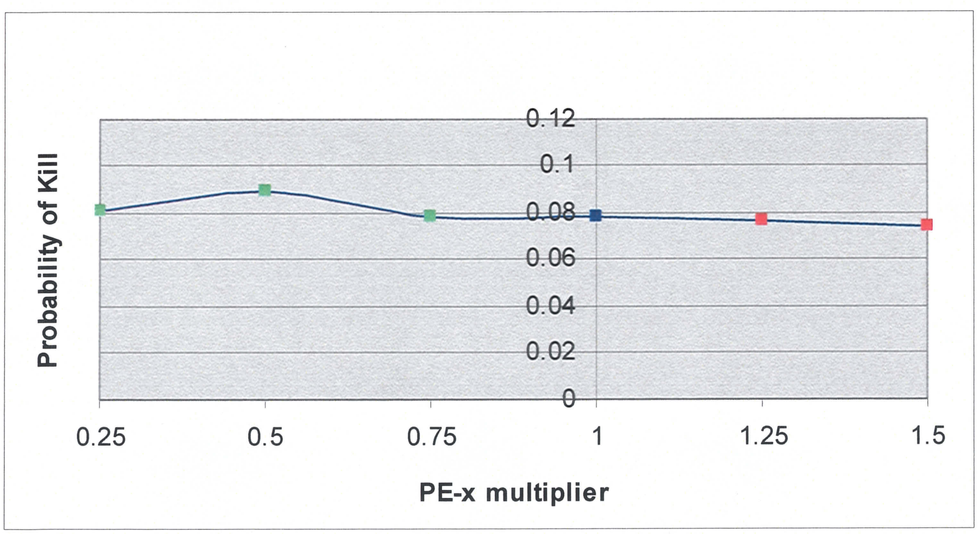


Figure 5.12 Isolated Metrics for X-direction PE Effects on Probability of Kill – Cannon, Normal

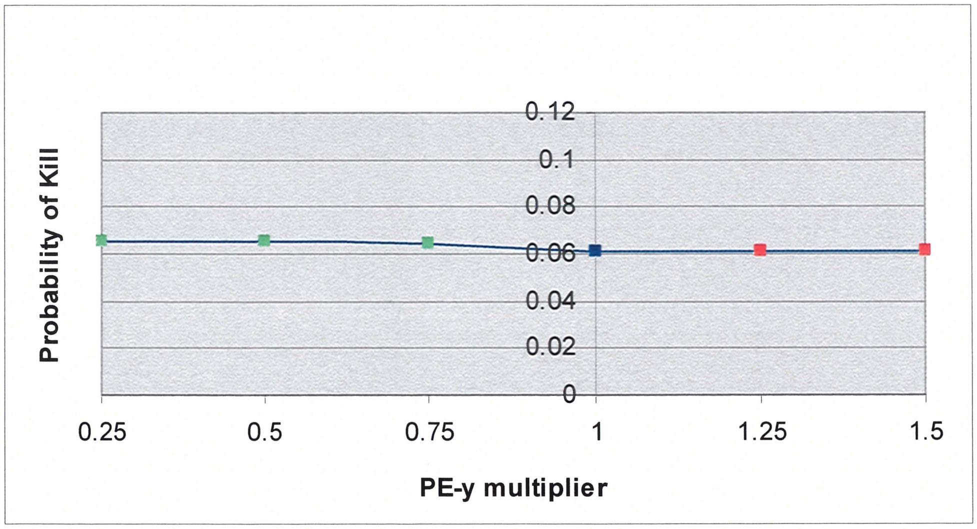


Figure 5.13 Isolated Metrics for Y-direction PE Effects on Probability of Kill – Cannon, Uniform

Figure 5.10 shows that when the PE-x is multiplied by 1.5, the result is very close to the base case. Figure 5.12 is a detailed look at the PE-x for the normal distribution. The slope of the line here is fairly flat, with little change occurring as the error is modified. Likewise, Figure 5.13 shows a gradual negative slope.

From the results of the Mortar and Cannon, the conclusion is drawn that the precision error is not significantly affected by scaling the error. The mean point of impact error tends to be more sensitive to modifications. So, in other words, the shooter error is more sensitive than the round error.

5.4 Long Range Cannon Error Factors

Finally, the individual factors which affect the error of the Long Range Cannon are analyzed. Again, Sensor 1 and the exposed soldier are used for each run while each of the MPI and PE errors are modified by 1.5 and 0.5 scaling factors. This is done for both the normal distribution and the uniform distribution as seen in Figure 5.14 and Figure 5.15, respectively.

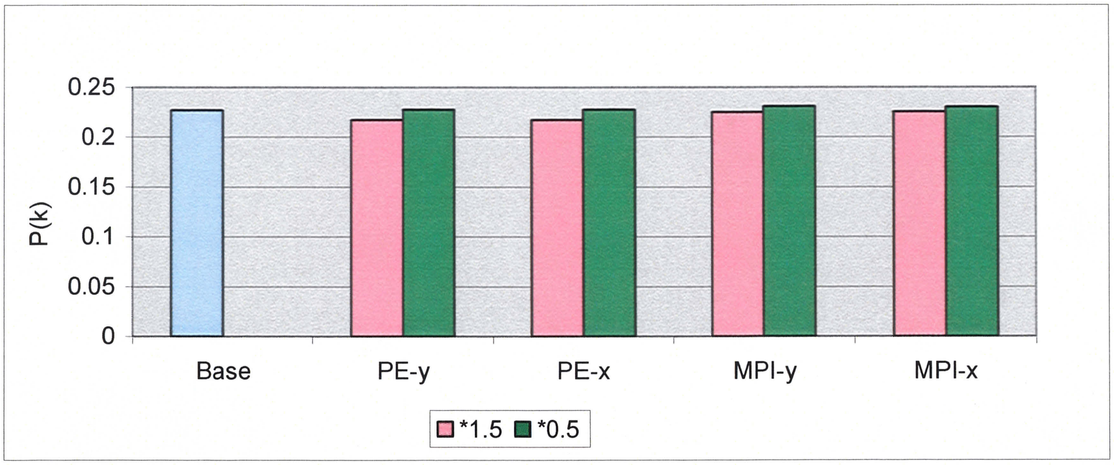


Figure 5.14 Probability of Kill for Isolated Long Range Cannon Variables using a Normal Distribution

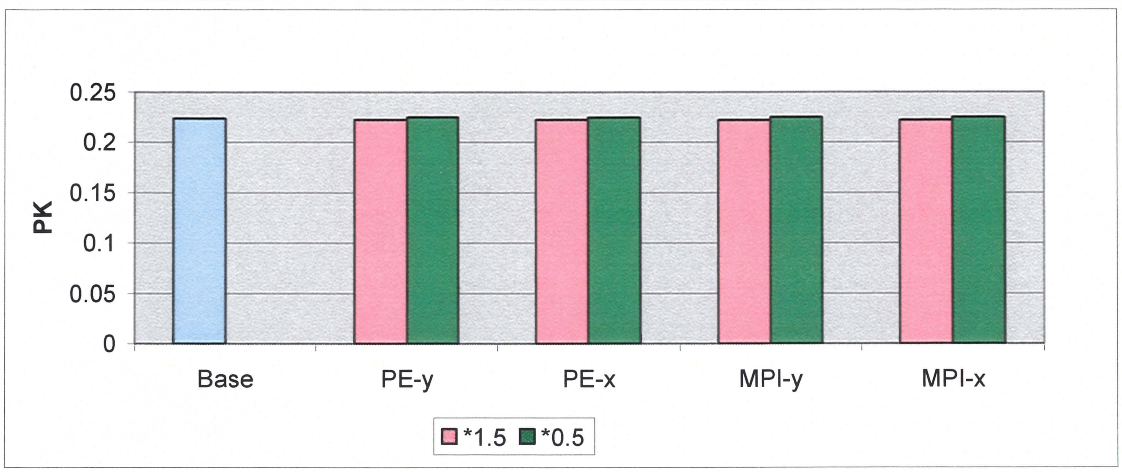


Figure 5.15 Probability of Kill for Isolated Long Range Cannon Variables using a Uniform Distribution

Figures 5.14 and 5.15 do not show any results conversely affected by scaling. This is attributed to the low error associated with the Long Range Cannon. The Long Range Cannon error for the MPI and PE is small in comparison to the Mortar and Cannon; therefore, when the error is multiplied by the same modifiers, there isn't much change in the resulting error. For each of the isolated variables, there is a slight increase in the resulting P(k) when the error is decreased and a decrease in the resulting P(k) when the error is increased.

5.5 Percent Difference of Isolated Error Factors

Since the isolated error contributors are analyzed for both the normal and uniform distribution, it is of interest to see the differences between the two distribution types. This is useful in determining whether or not the distribution type used in the Monte Carlo analysis has any effect on the resulting probability of kill. To do this, the percent difference is taken for each isolated variable. The percent difference is expressed in equation (5.1). This equation assumes that the normal distribution is the reference value since it is more common to use a normal distribution to represent munitions data. This process is completed for both the Mortar and Cannon. The Long Range Cannon was omitted due to the minute variation from the base case.

$$\text{Percent Difference} = (|\text{Normal} - \text{Uniform}|/\text{Normal}) * 100. \tag{5.1}$$

Figures 5.16 and 5.17 show the results of the percent difference calculations for the Mortar and Cannon, respectively. As seen in the figures, there is anywhere from a 5-28% difference in the Mortar results and a 22-68% difference in the Cannon results. For the Mortar, the largest percent differences tend to occur when the errors of the individual factors are increased, although this is not always true as in the case with the PE-y. These figures also indicate that there is a 12-22% difference present in the base cases for the Mortar and Cannon, respectively. The percent differences with the Cannon are greater than those of the Mortar; however, there is no clear pattern in the results.

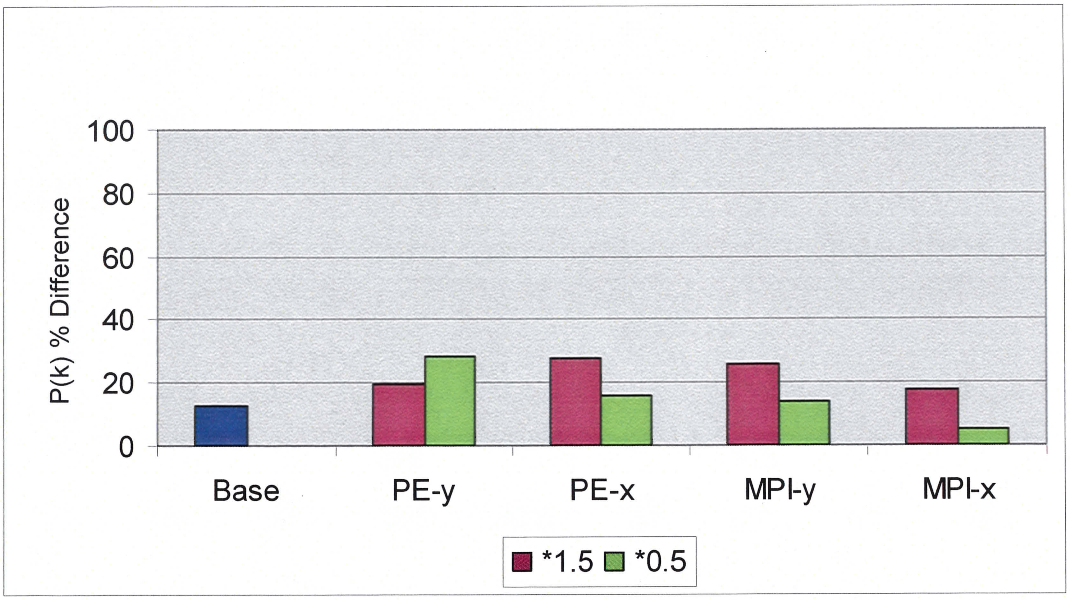


Figure 5.16 Distribution Type Percent Difference Results for Isolated Variables – Mortar

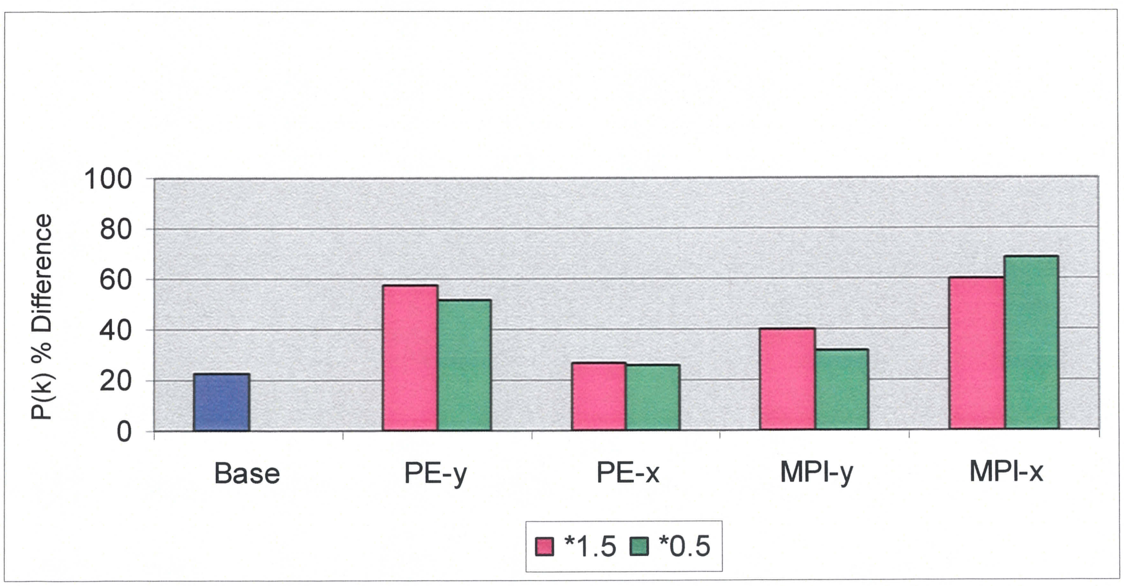


Figure 5.17 Distribution Type Percent Difference Results for Isolated Variables – Cannon

5.6 Dispersions

Since there is an apparent difference between the overall P(k) as the distribution type changes, it is of interest to view the shape of the dispersions for both distribution types. To do this, 10,000 points are plotted. Each data point represents one iteration generated through the

Monte Carlo simulation. The resulting P(k) used in the data above is an average of these 10,000 iterations. Figures 5.18 and 5.19 show the dispersions plotted for the normal and uniform distributions, respectively. For these plots, the following data set is selected: Sensor 1, Mortar, mid range, and exposed soldier target.

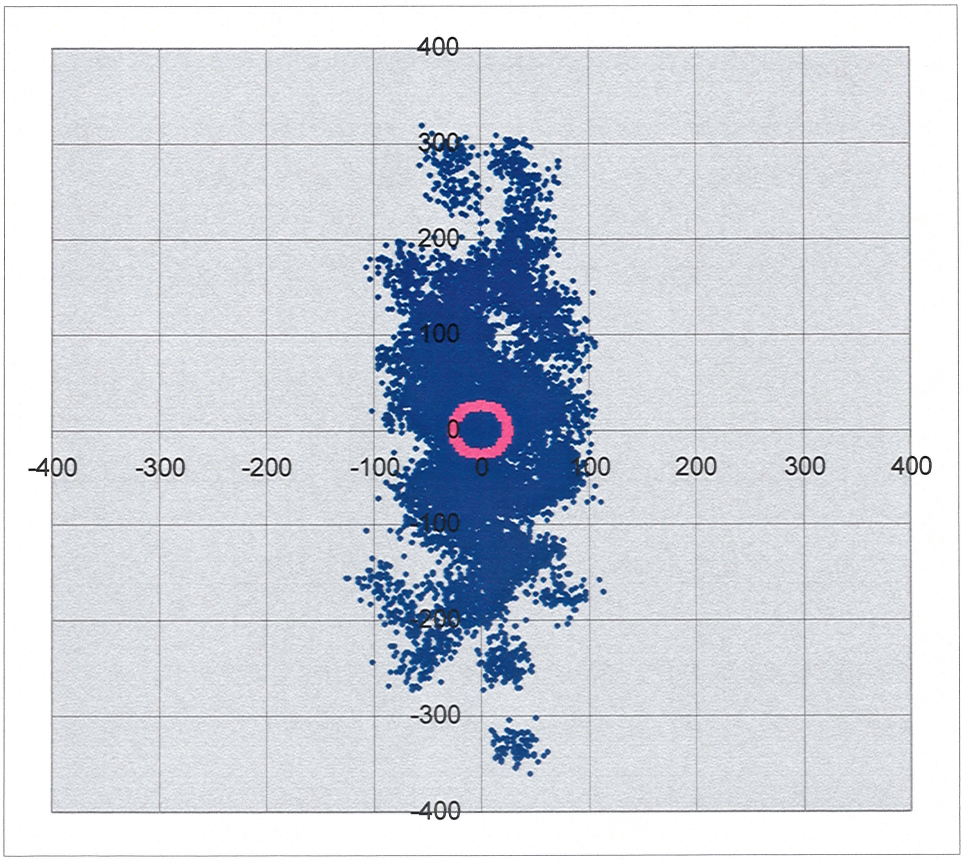


Figure 5.18 Normal Distribution Dispersion of Shots for Sensor 1, Mortar, Exposed Soldier

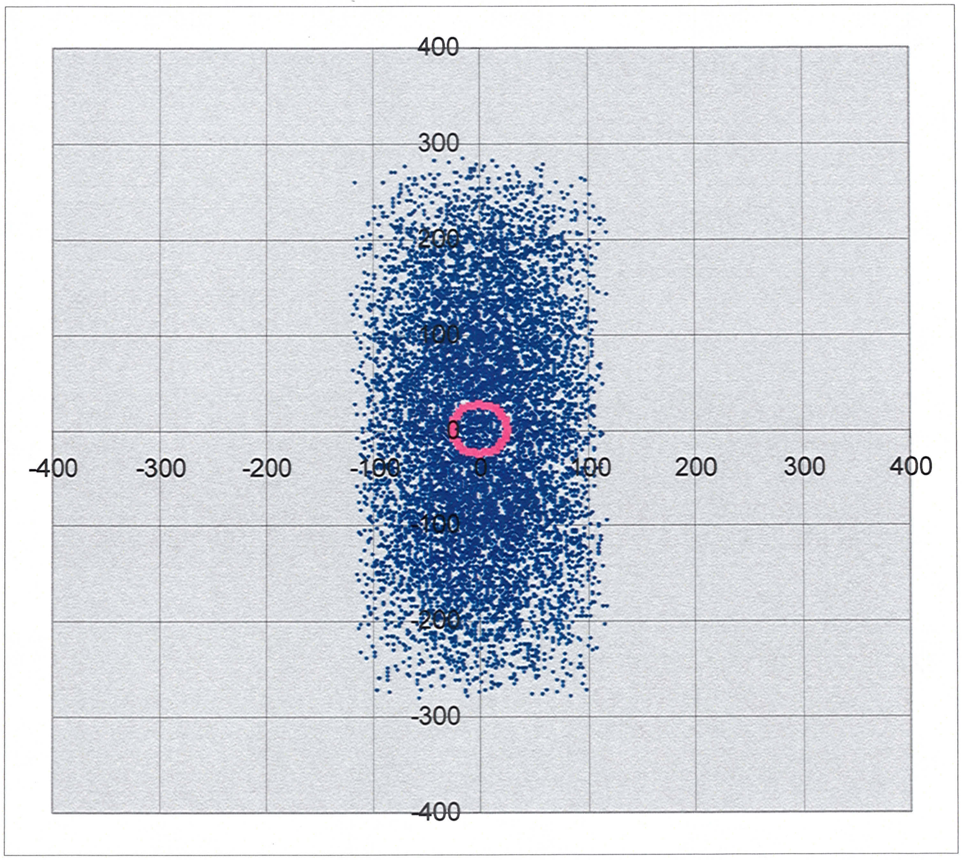


Figure 5.19 Uniform Distribution Dispersion of Shots for Sensor 1, Mortar, Exposed Soldier

5.7 Necessary Iterations

The majority of the analysis is conducted using 10,000 iterations. The generation of this many random numbers is time intensive, especially when the number of combinations to be investigated is large. Therefore, in order to speed up the process, an investigation was done to determine the maximum iterations needed for convergence. To do this, the calculated P(k) is plotted for each iteration from 1 to 10,000. Figure 5.20 shows that the P(k) converges at about 2000 iterations. Therefore, 2,000 iterations is sufficient to calculate a representative P(k). The data for the isolated variable histograms such as MPI-x, MPI-y, etc., for each individual shooting system is calculated using 10,000 iterations; however, to save computing time, the data for the isolated metrics displayed in the line graph is calculated using only 2,000 iterations.

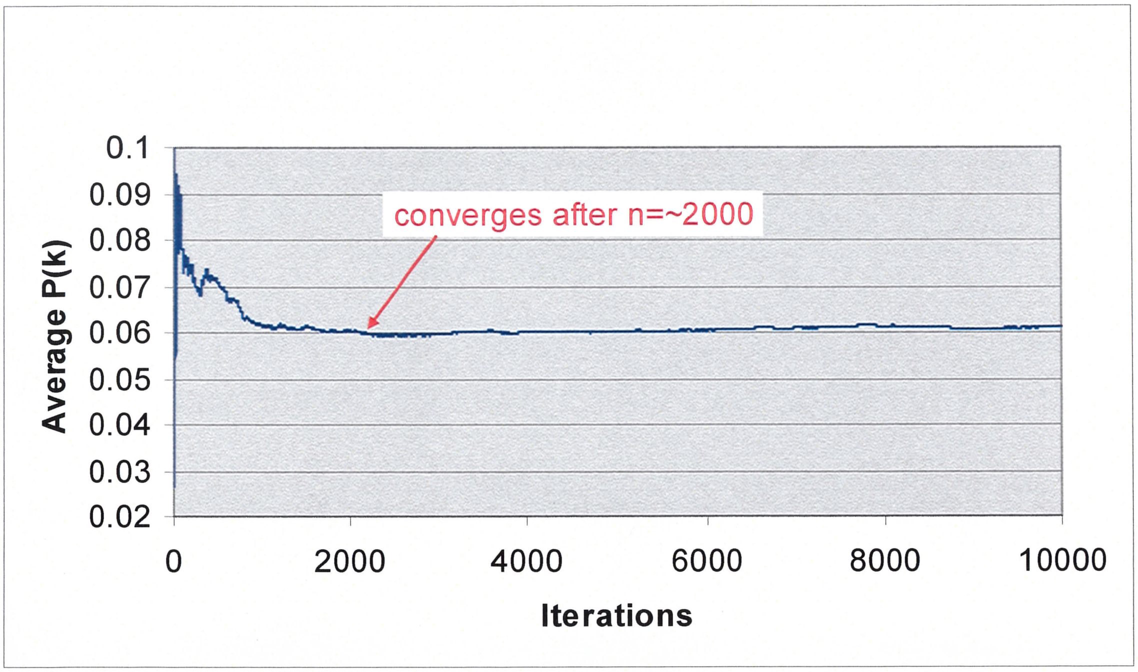


Figure 5.20 Number of Iterations Necessary for Calculating Probability of Kill

5.8 Results Summary

Table 5.2 summarizes the impact each individual error factor has on the probability of kill. The data indicates whether variations in the sensor error, shooter error, or round error influence the resulting probability of kill. The data is summarized for both the normal and uniform distributions and the error factors shaded green indicate that the probability of kill was impacted. A more detailed analysis of this table will be discussed in Chapter 6.

Table 5.2 Individual Error Factors Impact on Probability of Kill

		Normal Distribution	Uniform Distribution
Mortar	Sensor		
	MPI-x		
	MPI-y		
	PE-x		
	PE-y		
Cannon	Sensor		
	MPI-x		
	MPI-y		
	PE-x		
	PE-y		
Long Range Cannon	Sensor		
	MPI-x		
	MPI-y		
	PE-x		
	PE-y		
P(k) impacted		P(k) not impacted	
	Inconclusive impact		

CHAPTER 6

CONCLUSIONS AND RECOMMENDATIONS

The intent of this thesis was to analyze the uncertainty associated with the individual factors of an engagement. In the previous chapters, the individual uncertainties were analyzed with respect to the probability of success of the engagement. If a particular factor integral to the engagement is much more sensitive to error than another, the design for that component should be held to a higher tolerance than a less sensitive factor.

The data used for this analysis was representative of typical mortars and cannons. However, there are many variants of these weapons available throughout the world today. These concluding remarks are specific to the data samples used for this analysis; however, the calculations and processes used are easily replicated for other weapon systems.

6.1 Error Sensitivity Observations

After analyzing the results displayed in Chapter 5, some concluding observations are now made. The error of the sensor has a greater impact on the probability of kill (P(k)) when the shooting system errors (shooter and round) are low. If the accuracy of the shooter and round is high, the sensor error dominates the outcome of the probability of kill.

The mean point of impact error and precision errors are greater in the direction of range rather than in deflection. For the sensor, the range and deflection errors are the same due to the nature of the circular error probable definition.

When considering the Mortar, variations in the shooter error in the y-direction greatly influence the P(k) for both the uniform and normal distributions. The other shooting system errors are not consistently sensitive to modifications so no conclusions are drawn about these.

Modifications to the shooter error of the Cannon in both range and deflection influence the P(k). The error for the Long Range Cannon is too low to influence the probability of kill when modified by the scaling factors used. When the shooting system error is high enough (low accuracy), as with the Mortar and Cannon, modifications to the shooter error have a greater effect on the P(k) than modifications to the error of the round. Table 5.2 summarized the influences of error on P(k) for each factor.

6.2 Influence of Distribution Type

The type of distribution selected to represent the input data for the Monte Carlo Simulation has 5-60% difference on the probability of kill depending on the inputs selected. Therefore the distribution type significantly influences the outcome of the engagement.

The probability of kill calculated using the normal distribution is consistently higher than the probability of kill calculated using the uniform distribution. This can be attributed to the normal distribution having a higher concentration of shots near the origin or in other words, inside the lethal area. The "tails" are cut off for the uniform distribution and the shots are more evenly distributed over the entire area, so fewer shots actually land within the lethal radius around the target. In general, the percent difference between the uniform and normal distribution is slightly reduced as the performance of the sensor is reduced. Conversely, the percent difference between the uniform and normal distribution is greater when the target is better protected (smaller lethal area).

6.3 Impact on Design

The purpose of analysis is to enable a decision to be made. Frequently during the design phase of a new military system, decisions are made about where to focus the time and funding available. With limited resources, the designers want to concentrate on improving the technology in areas which provide the best performance return for the time and money spent. This thesis provides the process for analyzing the uncertainty of the factors integral to an engagement. The representative results of this analysis are useful in aiding future design efforts and decisions related to the lethality of a system.

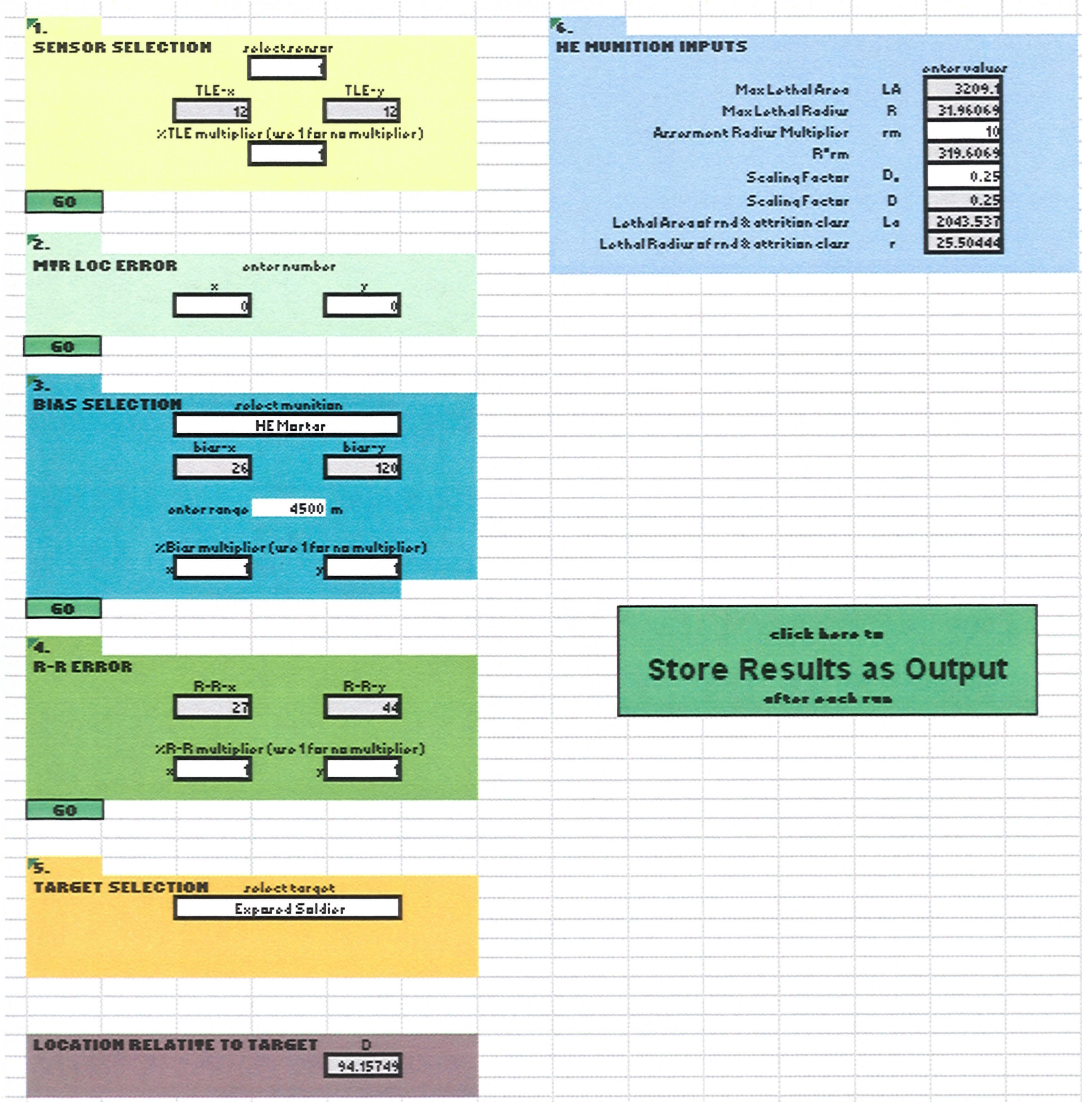
Specifically this analysis indicates that if the accuracy of the shooter and round are high, the sensor error dominates the resulting probability of kill, making the design of the sensor critical for the success of the engagement.

When the accuracy of the shooter and round is low, the error of the shooter has a greater impact on the P(k). The focus of the design effort should be placed on improving the design of the shooter rather than on the accuracy of the round. When this is true, sensor error does not have a profound influence on the probability of kill, although as the accuracy of the shooter and round improve, the sensor error will then have a greater influence on the resulting engagement.

And finally, the probability of kill can be increased by designing the sensor, shooter, and round to yield a normal probability distribution. When compared to making physical design changes, this is a relatively low cost solution for achieving a higher probability of kill.

APPENDIX
EXCEL IMPLEMENTATION OF RANDOM NUMBER GENERATION

EXCEL INPUT SELECTION TAB



EXCEL MACROS

```
Sub Sensor_Error()
'
' Sensor_Error Macro

' Computes a list of 10000 random variables for sensor error in the x and y direction
' Uses a normal distribution
'
'   Computes random variables in the x direction
    Worksheets("Sensor_Info").Select
    Range("SensorX, SensorY").ClearContents
    a = Range("C24") ' Standard Deviation
    b = Range("D24") ' Mean

    Application.Run "ATPVBAEN.XLA!Random", ActiveSheet.Range("SensorX"), 1 _
        , 10000, 2, , b, a
    ActiveWindow.SmallScroll Down:=-15

'   Computes random variables in the y direction
    Worksheets("Sensor_Info").Select
    a = Range("C24") ' Standard Deviation
    b = Range("D24") ' Mean

    Application.Run "ATPVBAEN.XLA!Random", ActiveSheet.Range("SensorY"), 1 _
        , 10000, 2, , b, a
    ActiveWindow.SmallScroll Down:=-15
    Worksheets("Calculator").Select

End Sub
```

```
Sub Bias_Error()
'
' System/Bias/MPI_Error Macro
' Computes a list of 10000 random variables for MPI error in the x and y direction
' Uses a normal distribution

' Computes random variables in the x direction

Worksheets("Munition_Info").Select

Range("SystemX, SystemY").ClearContents

StdDev_x = Range("L3")  ' Standard Deviation

Let max_row_x = Range("SensorX").Rows.Count

For x = 1 To max_row_x

  Mean_x = Worksheets("Sensor_Info").Cells(9 + x, 6)

  Application.Run "ATPVBAEN.XLA!Random", Worksheets("Munition_Info").Cells(3 + x, 12), 1 _
        , 1, 2, , Mean_x, StdDev_x

Next x

'Computes random variables in the y direction

StdDev_y = Range("N3")  ' Standard Deviation

Let max_row_y = Range("SensorY").Rows.Count

For y = 1 To max_row_y

  Mean_y = Worksheets("Sensor_Info").Cells(9 + y, 8)

  Application.Run "ATPVBAEN.XLA!Random", Worksheets("Munition_Info").Cells(3 + y, 14), 1 _
        , 1, 2, , Mean_y, StdDev_y

Next y

Worksheets("Calculator").Select


End Sub
```

```
Sub R_R_Error()
'
' R-R/PE_Error Macro

' Computes a list of 10000 random variables for PE error in the x and y direction
' Uses a normal distribution

'Computes random variables in the x direction
Worksheets("R_R_Info").Select

Range("R_R_X, R_R_Y").ClearContents

StdDev_x = Range("D3")  ' Standard Deviation

Let max_row_x = Range("SystemX").Rows.Count

For x = 1 To max_row_x

   Mean_x = Worksheets("Munition_Info").Cells(3 + x, 12)

   Application.Run "ATPVBAEN.XLA!Random", Worksheets("R_R_Info").Cells(3 + x, 4), 1 _
         , 1, 2, , Mean_x, StdDev_x

Next x

'Computes random variables in the y direction

StdDev_y = Range("F3")  ' Standard Deviation

Let max_row_y = Range("SystemY").Rows.Count

For y = 1 To max_row_y

   Mean_y = Worksheets("Munition_Info").Cells(3 + y, 14)

   Application.Run "ATPVBAEN.XLA!Random", Worksheets("R_R_Info").Cells(3 + y, 6), 1 _
         , 1, 2, , Mean_y, StdDev_y

Next y

Worksheets("Calculator").Select


End Sub
```

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