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Feasibility of a guided interceptor

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FEASIBILITY OF A GUIDED INTERCEPTOR

by

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A THESIS

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We, the undersigned members of the Graduate Faculty of the University of Alabama in Huntsville, certify that we have advised and/or supervised the candidate on the work described in this thesis. We further certify that we have reviewed the thesis manuscript and approve it in partial fulfillment of the requirements for the degree of Master of Science in Engineering with an option in Aerospace Engineering.

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ABSTRACT
The School of Graduate Studies
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Title Feasibility of a Guided Interceptor

This research examines the feasibility of a gun-launched projectile to intercept rocket, artillery and mortar threats. The interceptor is a 40*mm,* spin-stabilized squib-guided projectile guided by a model predictive controller. The six-degree-of-freedom model and guidance system were developed in C++. The linearized guidance model predicts the interceptor trajectory periodically during flight. The control system utilizes the linearized guidance model and a target prediction model to determine the nearest intercept. A guidance algorithm then sequences the firing of lateral squibs to remove the projected miss distance. Monte Carlo simulations that employed statistical variations in gun launch angle, projectile rotation bias, and target position error show the control system could increase the probability of intercept from 0.00 to 0.492 (up to 30 squibs) at a slant range of 1440m. Repositioning the squibs stations forward of the center of gravity along the spin axis destabilizes the control system due to excessive projectile moments.

Abstract Approval: Committee Chair

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LIST OF ACRONYMS

6DOF	=	Six Degrees of Freedom
CEP	=	Circular Error Probable
c.g.	=	Center of Gravity
DC	=	Direct Contact
DCM	=	Direct Cosine Matrix
HMMMV	=	High Mobility Multipurpose Military Vehicle
IPT	=	Integrated Product Team
mrad	=	milliradians
PBX	=	Polymer-bound explosive
PRODAS	=	Projectile, Rocket and Ordinance Design and Analysis Software
RAM	=	Rockets, Artillery and Mortars
UAH	=	University of Alabama in Huntsville

LIST OF SYMBOLS

C_T = thrust coefficient
I = moments of inertia tensor
I_{squib} = impulse
m = mass of the projectile, kg
M = Mach number
magnitude = distance of the target from the projectile at the nearest intercept
p = pressure
phase = angular position of the target with respect to the projectile
Pk = Probability of kill
$P(k)_{engagement}$ = Probability of kill per engagement
pulse_force_angle = the angular position of a single squib on the airframe about the x-axis. Rad.
sl = stationline, the location of a single squib on the airframe along the x-axis. M.
t_f = time of flight after a particular squib is fired
T = temperature
T_{eb} = transformation matrix from body to earth coordinates
T_{squib} = thrust of a squib at a moment in time
x_{cp} = center of pressure along the x-axis
x_{mag} = magnus pressure location along the x-axis
xyz_err = vector from the projectile to the target representing miss distance at closest approach
$\{\dot{x} \quad \dot{y} \quad \dot{z}\}$ = derivative of the position vector, m/s
$\{u \quad v \quad w\}$ = velocity vector, m/s
$\{\dot{u} \quad \dot{v} \quad \dot{w}\}$ = derivative of the velocity vector, m/s^2
$\{p \quad q \quad r\}$ = rotation rates, rad/s
$\{\dot{p} \quad \dot{q} \quad \dot{r}\}$ = derivative of the rotation rates, rad/s^2
$\{q_0 \quad q_1 \quad q_2 \quad q_3\}$ = quaternion
$\{\dot{q}_0 \quad \dot{q}_1 \quad \dot{q}_2 \quad \dot{q}_3\}$ = derivative of the quaternion
$\{X \quad Y \quad Z\}$ = summation of all the forces input to the projectile at any given time step, N
$\{L \quad M \quad N\}$ = sum of the moments about the projectile
ρ = density
γ = ratio of specific heats
θ_{pf} = pulse force angle of a squib

CHAPTER 1

Introduction

Rockets, artillery and mortars (RAM) are a constant threat on the battlefield. At the present there are few ways to mitigate or intercept these threats to human lives. The only viable recourse at present for many is to seek shelter until the air clears. RAM threats are cheap, prevalent, and can be quickly launched from hidden locations. While the threat is well-defined, a method to engage it has not yet found itself in wide distribution. The goal of this thesis is to discuss a method that could use existing equipment, travel well and integrate well with the current technology in use.

To assess conceptual designs for a mobile, ground-based system a detailed Concept Description Document has been developed and is shown in Appendix A. The projectiles are shot from an existing, mobile 40mm gun such as the Bofors L/70 or the Bofors MK-4. The intercept method is hit-to-kill with a desired 32 kJ of kinetic energy at the target. A 90% Pk per engagement is required in thirty shots or less, with 15 shots or less being desired. The system must be deployable from the US in one airlift C-130J and then airlifted into position by CH47 in three trips or less. The nominal intercept point for performance runs is 1000, 0, -1000 in the north-east-down coordinate system, 1440 meters slant range.

Many groups have worked on design iterations leading up to the design investigated in this study (Williams, et al.; Tournes, et al.). Design iterations have included fin-controlled models (Hewitt, et al.), squib-controlled models (Williams, et al.), and hybrid designs employing both squibs and fins (Hartlage, et al.). The present investigation is to retrofit a 40mm projectile with a squib-based linearized model predictive guidance system. Major entities investigated include the axial location of the squibs and the effect of phase error on system effectiveness. The major figure of merit used to evaluate system effectiveness is the number of shots required to achieve a 90% probability of kill.

CHAPTER 2

Literature Review

There are four systems – one that was designed but never realized, two existing, and a final notional system – that were reviewed in the design of the current system. These systems are all gun-launch air defense systems. Two of them have been evaluated against RAM threats. However none of the realized systems implemented a hit-to-kill strategy, instead opting for explosive warheads or sub-projectiles. Therefore the interest in these existing systems is to look at how other projects set up their systems, realizing that our projectile will be something new. Additionally, attention will be paid to how each projectile was programmed on the way from the hopper to the sky and how or if guidance is utilized.

2.1 Existing Systems

2.1.1 Sgt York

The Division Air Defense (DIVAD) Gun System, also known as Sgt. York was intended to provide all-weather, close-range air defense for forward area mobile tactical units against hostile fixed-wing aircraft, helicopters, and lightly-armored vehicles (Babbit, et al.). Figure 2.1 shows the DIVAD system, consisting of two 40mm Bofors L/70 automatic guns mounted on a self-propelled, armored tracked vehicle. The two independently operated guns had a combined firing rate of

600 rounds per minute. The vehicle also contained a search and track radar and a digital fire control system with threat prioritization, automatic ammunition type selection and burst schedule (Seven). DIVAD was to be operated by a crew of three. The projectiles were 40mm equipped with explosive warheads. There were several projectiles to be used for different unique situations that might be encountered.

After spending 51 billion dollars on the program and producing about 50 vehicles, the program was terminated. There were several problems that led to the cancellation of the program, including difficulties of the radar distinguishing low flying helicopters from ground clutter, along with the radar return from the gun barrel tips confusing the control system when tracking higher angle targets. Finally the use of a thirty-year-old hull design caused problems with the vehicle being able to keep pace with the modern tanks it was meant to protect. There were also human factor issues (Seven).

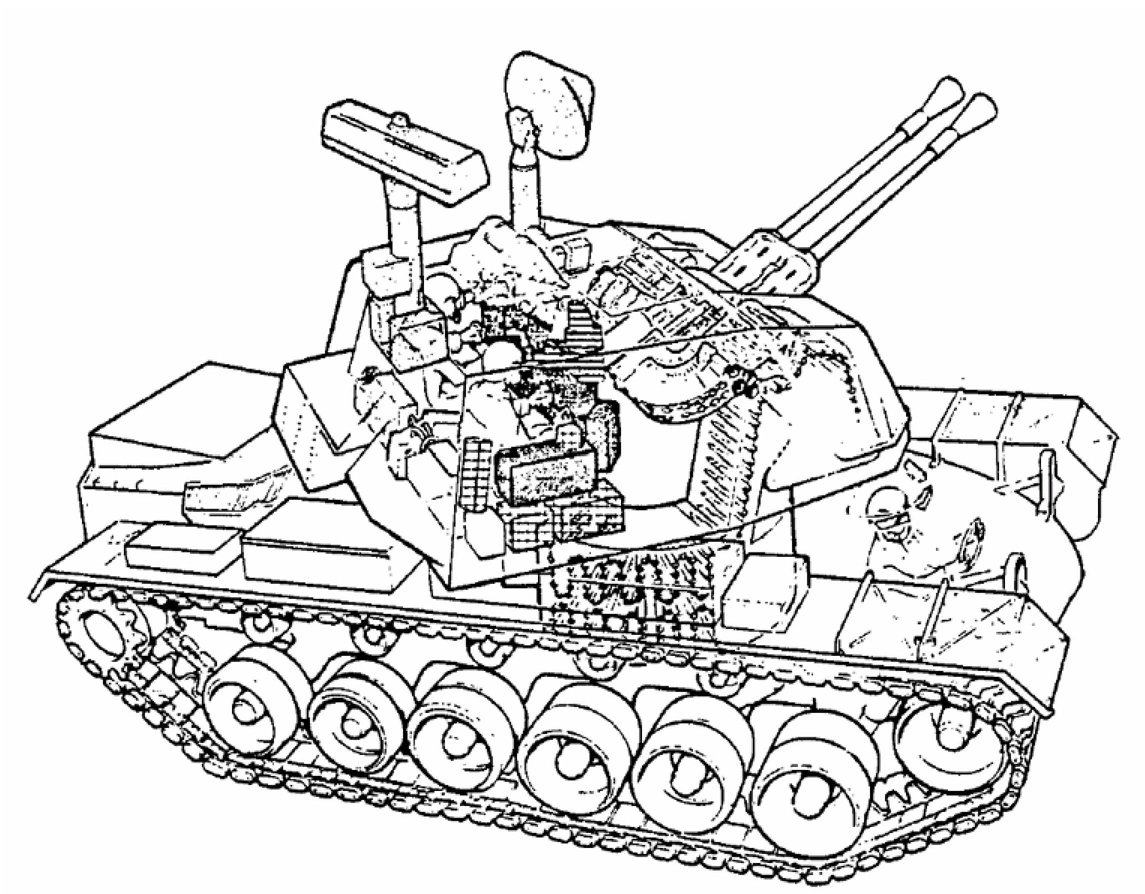


Figure 2.1 - Sgt York (Seven)

The guidance system is unknown. The DIVAD was never tested against RAM threats, although with the ability to swap out multiple projectile types, it is conceivable that adding an additional, hit-to-kill projectile would be feasible. Mobility was an issue due to the fact that an older hull design was used, and would not be as portable as a separate gun hitched to a High Mobility Multipurpose Military Vehicle (HMMMV) which could then be air-lifted via helicopter or aircraft.

2.1.2 Bofors 3P Multitarget Round

The Bofors 3P Multitarget round, shown in Figure 2.2 is designed to be an all-purpose interceptor (Borén). The projectile comes in two variants, 40mm and 57mm. It is shot from a Bofors L/70 gun and is programmed before firing by way of direct contact (DC). This also serves to power on the electronics onboard the projectile. The three "P's" stand for Prefragmented, Programmable Proximity fused ammunition. The projectile contains either 1100 or 2400 3mm tungsten pellets depending on the size. The explosive used to dismantle the projectile is PBX.

It has several modes of operation depending on the type of target engaged. It can be either timed to explode at a given time in flight or at a certain distance from the target. The projectile also has armor piercing and impact modes for slow-moving ground vehicles.

One distinct advantage of the 3P is the versatility of the projectile. Being able to defend against so many adversaries with a single programmable projectile is useful. The 3P is in service today as a defense system against fixed wing, rotary wing, and ground threats. The mobility of the system depends upon implementation: one implementation is a tank. Alternatively it can be implemented on a wheeled gun.

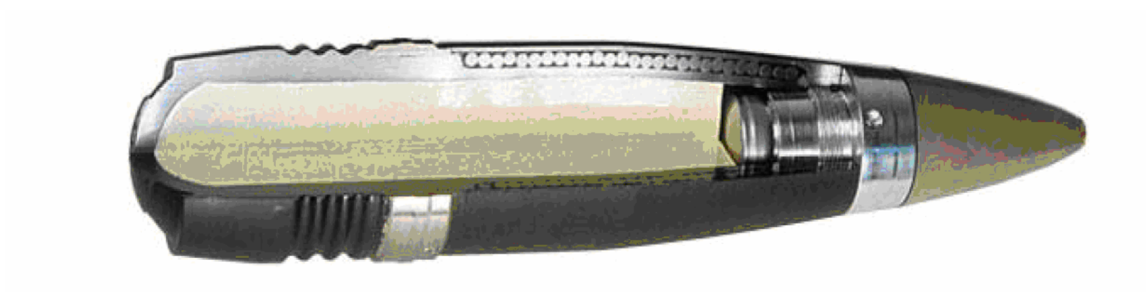


Figure 2.2 - Bofors P3 Multi-Targeted Round (Borén)

One design illustrated shows a standalone gun with a sensor pack. Such a gun could be easily transported. According to their own charts vs. a helicopter at 1km range (Borén), the Bofors 3P does not meet a 30 round Pk of 0.90. The system initially tracks the target but cannot maneuver in flight.

2.1.3 AHEAD and the Skyguard Air Defense System

The AHEAD projectile is a 35mm air burst munitions. While initially designed to defend against aircraft and missiles, it has also been evaluated to take out RAM threats in by the US, Germany and Israel. Each round contains 152 tungsten metal sub-projectiles weighing 3.3 grams apiece (as shown in Figure 2.3) and a detonation system programmed by an electromagnetic inductor located outside the gun. As the projectile leaves the gun barrel, it is programmed by the inductor as to when it should detonate. The projectile then travels the defined time before detonating, releasing a cone of lethal projectiles in the intercept area. Each engagement a double-barreled gun fires 24 such projectiles, and the manufacturer claims a response time as low as 4.5 seconds from target detection to firing.

The advantage of this system is in the sheer simplicity of the system – after programming the projectile on its way out the barrel, it is a fire-and-forget weapon. It should be rather cheap to manufacture and not have any implications on the detection system, as it does not require further messaging after leaving the gun. With a large enough lethality cone and 24 projectiles, given good enough initial guidance information about the target, a meaningful area should be filled with a spray of high kinetic energy sub-projectiles that will batter the incoming RAM threat. The primary

disadvantage of the system is the number of shots coupled with the sub-projectiles will lead to a large amount of debris which will need to land somewhere. Collateral damage potential will need to be assessed prior to deployment of such a system.

Pk could not be assessed for this system due to lack of data. The system is mobile, consisting of a gun unit and a fire control/radar unit. The system initially tracks the target but cannot maneuver in flight. Based on this author's estimates of the trajectory of the submunition drag of the pellets, 32 subprojectiles must intersect the target in order to meet the kinetic energy requirements of 32 kJ at on a target.

2.1.4 Phalanx

The Phalanx is an anti-fixed wing, aircraft, and anti-cruise missile terminal defense system produced by Raytheon Systems Company. The system, shown in Figure 2.4 consists of search radar, tracking radar, a six-barrel gatling gun and an ammunition drum. The ammunition consists of 15mm armor piercing rounds – either depleted uranium or tungsten – in a 20mm subcaliber plastic sabot.

Figure 2.3 - AHEAD 35mm Projectile (Olerikon Contraves)

The Phalanx system “automatically engages functions usually performed by separate, independent systems such as search, detection, threat evaluation, acquisition, track, firing, target destruction, kill assessment and cease fire.” It utilizes a closed loop control system that both tracks the incoming target and the outgoing projectiles. The projectiles have no capability to maneuver in flight and no warhead. Depending upon the model, the Phalanx can shoot 3,000 – 4,500 rounds per minute although the rounds per engagement are limited by the time of engagement and the number of rounds in the hopper, nominally 989 – 1550. The range is not published. Typically the Phalanx is mounted on ships as a terminal defense system, but there exists land-based versions which are used to counter RAM threats. The land-based Phalanx shoots a high explosive incendiary projectile which explodes on impact. In case of a miss, the projectiles explode in midair leaving low speed shrapnel to fall instead of a high-speed, high kinetic energy explosive projectile (Raytheon Missile Systems).

The range and Pk data are unpublished, although one source claimed a set of tests showed the Phalanx was able to attain a 60 to 70 percent shoot-down capability (Globalsecurity). The land-based Phalanx is mounted on a flatbed truck and thus would be difficult to move about relative to the other more compact and modular designs,

Figure 2.4 - Flatbed-mounted Phalanx (Raytheon)

although a palletized version appears to be in the works. The system is attractive as it is an all-in-one solution and has been used successfully at sea for many years. And the mechanism for dealing with misses is very attractive. One of the problems with a hit-to-kill munition is all of the projectiles that miss are traveling at a high rate of speed and end up colliding somewhere, whether it is an open space or a friendly asset. One of the negatives of this system is the extreme number of shots that would be taken to engage a threat.

2.2 Notional Designs

Over the past several years, the subject of a guided interceptor has been the topic of the senior design class at UAH. The senior design class takes the form of an IPT, working with AMREDEC through the initial phases of development of a guided projectile. Over the years there have been a multitude of proposed designs, but for the most part, the designs can be separated into three categories: fin controlled, squib controlled, or a combination thereof.

2.2.1 Fin Control

Fin control utilizes either fins at the rear of the projectile (Sigurmundsson, et al.; Hewitt, et al.) or canards (Heflin, et al.) in the front. This causes a packaging issue; the fins must either stow within the projectile and be released after firing or the projectile must be shot with a sabot. A second issue is conventional guns have rifling that can impart a spin rate of 800Hz on the projectile. The fins will despin the projectile, which is desired for the fin control mode, but the fins themselves must be strong enough to withstand the shearing forces as they slow the projectile down.

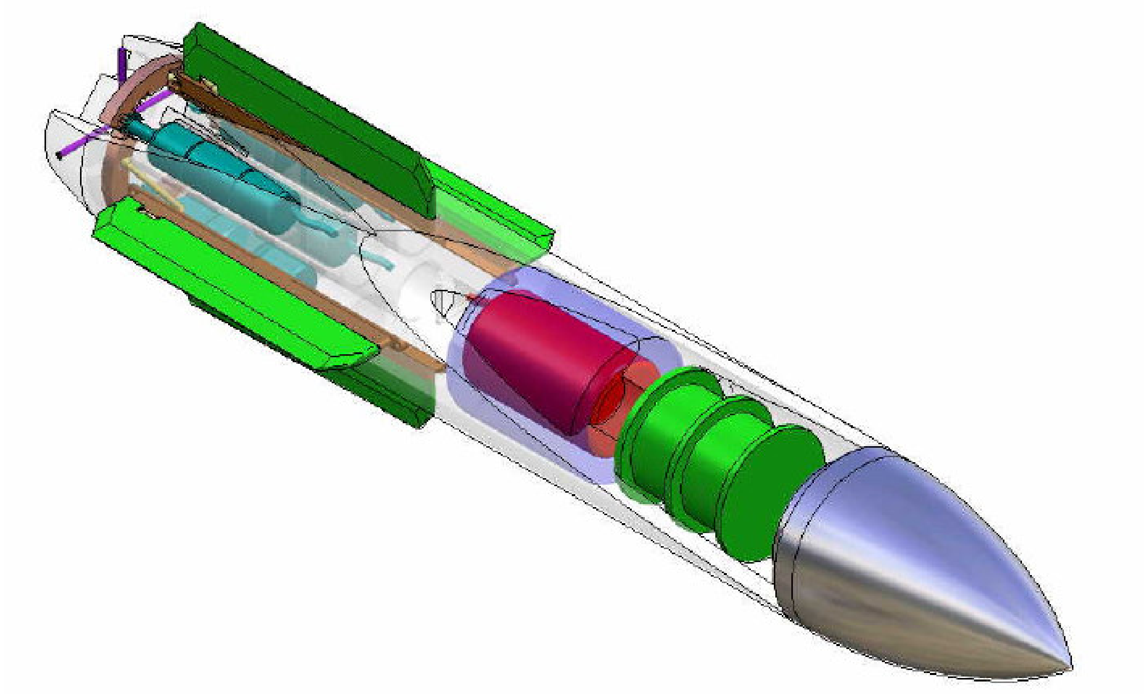


Figure 2.5 - Shark 1A (Hewitt)

One notable design was the Shark 1A depicted in Figure 2.5. The Shark used stowed fins (Hewitt, et al.) which were deployed immediately after leaving the gun barrel. This rapidly despun the projectile at which point the fins were used for active control.

The Shark was retrofitted with a guidance system and is documented in an AIAA paper (Tournes, et al.). The guidance system utilized second order sliding and adaptive mode control. It was shown to work in theory but required a data rate of 5kHz, not yet feasible with existing hardware. In addition there was some question as to whether or not the actual mechanical linkages would be able to withstand the stresses of deployment and the initial despinning of the projectile.

2.2.2 Squib Control

Squib control (Burnett, et al.; Williams, et al.) utilizes a set of squibs mounted around the axis of the projectile. The projectile velocity, spin rate, and moments of inertia were designed to achieve gyroscopic stability without the use of fins. This makes it mechanically simple. The squibs translate the projectile laterally. Generally squib controlled vehicles are axisymmetric and do not feature fins. A spin rate is desired to make a squib available in any given direction in a short possible time. The nominal spin rate with an unmodified gun barrel is 800 Hz which is very fast, and depending on the firing duration of the squib, possibly too fast as the sweep that occurs during firing will sweep the force vector, reducing the effective force in the desired direction.

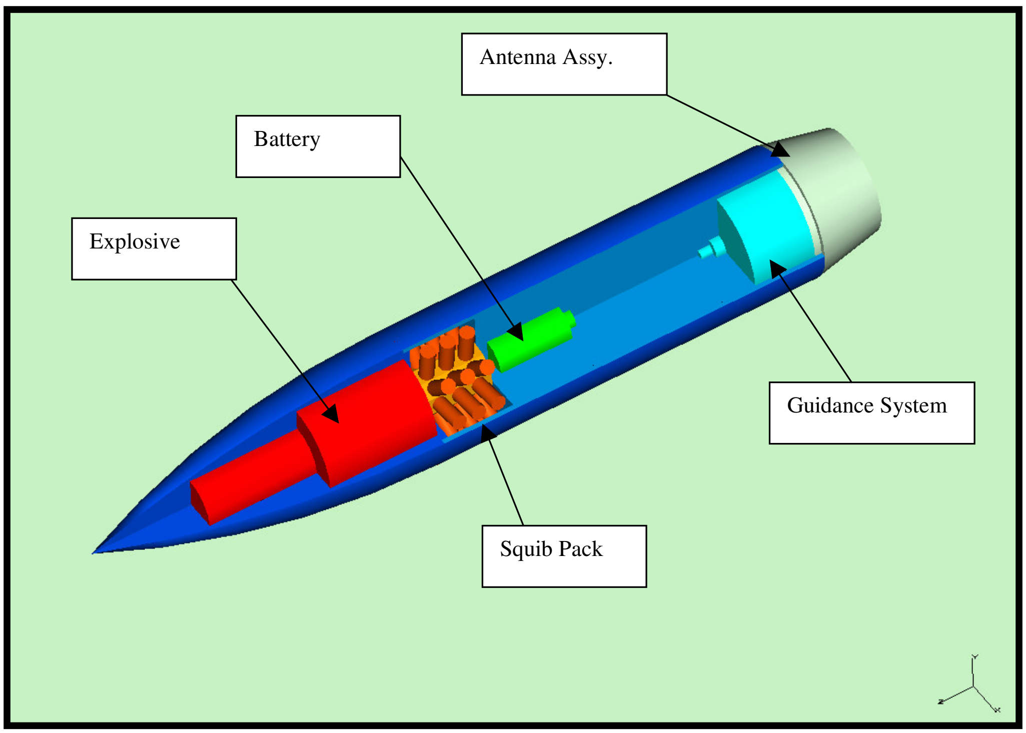


Figure 2.6 – Hammerhead Design (Williams, et al.)

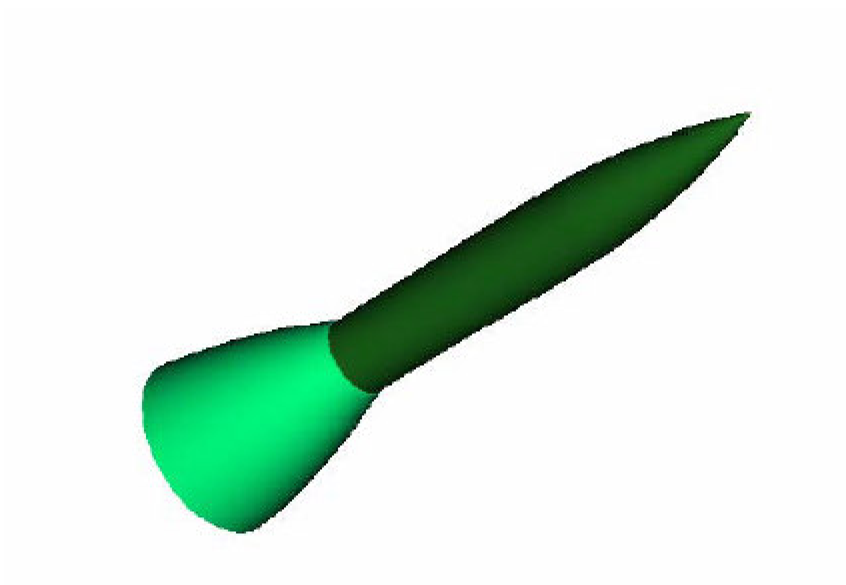


Figure 2.7 – LP3 Design (Hartlage, et al.)

The unique aspects of the system as compared to existing solutions are the fact that it uses squib control with a very high rotation speed along with attempting a tactical interception. The existing gun-launched solutions have no control authority after leaving the gun, and previously proposed solutions generally propose a slow rotation with squib control or no rotation with fin control.

2.2.3 Hybrid Control

Some hybrid designs included a combination of fins and squibs (Davis, et al.; Gladden, et al.) which slowed the projectile down to a reasonable rate, a dual-spin projectile with a moveable flared tail (Hartlage, et al.), and a design which utilized control pins and shock interactions to create forces for minor diversion along with squib forces for major motion (Howard, et al.).

2.3 Critical Assessment

Table 2.1 shows an assessment of potential design solutions. Focusing on the notional systems (LP5, Shark1A, Hammerhead), we see that, assuming the solution can be realized, any one of these solutions can deliver in excess the kinetic energy requirements over the required range, can be transported easily, and uses an existing, unmodified gun system. However taking a look at some of the details will bring to light why the Hammerhead was chosen as a base and why the guidance system in particular was chosen to fit this projectile. The Shark1A can be made to work; however, there are very real concerns about the mechanical linkages on the stowed fins. Also, the control system designed by Tournes, et al. cannot be realized with existing hardware, the data rates exceed the capabilities in existence. The LP5 is in a similar situation of requiring a high data rate to maneuver its tail. The Hammerhead,

on the other hand, did not fight the spin rate but rather embraced it, and worked with it – utilizing squibs which would spin with the projectile and thus sweep a plane in which the projectile could divert. The control system, a model predictive controller utilizing a modified linearized guidance system, was excellent for this task as the short, impulsive nature of the squibs quickly influenced the mechanics of flight, leaving very few transients input to the guidance system.

Table 2.1- Assessment of Potential Design Solutions

	Range	Mobility	Kinetic Energy on Target	Existing Gun System	Accuracy	Reference
System	Excellent	Meets Req	~100 kJ	Bofors L/70	Excellent	Hahn
Sgt. York	Unknown	Questionable	~100 kJ	Bofors L/70	Excellent	Seven, Babbit
Bofors 3P	Excellent	Meets Req	~1 kJ[1]	Bofors MK3	Good[2]	Bofors
AHEAD	Excellent	Meets Req	~1 kJ[1]	Skyshield 35mm	Good[2]	Oerlikon Contraves
Phalanx	Excellent	Unknown	Unknown[3]	M61A1	Good	Raytheon
LP5	Excellent	Meets Req	~100 kJ	Bofors MK44	Excellent	Hartlage
Shark 1A	Excellent	Meets Req	~100 kJ	Bofors L/70	Excellent	Hewitt
Hammerhead	Excellent	Meets Req	~100 kJ	Bofors L/70	Excellent	Williams

[1] Per Subprojectile
[2] Although it uses the same gun as other systems rated excellent, this system uses scattering subprojectiles.
[3] Mass is unknown; also round is explosive imparting additional energy to detonate the RAM threat.

CHAPTER 3

Approach

3.1 Notional Projectile for the Study

The conceptual projectile for this study is based on the Hammerhead Design presented earlier. It is a 40mm diameter round. It is 235mm long. The projectile is a body of revolution symmetric about the x-axis consisting of a tangent ogive nose, a cylindrical body section and a boat tail. The mass of the projectile is 1.495 kg. The projectile is shot from a gun with a muzzle velocity of 500 m/s and a spin rate of 2415 rad/s.

The control system consists of thee rows of squibs located axisymmetrically about the *x*-axis. Each row of squibs contains 10 squibs equally spaced, symmetric about the *x*-axis. The three squib banks are located forward of the center of gravity by 5, 15 and 25mm respectively as opposed to the original design which was centered around the center of gravity. Each squib provides 0.124N of thrust, providing a total system impulse of 3.72 N. The guidance model implemented is a linearized guidance model, which is utilized by a model predictive controller to determine the guidance of the projectile. Twenty times a second, the guidance model predicts the future trajectory of the projectile based on the current states of the projectile. Using the predictive trajectory and target data, the model predictive controller generates a

guidance signal and if necessary squibs are fired to intercept the target. A maximum divert of 3.7 meters is realized by firing squibs in a single direction.

One of the unique aspects of the projectile is the use of an onboard modified linear projectile model. The modified model does not make a flat fire assumption, which results in a more accurate prediction. This model predictive controller is used to make a tactical intercept with a mortar at the notional intercept point.

3.2 Modeling and Simulation

The task at hand is developing a guidance scheme for a gun-fired anti-RAM projectile. In order to test the guidance system, either a commercial simulation package must be used or our own six degree of freedom (6DOF) code must be developed. It was decided to develop our own 6DOF code, validated to a commercial simulation package.

A 6DOF model consists of three translational degrees of freedom and three rotational degrees of freedom. Coupled with their derivatives, there are a total of 12 states in a 6DOF that must be integrated. These 12 states plus their associated derivatives are the core of the 6DOF model.

3.2.1 Relating Derivatives of Vectors Across Reference Frames

The time derivative of a vector is related to a vector in a different reference frame by Equation 3.1 (Zipfel)

$$\frac{{}^{I}dh}{dt} = \frac{{}^{B}dh}{dt} + \omega_{B/I} \times h \,, \tag{3.1}$$

where h is the vector and $\omega_{B/I}$ is the angular velocity of the projectile, with respect to the inertial reference frame. This definition is useful in the derivation of the velocity derivative and the rotation rate derivative. The first term on the right-hand side is the inertial speed, a time derivative. The second term is referred to as the tangent acceleration.

3.2.2 Velocity Derivative

Starting with Equation 3.1 and substituting the velocity vector for h:

$$A = \frac{{}^{B}dv}{dt} + \omega_{B/I} \times v \,. \tag{3.2}$$

The quantity of interest is the time derivative of velocity in the body axes, $\frac{{}^{B}dv}{dt}$.

Rearranging and solving for $\frac{{}^{B}dv}{dt}$,

$$\frac{{}^{B}dv}{dt} = +\omega_{B/I} \times v + A \,. \tag{3.3}$$

The vector A is the acceleration being applied to the vehicle, and consists of acceleration due to gravity, aerodynamics and squib force injection. All three of these quantities are calculated as forces in the code; therefore, they must be divided by mass to calculate the acceleration.

3.2.3 Rotational Derivative

Starting with the general equation for moments

$$\sum M = \frac{{}^{I}d}{dt} H_{B/I}, \tag{3.4}$$

where M is the sum of the moments and H is the angular momentum of the projectile. H can be expressed as

$$H_{B/I} = h_{XB} \cdot \overrightarrow{I_B} + h_{YB} \cdot \overrightarrow{J_B} + h_{ZB} \cdot \overrightarrow{K_B} = \begin{bmatrix} Ixx & 0 & 0 \\ 0 & Iyy & 0 \\ 0 & 0 & Izz \end{bmatrix} \cdot \begin{Bmatrix} p \\ q \\ r \end{Bmatrix}. \tag{3.5}$$

Using Equation 3.1 and the resulting total angular momentum from Equation 3.5, the time derivative of the angular momentum of the projectile is

$$\frac{{}^{I}dH_{B/I}}{dt} = \frac{{}^{B}dH_{B/I}}{dt} + \omega_{B/I} \times H_{B/I}, \tag{3.6}$$

where $\frac{{}^{B}dH_{B/I}}{dt}$ is simply the moment of inertia tensor times the derivative of the angular momentum vector, since the moments of inertia are a constant at any given moment in time. Rearranging and solving for the time derivatives of angular momentum,

$$\frac{{}^{B}dH_{B/I}}{dt} = \frac{{}^{I}dH_{B/I}}{dt} - \omega_{B/I} \times H_{B/I}, \tag{3.7}$$

where $\frac{{}^{I}dH_{B/I}}{dt}$ is the sum of the moments input to the 6DOF. There are two sources of moments: the projectile aerodynamics, and squibs that are not coplanar with the center of gravity and the Y-Z plane.

3.2.4 Position Derivative

The derivative of the position vector is the velocity vector. The position is in the inertial frame whereas the velocity is in the body frame; therefore, a transformation matrix, T_{eb}, must be employed. This transformation matrix is commonly referred to as the Direct Cosine Matrix (DCM). The DCM will be derived later in the chapter. The result is Equation 3.8.

$$\begin{Bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{Bmatrix}_E = [T_{EB}] \cdot \begin{Bmatrix} u \\ v \\ w \end{Bmatrix}_B . \tag{3.8}$$

3.2.5 Orientation Derivative

The orientation derivative is represented by a quaternion. A quaternion is a four-dimensional vector representing an orientation in space. Instead of integrating three Euler angles, ϕ, θ, and ψ, a four-dimensional vector is integrated. While this may seem counter-intuitive as it adds an additional integration, it removes all of the sin and cosine calculations of the Euler DCM which are computationally taxing. Additionally, it removes a singularity which occurs in the Euler DCM at $\theta = 90°$: since the quaternion has no angular limits due to the fourth dimension. The only consideration for the quaternion is to normalize it regularly.

The derivative routine for the quaternion is a function of the quaternion itself and the rotation rates of the projectile.

$$\begin{Bmatrix} \dot{q}_0 \\ \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{Bmatrix} = 0.5 \cdot \begin{bmatrix} 0 & -p & -r & -q \\ p & 0 & r & -q \\ q & -r & 0 & p \\ r & q & -p & 0 \end{bmatrix} \cdot \begin{Bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{Bmatrix}. \tag{3.9}$$

The transformation matrix from body to inertial coordinates, or the Quaternion DCM

$$[T_{eb}] = \begin{bmatrix} q0^2 + q1^2 - q2^2 - q3^2 & 2\cdot(q1\cdot q2 - q0\cdot q3) & 2\cdot(q1\cdot q3 + q0\cdot q2) \\ 2\cdot(q1\cdot q2 + q0\cdot q3) & q0^2 - q1^2 + q2^2 - q3^2 & 2\cdot(q2\cdot q3 - q0\cdot q1) \\ 2\cdot(q1\cdot q3 - q0\cdot q2) & 2\cdot(q2\cdot q3 + q0\cdot q1) & q0^2 - q1^2 - q2^2 + q3^2 \end{bmatrix}. \tag{3.10}$$

T_{eb} is used in Equation 3.8 to transform the velocity vector from body to inertial coordinates. T_{eb} is also used to transform the gravity vector from inertial to body coordinates in Equation 3.10.

The final resulting equations are Equation 3.11 through Equation 3.14.

$$\begin{Bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{Bmatrix}_E = [T_{EB}] \cdot \begin{Bmatrix} u \\ v \\ w \end{Bmatrix}_B \tag{3.11}$$

$$\begin{Bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{Bmatrix} = \begin{bmatrix} 0 & r & -q \\ -r & 0 & p \\ q & -p & 0 \end{bmatrix} \times \begin{Bmatrix} u \\ v \\ w \end{Bmatrix} + \frac{1}{m} \begin{Bmatrix} X \\ Y \\ Z \end{Bmatrix} \tag{3.12}$$

$$\begin{Bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{Bmatrix} = \begin{bmatrix} Ixx & 0 & 0 \\ 0 & Iyy & 0 \\ 0 & 0 & Izz \end{bmatrix}^{-1} \cdot \left[\begin{Bmatrix} L \\ M \\ N \end{Bmatrix} + \begin{bmatrix} 0 & r & -q \\ -r & 0 & p \\ q & -p & 0 \end{bmatrix} \cdot \begin{bmatrix} Ixx & 0 & 0 \\ 0 & Iyy & 0 \\ 0 & 0 & Izz \end{bmatrix} \cdot \begin{Bmatrix} p \\ q \\ r \end{Bmatrix} \right] \qquad (3.13)$$

$$\begin{Bmatrix} \dot{q}_0 \\ \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{Bmatrix} = 0.5 \cdot \begin{bmatrix} 0 & -p & -r & -q \\ p & 0 & r & -q \\ q & -r & 0 & p \\ r & q & -p & 0 \end{bmatrix} \cdot \begin{Bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{Bmatrix}. \qquad (3.14)$$

Equation 3.11 through Equation 3.14 contain the core 6DOF model. Integrating the six states with their six derivatives, given the appropriate initial conditions and inputs in the form of acceleration and moments, will result in a working 6DOF model of a projectile.

The initial conditions to the system are the starting position, velocity, orientation, rotation rates and mass. There are two additional inputs to the system – the aerodynamics and the guidance system. The aerodynamics are calculated each update as a function of position, velocity and rotational velocity and air conditions. The guidance system will be discussed further in the next section.

3.2.6 Atmosphere

In order to calculate the aerodynamic forces acting on the projectile, the atmosphere at the altitude of the projectile must first be calculated. The model utilized is a standard atmosphere model.

$$T = 15.04 - (0.00649 \cdot |z|) \qquad (3.15)$$

$$p = 101.29 \cdot \frac{T + 273.1}{288.08}^{5.256} \tag{3.16}$$

$$\rho = \frac{p}{0.26869 \cdot (T + 273.1)} \tag{3.17}$$

$$M = \frac{\left| \dot{x} \quad \dot{y} \quad \dot{z} \right|}{\sqrt{\gamma \cdot R \cdot T}}. \tag{3.18}$$

By proceeding through Equation 3.15 through Equation 3.18 the standard atmosphere can be calculated, given the projectile's vertical position, z, and total velocity, which is the magnitude of the velocity vector. Equation 3.17 gives the local density and Equation 3.18 gives the local Mach number, both of which will be used in the calculation of the aerodynamics.

3.2.7 Aerodynamics

The aerodynamics model is a table-based coefficient model based on McCoy. Each of the aerodynamic coefficients is table interpolated as a function of Mach number. Each update of the 6DOF new coefficients are interpolated from a table and the aerodynamic force and moment is recalculated.

$$X_a = -qS \cdot (C_{x0} + C_{x2} \cdot \frac{v^2 + w^2}{V^2}) \tag{3.19}$$

$$Y_a = -qS \cdot (C_{y0} + C_{nA} \cdot \frac{v}{V}) \tag{3.20}$$

$$Z_a = -qS \cdot (C_{z0} + C_{nA} \cdot \frac{w}{V}) \tag{3.21}$$

$$L_a = qS \cdot D \cdot \frac{p \cdot D \cdot C_{lp}}{2 \cdot V} \tag{3.22}$$

$$M_a = qS \cdot D \cdot \frac{q \cdot D \cdot C_{mq}}{2 \cdot V} \tag{3.23}$$

$$N_a = qS \cdot D \cdot \frac{r \cdot D \cdot C_{mq}}{2 \cdot V} \tag{3.24}$$

$$Y_{mag} = qS \cdot \frac{p \cdot D}{2 \cdot V} \cdot C_{ypA} \cdot \frac{w}{V} \tag{3.25}$$

$$Z_{mag} = -q_{\infty} S \cdot \frac{p \cdot D}{2 \cdot V} \cdot C_{ypA} \cdot \frac{v}{V}\,. \tag{3.26}$$

Equation 3.19 through Equation 3.26 describe the forces acting on the body. Equation 3.19 through Equation 3.21 are axial forces acting in the *X*, *Y* and *Z* coordinate directions, respectively, acting at the center of pressure. The remaining components are acting at different pressure locations and contribute to the moment of the projectile. The center of pressure is defined by Equation 3.27

$$x_{cp} = \frac{C_{ma}}{C_{nA}} \cdot D. \tag{3.27}$$

Equation 3.22 through Equation 3.24 describe the moments imparted on the projectile, acting at the center of mass. Equation 3.25 and Equation 3.26 are the Magnus forces, which occur due to the spin rate of the projectile. These forces act at a different location and provide both a force and a moment on the vehicle depending on the attitude. The Magnus pressure location is defined by Equation 3.28.

$$x_{mag} = \frac{C_{npA}}{C_{ypA}} \cdot D. \tag{3.28}$$

Summing forces and moments

$$\begin{Bmatrix} X \\ Y \\ Z \end{Bmatrix} = \begin{Bmatrix} X_a \\ Y_a + Y_{mag} \\ Z_a + Z_{mag} \end{Bmatrix} \tag{3.29}$$

$$\begin{Bmatrix} L \\ M \\ N \end{Bmatrix} = \begin{Bmatrix} L_a \\ M_a + x_{cp} \cdot Z_a + x_{mag} \cdot Z_{mag} \\ N_a - x_{cp} \cdot Y_a - x_{mag} \cdot Y_{mag} \end{Bmatrix}. \tag{3.30}$$

Equation 3.29 is the aerodynamic force vector and Equation 3.30 is the aerodynamic moment vector. These equations are updated every iteration and injected into the 6DOF mode. All of the coefficients were derived from the PRODAS aerodynamic predictor.

3.3 Many Objects

Each of the objects discussed is implemented as a class in C++. Figure 3.1 shows the relationships between the classes. The guidance and squib classes will be discussed in the next section. The flexibility of using classes allows us to have any number of squib objects; it is trivially easy to change the number of squibs from one run to the next. It also allows future users to swap out models, providing higher fidelity models without changing any other code so long as they present the same interface to the code.

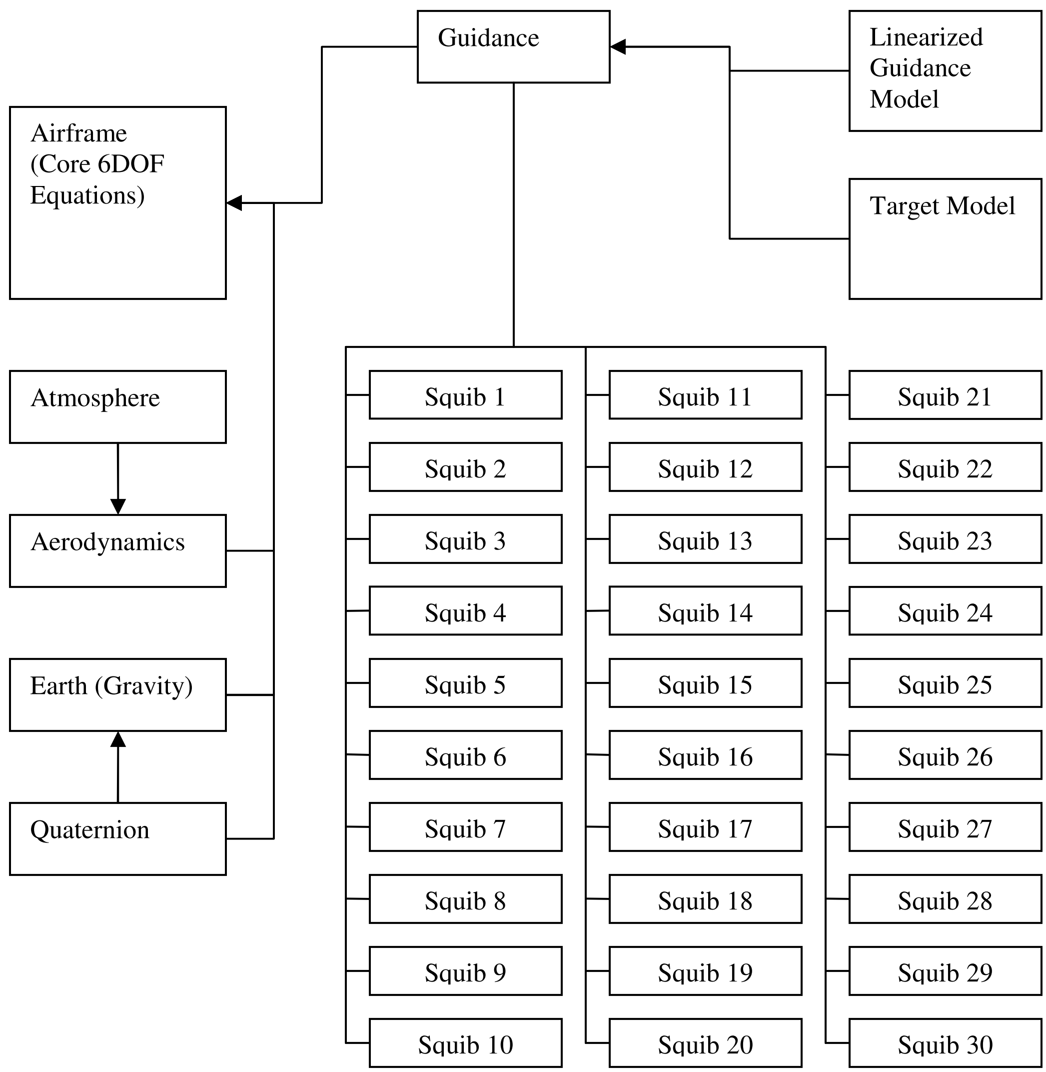


Figure 3.1 - Model Configuration of the 6DOF Simulation

3.4 Model Verification

Prior development of the projectile was done in a modeling and simulation package called PRODAS – Projectile Rocket and Ordinance Design and Analysis Software. This software package allows for complete end-to-end development of a projectile, from graphical design of the projectile to 6DOF simulation runs complete with control systems. Due to the design requirements of the guidance model being developed, PRODAS was not an option; however, the base 6DOF without guidance could be validated to PRODAS since the projectile was being flown against a flat, inertial frame similar to PRODAS.

The results shown in Figure 3.2 through Figure 3.5 show a good correlation between the 6DOF model and PRODAS. Figure 3.3 shows a slight discrepancy in the cross range of approximately 4 centimeters at 3.3 seconds. The good correlation makes sense as the 6DOF derives its aerodynamic coefficients from PRODAS, share the same initial conditions as PRODAS and makes the same flat-earth assumption as PRODAS.

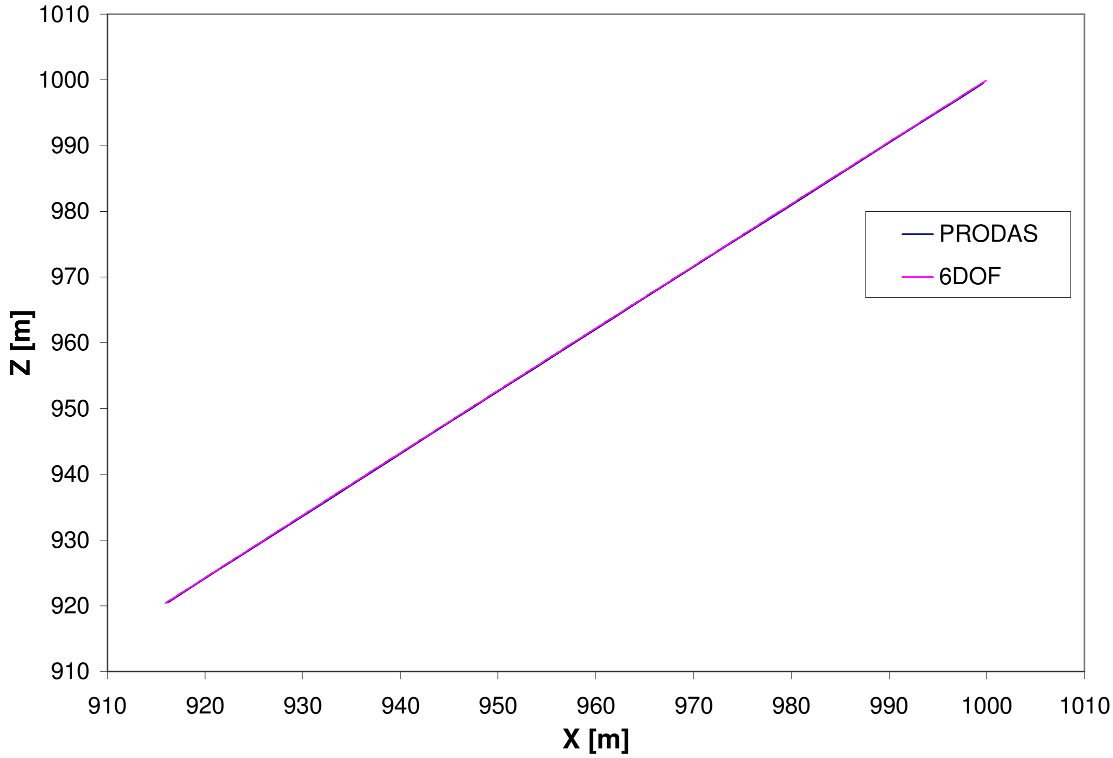


Figure 3.2 - Z vs. X, Last 0.3 Seconds of Engagement

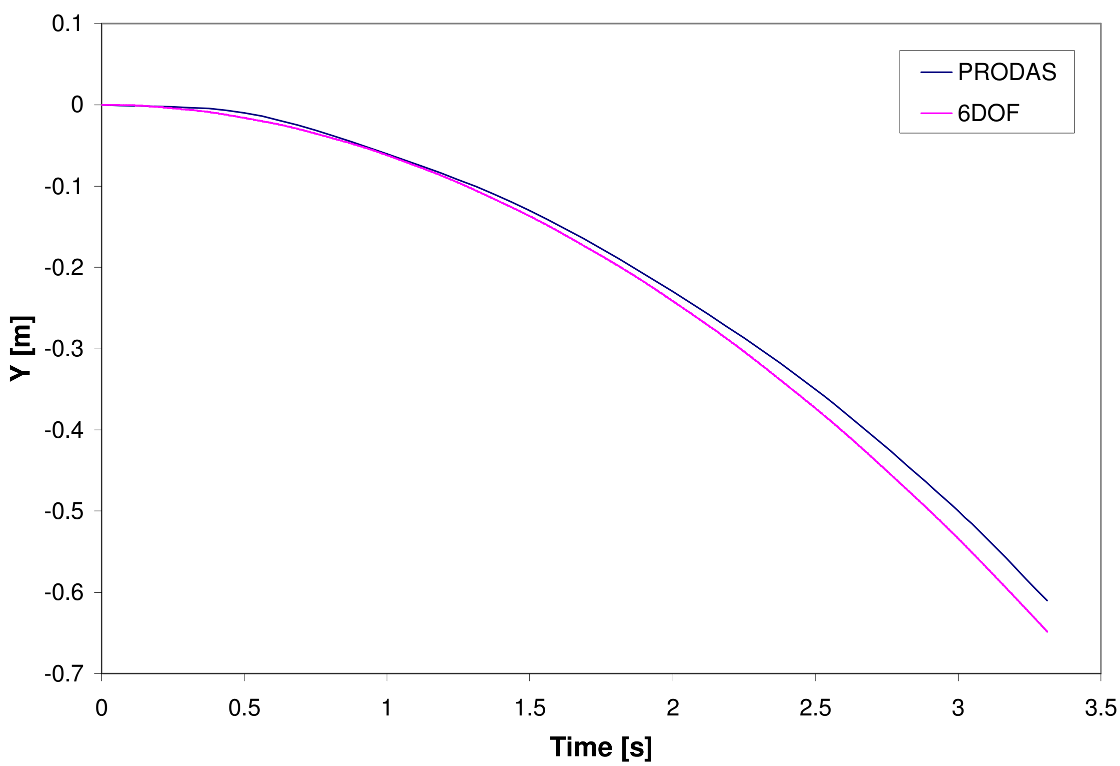


Figure 3.3 - Crossrange vs. Time, Entire Flight

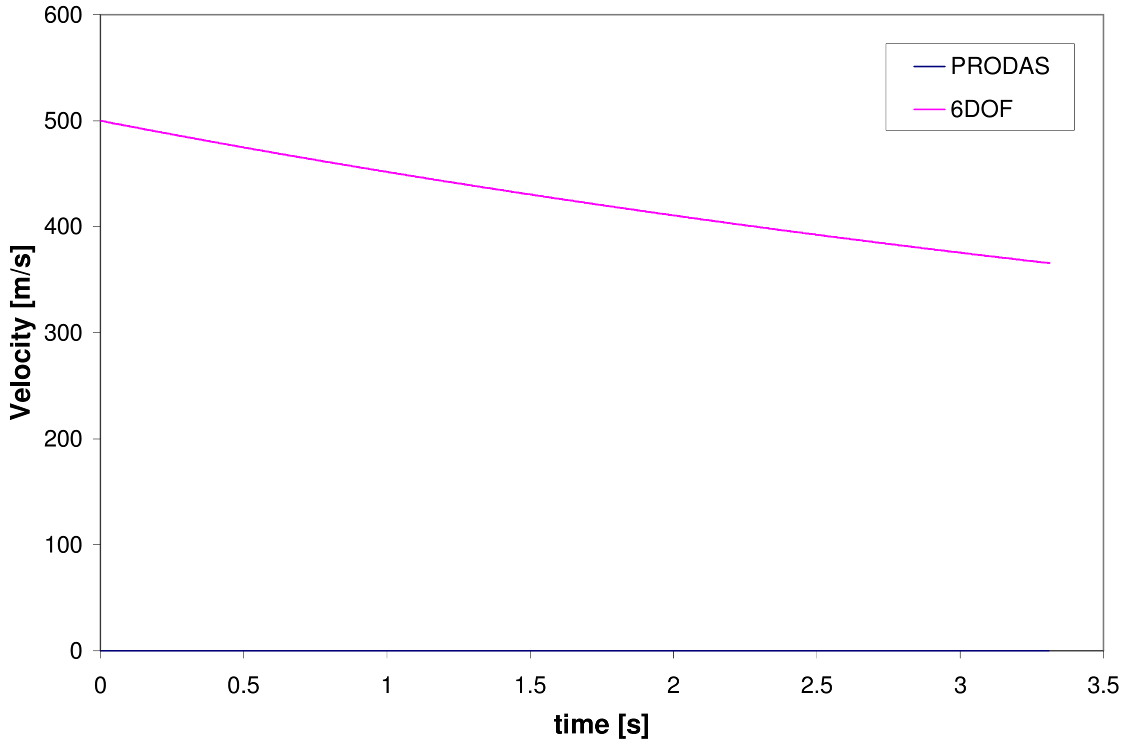


Figure 3.4 - Velocity vs. Time, Entire Flight

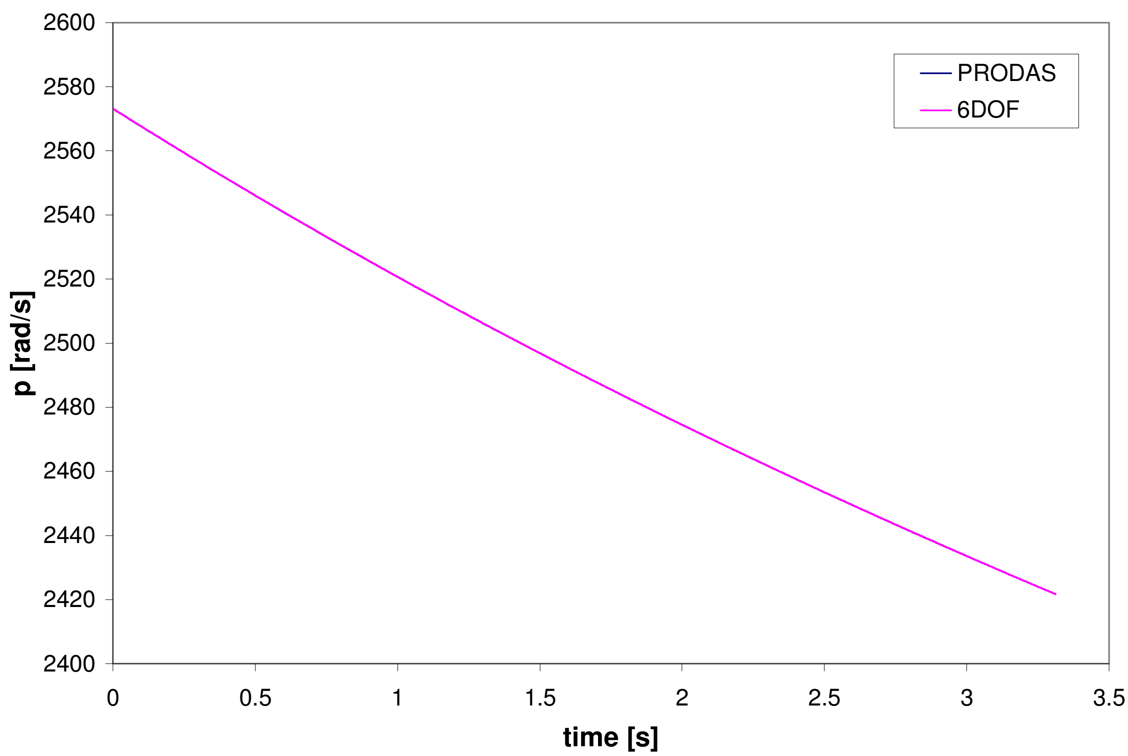


Figure 3.5 - Spin Rate vs. Time, Entire Flight

CHAPTER 4

Development of a Guidance Model

The guidance model consists of four distinct components: a squib model, a linearized guidance model, a target model and a controller. The squib model simulates a single squib, capturing its location on the projectile airframe along with a thrust curve. It is a simple device that can be triggered to fire at any point in time, after which point it is a dead object. The guidance model takes inputs from the 6DOF and generates a trajectory into the future, predicting where the projectile will go should no squibs be fired. The target model is a table-based trajectory model with available error corruption, predicting where the target will be in the future. The controller uses data from the guidance model and the target model to make decisions on when to fire any number of squibs.

4.1 Squib Model

The squib model encapsulates all of the details of a single squib. Upon initialization, four unique pieces of information need to be passed to the squib:

- The angular position of the projectile on the body, about the axis of revolution
- The location of the squib along the axis of revolution, relative to the c.g.
- Two error control terms:

- Ability for the squib to misfire, producing no net thrust.
- Ability for the squib to fire with less-than-perfect thrust.

The first two initialization options are the locations of a unique squib on the body. The third option, if enabled, allows for dead squibs – squibs which do not fire when commanded. The fourth option, if enabled, allows for squibs with less-than-perfect thrust, by varying the coefficient of thrust applied to the thrust model.

The thrust model is a table lookup model. Once the model is triggered, the model checks the two error control terms and, if either are set, will compute the required modifications to the thrust curve. Each iteration of the simulation until the thrust curve ends the model outputs both the thrust and the moment contributions of the squib to the 6DOF. Equation 4.1 is the calculation of the squib force in the body frame. Equation 4.2 is the calculation of the moment force, in the body frame.

$$\begin{Bmatrix} X \\ Y \\ Z \end{Bmatrix}_B = \begin{Bmatrix} 0 \\ C_t \cdot T_{squib} \cdot \cos(\theta_{pf}) \\ C_t \cdot T_{squib} \cdot \sin(\theta_{pf}) \end{Bmatrix} \tag{4.1}$$

$$\begin{Bmatrix} L \\ M \\ N \end{Bmatrix}_B = \begin{Bmatrix} 0 \\ -sl \cdot Y \\ sl \cdot Z \end{Bmatrix}. \tag{4.2}$$

The thrust curve is shown in Figure 4.1. Each squib has an impulse of 0.125 N/s. The duration of firing is 0.4ms. The short firing time is crucial due to the high rate of spin of the projectile.

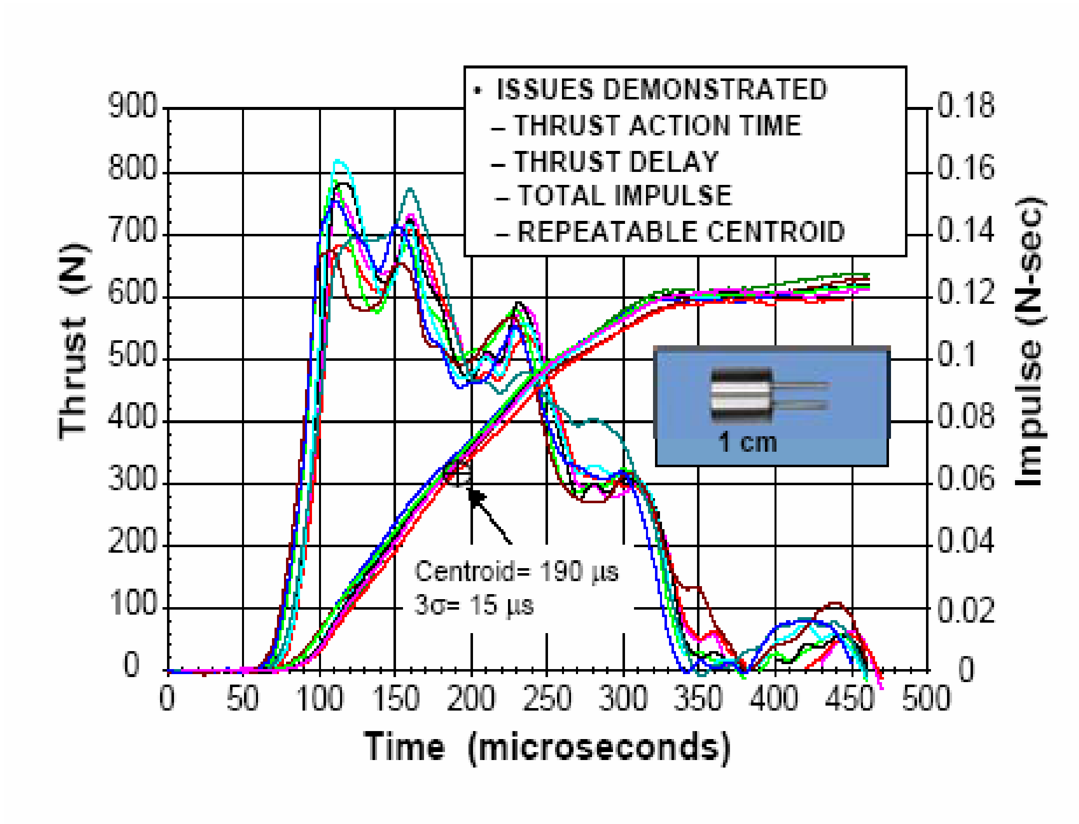


Figure 4.1 - Squib Thrust Curve (Williams)

4.2 Linearized Guidance Model

The linearized guidance model is based on the model presented in a paper by Hainz and Costello (Hainz, et al.). This model is a modified linearized model. A modified linearized guidance model does not make a flat fire assumption.

A linearized guidance system makes a host of assumptions in order to simplify the calculations, resulting in a speedy predictor. The major assumptions include

1. Forward velocity is large compared to side velocities.
2. Roll rate is large compared to pitch and yaw rates.
3. Yaw angle is small, therefore small angle theorem is applied.
4. Magnus force is negligible, Magnus moment is not.
5. Projectile is a body of revolution; therefore,
    - The moment of inertia tensor is symmetric
    - The aerodynamics can be simplified
6. Aerodynamic coefficients are constant over the entire trajectory.
7. Products of small values and derivatives are negligible.
8. The velocity and roll rate are slowly changing with respect to time.

The procedure for solving the linearized model starts by inputting the 12 states from the 6DOF. Using data from the states, the current aerodynamics are looked up. These aerodynamics are held constant during the prediction. The independent variable is transformed from time to arc length in units of caliber. Using the assumptions listed, the equations of motion can be roughly decoupled into 5 groups of

equations: the total velocity, roll rate, epicyclic, Euler angle, and swerve equations. Ultimately we are only interested in the swerve equations as these are the displacements of the projectile; however, the swerve solution requires eigenvalues from the epicyclic solution for its solution.

While Hainz and Costello included all of the equations necessary to create the linearized model, the paper itself lacked some of the coefficients. Upon contacting Costello to attempt to acquire those coefficients, he had Doug Ollerenshaw send me a copy of the code they used in the paper. It was written in FORTRAN 95. I proceeded to convert the code to C++, made it into a class and re-tooled it into a single function that, given the states of the 6DOF model, would return a map containing future times paired with future positions.

4.3 Target Model

The target model is a table-based trajectory model with Gaussian error. Every iteration, a new set of errors is generated based off of a pre-determined standard deviation set at the beginning of the run. A function in the target model returns the target's position as a function of any future time. The function interpolates the table using the future time, and then corrupts the data with the Gaussian error. This function is used by the controller and will be discussed in the next section.

The table-based model with error corruption was chosen to simplify the analysis process. Instead of making assumptions in the design of a radar or acquisition system, meaningful results could be presented by running several sets of Monte Carlo simulations, varying the standard deviation of the error. Results can then

be displayed parametrically, so a future designer could pick out the appropriate error from a chart based on relevant radar data and determine the accuracy of the projectile.

4.4 Controller

The controller manages the squibs, controls when the linear guidance model is updated and uses guidance information from the linearized guidance model and the target model to make firing decisions.

Upon instantiation the controller creates the linearized guidance model and thirty squibs, giving each squib the same thrust curve along with unique locations on the airframe. The target model is an independent model and is instantiated independently of the controller.

The flow of the controller is laid out in Figure 4.3. The controller’s functions can be broken up into two main functions, updating the guidance model and then controlling the squib firings.

4.4.1 Updating the Guidance Model

Entering the guidance loop, the first decision is whether or not to update the linearized guidance model. The linearized guidance model is updated 20 times per second. The linearized guidance model then returns a map of data to the controller, correlating future locations of the projectile to future times. An if-loop is entered, comparing each entry in the map to target model’s error-corrupted prediction of where the target will be at the same future time. The loop proceeds until the nearest approach is detected, at which point the target model coordinates minus the projectile’s position coordinates from the map are stored into a local variable, a vector (xyz_err) from our future position to the target future position in the inertial frame, as

best we can predict it at this point in time. The x coordinate of the vector is set to zero; since the squibs are mounted perpendicular to the *x*-axis, an error in the *x* direction cannot be corrected; there is no point in introducing an error we cannot hope to control. The value of xyz_err is then used as our guidance data until the next guidance update.

4.4.2 Squib Fire Control

The core concept behind the controller is two properties, phase and magnitude. Phase is the angular position of the target about the *x*-axis in the body frame. Magnitude is the distance to the target from the body at the closest approach, in the body frame. Both quantities can be calculated directly from xyz_err, after converting xyz_err to the body frame (Equation 4.3) as shown in Equation 4.4 and Equation 4.5.

$$\begin{Bmatrix} x \\ y \\ z \end{Bmatrix}_B = T_{eb}^{-1} \cdot \{xyz\_err\} \tag{4.3}$$

$$phase = \tan^{-1}\left(\frac{x}{z}\right) \tag{4.4}$$

$$magnitude = \sqrt{y^2 + z^2} \ . \tag{4.5}$$

The phase and magnitude are depicted graphically in Figure 4.2. Looking from the rear of the projectile, the sense of the phase is clockwise about the *x*-axis of the projectile from the positive *z*-axis. The magnitude is simply the distance to the target.

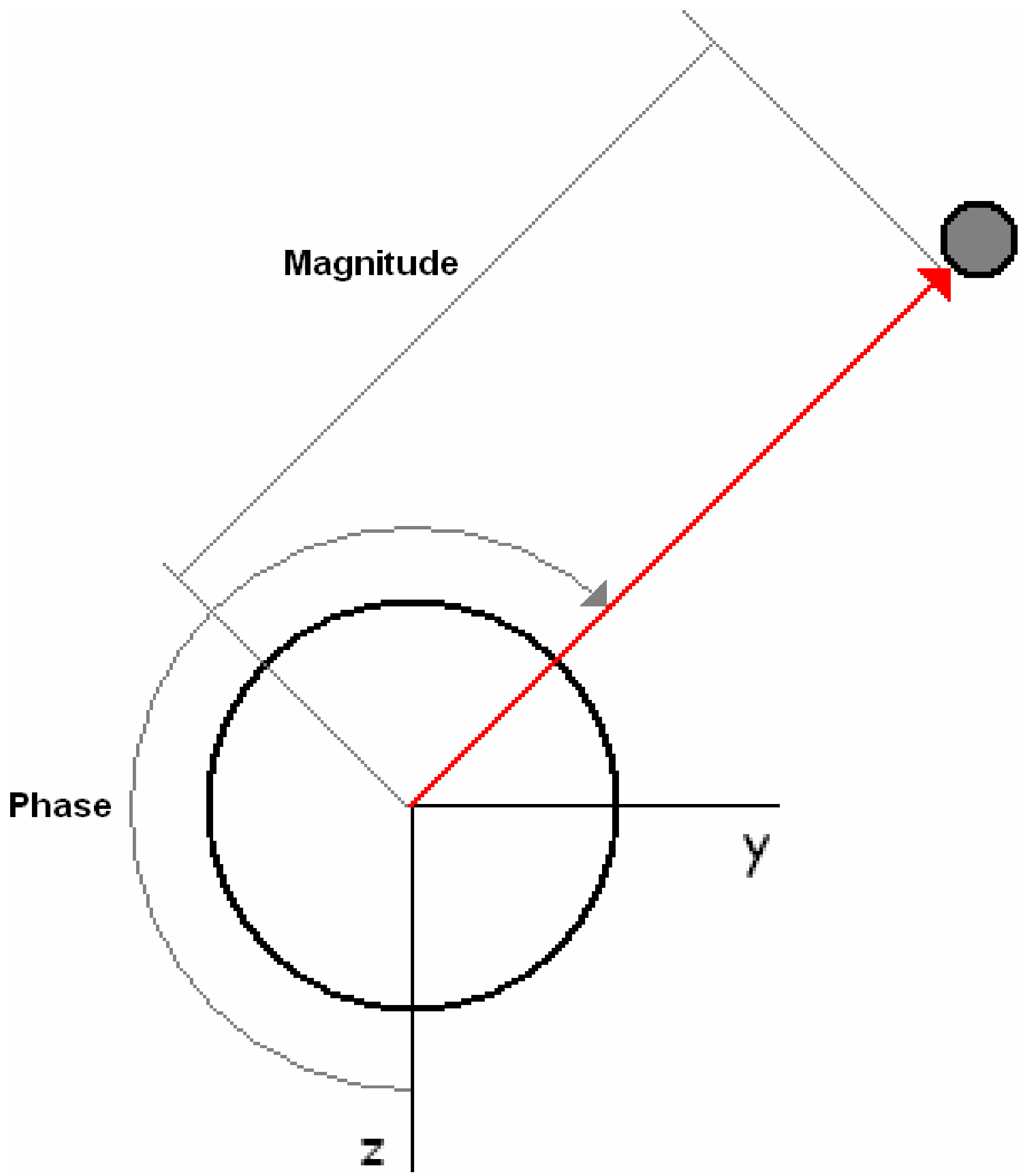


Figure 4.2 - Phase and Magnitude

Two more pieces of information are introduced to the guidance loop to regulate the rate at which squibs are fired; a firing hold and a firing delay. The firing hold prevents the projectile from firing squibs until a certain time, whereas the firing delay prevents the projectile from firing a squib until a certain amount of time passes after firing the squib prior. The hold represents any time required for the projectile to effectively get its bearings and charge the firing circuits, whereas the delay models any computational or charging delays in between squib firings. For the baseline case, the hold value is 1.0 second and the delay value is 0.05 seconds.

The squib event loop checks three of the four states for the following conditions:

- The magnitude exceeds the capability of a single squib
- No firing hold
- No firing delay

The magnitude must exceed the capabilities of the squib, otherwise the squib will overshoot, causing additional error which would have to be corrected by another squib firing. The formula used in the code is an empirical curve fit of simulation data. Truth relationships can be made between the impulse of the squib and the distance the projectile will travel as shown in Equation 4.6

$$\frac{I_{squib} \cdot t_f}{m}, \tag{4.6}$$

the derivation of which is documented in Appendix B. However due to nonlinearities in the 6DOF, this equation does not hold and the scale factor fit does a better job.

If these three conditions are met, then the integration rate is increased by a factor of 100 in order to accurately capture the last condition, the alignment of a squib with the phase angle of the target.

The procedure to determine squib alignment is the same for every squib although each squib has a unique position on the airframe of the projectile. First the pulse force angle, the angular position of the squib on the body, is subtracted from the phase. Next the phase warp must be determined. A body with a high roll rate will experience an interaction between the thrust vector and the gyroscopic forces causing the projectile to swerve; this is referred to as phase warp and to compensate, a table is implemented with a correction factor for each station line location along the axis of revolution. The derivation of the pulse force table is outlined in Appendix C. The phase warp is subtracted from the phase. After performing all of these operations, we are left with the angle between the vector pointing to the target and a squib.

In order to maximize thrust in the direction of travel, the squib must be fired such that centroid of the thrust occurs when the squib is aligned with the firing direction. Referring back to Figure 4.1 the centriod of the thrust curve is located at approximately 0.00019 seconds. Multiplying the centroid offset time by the rotation rate determines the phase criteria. If the phase is equal to the phase criteria, allowing for a little bit of error, then the squib is in line to fire.

This process is repeated for every active squib until a squib is found that meets all of the above criteria. Once all criteria are met, the guidance model commands the aligned squib to initialize the firing routine, and with every update thereafter continues to fire the squib until the squib stops thrust production. Squibs

are processed in the order that they were created. Therefore, squibs with a short station line component are processed first, and within that group, squibs are processed in order of their angular position, smallest to largest.

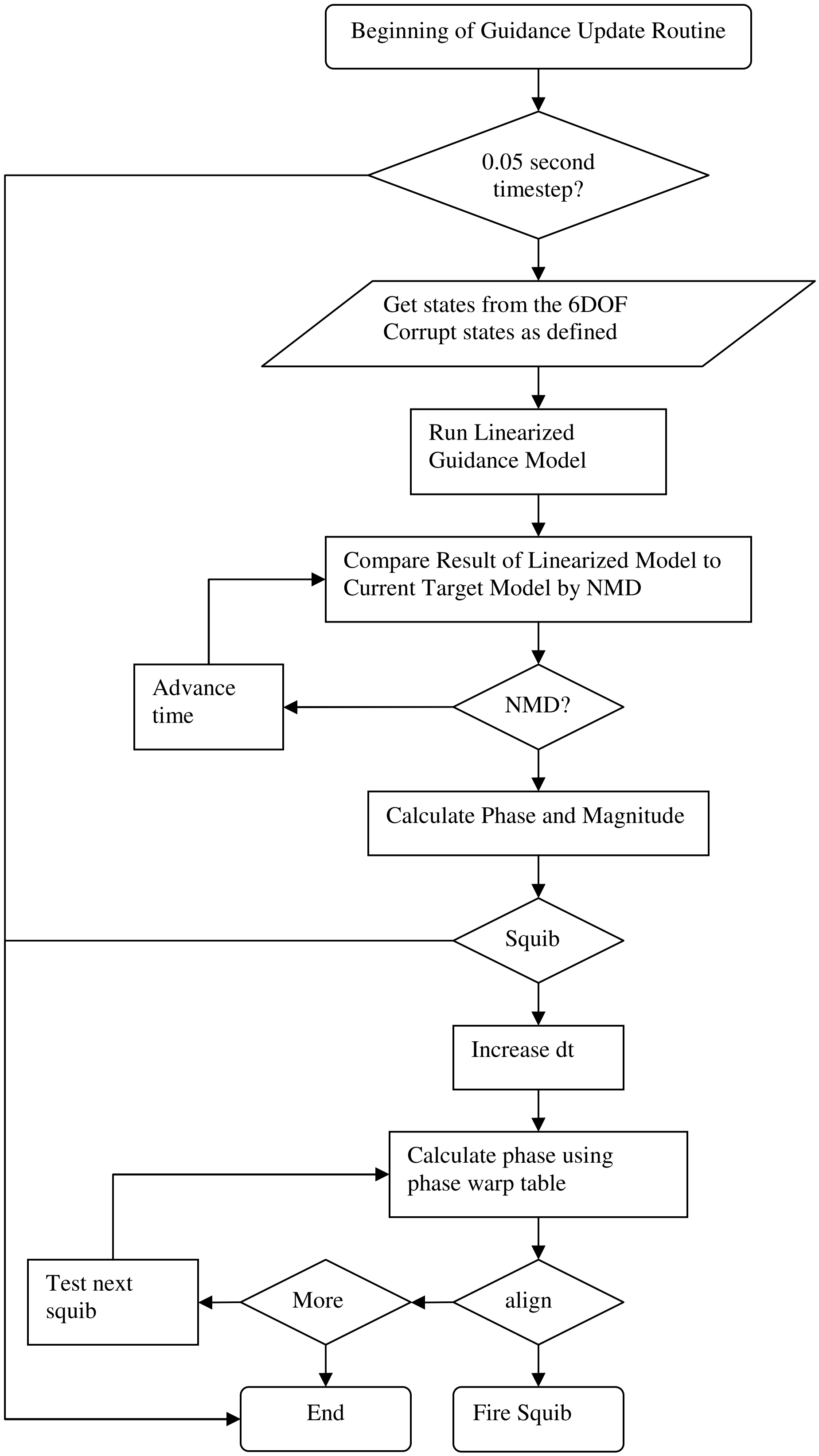


Figure 4.3 - Controller Flow Chart

CHAPTER 5

Results

As previously mentioned, the bank of squibs consists of three rows of squibs located forward the center of gravity at 0.005m, 0.015m and 0.025m respectively. The initial thought was that that placing the squibs forward the c.g. would result in additional side forces which would move the projectile while not creating excessive pitching or yawing which would introduce errors that would overpower the control system. As will be seen in Section 5.5, this was not the case. It turns out that for this particular configuration and guidance system, any amount of pitching and yawing added to the system will cause errors. Ideally the squibs should be moved back to the center of gravity; however, for the case presented, the losses are not very extreme.

5.1 Control Authority

The first task is to define control authority, that is, the envelope in which our control system is useful. A working knowledge of how much error the control system can correct is invaluable in the analysis of the system. This will aid in determining the gun alignment error that is acceptable along with creating an error budget for the projectile, which in turn will help define the number of projectiles needed per engagement.

A series of four run sets was taken to determine the open-loop control authority as a function of the squib hold and squib delay constants. Table 5.1 displays the run matrix. Figure 5.1 displays the results of the four runs. This figure was generated by firing all thirty squibs in a single direction during one run, then sweeping the firing direction by five degrees each run over the course over 72 runs to complete a circle. As would be expected, by decreasing the squib hold, the projectile has capabilities to divert further. Likewise, by decreasing the squib delay, additional diversion can be realized. If the system designer can reduce delays in the system, diversion can be gained essentially for free, without adding additional hardware to the projectile. This can be useful depending upon the situation. Since gun pointing errors are relatively small compared to tracking errors and the “picture” of the engagement gets better with time, most likely many of the squibs will be reserved to be fired late in the flight. Being able to fire the squibs sequentially with little or no time delay in between firings will be essential to maximizing the performance of the projectile.

Table 5.1 - Control Authority Run Matrix

Run	Hold	Delay
1	1.0	0.05
2	1.0	0.01
3	0.5	0.05
4	0.5	0.01

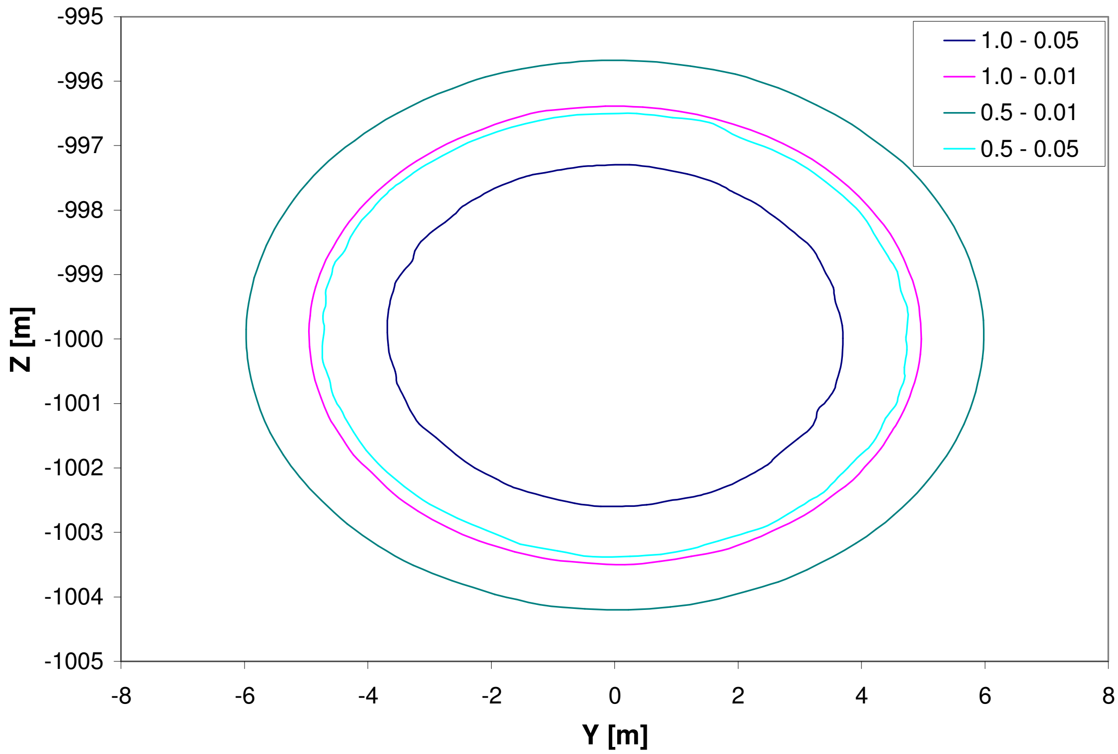


Figure 5.1 - Maximum Divert Capability

5.2 Test Case: Perturbed Trajectory

The next case study illustrates the model predictive control system in action. The gun is fired with a misalignment of 0.10 radians in the azimuth, which causes a diversion in the positive Y direction as shown in Figure 5.2. The control system starts firing squibs at t=1.0 and by t=1.35 has offset the initial error. Four squib corrections later at t=1.55, the projectile is headed on a course collision for the target. There are a total of five mid-course corrections between t=1.35 and t=3.00 seconds, ending with 5 more corrections in the last 0.3 seconds. The mid-course corrections could be reduced or eliminated if the phase criteria were better tuned. The XZ trajectory is shown in Figure 5.3 and is nominal. Figure 5.4 and Figure 5.5 show the velocity and roll rate with respect to time, respectively. These curves show that the velocity and roll rate are not influenced to any great degree by the control system. Figure 5.6 and Figure 5.7 show the pitch and yaw with respect to time, respectively. The control system causes both pitch and yaw to become very unstable when the control system starts to engage at t=1.00 seconds.

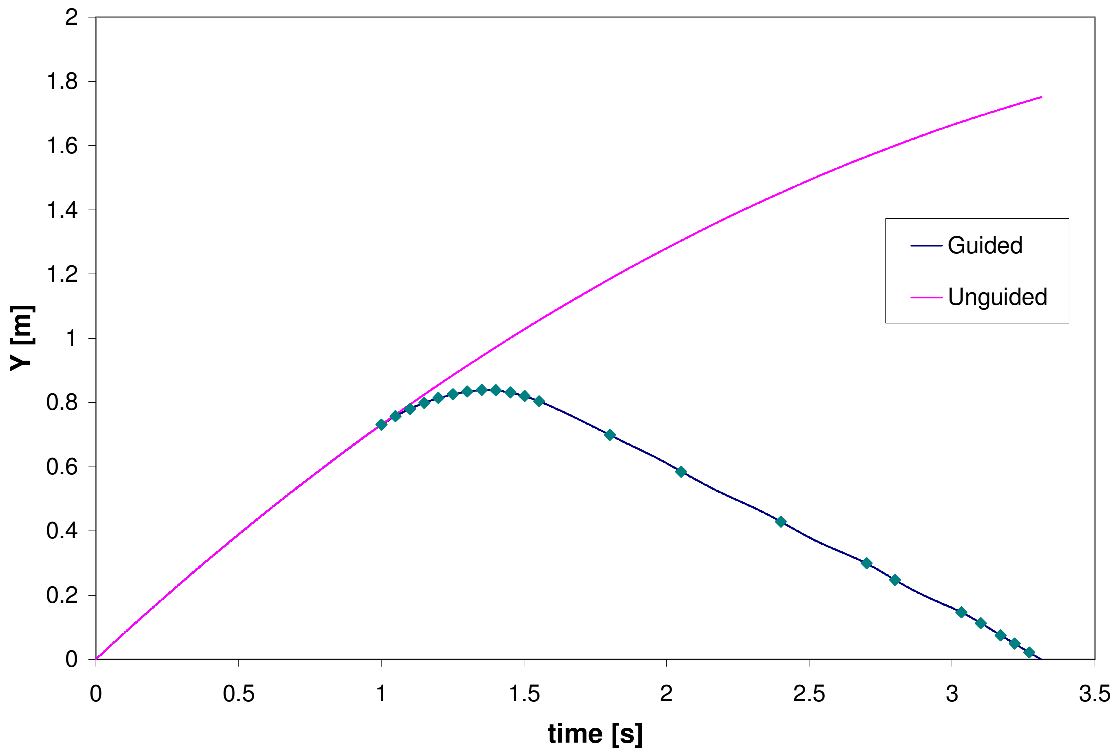


Figure 5.2 – Crossrange with 0.10 radians of error in the azimuth (▲) indicates a squib firing

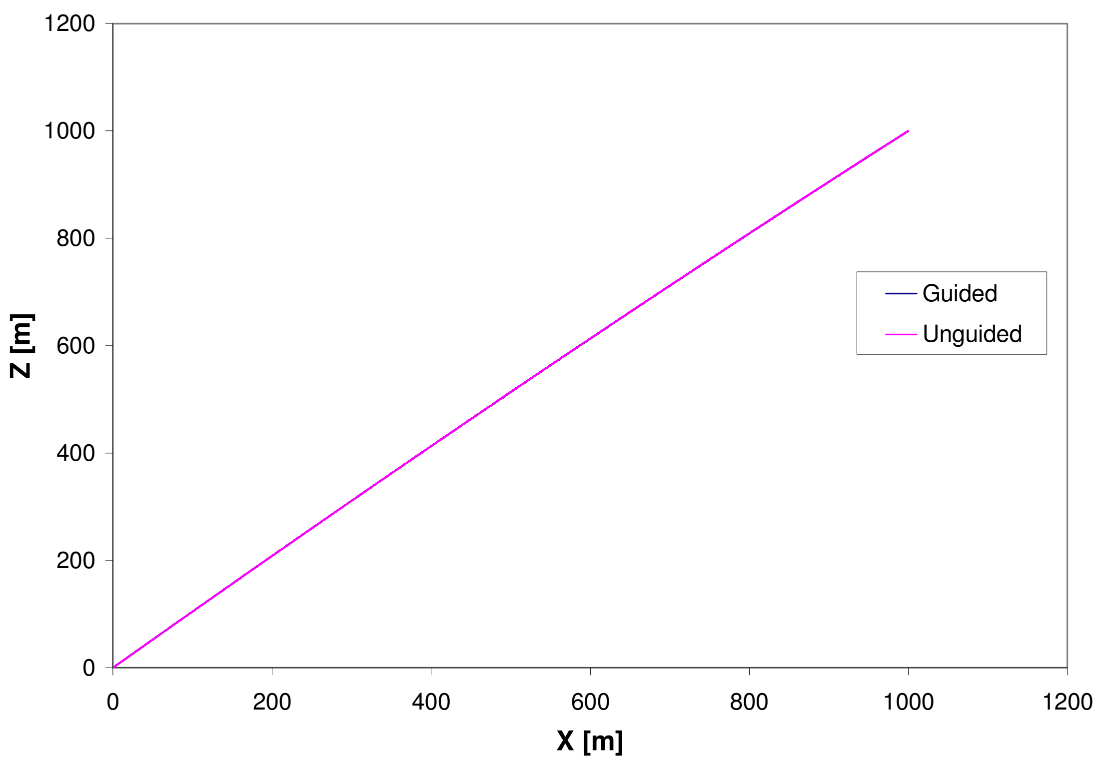


Figure 5.3 - XZ Plane with 0.10 radians of error in the azimuth

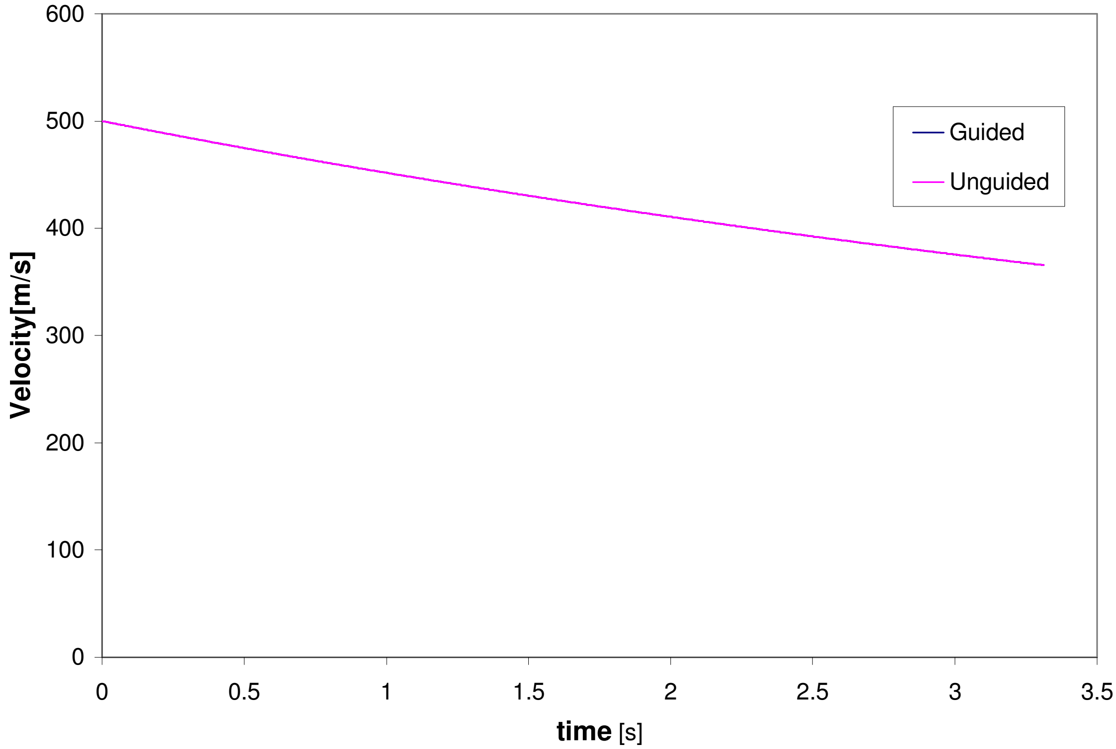


Figure 5.4 – Velocity with 0.10 radians of error in the azimuth

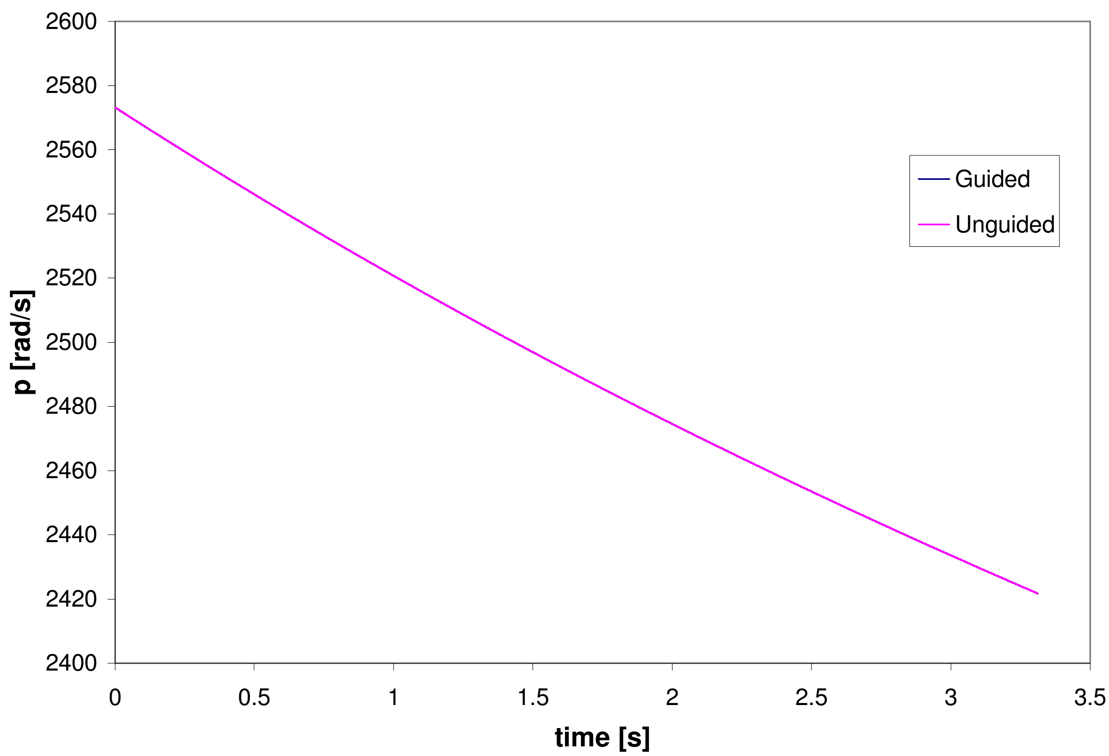


Figure 5.5 - Roll Rate with 0.10 radians of error in the azimuth

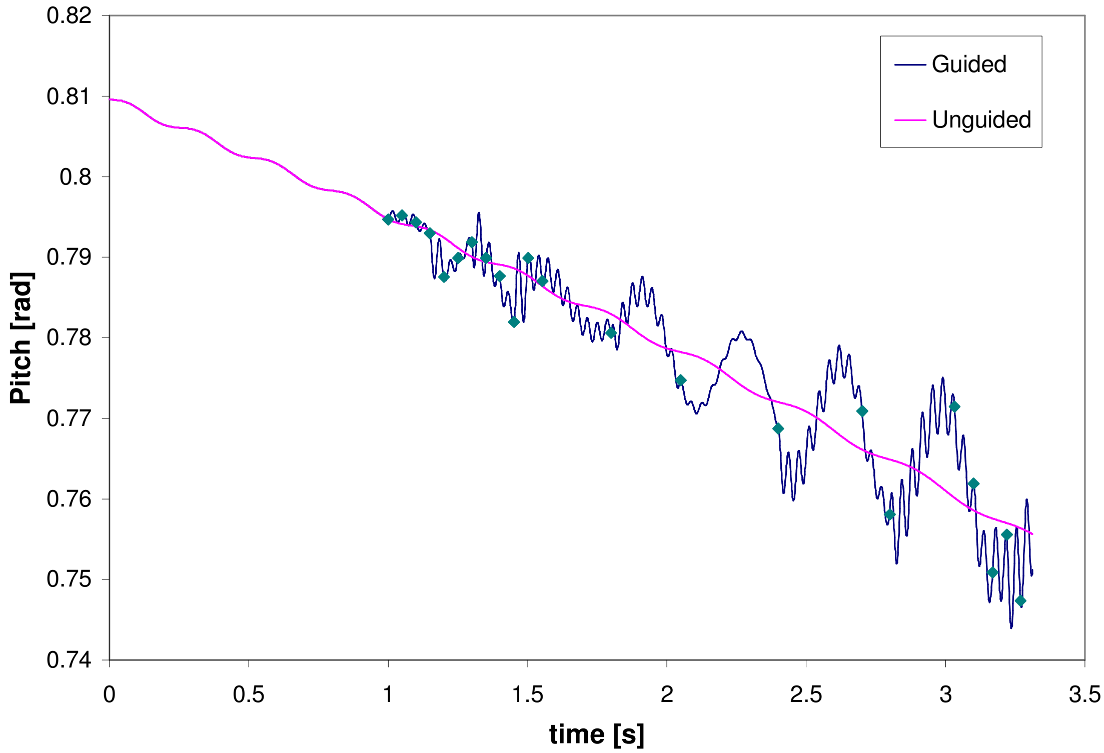


Figure 5.6 – Pitch with 0.10 radians of error in the azimuth (▲) indicates a squib firing

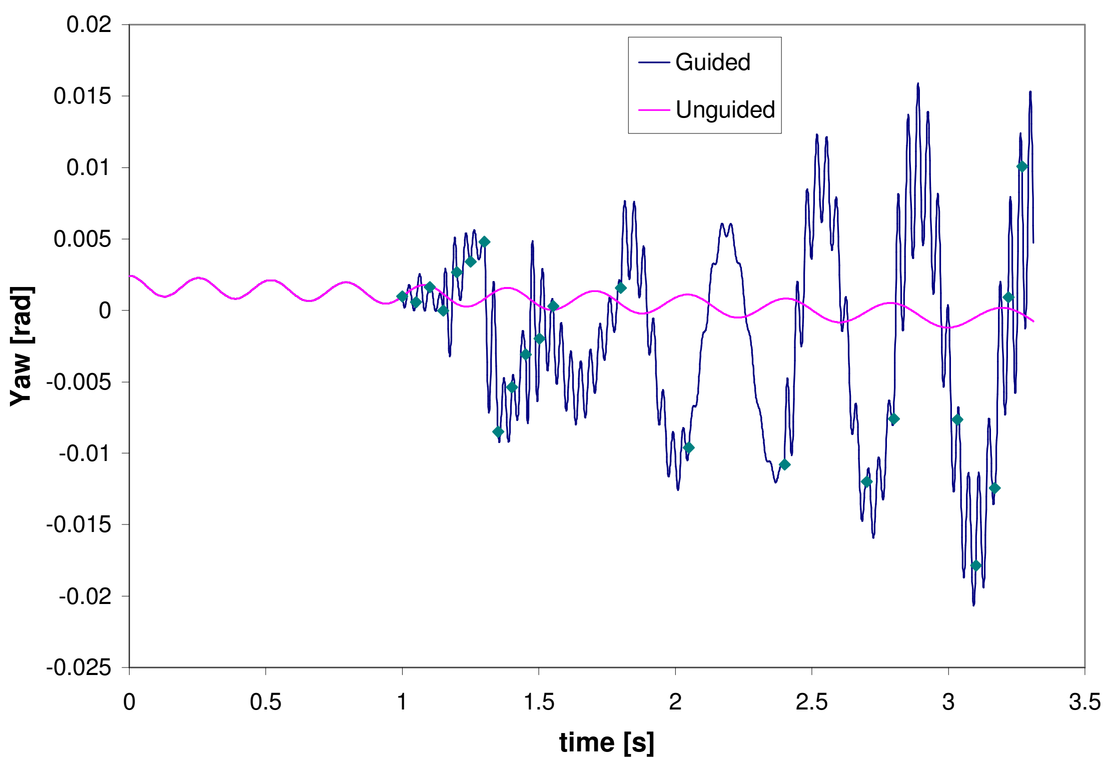


Figure 5.7 – Yaw with 0.10 radians of error in the azimuth (▲) indicates a squib firing

5.3 Test Case: Control System vs. No Control System

Figure 5.8 and Figure 5.9 show a comparison between a Monte Carlo run set of 500 runs with the guidance system disabled in the first case and enabled in the second. The azimuth and altitude angles on the gun were allowed to vary in a Gaussian distribution with a standard deviation of 0.002 radians. A bias of 0.25 radians was applied to the phase along with a Gaussian distribution with a standard deviation of 0.25 radians. A Gaussian distribution of 0.10 meters was applied to the target.

The control system does a decent job of controlling the projectile. The probability of kill of the uncontrolled projectile is 0.000 whereas the controlled case is 0.152. If the target noise is turned off, the Pk increases to 0.492. The Pk of the system with a constant phase bias is 0.556. The knowledge of target position is crucial for a hit-to-kill weapon to be successful, and is the largest source of error in this test case.

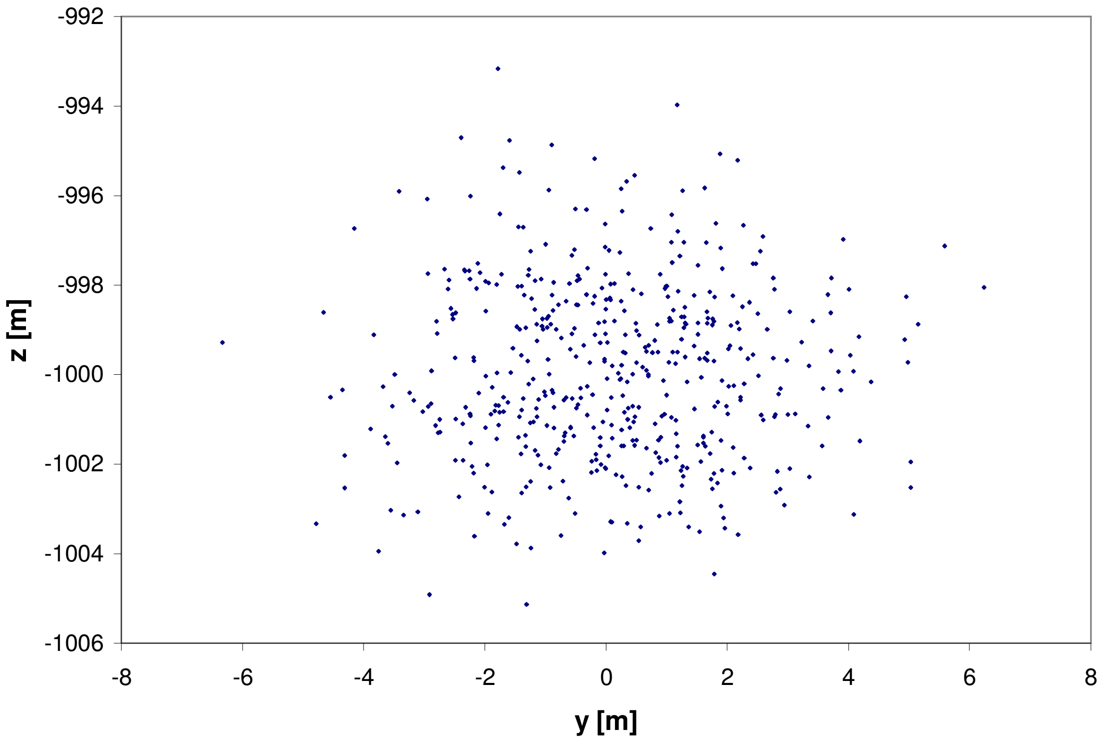


Figure 5.8 - Unguided Dispersion

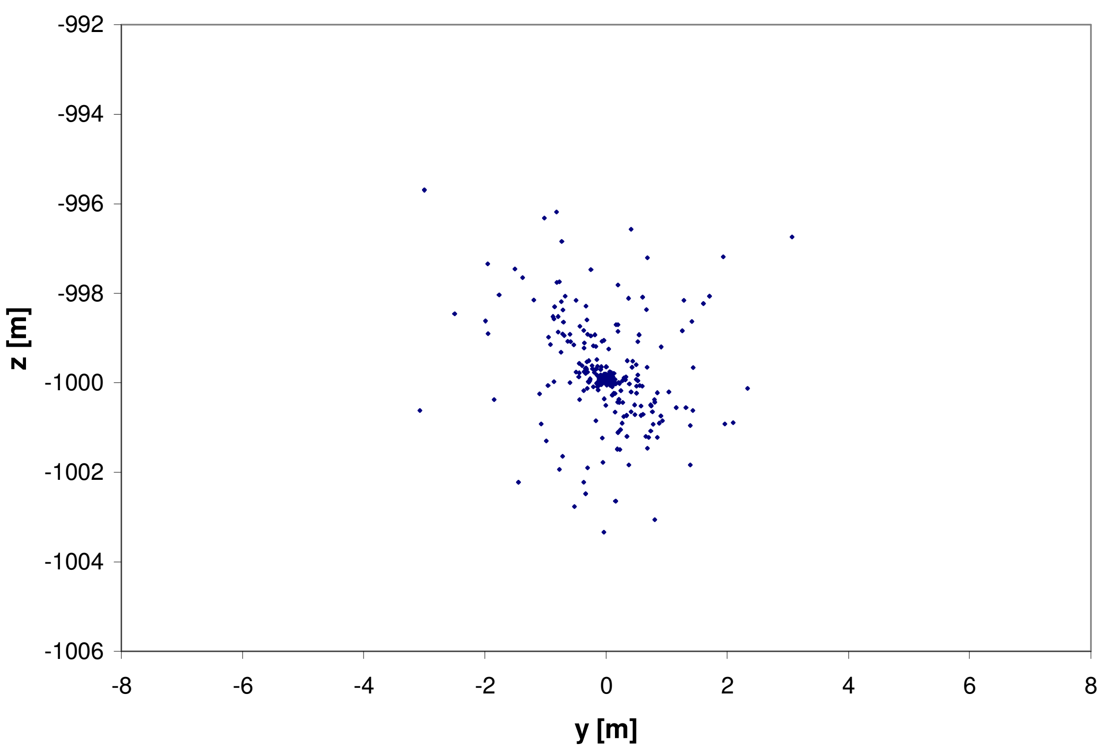


Figure 5.9 - Guided Dispersion

5.4 Test Case: Phase Bias Error with Normal Gun Pointing Error and Target Noise

With a high spin projectile it would be anticipated that regular errors would occur in the angular position of the projectile relative to the commanded position when firing. This would be due to both imperfect knowledge of the orientation of the projectile and delays in the firing system and squib ignition system. All of these errors can be simply represented by adding a bias to the phase.

In this set of runs, each line represents a constant gun pointing error and is constituted of 5 Monte Carlo run sets of 500 runs. Each Monte Carlo run set has a constant phase bias and normal distributed gun pointing error. Each run set sharing a similar gun pointing error used the same seed for gun pointing errors in order to obtain consistent results. The results are displayed in Figure 5.10. Additionally, the Circular Error Probable (CEP), that is, the circle that contains 50% of the hits, is displayed as a function of phase bias in Figure 5.11. While CEP is not a useful figure of merit for a hit-to-kill vehicle, it is useful if a weapon designer decides to employ an explosive warhead.

Figure 5.10 shows that the projectile under most operating conditions is relatively insensitive to phase errors up to 0.50 radians.

Figure 5.10 is very useful in the initial design iterations of a new concept. It is desired to reduce the shots per engagement; however, if the cost of increasing the fidelity of say an IMU to reduce bias errors is so great that the alternative of firing a few more shots with a lower resolution IMU is cheaper, perhaps the trade will swing

in the other direction. This is where a Pk chart is very useful. Now, Pk per engagement is defined as

$$P(k)_{engagement} = 1-(1-Pk)^{n} \qquad (5.1)$$

where n is the number of shots taken. Analysis of this kind is heavily dependant on available technology and beyond the scope of this document. However if the gun pointing errors are below 0.005, the projectile can have a firing error of ±0.5 radians (nearly 60 degrees sweep) and engage with a $P(k)_{engagement} > 0.90$ with less than 15 shots, which is half of the number of shots that were required in the IPT requirement documents. Table 5.2 enumerates a number of scenarios for a Pk(e) of 0.90.

Target error was introduced in three additional Monte Carlo run sets. The run sets were identical to the sets used to generate Figure 5.10, except for the addition of target error. These are shown in Figure 5.12 through Figure 5.14. There is a steady decrease in the Pk of the projectile as target error is introduced; however, there are no real surprises, the curves still extend out to about 0.5 radians before making a steep descent.

Figure 5.15 shows an inversion of Figure 5.10, plotting the gun pointing error on the ordinate and showing lines of constant Pk in the chart region. This chart is very useful in system design and evaluation: the curve will contain a solution space of pointing error and phase biases that are acceptable to operate within. If a designer picks a line on the graph, so long as they stay under and to the left of that line they will have a Pk greater than or equal to the line itself. Additionally the graph is

mirrored across the *x*-axis, meaning negative phase biases are bound by curves of roughly the same geometry.

Figure 5.16 shows the Pk chart for the Bofors L/70. The Bofors L/70 has a pointing error of 0.1 mrad. This means that with the guidance system, a Pk of 0.95 can be achieved with just 1-2 shots with even 0.8 mrad of phase bias. The dot represents the Pk of an unguided projectile shot from a Bofors L/70, and is precisely 0.1. This Pk translates into 28 shots for a Pk(e) of 0.95. A projectile with no divert requirements, therefore, could technically meet the Pk(e) requirements. However it is desired that the projectile be able to divert after leaving the barrel and it is possible that several smart bullets may be cheaper than thirty dumb bullets.

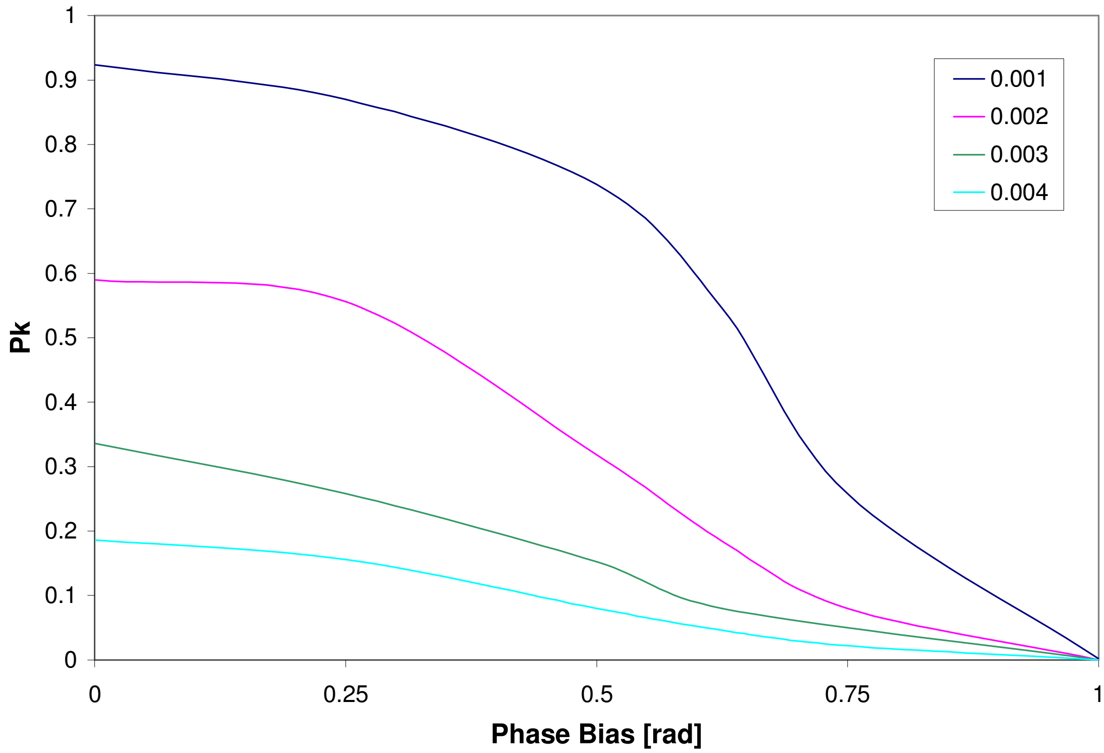


Figure 5.10 - Probability of Kill with a Phase Bias

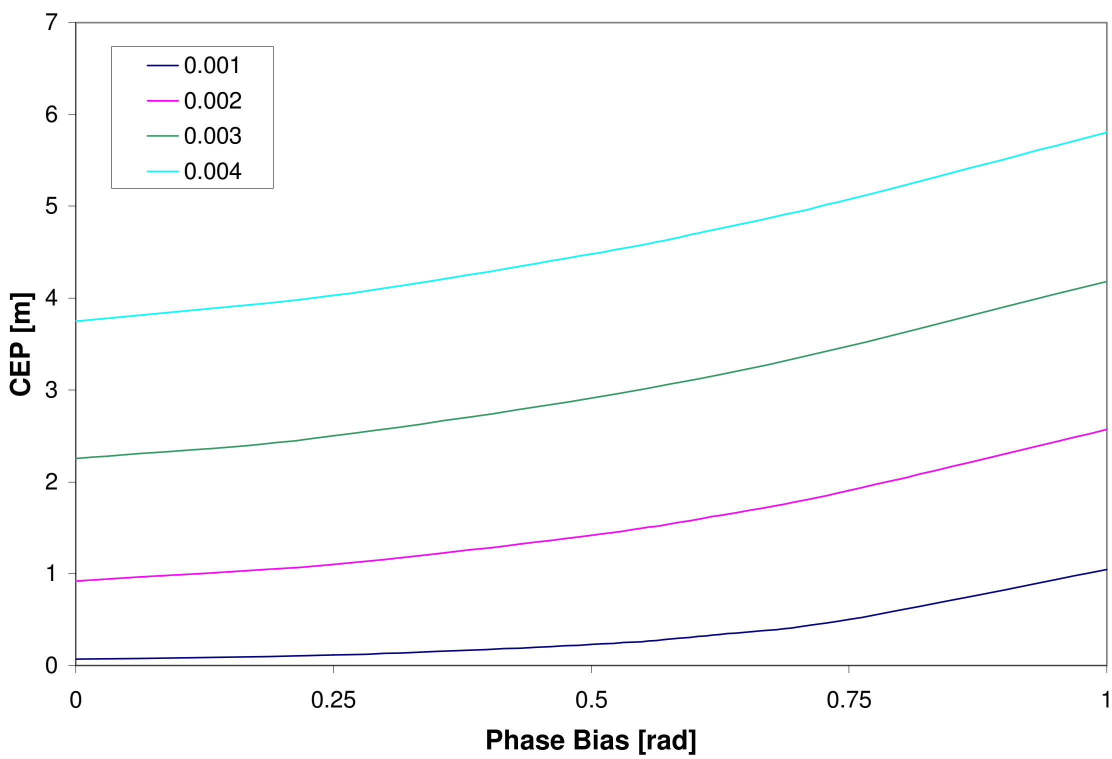


Figure 5.11 - Circular Error Probable with a Phase Bias

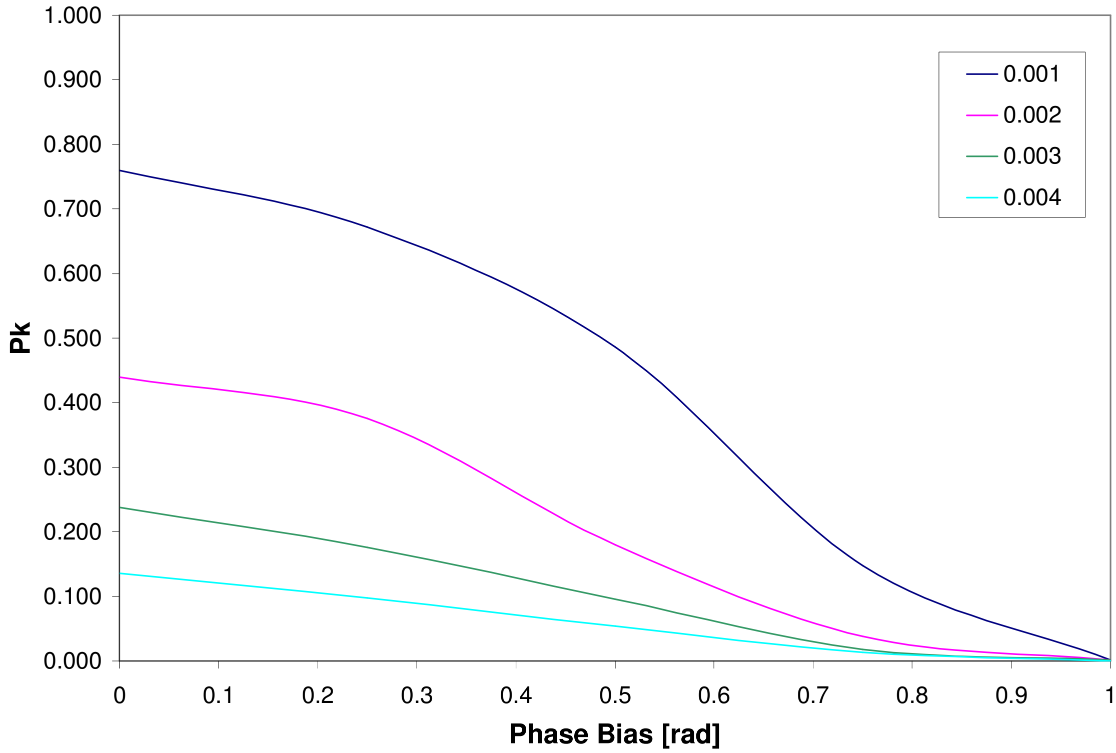


Figure 5.12 - Pk, Phase Bias with 0.05m Target Error

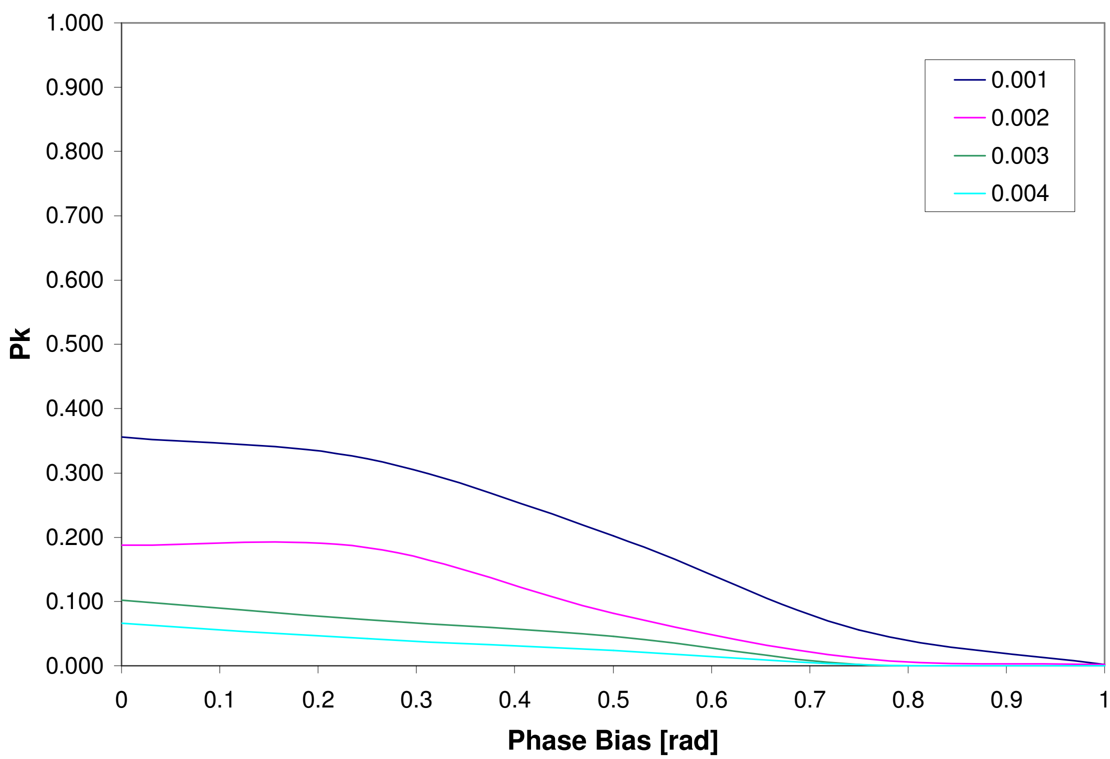


Figure 5.13 - Pk, Phase Bias with 0.10m Target Error

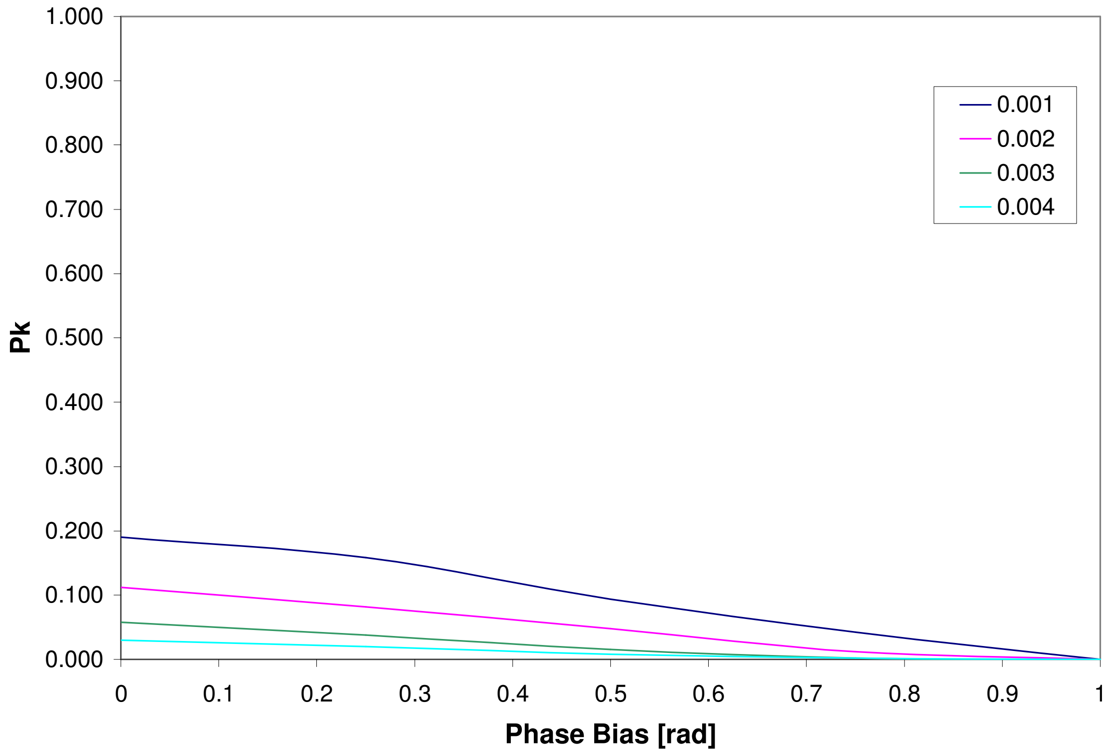


Figure 5.14 - Pk, Phase Bias with 0.15m Target Error

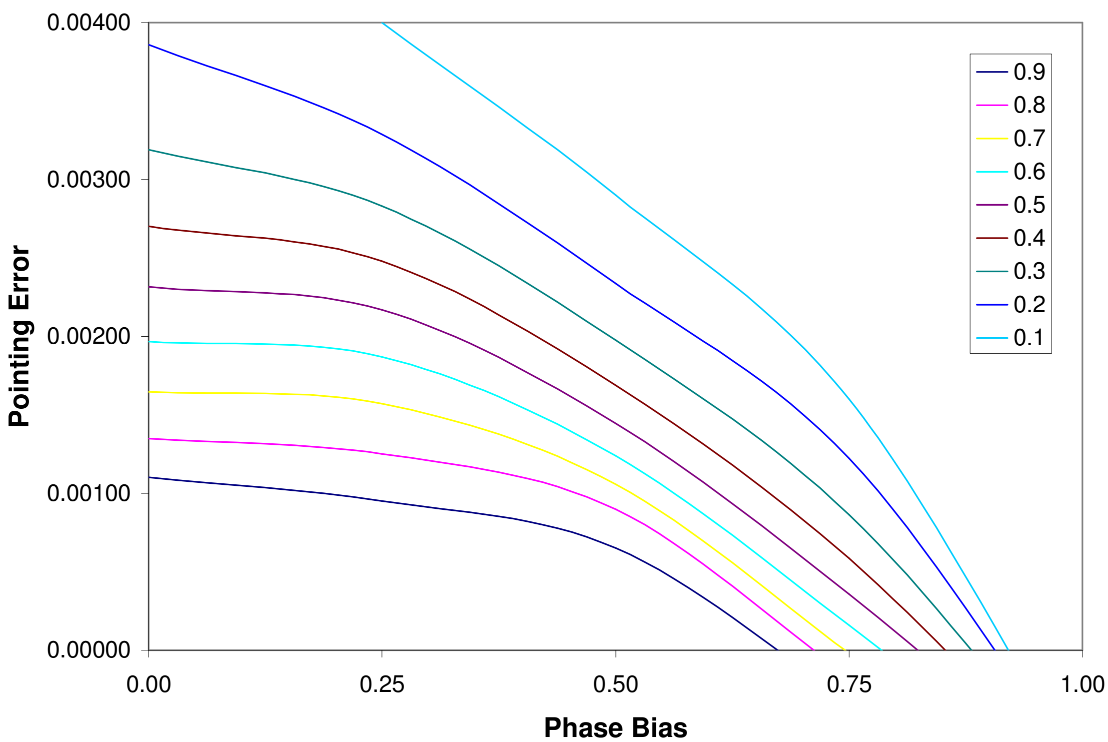


Figure 5.15 – Probability of Kill Map, No Target Error

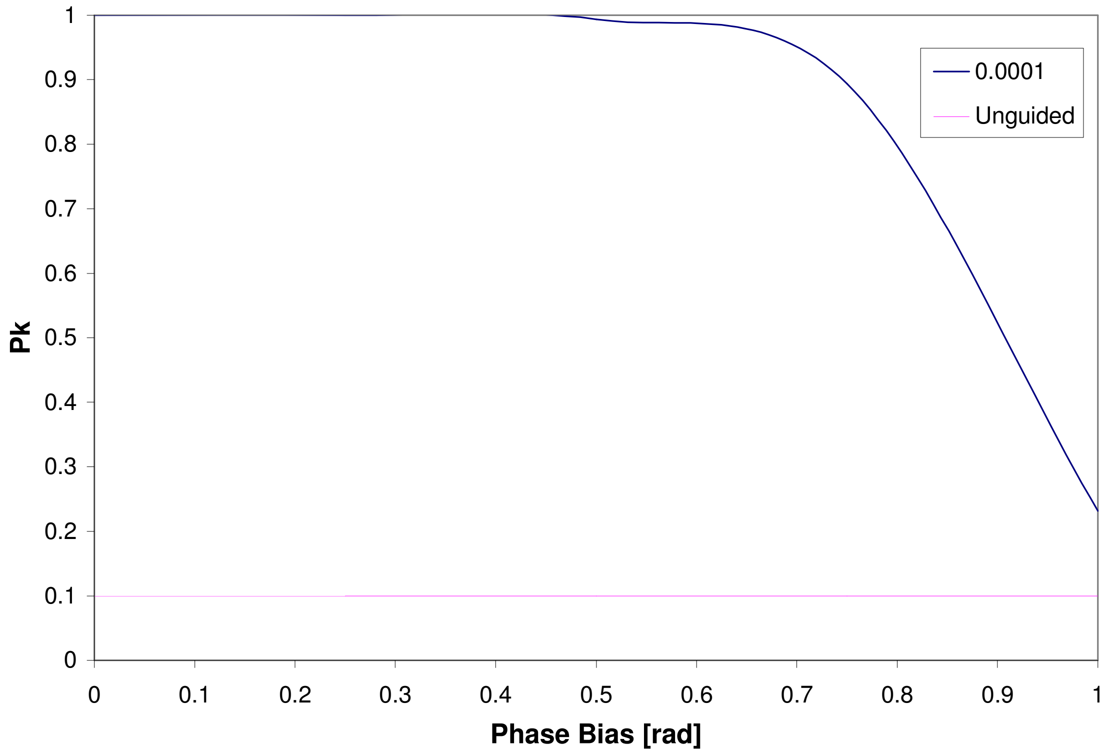


Figure 5.16 - Pk, 0.1 mrad Pointing Error

Table 5.2 – Pk and Number of Shots resulting in a Pk(e) of 0.90

Pk	Number of Shots
0.90	1
0.37	5
0.21	10
0.15	15
0.11	20
0.09	25
0.08	30

5.5 Test Case: Moving Squibs Along the Station Line

The location of the squibs on the body of the projectile greatly affects the influence a squib has on the projectile. Interactions with the gyroscopic forces, if the squibs are not located coplanar to the center of gravity, will introduce additional pitching and yawing forces and will also introduce angular error, which is further discussed in Appendix C. One of the benefits of moving the squibs up the station line is an increase in cross range, as shown in Figure 5.17. The squib was fired at t=1.00 seconds and the simulation was stopped at t=3.00 seconds. It can be seen that the divert distance does increase nonlinearly with stationline position.

Figure 5.18 displays the phase warp angle as a function of station line position. Although the phase warp angle is itself a function of angular position, this gives a sense of magnitude: as the squib is moved along the projectile, the magnitude of the corrective phase warp angle increases.

It was initially anticipated that placing the squibs forward the c.g. would produce extra forces without any penalties to the control system. It was realized by inspection that placing them a significantly large distance forward the c.g. resulted in excessive pitching and yawing moments which caused errors in the linearized model. Successive predictions in the linearized guidance model would vary by several meters even when a squib had not fired between predictions, due to the excessive pitching and yawing. Figure 5.19 shows the results of a series of Monte Carlo runs with a Gaussian pointing error of 0.002 radians and no other error sources. The squibs were slid along the station line and the effect on the probability of kill was noted. The optimal location of the squib pack for this projectile is at the center of gravity. This is

most likely due to errors entered into the linearized guidance system by excessive pitching and yawing which can be seen in Figure 5.20.

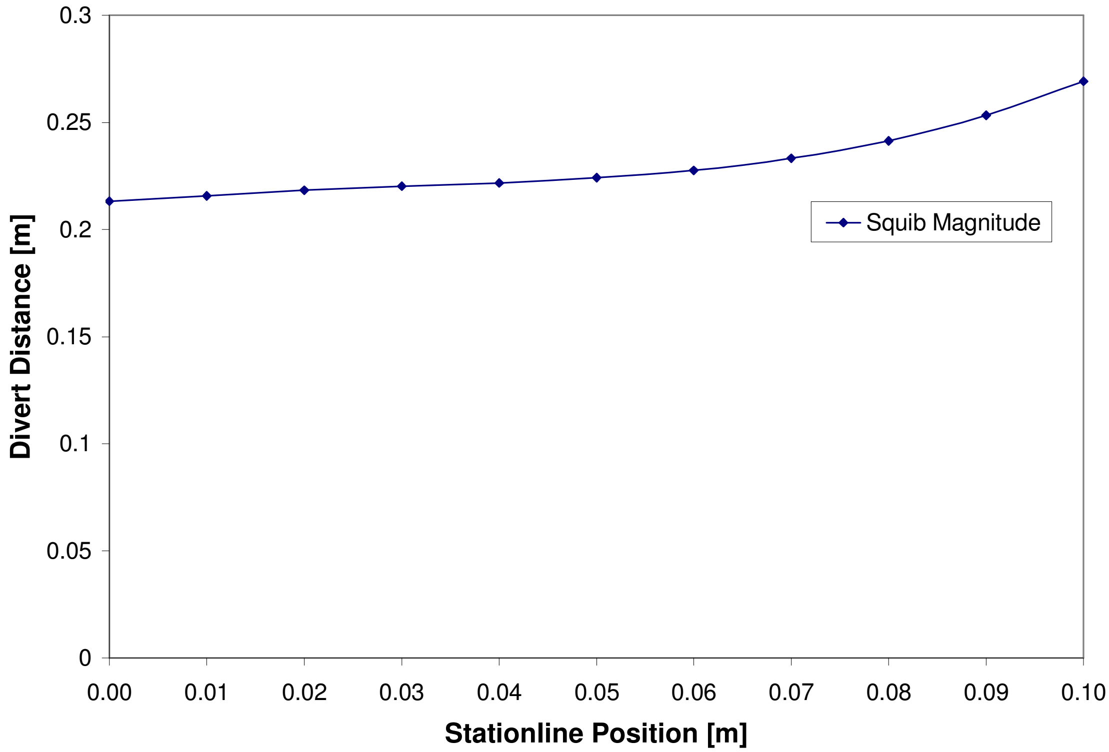


Figure 5.17 - Divert distance as a function of squib station line position

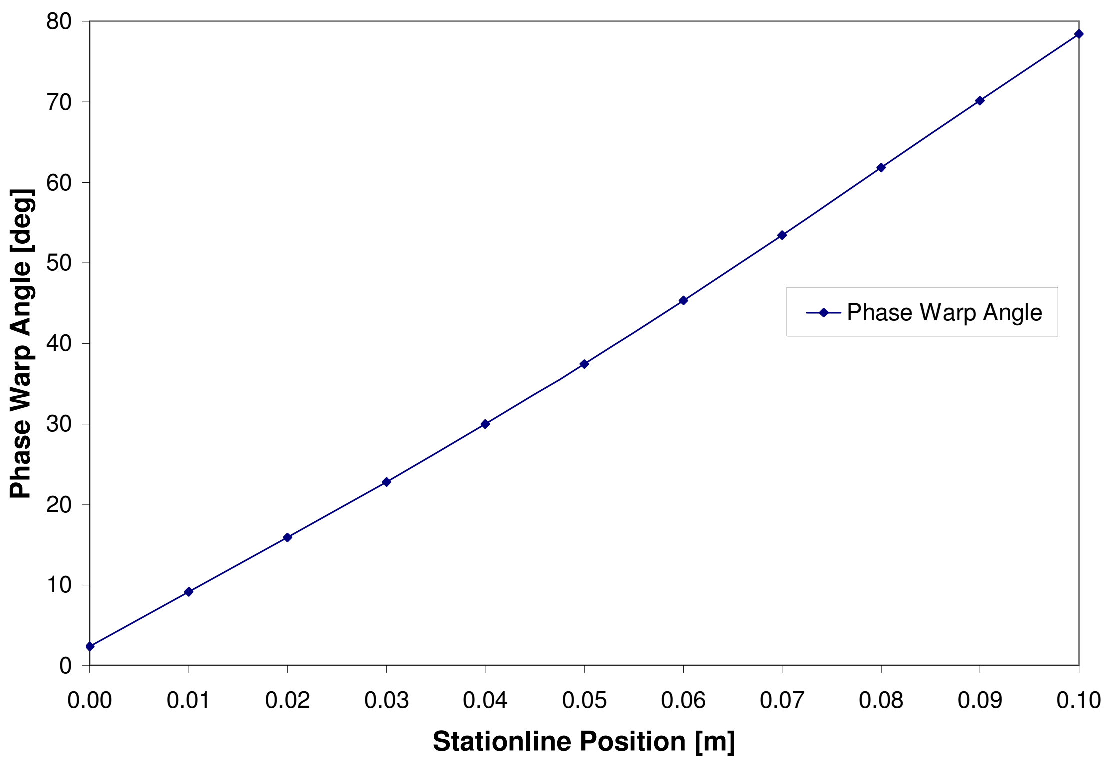


Figure 5.18 - Phase warp angle as a function of station line position

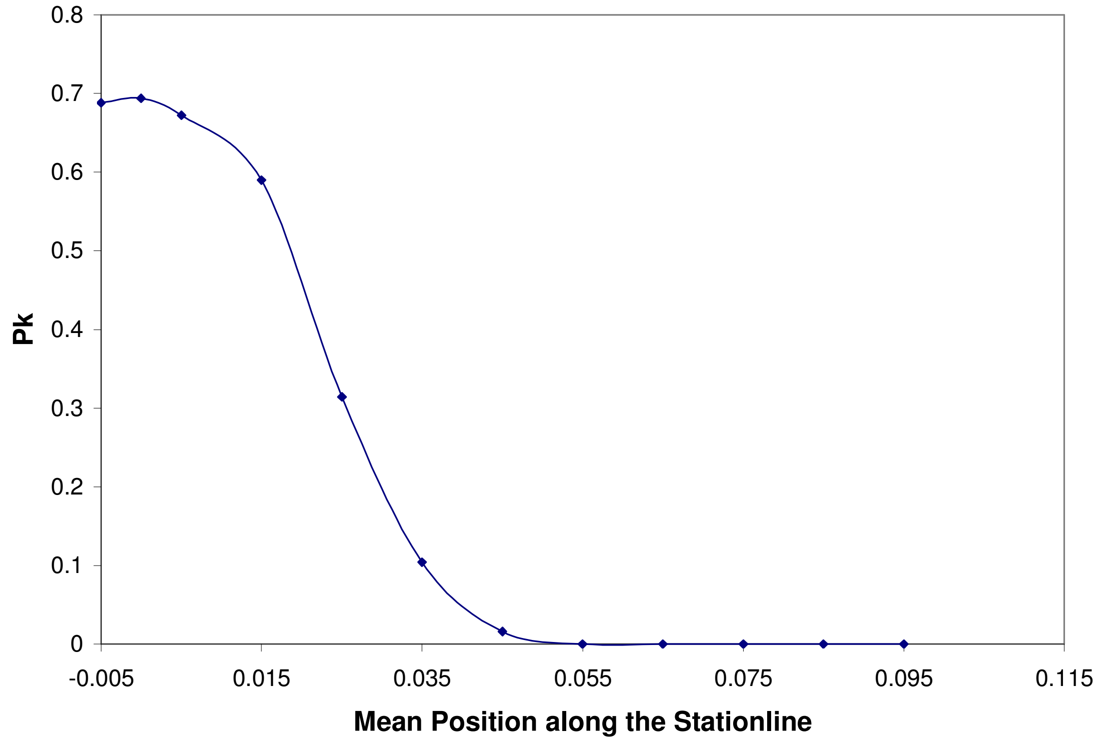


Figure 5.19 - Probability of Kill as a function of Mean Position along the Station line

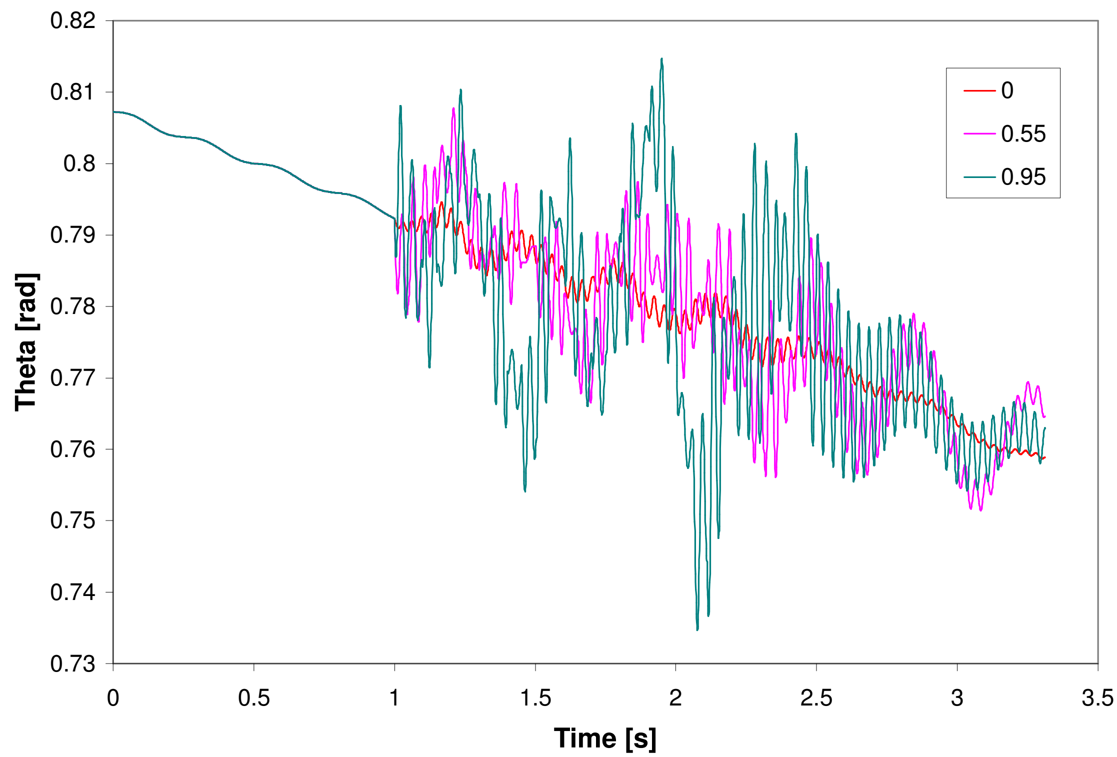


Figure 5.20 - Theta as a function of Time(▲) indicates a squib firing

CHAPTER 6

Discussion and Conclusions

The system presented is a linearized guidance system that runs quickly and accurately to predict the future position of the projectile. The linearized guidance system takes the 12 states from the 6DOF and projects forward into time a future trajectory that the guidance system can use to make decisions concerning the firing of squibs.

The linearized guidance model coupled with the controller greatly improves the projectile's performance. It does an excellent job of reducing error to an end goal of a hit-to-kill projectile, even when the scale is a 40mm projectile and a 100mm mortar. Even at a high rate of spin, with Magnus forces pulling the projectile away from its target, the control system overcomes in its envelope of control authority.

The results from Section 5.3 show quite clearly that a gun without a guidance system is useless against RAM threats. On the other hand, a Pk of 0.90 can be realized with 20-30 projectiles, even with a phase bias, pointing error, and target noise.

While trade studies were performed on variables that were felt to have the greatest impact on a successful flight, it is possible that there are other parameters that

are meaningful that must be considered in greater detail. For example, a variation of muzzle velocity between projectiles will have an effect of moving the target relative to the projectile. While the projectile can make lateral corrections to compensate, incoming RAM threats are approaching with velocities in the hundreds of meters per second, several meters per second of muzzle velocity can potentially cause an initial error in excess of the capabilities of the projectile.

Although the open-loop divert range of just a few meters leaves much to be desired, considering the gun pointing error on a Bofors 40 MK 3 is 0.9 mrad (Frederick, et al.) and the more recent Bofors L/70 is 0.1 mrad the projectile as it stands is more than capable to compensate for the gun pointing error and still have several meters of divert to spare.

There are many other issues that need to be addressed. Many assumptions were made to produce this thesis and slowly, one by one they must be stripped away if a counter-RAM system is to become a reality.

APPENDICES

Appendix A

CDD (REPRINT)

CONCEPT DESCRIPTION DOCUMENT (CDD)
Enhanced Counter Air Projectile (ECAP)

Prepared by
IPT Project Office
The University of Alabama in Huntsville
Huntsville, AL 35899
www.iptmadness.com

1. SCOPE

This specification establishes the performance, operational, environmental requirements, and study reporting requirements for the Enhanced Counter Air Projectile (ECAP) Weapons System. The mission of the ECAP Weapon System is to intercept rockets artillery and mortar (RAM) which are armed with mechanical fuses.

2. REQUIREMENTS

An ECAP Weapons System, shown in Figure 8, is defined as multiple ECAP Rounds, a Gun Platform, and an Acquisition Unit. An ECAP Round consists of an ECAP Projectile and an ECAP Shell. The Gun Platform consists of the Gun, Gun Pointing System, Ammunition Feed System, and any other components critical to the operation of the Gun Platform. The Acquisition Unit consists of the sensors that identify and track targets, Fire Control Systems, and any other component critical to tracking of targets and the Fire Control of the ECAP Weapons System. An mobility platform for use with Army ground troops is desired but not required. An existing Gun Platform is preferred but not required.

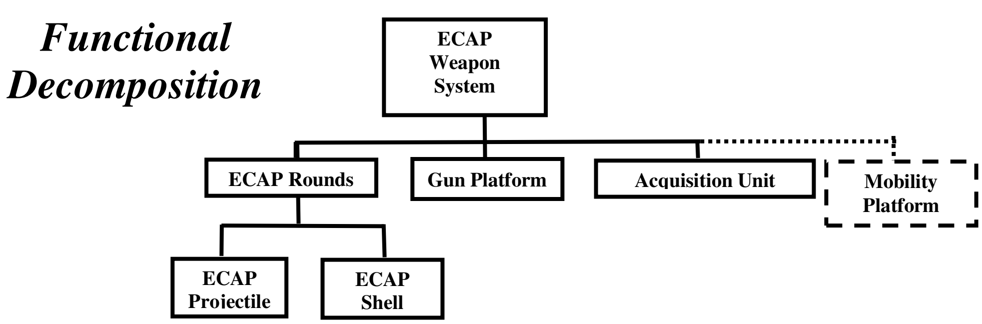


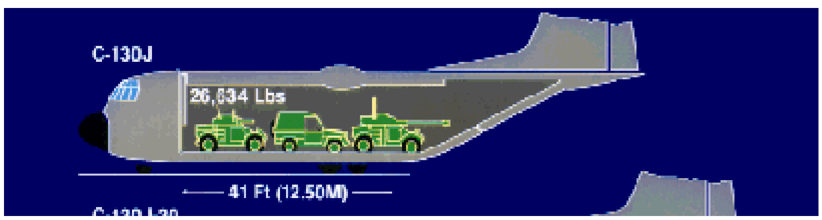
Figure 8 – ECAP Weapons System

The ECAP Weapons System shall have a Defense Volume defined as a hemisphere that has a radius of 0.5 km positioned above a Defensive Volume Origin (DVO). The

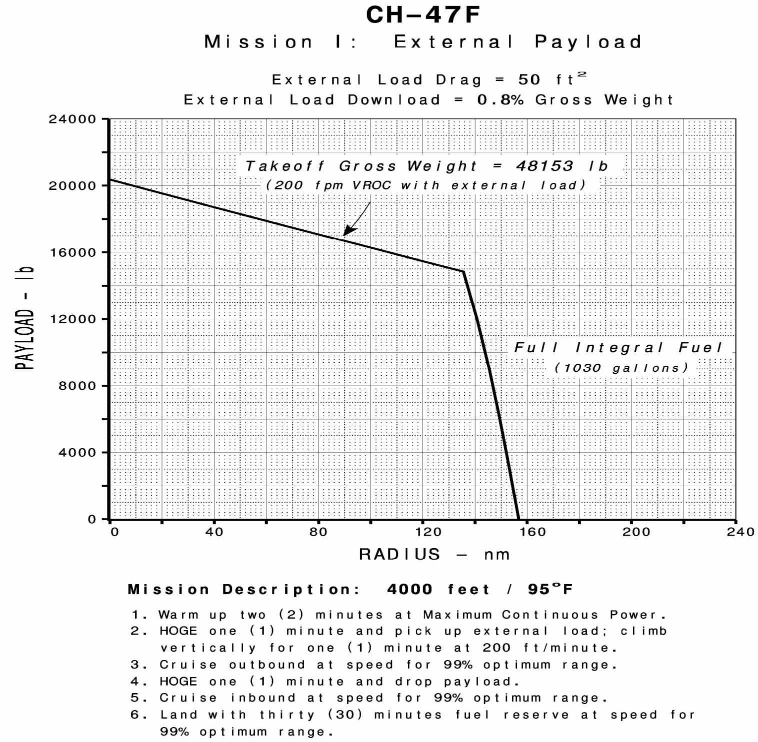
X-coordinate is the horizontal downrange from the DVO, the Y-Coordinate is cross range to the left of the Defensive Volume Origin, and the Z-Coordinate is the vertical altitude above the DVO and opposite the gravity vector.

A Successful Intercept is defined as fully intersecting the volume of a 100 mm diameter, spherical threat with an ECAP Round (or sub round) that contains a minimum of 32 kJ of kinetic energy (energy relative to the X-Y-Z coordinate system). An Engagement is defined as a the total series of shots from the ECAP system to attempt a Successful Intercept on one threat before that threat enters the Defense Volume. The ECAP shall have a 90% probability of a Successful Intercept per Engagement. Multiple shots and/or sub-projectiles are allowed.

An ECAP system with the capacity for 20 engagements will be transportable from Ft. Knox, Kentucky to Kabul, Afghanistan Airport in one C-130J aircraft; then transportable from Kabul 100 nm in any direction sling loaded (up to three trips) by a CH-47F Chinook Aircraft and be ready for Successful Intercepts in 72 hours from call out. A CH-47F External Payload versus range curve is shown in Figure 10. After deployment the ECAP WS is required to be movable and desired to be mobile. ECAP Rounds shall be storable for 15 years without maintenance.



Hercules Model	Cargo Length	463L Pallets	Medical Litters	CDS Bundles	Combat Troops	Para Troops
C-130J	41 ft	5	74	16	92	64



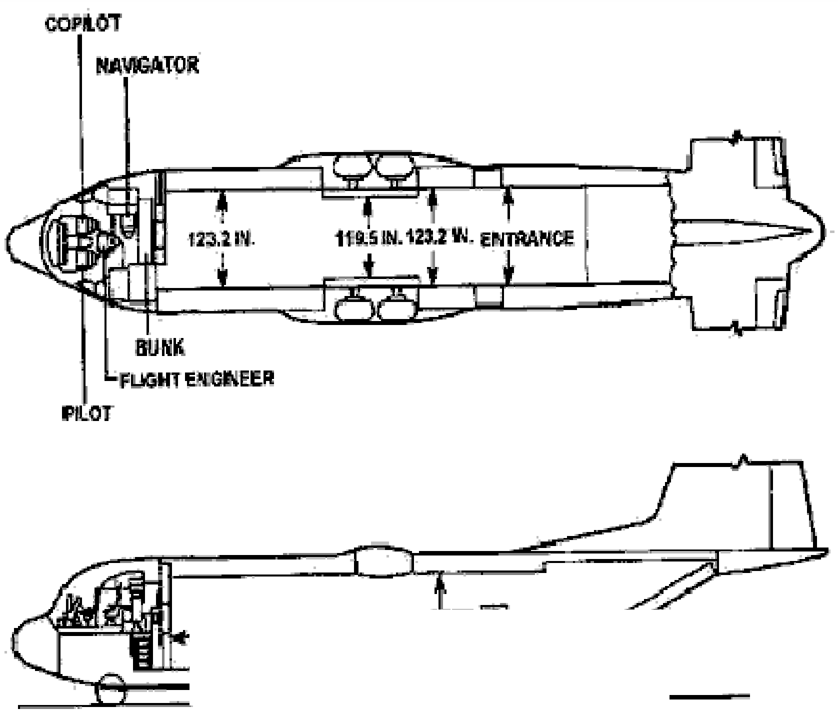


Figure 9 C-130J Specifications[5]

Figure 10 – CH-47F Mission Radius as a Function of Payload[6]

Static Performance

The Baseline Intersection Point is 1000*m* downrange and 1000*m* elevation. The ECAP (sub) projectile shall be able to maneuver *after launch* to intercept a target at

[5] http://www.fas.org/man/dod-101/sys/ac/c-130.htm
[6] Personal Communication, J. Winkeler, January 31, 2005.

any point inside of a 30 meter radius circle on the Baseline Intersection Plane. The Baseline Intersection Plane is perpendicular to a line connecting the launch point and the Baseline Intersection Point. The origin of the circle is the point where an unguided round would hit the Baseline Intersection Point. The ECAP shall meet static performance requirements in the following three ICAO Atmospheres.: 1) Cold, 2) Standard, and 3) Hot

Table 2 ICAO Standard Atmosphere Table[7]

Atmosphere	Source	Temperature	Pressure	Density
		Deg. C	mbars	kg/m^3
Cold	MIL-STD-210A	-51.3	1013.25	1.592
Standard	ICAO	15.0	1013.25	1.225
Hot	MIL-STD-210A	39.3	1013.25	1.129

The ECAP shall be able to demonstrate compliance to static performance on stationary targets at 0, 180, 270, and 90 degrees in a period of 10 seconds (in the order shown). That is to say the ECAP Weapons System will complete a Successful Intercept on static targets (100mm diameter) at (1000, 0, 1000), (-1000, 0, 1000), (0, 1000, 1000) and (0, -1000, 1000). (X,Y,Z).

[7] PRODAS V3 Help Files

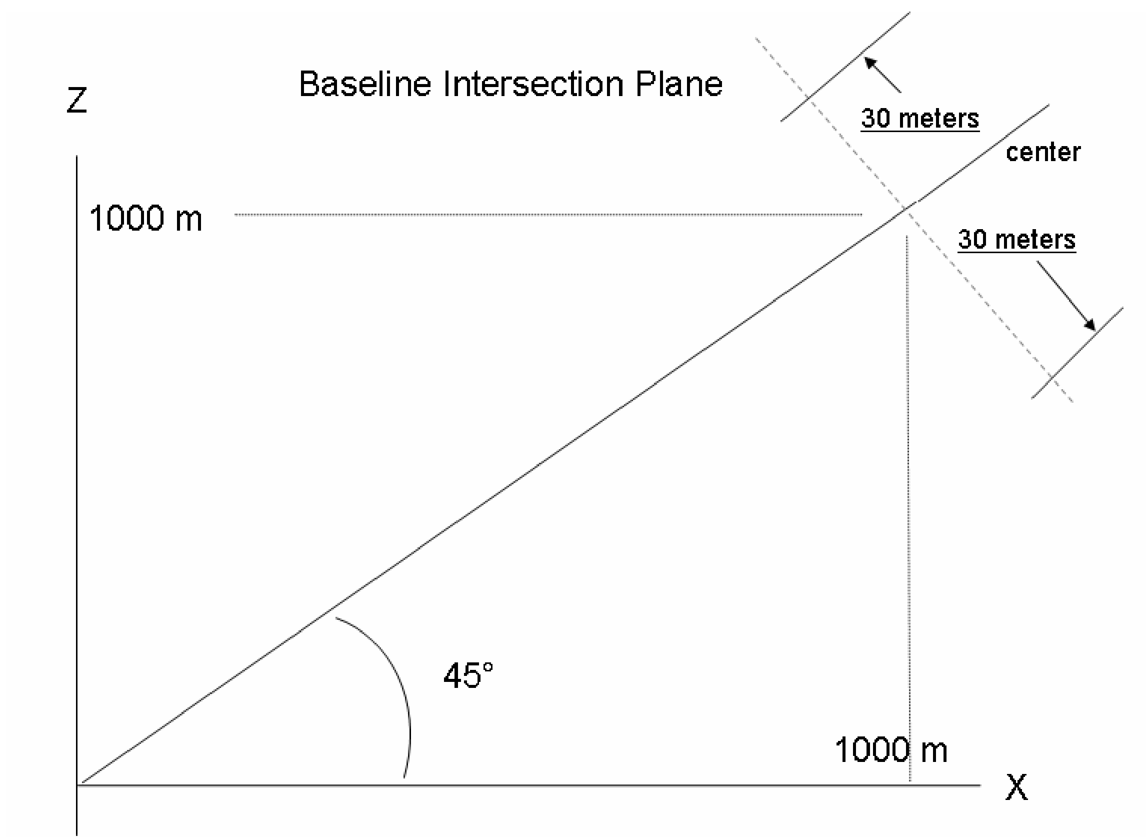


Figure 11 - Geometry for the Static Performance

Dynamic Performance:

Threat 1 - Mortar The Baseline Mortar dynamic threat is a mortar shown in Figure 1. The data points on Threat 1 are shown at 1 second intervals. The threat is launched from a coordinate (3000, -500, 0) with a ground intercept point of (0, -500, 0). The Customer will provide a digital description of the trajectory to the IPT Project Director. Minimum kill altitude for Threat 1 is 100 meters.

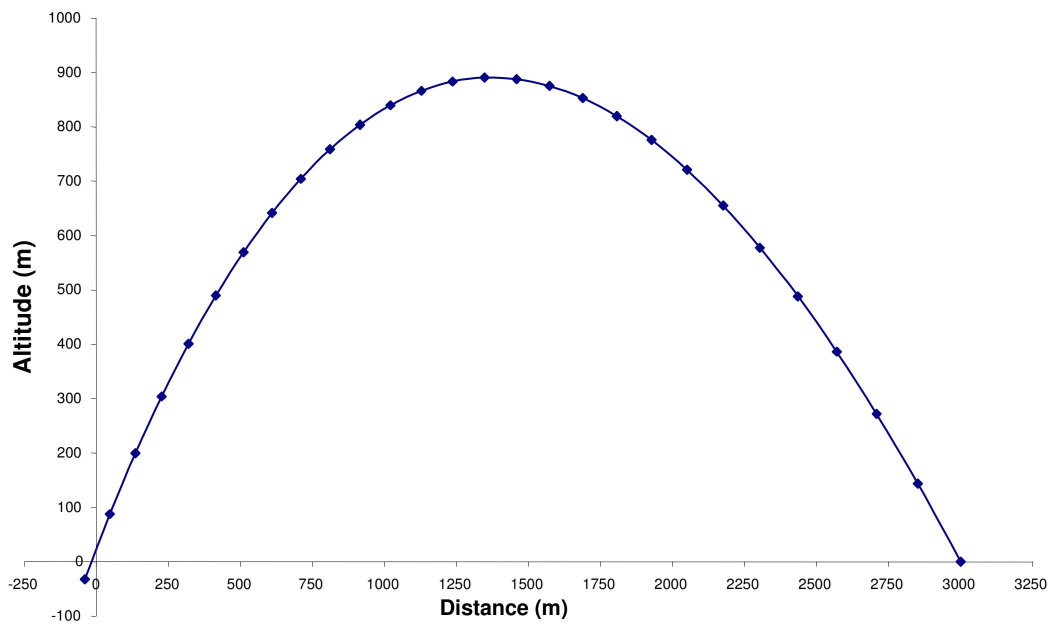


Figure 2 – Baseline Mortar Threat[8]

Threat 2 - Rocket The Threat 2 rocket threat is shown in Figure **3.** The data points on the threat are shown at 1 second intervals. The threat is launched from a coordinate (4000, -350, 0) with a ground intercept point of (0, -350, 0). Minimum kill altitude is 50 meters for Threat 2. The Customer will provide a digital description of the trajectory to the IPT Project Director.

[8] Holder, Jeff, AIAA Tactical Interceptor Technology Symposium, January 2005, Huntsville, AL.

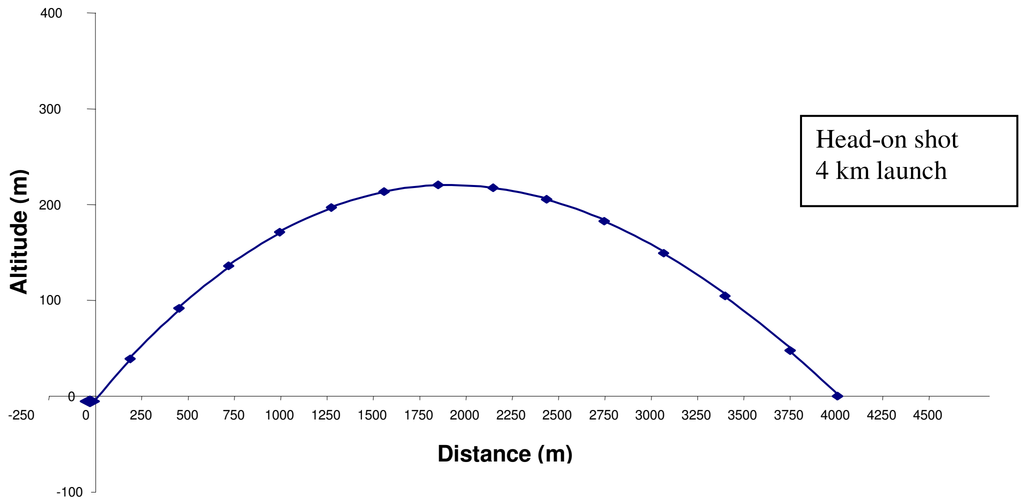


Figure 3 - Baseline Rocket Threat[8]

Reporting Requirements

A Final Data Package is required that allows reproduction of results by a third party. Specifically the reporting will include:

A. ENGINEERING ASSESSMSNT OF ECAP WS AGAINST CDD REQUIREMENTS

- Justification of All Assumptions
- Comparison of Predicted Performance against CDD Requirements
- Cross-Sectional Diagram and Three View Assembly Drawing of ECAP Projectile
- Shots per Engagement (Desired < 30)
- Cost Estimate per Engagement (Desired < $10,000) (Ammunition Cost)
- X-Y location of landing points of non-intercepting shots for Threat 1 and Threat 2
- Identification of Any Limiting Technologies
- Documentation of procurement information for any off-the-shelf components
- Paper report with Findings

B. ELECTRONIC DOCUMENTATION

- Electronic Copies of PRODAS Files
- Electronic Copies of IDEAS CAD Files
- Electronic Copies of CAM Files
- Electronic Copy of CFD Files
- Electronic Copy of Final Report
- Electronic Copy of Filmed Final Review
- CD ROM for Electronic Materials

C. HARDWARE

- Full-Scale Prototype wind tunnel Model
- Carrying Case for Model

All reporting requires metric SI units with English engineering system units reported parenthetically as a desired enhancement.

GLOSSARY

AL – Alabama
CDD – Concept Description Document
CFD – Computational Fluid Dynamics
DVO – Defense Volume Origin
ECAP – Enhanced Counter Air Projectile
ICAO – International Civil Aviation Organization
IDEAS –Produced by Structural Dynamics Research Corporation (SDRC), is CAE/CAD/CAM software used to develop digital master models.
IPT – Integrated Product Team
PRODAS - PROjectile Design & Analysis System
RAM – Rocket, Artillery, and Mortar
RW –Rotary Wing
UAH –The University of Alabama in Huntsville
UAV – Unmanned Aerial Vehicle
WS – Weapons System

Appendix B

Squib Force

Acceleration is the time derivative of velocity

$$a = \frac{dv}{dt_i} \tag{B.1}$$

$$\int a * dt_i = dv \tag{B.2}$$

$$a * t_i = v\,. \tag{B.3}$$

Velocity is the time derivative of position

$$v = \frac{dv}{dt_f} \tag{B.4}$$

$$\int v * dt_f = dx \tag{B.5}$$

$$v * t_f = x\,. \tag{B.6}$$

The subscript “i” denotes impulse (time while the squib is firing) whereas “f” denotes time of flight, that is, time between completion of the squib firing and impact with the target.

We can substitute Equation B.3 into Equation B.6

$$a * t_i * t_f = x \quad \text{(B.7)}$$

And substitute Newton’s second law

$$\frac{F}{m} * t_i * t_f = x, \quad \text{(B.8)}$$

where F is the force required of the squib and m is the mass of our projectile. We can further simplify this equation by remembering that an impulse is a force times a time, namely $F * t_i$. This is good for now because it gives us one less constraint to worry about.

$$\frac{I}{m} * t_f = x. \quad \text{(B.9)}$$

Appendix C

Phase Warp Table

When a squib is fired from a location not collocated with the center of gravity, it produces a moment about the center of gravity. When the projectile is also spinning at a high rate of speed, the coupling between the squib force and the gyroscopic forces produces an off-axis force that is not accounted for by the force and moment calculations of the squib model. The off-axis force can be pre-calculated and accounted for via a table lookup.

The simulation is run with a single squib at a single axial location. A fixed target is generated at a distance of 500 meters perpendicular to the x-axis of the projectile. Each update, the phase and magnitude are calculated and the same squib guidance loop is used as was presented in the section titled , the only exception being no phase warp table is applied as it has yet to be generated. The run is terminated at a set time.

After each run, the final intercept point is saved. The nominal intercept point, the point that would be reached if no squibs were fired, is already known. To calculate phase warp, the final intercept point is subtracted from the nominal intercept point. The resultant is the distance that the projectile moved due to squib forces. The direction the squib forces acted in is then calculated by taking the inverse tangent of y/z. Subtracting this angle from the intended firing direction, the phase warp angle is realized.

This process is repeated by moving the target by 5 degrees about the projectile and is repeated until a full set of phase warp data is generated.

This process must be repeated for every position along the station line where squibs will be located.

Appendix D

Input to the 6DOF

```
[Projectile]
phi   = 0.0           [deg]
theta = 46.38875      [deg]
psi   = 0.0375        [deg]

x = 0                 [m]
y = 0                 [m]
z = 0                 [m]

u = 500               [m/s]
v = 0                 [m/s]
w = 0                 [m/s]

p = 2573.15           [rad/s]
q = 0                 [rad/s]
r = 0                 [rad/s]

Ixx = 0.000311        [kg*m2]
Iyy = 0.00488         [kg*m2]
Izz = 0.00488         [kg*m2]
```

```
m = 1.495               [kg]

D = 0.039               [m]


[Guidance]

num_squibs  = 30        [number of squibs]

squib_delay = 1.0       [delay until control system starts]

squib_wait  = 0.05      [pause between squib firings]


[Target]

error   = 0             [bool, toggle error corruption]


[LinearizedGuidance]

DS      = 1

UPDATEINT = 500

SF      = 40000


[Output]

fname = data.dat  [output file name]

pt = 0.001                    [timestep to record to file]
```

Appendix E

Aerodynamic Tables

```
[Cx0]
21
Mach  CX0
0.01  0.149073
0.4   0.145378
0.6   0.14601
0.7   0.146968
0.75  0.149461
0.8   0.154876
0.85  0.158648
0.875 0.171478
0.9   0.190377
0.925 0.213274
0.95  0.251111
0.975 0.299294
1     0.351898
1.025 0.384818
1.05  0.405957
1.1   0.407287
1.2   0.401909
1.35  0.383834
1.5   0.366869
1.75  0.343209
2     0.324178
```

```
[Cx2]
21
Mach  CX2
0.01  2.5409
0.4   2.5409
0.6   2.55075
0.7   2.82403
0.75  2.94171
0.8   3.04537
0.85  3.24365
0.875 3.37223
0.9   3.49281
0.925 3.6438
0.95  3.8325
```

```
0.975 4.09109
1     4.37718
1.025 4.64823
1.05  4.88593
1.1   5.48135
1.2   6.12915
1.35  5.58289
1.5   5.0634
1.75  4.51718
2     3.96081




[Cy0]
2
Mach  Cy0
0.01  0
2     0


[Cz0]
2
Mach  Cz0
0.01  0
2     0


[CnA]
21
Mach  CNa
0.01  1.74199
0.4   1.7485
0.6   1.72962
0.7   1.71453
0.75  1.69941
0.8   1.72929
0.85  1.74625
0.875 1.78421
0.9   1.83954
0.925 1.91833
0.95  1.99715
0.975 2.05044
1     2.07639
1.025 2.12175
1.05  2.16544
1.1   2.23509
1.2   2.3127
1.35  2.41494
1.5   2.5253
1.75  2.66395
2     2.78072


[CypA]
21
```

```
Mach  CYpa
0.01  -0.879877
0.4   -0.879877
0.6   -0.879877
0.7   -0.889503
0.75  -0.897316
0.8   -0.912941
0.85  -0.941192
0.875 -0.960631
0.9   -0.982069
0.925 -1.10589
0.95  -1.25496
0.975 -1.1127
1     -1.03076
1.025 -0.987882
1.05  -0.948005
1.1   -0.88369
1.2   -0.801936
1.35  -0.720995
1.5   -0.694744
1.75  -0.667492
2     -0.626616


[Clp]
21
Mach  Clp
0.01  -0.025646
0.4   -0.0259013
0.6   -0.0257406
0.7   -0.0255861
0.75  -0.0253765
0.8   -0.0252472
0.85  -0.0250853
0.875 -0.0250402
0.9   -0.0250491
0.925 -0.0250948
0.95  -0.0253803
0.975 -0.0259136
1     -0.0265252
1.025 -0.026323
1.05  -0.0262448
1.1   -0.0259207
1.2   -0.0254061
1.35  -0.024835
1.5   -0.0241576
1.75  -0.0229295
2     -0.021875


[Cmq]
21
Mach  Cmq
0.01  -16.2413
0.4   -16.1634
0.6   -15.8778
0.7   -15.8432
```

```
0.75  -15.9607
0.8   -16.4744
0.85  -16.75
0.875 -17.5664
0.9   -18.249
0.925 -21.3408
0.95  -20.2793
0.975 -22.562
1     -24.8877
1.025 -28.9081
1.05  -32.6165
1.1   -39.1649
1.2   -46.0873
1.35  -46.2112
1.5   -46.151
1.75  -42.5993
2     -39.9336

[CnpA]
21
Mach  Cnpa
0.01  -1.96316
0.4   -1.96316
0.6   -1.97859
0.7   -1.99279
0.75  -1.92762
0.8   -1.77275
0.85  -1.48914
0.875 -1.13842
0.9   -1.04774
0.925 -0.778995
0.95  -0.359536
0.975 0.0575965
1     0.355969
1.025 0.720668
1.05  0.847739
1.1   0.986578
1.2   1.03735
1.35  1.11103
1.5   1.16385
1.75  1.18756
2     1.17723


[Cma]
21
Mach  Cma
0.01  3.45928
0.4   3.47005
0.6   3.4632
0.7   3.45662
0.75  3.45107
0.8   3.51163
0.85  3.69305
0.875 3.7912
0.9   3.87362
0.925 3.97064
```

0.95	3.95083
0.975	3.88218
1	3.74818
1.025	3.55747
1.05	3.47707
1.1	3.49014
1.2	3.45061
1.35	3.41331
1.5	3.23563
1.75	3.018
2	2.93654

Appendix F

Parameterization of Inputs to the Linearized Guidance Model

One concern is that it might not be possible to feed 12 states into a linearized guidance model in real time. The possibility of parameterization of input parameters was briefly investigated. First and foremost, the position states (x, y, z) cannot be parameterized, for in doing so positional authority of the projectile is lost. An attempt was made to parameterize the orientation of the projectile. This did not work, and should make sense, for as soon as the model predictive controller starts firing squibs the orientation of the projectile, depending on the state of the projectile, will assume a unique orientation which will influence further flight.

It may be recalled from Section 0 that the total velocity and roll rate were both invariant to a large degree regardless of whether squibs were fired or the projectile was allowed a free flight. The next set tried was the velocity (u, v, w). This did not work well, resulting in errors on the order of a few meters. However it was an improvement over the orientation parameterization. An attempt to parameterize the body rates (p, q, r) did work well. The rates q, r were set to zero. The rate p was set to

$$(time / 3.312) * (2573.16 – 2421.68) + 2421.68,$$

where time is the current simulation time. This worked acceptably for gun pointing errors up to 2 mrads, at which point in time a substantial loss of Pk was noted.

Additionally, the linearized guidance model was investigated to see if any simplifications could be made based on this curve fit. Unfortunately in the epicyclic equations there is a tight coupling between the body rates and the body orientation. There is no real way to take advantage of a curve fit to further simplify the solution unless the orientation can be parameterized.

The traditional IMU consists of a gyroscope and an accelerometer, which detects body rates and accelerations, respectively, and integrates down to position and velocity. It is highly unlikely that the designer would be in a position where some of these states would be unknowns.

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