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Swarnalatha Kathalagiri Vasantha kumar

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EXPERIMENTAL INVESTIGATION OF SPRAY CHARACTERISTICS FOR DIFFERENT GEOMETRICAL MISALIGNMENT CASES OF LIKE DOUBLET IMPINGING INJECTORS

by

SWARNALATHA KATHALAGIRI VASANTHA KUMAR

A THESIS

Submitted in partial fulfillment of the requirements for the degree of Master of Science Aerospace Systems Engineering in The Department of Mechanical and Aerospace Engineering to The School of Graduate Studies of The University of Alabama in Huntsville

HUNTSVILLE, ALABAMA

2019
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THESIS APPROVAL FORM

Submitted by Swarnalatha Kathalagiri Vasantha kumar in partial fulfillment of the requirements for the degree of Master of Science in Aerospace Systems Engineering and accepted on behalf of the Faculty of the School of Graduate Studies by the thesis committee.

We, the undersigned members of the Graduate Faculty of the University of Alabama in Huntsville, certify that we have advised and/or supervised the candidate on the work described in this thesis. We further certify that we have reviewed the thesis manuscript and approve it in partial fulfillment of the requirements of the degree of Master of Science in Aerospace Systems Engineering.

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ABSTRACT

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Degree Master of Science College/Dept. Engineering/Mechanical and Aerospace Engineering

Name of Candidate Swarnalatha Kathagalir Vasantha kumar

Title Experimental Investigation of Spray Characteristics for Different Geometrical Misalignment Cases of Like Doublet Impinging Injectors

Liquid rocket engines sometimes use jet impinging injection streams to atomize liquid fuels and oxidizers in the combustion chamber. The work examines the effect of injector steam velocity, injection angles, and injection stream misalignment on the resulting sheet angles and sheet breakup lengths for like-double injectors. A set of cold flow, water simulant experiments at atmospheric pressure were conducted at three different geometric conditions. Angularly skewed impingement conditions investigated nine impingement angles from 30° to 90° with each injector having identical or different impingement lengths. Linearly-skewed impingement conditions included a set of five offsets where the impingement length of the first injector was different from the second by 0 in, 0.5 in, 1 in, 1.5 in, and 2 in. Partial impingement conditions included alignment offsets in the injection plane of +0.0138 in, 0 in, and -0.0138 in. All these conditions were tested for two jet velocity conditions (49.21 ft/s and 82.02 ft/s) for the 0.04 in diameter jets. A Phantom V711 high-speed camera captured spray images at 10,000 frames per second. Phantom Cine Viewer software algorithms were tailored to estimate the spray sheet angles. MATLAB image processing tools were tailored to determine the sheet breakup lengths. Increased impingement angles resulted in increased spray sheet angles from 40° to 110° and decreased spray sheet breakup lengths from 2.16 in to 1.15 in. Sheet angles are relatively more sensitive and sheet breakup lengths are
relatively less sensitive at angles of impingement between 55° to 65°. The jet velocities and y-offsets affect the visual sheet characteristics. Narrow sheets with diagonal ligaments are formed for the partial jet impingements and this results in the decreased sheet angles, and the increased sheet breakup lengths. The results of these experiments can be used as preliminary data for further hot firing tests, for comparison with numerical predictions, and the results show the possible effects of manufacturing inaccuracy on the spray patterns for the conditions investigated.
ACKNOWLEDGEMENTS

I would like to thank my committee for their time. I would like to thank my advisor Dr. Frederick for all the support and the time spent on reviewing my draft. I would like to thank Propulsion Research Center, Mr. Tony Hall, Evan Unruh, James Venters and Dr. Lineberry for all the help in conducting the experiments. Last but not the least, I would like to thank my parents for their constant moral support.
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</tr>
<tr>
<td>$d_0$</td>
<td>Injector Orifice Diameter</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Half Angle of Impingement</td>
</tr>
<tr>
<td>$l_b$</td>
<td>Jet Breakup Length</td>
</tr>
<tr>
<td>$l_i$</td>
<td>General Impingement Length</td>
</tr>
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<td>$l_s$</td>
<td>Sheet Breakup Length</td>
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<td>$\vartheta$</td>
<td>Half Cant Angle</td>
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<tr>
<td>$P_l$</td>
<td>Upstream Pressure</td>
</tr>
<tr>
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<td>Wave Number</td>
</tr>
<tr>
<td>$\Omega_n$</td>
<td>Frequency shift</td>
</tr>
<tr>
<td>$\alpha_n$</td>
<td>Growth Rate or Decay Rate Constant</td>
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<tr>
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<tr>
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</tr>
<tr>
<td>$c$</td>
<td>Speed of Sound</td>
</tr>
<tr>
<td>$T$</td>
<td>Time Period of Acoustic Oscillation</td>
</tr>
<tr>
<td>$\hat{\omega}_l$</td>
<td>Source Term of Fourier Expansion of $\alpha_n$ and $\Omega_n$</td>
</tr>
<tr>
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<td>Time lag</td>
</tr>
<tr>
<td>$n$</td>
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<tr>
<td>$U$</td>
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<tr>
<td>$\delta$</td>
<td>Reciprocal of Square Root of Acoustic Reynolds Number</td>
</tr>
<tr>
<td>$\delta_d$</td>
<td>Superimposed Wave of Compressibility</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Vorticity</td>
</tr>
<tr>
<td>$F$</td>
<td>Body Force Term of Energy Density Equation</td>
</tr>
<tr>
<td>Re</td>
<td>Reynolds Number</td>
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</table>
\( C_d \) Discharge Coefficient

\( L_L \) Impingement Length of Left Jet

\( L_R \) Impingement Length of Right Jet

\( L_L^1 \) Impingement Length of Left Jet for Angularly Skewed Impingements

\( L_R^1 \) Impingement Length of Right Jet for Angularly Skewed Impingements

\( L_L^2 \) Impingement Length of Left Jet for Linearly Skewed Impingements

\( L_R^2 \) Impingement Length of Right Jet for Linearly Skewed Impingements

\( L_L^{21} \) Impingement Length of Left Jet for Combination of Linearly and Angularly Skewed Impingements

\( L_R^{21} \) Impingement Length of Right Jet for Combination of Linearly and Angularly Skewed Impingements

\( \theta_L \) Impingement Angle of Left Jet with Vertical Axis

\( \theta_R \) Impingement Angle of Right Jet with Vertical Axis

\( y \) Distance Moved by Right Injector Along Its Axis (Linearly Skewed Impingements or Y-offsets)

\( z \) Distance Moved by Left Injector Along Z-axis (Partial Impingement Distance)

\( v_j \) Jet Velocity

\( \dot{m} \) Average Mass Flow Rate

\( K_{orifice} \) Orifice Cavitation Number

\( P_v \) Vapor Pressure of Simulant

\( P_2 \) Orifice Downstream Pressure

\( K_{injector} \) Cavitation Number of Injectors

\( P_{injector} \) Upstream Pressure of Injector Orifice
CHAPTER 1. INTRODUCTION

Liquid Rocket Engines (LREs) are used in low-thrust applications, spacecraft attitude control thrusters, and high-thrust booster engines. The advantages of LREs over the other propulsion systems are component redundancy, performance, ease of throttling and restarting and ease of control \[^{[1]}\]. If the propellants carried are in liquid phase, then the system is called as a liquid propellant rocket engine \[^{[2]}\] or Liquid Rocket Engine (LRE).

LREs consist of propellant tanks to store the fuel and oxidizer. Pressure fed (low thrust applications) or pump fed (high thrust applications) feed systems include all the control devices to initiate and regulate propellant flow from the tank to the combustion chamber. Injectors introduce the propellants into the combustion chamber or the thrust chamber. The thrust chamber is where chemical reactions of the propellants form the hot gases which is then expanded and accelerated by the supersonic nozzle and ejected at a high velocity \[^{[3]}\].

The combustion chamber is a critical subassembly, where the injected propellant components atomize, evaporate, and mix with each other and combust. The chemical energy of the reaction is converted into heat producing the high temperature and the high-pressure gas in the chamber. This combustion gas is then expanded and accelerated through the nozzle producing thrust. The combustion chamber subassembly includes, the injectors, the chamber, the nozzle, and the chamber cooling systems.

Figure 1.1 demonstrates the sub processes occurring in the two-phase flow in the combustion chamber. The combustion processed complex phenomenon, which contains different physical/chemical sub processes of different temporal and spatial scales and multiple characteristics, that occur simultaneously and strongly coupled. As it can be seen in the Figure 1.1, in a LRE, once the propellants are injected into the chamber, it undergoes atomization,
vaporization, mixing and finally, the combustion occurs. Propellant injection is the first important process in the combustion chamber of a LRE. It controls the propellant spray characteristics and thereby affects all the subsequent processes that occur in the combustion chamber. Therefore, propellant injection and types of injectors used are directly related to the combustion efficiency and potential combustion instabilities in a LRE.

In the combustion chamber, temperature gradients, pressure gradients, and propellant concentration gradients are significantly high in the vicinity of the injectors. The residence time of the propellant in the combustion chamber is low, which lowers the combustion efficiency. These factors make the combustion process difficult to characterize accurately \[4\].

Figure 1.1: Schematic of the combustion process in the liquid rocket engine \[4\]
Combustion instabilities are also prone to occur in the LREs. Combustion instabilities are the severe pressure oscillations due to the coupling of acoustic modes of the chamber with the combustion processes. The growth of these combustion instabilities can destroy the engines and damage test facilities.

Various passive combustion damping techniques involve symmetric and asymmetric fuel injector distributions and different types of injectors in the same rocket engine to damp the combustion instabilities. A detailed discussion on the combustion instabilities, the relevance and importance of the injection and the types of injectors, in the combustion process and in damping the combustion instabilities and some case studies are presented in Chapter 2.

The present work focuses on the liquid spray characteristics produced by the LRE injectors. In the next section, the different kinds of injectors available and the reason for selecting the like doublet impinging injector for the current research are described.

1.1. Types of Injectors

There are many different configurations of injector elements that are used in LREs. Selection of a particular type of injector element to a LRE is based on few main parameters. The parameters are, propellant type (storable, hypergolic or cryogenic), state of the propellants (liquid, gas or gel), chamber length, chamber wall conditions (ablative, uncooled or regenerative cooled), chamber pressure, mixture ratio, pressure drop requirements, throttling requirements and engine life.

Depending on the application, the injector elements are selected. No single element design can achieve all the parameters mentioned above. Table 1.1 shows the various injector element configurations \[^{[25]}\]. The unlike doublet configuration has two injector elements, one of which, injects the fuel and the other injects the oxidizer. The unlike triplet has three injector elements, of
which the center injector injects the fuel and the other two inject the oxidizer. The unlike quadlet has 4 injector elements, two of which inject fuel and remaining two inject oxidizer. Unlike pentad has 5 injector elements, of which, only the center one injects fuel and remaining inject the oxidizer. The concentric tube injectors consist the oxidizer tube inside the fuel injectors and concentric to it. The center swirler, if used in the concentric tube injectors, swirls the oxidizer and helps in better

Table 1.1: Typical Injector Element Configurations\textsuperscript{[25]}

<table>
<thead>
<tr>
<th>Element configuration</th>
<th>Element Designation</th>
<th>Element configuration</th>
<th>Element Designation</th>
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<tbody>
<tr>
<td>Unlike Doublet</td>
<td>(1 on 1)</td>
<td>Unlike Quadlet</td>
<td>(2 on 2)</td>
</tr>
<tr>
<td>Concenctric Tube</td>
<td>(with swirler)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Like-doublet</td>
<td>(1 on 1)</td>
<td>Variable Area</td>
<td>(Pintle)</td>
</tr>
<tr>
<td>Showerhead</td>
<td></td>
<td>Splash Plate</td>
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mixing and atomization. The like-doublet configuration has two injector elements, both inject either fuel or oxidizer. The showerhead configuration has parallel injector elements that inject fuel and oxidizer through the alternative injector elements. The showerhead configuration was used in Aerobee sustainer, X-15 and Pioneer engines. The variable area injector configuration has a movable Pintle that varies the oxidizer flow, by changing the area of the oxidizer annulus. The variable area injectors were used in LEM descent engine and Lance sustainer. In the splash Plate configuration, the fuel and oxidizer are injected at a point on the splash plate inside the combustion chamber. The splash plate configuration was used in Lance booster (early version), Saturn SIVB ullage control, Apollo CM RCS (SE-8) and Gemini SC maneuvering, attitude control, and reentry engines.

1.2. Like-Doublet Injector

Like-doublet injectors are selected for the current research. The like-doublet impinging injectors are designed to inject the two liquid jets of same propellant with similar geometric and dynamic conditions. Their major advantages are that they are easy to manifold, show good mixing, are very stable, allow deliberate control of spray for wall compatibility, are mechanically simple and dependable, have proven dependability, and are a well understood type of injectors. Greater mixing efficiencies and propellant atomization make the like-doublet injectors better than the co-axial type of injectors. Also, these injectors’ fabrication, maintenance and inspection costs are less compared to the other types of injector designs. The disadvantages of these injector elements are sensitivity to design tolerances and relatively longer axial distance requirements to mix the fuel and the oxidizer. The like-doublet impinging injectors were used in Gemini LV first
stage, and Titan I and II first stage, booster engines of Redstone, Jupiter, Thor and Atlas, F1 engine of Saturn V rocket, H1 engine of Saturn I and Saturn IB rockets, and upper stage of VEGA \[25\].

![Figure 1.2: Schematic of Single Like-doublet Injector Element][27]

Figure 1.2 shows a schematic of a single like-doublet element. In the Figure 1.2, \( l_0 \) is the orifice length and \( d_0 \) is the orifice diameter. The angle, \( 2\theta \) is the impingement angle and it is defined as the angle between the axes of the two impinging jets. \( l_i \) is the impingement length. The impingement length is defined as the length of the freestream jet from the exit point of the orifice to the impingement point. The impingement point is also called as the stagnation point.

![Figure 1.3: Streamlines in the Jets and the Sheet Formed from the Front View][28]

Figure 1.3 shows the streamlines in the jets and the sheet formed in the region of impingement; the impact pressure deflects all the streamlines of the jets in the plane perpendicular...
to the plane of impingement. Only the axis of the jets, which is also called as ‘the separation streamline’ impinge at a point and the point is called as the impingement point or the stagnation point. A flat sheet (spray fan) is formed when the impact pressure deflects the liquid stream laterally. The liquid velocity of the fan is assumed to accelerate back to the jet velocity, after the impingement point. Because of viscous, turbulence, tensile, aerodynamic and inertial forces, the liquid sheet is fragmented into ligaments and large droplets. This is called as the primary atomization.

![Diagram of spray fan impingement](image)

**Figure 1.4: Spray Fan Impingement Recommended for Two Like-Doublets** [25]

Further fragmentation of these ligaments and the large droplets into finer droplets are called as the secondary atomization. \( l_s \) is the breakup length of the sheet. The breakup length of the sheet is defined as the distance from the impingement point to the point where sheet disintegrates into ligaments and large droplets (primary atomization point) [26]. In a like-doublet impinging injector engine, effective mixing depends on the geometric orientation of fuel doublets and oxidizer doublets. It is observed that the highest performance and excellent mixing uniformity is achieved when the fuel sheet and the oxidizer sheet impinge edge to edge, as shown in the Figure 1.4. The cant angle \( \theta \) has a significant effect on mixing uniformity. The literature says that the mixing uniformity can be increased by increasing the cant angle from \( 0^\circ \) to \( 41^\circ \) [25]. In a typical rocket
engine cant angle is generally between 20° to 41° [26]. Figure 1.4 shows the recommended fuel and oxidizer spray fans alignment for the like-doublets.

Figure 1.5 shows an example of spray sheet angle. The present research also deals with one more term called sheet angle. The sheet angle is nothing but the angle between two tangents passing through outer boundary of the spray ligaments formed before the primary atomization point.

![Sheet angle](image)

**Figure 1.5: Sample Spray Sheet Angle.**

A typical rocket engine that uses the like-doublet injector design, contains a faceplate at the chamber headend, drilled with alternating fuel and oxidizer rings. The injectors near the chamber wall consists of fuel elements for wall cooling and compatibility. For typical operating conditions, the Weber number is generally around $10^5$, Reynolds number is between $10^5$ and $10^8$, the range of pressure drop through the injectors is between 5 to 30 atm, the ratio of orifice length to the orifice diameter are between 2 and 5, and discharge coefficient is generally between 0.61 and 0.9 [26]. The droplet sizes are approximately proportional to the square root of the orifice diameter and smaller orifices are observed to result in better vaporization, mixing and higher performance. Round-edged or contoured orifice inlets are observed to have higher discharge coefficients and lower cavitation issues compared to the sharp edge orifice. However, because of fabrication difficulties and high cost, sharp-edged doublet orifice inlets have been used in most of
the rocket engines \cite{25}. Chapter 2 of this thesis will summarize many experimental results about the injector elements.

1.3. **Objective and Approach**

The objective of the present study is to measure spray characteristics, mainly the sheet breakup lengths and the sheet angles, for asymmetrical orientations and misalignments of the like-doublets injectors as a function of the geometric alignments and misalignments of the incoming spray streams. The work is carried out at atmospheric pressures using water as a liquid simulant.

Three types of geometrical orientations of the injectors and their combinations in the like-doublet impinging injector setup were chosen and the spray sheets were systematically studied. The three types of geometrical orientations chosen to study are called as Angularly Skewed Impingement, Linearly Skewed Impingements and Laterally Skewed Impingements or Partial Impingement of jets.

1. **Angularly Skewed Impingement**, is a type of impingement, where one injector is rotated, and the other injector is kept constant. This changes the impingement length of both the right and left injectors.

2. **Linearly Skewed impingement** is obtained by moving one injector along its flow axis while the other injector is kept fixed. In this method, only the impingement length of moving injector changes and the impingement length of stagnant injector remains fixed.

3. **Laterally Skewed Impingement or Partial impingement of jets** is a method, where one injector is moved back and forth along the plane of the sheet formed, to obtain partial impingements.
The present research is carried out in three steps and they are, 1) mass flow rate calibration experiments, 2) single jet experiments, and 3) impinging jet experiments. For the mass flow rate calibration experiments, water is throttled using an orifice of diameter 0.038in. The pressure upstream the orifice is referred as upstream pressure ($P_1$) in this research. The mass flow rates of the orifice, right and the left injectors are calibrated separately for 7 upstream pressures, and they are 105, 202, 303, 400, 505, 606, and 700 psig. The corresponding jet velocity of both right and left injectors obtained were 34.45, 49.21, 60.70, 70.54, 78.74, 86.94, and 93.50 ft/s. For the same jet velocities single jet experiments were conducted and the high-speed videos were obtained. For the impinging jet experiments, two jet velocities, nine angular skewed impingement cases, five linearly skewed impingement cases and three partial impingement cases and their combinations were chosen to study. Detailed test matrix is presented in Chapter 3. At each setpoint, the spray images are captured using Phantom v711 high-speed camera. The Data Acquisition system (DAQ) and LabVIEW software were used to obtain the pressure and temperature data of the system. MATLAB image processing techniques and Phantom PCC software are used for the analysis of the spray images. Based on the analysis, the empirical sheet breakup length and sheet angles were determined. A systematic study of combination of these geometrical alignments of injectors are not to be seen in the previous literatures.

Different experimental efforts using the impinging injectors, are also reviewed in the Chapter 2. Chapter 3 consists of the detailed experimental setup, instrumentations, and analysis techniques used for the current research. Chapter 4 includes the discussion of results and uncertainty calculations. Chapter 5 contains the final remarks and the recommendations for the future work. Appendix A provides the details of MATLAB image processing tools used for the analysis, Appendix B provides all the results and plots and Appendix C gives the details of
uncertainty analysis. The combustion instability, important role of the injectors in combustion instability and few case studies that show the importance of injector designs in damping the instabilities are reviewed in the Appendix D.
CHAPTER 2. LITERATURE REVIEW

2.1. Experimental Studies on Injection Parameters

Like doublet impinging injectors are selected for the present research. It is important to review the relevant works. Many experimental studies on the spray characteristics of impinging jets, have been carried out in many parts of the world. The next sections review past experimental studies about the spray characteristics of impinging jets and related works.

2.1.1. Sprays Formed by Laminar and Turbulent Impinging Jets

Dombrowski and Hooper conducted some experiments in 1963 that hypothesized that sheet breakup caused by the presence impact waves as a function of Weber number and independent of Reynolds number. Two precision bore glass tubes of 20 cm long and 0.05 cm inner diameter were used as injectors to obtain the fully developed flows. Surge chambers were used to obtain the laminar jets up to Re = 12,000. Wires were inserted at the entry of each tube to obtain turbulent jets. The equipment allowed for the rotation of injectors to get the required impingement angles. Water mixed with 0.5% Nigrosine dye was used for the experiments. Impingement angles of 50°, 80°, 110°, and 140° and 730, 1160, 1600 and 1950 cm/s velocities were tested. Microsecond flash photos were used for droplet size measurements and qualitative analysis of the sheet and 8000 frames per second high-speed cine film was used to measure the sheet velocities [35].

For turbulent jets impact waves were produced under all operating conditions tested except at the lowest impingement angle and jet velocity. At lower impingement angle, the volume of liquid was concentrated at the sheet axis and at higher impingement angle, liquid was spread uniformly about the sheet axis, but the disintegration was observed due to the impact waves. For laminar jets, the images illustrated that, the sheet was more unstable at the axis of the sheet. At lower velocity and higher impingement angles, stable and unruffled sheets were observed. The
sheet breakup was affected by impingement angle and the jet velocities in the laminar jet cases. This proved that the sheet disintegration was independent of Reynolds number. The sheet velocities were found using the cine films, by measuring the movement of irregularities on the sheet. It was observed that, central sheet formed from the laminar jets moved faster than that of the sheet formed from the turbulent jets and at lower impingement angle, where the impact loses are minimum, the sheet velocities are higher than the mean jet velocities. It was found that the impact waves are produced in case of turbulent jet impingements for a critical Weber number between 65 and 165.

Droplet measurements done using the still images, were plotted against the jet velocities. It was observed that, for the turbulent impingement cases, the droplet size decreased with the increase in the jet velocity and the impingement angle. For laminar impingement cases, a rapid initial decrease and then increase in droplet sizes was observed for the increasing jet velocities, depending on impingement angle \textsuperscript{[35]}. Final conclusions drawn from these experiments are, in the absence of internal and external flow disturbances, an equilibrium state is attained by the edge of the sheet that forms a thick rim from which the droplets are formed (laminar impingement cases) and droplets are formed due to the presence of impact waves in all turbulent impingement cases. It was concluded that the inertial forces of the jet, determined solely from the jet Weber number, that drives the impact waves and impact waves are independent of the Reynolds number.

2.1.2. Atomization of the Spray

Anderson et al. studied the sheet breakup lengths, the droplet sizes and the surface wave measurements for laminar and turbulent jets impingements and compared the experimental observations to the known analytical models. Three different diameters, precision bore glass tubes
of same length were used for the study of turbulent jets impingements and a longer glass tube with a much smaller diameter ($l_0/d_0 = 375$) is used for the study of fully developed laminar jets impingements. The experimental setup allowed the different sizes glass tube fixtures and the variation in the impingement angle. Precise injector alignment and minor spatial adjustments were done using a micrometer stage, used in the setup.

In this cold flow experiment, compressed Nitrogen gas was used to pressurize the water tank. A rotameter was used to control the flow rate and pressure gauges were used to measure the pressures in the system. Three impingement angles (40°, 60° and 80°) and velocities from 5-20 m/s were tested. Sheet breakup lengths were studied using the instantaneous images captured by a CID solid state camera. Seventeen images were captured at each operating condition. Droplet size measurements and sheet velocity measurements were done using an Argon-ion laser based, Fast Fourier Transform version of the PDPA. Complete details of the experimental setup and the test matrix can be seen in Ref. 26 and 29.

The main findings of these tests are discussed here. In the turbulent regime, both the detaching ligaments and the disturbances on the sheet surface are periodic in nature. The sheet breakup lengths of laminar jets are longer than that of the turbulent jets’ impingement. The sheet breakup length of the turbulent jets’ impingement decreases as the angle of impingement increases. But, in case if laminar jets impingement, the sheet breakup length increases as the angle of impingement increases. This shows that breakup mechanisms are different for both the cases. The observations showed that the sheet breakup lengths of the laminar jets’ impingement, increases to maximum and then decreases as the Weber number increases. These results agree with
Dombrowski and Hooper's analytical studies, done earlier [29]. Figure 2.1 shows the results discussed above and the different breakup mechanisms for the two cases.

![Figure 2.1: Non-Dimensional Sheet Breakup Length Ploted Against Weber Number a) Laminar Jets Impingements b) Turbulant Jets Impingements [26]](image)

The sheets formed by the high velocity laminar jets had the impact waves concentrated at the center of the sheet and it is observed that the sheet tears at the center where impact waves are formed. All the low velocity turbulent jet impingements are observed to have the impact waves in the sheet as well. This showed that the impact wave theory put forward by Dombrowski and Hooper was laudable.

![Figure 2.2: Arithmetic Mean Diameter of Drop Size D_{10}, Verses Horizontal Position in the Plane Away from Jet Axis Plane a) 16 mm Down the Impingement Point b) 41 mm Down the Impingement Point [26]](image)

Droplet size measurements were done using a PDPA at the locations, 16 mm and 41 mm downstream the impingement point. The angle of impingement, and the diameter of the precision
glass tube had a huge effect on the mean droplet size. Figure 2.2 shows the arithmetic mean droplet size along the horizontal axis away from the jet axis plane, at locations 16 mm and 41 mm downstream of the impingement point.

The results illustrate that the mean droplet sizes decrease as the angle of impingement increases and the mean droplet sizes are high for the higher diameter injectors. It was observed that the mean droplet velocity, increases almost linearly with increase in the jet velocity and decreases slightly with increase in the angle of impingement \cite{26}. The results of droplet sizes were reasonably in agreement with the linear stability-based theory.

Further the turbulent cases were studied for the surface wave and the periodic ligaments formations. The wavelengths of these periodic waves and ligaments seemed to be directly proportional to the diameter of the injector and were independent of impingement angle and velocity of the jets. These results do not clearly define the velocity profile of the sheet, turbulence intensity effects on the sheet and flow turning effects that occurs in the injector manifold. Anderson et al. suggest focusing further on generation and growth of the impact waves and their association with the following atomization process \cite{29}.

Many theoretical models existed earlier to these experiments. But this is one of the early efforts to make a comparative study of the spray characteristics of the impinging jets by cold flow experiments. In these experiments, a careful injector alignment has been done and the main factors varied to study the spray characteristics are the velocity of jets, the angle of impingement and the diameter of the injector.

### 2.1.3. Effect of Jet Breakup Length to Impingement Distance Ratio

The cold flow experiments using water at atmospheric pressure, were conducted at the University of Alabama in Huntsville by Sweeny et al., to study the effects of jet breakup length to
impingement distance ratio \( \left( \frac{l_b}{l_i} \right) \) for a like doublet impinging injector setup. Four different jet velocities \( (5 \text{ m/s}, 10 \text{ m/s}, 15 \text{ m/s}, \text{ and } 20 \text{ m/s}) \) were tested for both single jets and impinging jets. The single jets are studied to determine the average jet breakup lengths and breakup characteristics as the function of jet velocity. Three angles of impingement \( (30^\circ, 60^\circ \text{ and } 90^\circ) \) were tested and for each angle of impingement and each jet velocity, four \( \frac{l_b}{l_i} \) ratios \( (2, 1.5, 1 \text{ and } 0.5) \) were tested. To vary the \( \frac{l_b}{l_i} \) ratios, the angle of impingement and the spacing between the orifices are changed.

Smooth-bore, sharp-edged borosilicate glass tubes were used as injectors. The injectors were mounted on a rotation platform and the subassembly was mounted on a horizontal rail guide. The horizontal rail guide was mounted on two vertical rails that allowed the movement of the injectors in vertical direction. The setup included a pressure-fed feed system with control valves, regulators, pressure transducers to measure the pressures and an Omega \( k \)-type thermocouple to measure the temperature. Upstream of the injector, PCB 106B high-frequency Integrated Circuit Piezoelectric microphones were mounted to measure the high frequency pressure oscillations present in the feed system. A Phantom v711 high-speed camera was used to obtain the high-speed images of the spray. The images were analyzed to determine the breakup characteristics, the jet and sheet breakup lengths, and the wave lengths of the ligaments. A TSI two-component Phase Doppler Particle Analyzer (PDPA) used to measure the droplet diameter, two-dimensional droplet velocity and turbulent intensity. Complete experimental setup and the techniques can be obtained from Ref. 27 and Ref. 30. The results of these experiments are discussed in brief, below.

Each single jet observed to follow the turbulent primary breakup process. Jet breakup lengths are obtained by analyzing the 150 sequential frames of the high-speed video for each operating condition. The jet breakup length was found to increase with the increase in Weber number and fluctuate randomly with time for all the operating conditions tested. Still images from
the high-speed videos are used to analyze the large droplet diameters and the axial spacing between the large droplets of the single jets. The numerical average Waddel disk diameter of the turbulent jet for all flow conditions was found to be $1.81 \, mm$ with standard deviation of $0.25 \, mm$. The numerical average spacing for all operating conditions of single jets were found to be $4.52 \, mm$ with the standard deviation of $0.22 \, mm$.

The spray characteristics for impinging jets were observed as follows. As the velocity of the jets increased, more and more impact waves were observed on the surface of the sheets and smaller droplets shed from the edges of the sheet. For breakup length of jets equal to impingement length of jets ($l_b/l_i = 1$), the sheet formed was unsteady and the sheet was segmented at multiple places, down the length of the sheet. For $l_b/l_i < 1$, no flat sheet was formed. The intermittent droplet collision formed unsteady ligaments that further integrated into smaller droplets. The mean sheet breakup length is obtained from the analysis of every fourth frame of 200 frames, high-speed video. Broader and shorter sheet were seen as the impingement angles increased. This is expected due to the greater impact forces at higher angle of impingement. At lower angle of impingement, the sheet breakup length increased with the increase in Weber number. But, at higher angle of impingement, the sheet breakup lengths increased to certain point and then decreased as the Weber number increased.

![Figure 2.3: Dimensionless Sheet Breakup Lengths Versus Weber Number](image)

Figure 2.3: Dimensionless Sheet Breakup Lengths Versus Weber Number$^{[30]}$
number increased. Figure 2.3 shows the dimensionless sheet breakup lengths as the function of weber number and the angle of impingement at different $l_b/l_i$ ratios. The axial distance between two distinct ligaments is called as the spatial ligament wavelength. For every set point, depending on number of distinct ligaments shed from the end of the sheet during the high-speed video length, about 20 to 50 ligament wavelengths were measured. The observations manifested that the ligament wavelengths are not affected by the impingement angle and the Weber number.

Also, similar results were observed for all the three $l_b/l_i$ ratios ($l_b/l_i = 1, 1.5$ and 2) tested, proving that the ligament wavelength is independent of $l_b/l_i$ ratios when the sheet is formed. The droplet size measurements are made by the PDPA at a location 75 mm down the impingement point. Around 10,000 measurements were sampled for each set point. Figure 2.4 shows the mean droplet variation with the Weber number. The graph illustrates that the mean droplet diameter decreased with the increase in Weber number and impingement angle. $l_b/l_i$ ratios seems to have very little effect on the droplet diameter when the sheet is formed ($l_b/l_i \geq 1$) [30].

![Figure 2.4: Numerical Mean Droplet Diameter Versus Weber Number; $l_b/l_i = 1$ and $> 1$](image)

In this cold flow experiments, different $l_b/l_i$ ratios are obtained by changing the horizontal spacing between the two injectors. It was observed that for $l_b/l_i < 1$, no sheet was formed. However, intermittent collision of droplets formed the unsteady ligaments which subsequently disintegrated.
into smaller droplets. For $l_b/l_i = 1$, an unsteady sheet was formed, that had higher tendency to break at multiple points and for $l_b/l_i > 1$, a flat liquid sheet is formed in the plane perpendicular to the axis of impingement. From these experiments, the conclusion can be drawn as followed. After a liquid sheet is formed ($l_b/l_i \geq 1$), $l_b/l_i$ ratios has little to do with the spray characteristics. The results show that sheet breakup length, ligament wavelengths and the droplet sizes are independent of $l_b/l_i$ ratios when a sheet is formed.

### 2.1.4. The Sheet Instability in Presence of Acoustic Forces.

As we have seen in the introduction chapter, the combustion instability in LREs is due to the coupling between the acoustic pressure oscillations and the atomization process. A sudden increase in the rate of combustion could be explained as the breakup of larger propellant droplets into smaller droplets due to the influence of the acoustic pressure oscillations. Mulmule et al. conducted few experiments at Indian Institute of Technology – Bombay that helped to understand the effects of external acoustic waves on the dynamics of flat sheet produced from the impinging jets [31]. It is a cold flow experiment where two water jets were impinged head-on to each other in presence of the acoustic waves, like the pressure waves in the combustion chamber. The variation in the diameter of the sheet is studied using the high-speed images. Two borosilicate glass tubes of length 20 mm and diameters 2 mm were mounted head-on, on two vertical posts which could move in all the three directions and rotate to allow the injector’s angle change. A graduated scale placed above the jets helped in pixel to mm conversions for measuring diameter of the jets and the sheet. The injectors were placed exactly 4mm away from each other, to ensure the laminar jet impingement. Rotameters were used to measure the flow rate of water from the overhead tank. Redlake Motion Pro high-speed camera was used to capture the spray images.
Acoustic waves were generated using 200 W RMS, high decibel (Max SPL of 117 dB) speaker. The camera and the speaker were placed opposite sides of the sheet and a great care was taken in their alignment with the center of the sheet. Before the experiments were conducted, the waveforms and FFT of both recorded and original sounds were generated. The analysis confirmed that the distortions in the acoustic waves were negligible after 110 Hz. Two types of tests were conducted to study the sheet diameter. One without the acoustic waves and one, in presence of the acoustic waves.

The experiments without the acoustic waves confirmed the accuracy of the diameter measurements, by comparing the measured diameter of sheets with the previous works. Error analysis was done, to account for the uncertainties in rotameter measurements and image analysis. Then the acoustic waves of frequency, varying from 100 Hz to 1200 Hz at 100 dB was introduced to the sheet. It was observed that, the sheet diameter decreased suddenly with increase in the number of droplets. Further, the droplet size was reduced in presence of the acoustic waves. Experimental repeatability was tested for ten different Weber numbers, at 100 dB sound wave pressure level. Figure 2.5 illustrates the effect of the acoustic waves on the sheet formed.

![Figure 2.5: The Sheet at We = 875 a) Without Acoustic Waves b) In Presence of Acoustic Waves of Frequency 160 hz at 100 db][31]

To test the response of sheet to the acoustic frequencies, two Weber numbers below flapping regime (We = 492 and 608) and two Weber numbers in the flapping regime (We = 875 and 1350) were selected. It was observed that, for the lower We, significant response of the sheet...
was noticed at lower wave frequencies. At $We = 492$, the significant sheet response was at 110 Hz and not much variation in the sheet was observed at all other higher frequencies tested. Similarly, at $We = 608$, highest response of the sheet was only at 110 Hz and 160 Hz of the acoustic waves. But in the flapping region, when $We = 875$, the response of the sheet was high up to 360 Hz and no response was seen after 440 Hz. The trend was different for $We = 1350$. The sheet response or the reduction in the diameter of the sheet was noticed at very few frequencies and the sheet responses were not significant as in the previous cases.[31]

From these experiments, conclusion could be drawn as, the liquid sheet responds to the acoustic waves of lower frequencies and as the Weber number of the jet increases, the frequencies to which the sheet responds increases as well. At higher Weber numbers of the flapping regime, the sheet response is exceptionally low to any acoustic frequencies. In this cold flow experiments the geometry of the injectors were kept constant and the sheet diameters in presence of different frequency acoustic waves were studied.

2.1.5. Orifice Internal Flow Effects on Sheet Characteristics

An effort was made by Jung et al. at Seoul National University of Korea, to study the effects of the internal flow of sharp and round-edged orifice on the sheet breakup characteristics. A pair of round-edged orifice and a pair of sharp-edged orifice were tested in the form of like doublet injectors. All the orifices were of diameter 0.11 cm and length to diameter ratio of 17. The entrance of the round-edged orifice was rounded to a curvature of radius equal to that of the orifice diameter. Tests were conducted for 50° to 90° angle of impingement and the 950 cm/s to 3000 cm/s jet velocities. The impingement point was set to five times the orifice diameter, from the orifice exit. Two fuel simulants, water and Kerosene were tested. It was observed that the jets flowing through the smooth, round-edged entrance orifice had no perturbations. Whereas the flow
through sharp-edged entrance orifice was turbulent in nature, as it had to change the direction rapidly at the entrance of the orifice [32].

The discharge coefficients were plotted against the injection pressure. It was observed that the $C_d$ of round-edged orifice is higher than that of the sharp-edged orifice. Also, the $C_d$ of the sharp-edged orifice increased initially and decreased with the increase in injection pressure. This was explained by the hydraulic flip phenomenon, where at higher jet velocities, cavitation occurs at the entrance of the sharp-edged orifice due to sudden change in the flow direction. Both the water and kerosene flow through the round and sharp orifice were snapped at three different jet velocities (980 cm/s, 1690 cm/s and 2180 cm/s). For the water flow tests, at lower jet velocities, regardless of the orifice entrance, the jets seemed to be smooth. But, as the velocity increased, turbulence strength of the flow through sharp-edged orifice increased rapidly. For the kerosene flow tests, similar trends were observed for both round and sharp-edged orifice, but the turbulence strength of the jets were much higher than that of water. This was explained by the lesser surface tension of kerosene. The round-edged orifice showed a laminar flow for both water and kerosene at lower velocities, and at higher velocities, the flows were semi turbulent. The dimensionless jet diameter was plotted against the Weber number for round and sharp-edged orifices and for both, [32]

![Figure 2.6: Dimensionless Jet Diameter of Sharp and Round-Edged Orifice Against Weber Number](image)

23
water and kerosene tests. That can be seen in Figure 2.6. The jet diameter of round-edged orifice increases slightly with the Weber number, but the jet diameter of sharp-edged increases rapidly with $We$. Also, the kerosene jets had a higher jet diameter than that of water $^{[32]}$.

One hundred instantaneous spray images, obtained by stroboscopic light, were analyzed at each test condition, for the study of sheet and ligament breakup lengths. A general trend of decreased sheet breakup length with the increase in Weber number is observed for both round and sharpened edged orifice and the water and kerosene flow tests. But the sheet breakup lengths of kerosene were higher than that of the water for both round and sharp-edged orifice cases. This was explained by the higher turbulence strength of kerosene, because the impact forces are same for water and kerosene at a given Weber number. It could be seen that the sheet breakup length of round-edged orifice is higher than that of the sharp-edged orifice for both water and kerosene tests. For the water simulant tests, at higher Weber number the sheet breakup lengths become almost same for both round and sharp-edged orifices. In the round-edged orifice, water flow becomes more sensitive to the increase in Weber number because, the turbulence strength of the jet relaxes the impact force effects on the sheet at higher Weber number. The distance for the liquid ligament to break into fine droplets are called as the ligament breakup length in this work. The ligament breakup length follows the same trend as the sheet breakup length except for the water flow tests through the round-edged orifice. The ligament breakup lengths of water through the round-edged orifice increases with the increase in Weber number and that is due to the fact that the turbulence strength of water jets through the round-edged orifice do not increase greatly.

To observe the effect of impingement angle, the sheet breakup lengths and the ligament breakup lengths for the water simulants were plotted against the sine of half angle of impingement. The sheet breakup lengths were decreased slightly with the increase in impingement angle. But the
ligament breakup lengths were mostly not affected by the impingement angle. Also, the sheet and ligament breakup lengths of the round-edged orifices were greater than that of the sharp-edged orifice flows. Breakup wave lengths of the sheet and ligaments were plotted against Weber number and sin of half angle of impingement. The trend was very much like the sheet breakup lengths explained above [32].

In conclusion, these tests showed that the breakup characteristics are affected by both the impact forces and the jet turbulence. The jet turbulence is greatly affected by the orifice entrance shape, injection velocity and liquid properties.

2.1.6. Study of Sheet Velocity Distribution of Gelled Fuel Using PIV

Yang et al. carried out an experimental study of velocity distribution of the sheets, formed by like doublet impinging jets and axisymmetric like triplet impinging jets. Gelled gasoline, prepared by mixing gasoline and 5% Nano-silica in a mechanical stirrer and using an ultrasonic shaking instrument, was used for the experiments. Relieve valve operated compressed Nitrogen drove the gelled fuel from the fuel tank to the injectors. The injectors used, were of diameter 0.8 mm and the length of 4.8 mm. The impingement point was 9.6 mm down the orifice exit. The liquid sheet formed was captured by PIV system that included a 532 nm wavelength, double-pulse laser that operated at 200 MJ/pulse power and 15 Hz frequency. High frame CCD camera with Nikon 532 nm narrow band filter and Zeiss 50 mm f/1.4 lens was used to capture the image of the sheet. Two angle of impingements (90° and 120°) and two injection pressures (0.5 MPa and 1 MPa) were tested [33].

Results of the experiments are as follows. To check the injection pressure effects, PIV measurements were taken at 2θ = 90° and the injection pressures were varied. At low injection pressure (0.5 MPa), there was no significant difference in the velocity distribution of the sheet
formed by like doublet and like triplet injectors. However, there was a small high-speed particle zone, symmetric about the sheet axis, right below the impingement point. At high injection pressure (1 MPa), significantly different high velocity zones, existed on the sheets of like doublet and like triplet injectors. The high-velocity zone on the sheet of like triplet injector was seen to deflect towards the left of central axis and the high velocity zone on the sheet of like doublet injector was evenly distributed about the central axis.

Also, a high velocity zone belt was observed on like doublet injection sheet, which was not seen on the like triplet injection sheet. To observe the effect of the impingement angle, $2\theta = 90^\circ$ and $120^\circ$ were tested at injection pressure of 1 MPa. A general trend of decrease in the high velocity zones were observed as the impingement angle was increased, for both like doublet and like triplet conditions. This is because, as the impingement angle is increased, the vertical velocity component is decreased. However, the highest velocity of particle on sheet remained the same for both the angles, with the triplet sheet having a little higher velocity particle than that of the doublet sheet. As the angle of impingement increased, two high-speed belts, tilted towards the left of the central axis was observed on the sheet of like triplet injector and the high-speed belt around the central axis on the sheet of like doublet injector still remained as it is [33].

They concluded, high velocity zones are observed on the sheets at higher injection pressures and as the impingement angle increases, the high velocity zones decrease, but the highest particle velocity remains the same. A symmetrical high velocity belts were observed on the sheets of the like doublet impingements at higher injection pressure and this belt does not vanish with the increment in the impingement angle. Velocity distributions on the sheets were observed more clearly because of the use of gelled simulants. Gelled fuels could be easily stored for a longer time.
and have safety benefits. One needs to concentrate on methods to obtain the high quality atomization of gelled fuels.

2.1.7. Sheet Characteristics and Reliability Study of Partial Impinging Jets

To study the effects of partial impingement of jets on sheet characteristics, Subedi et al. conducted an experiment at Korea Aerospace University. A deviation or an offset of the jet axis from the perfect impingement condition is called as skewed impingement and the ratio of the offset between the jet axis and the jet diameter is called as fraction of skewness, in this experiment. Pressure fed system with water simulant was used for the experiments. Two steel rod like doublet injectors were used. The setup allowed the required separation of injector tips, impingement angle change, and pre-impingement length change. Load cell was used to measure the simulant mass discharge. Consistent $C_d$ of 0.5 to 0.64 was maintained throughout the experiment. Fraction of skewness from 0 to 0.9, impingement angles 40°, 60° and 90°, and injection pressures from 1 bar to 5 bar were tested. The shadowgraph images were used to study the qualitative and quantitative spray characteristics. A laser diffraction apparatus was used to study the droplet sizes. Details of the experimental setup and procedures could be seen in Ref. 36.

Qualitative analysis of the images illustrated that, for a perfect impingement, the sheet with a distinct rim that disintegrated into ligaments and droplets was observed. As the fraction of skewness increased, the rim became short and slim and the ligaments broke early and rapid. A patternator was used to analyze the mass distribution at the point of impingement. For a perfect impingement, a single region of highly concentrated mass was observed. The high concentration region starts to split as the fraction of skewness increases and two regions of concentrated mass were observed for a complete miss impingement case.
Breakup lengths of the sheet were plotted against the fraction of skewness. It was observed that the breakup length of sheet decreased as the fraction of skewness increased and this trend remained same for all the impingement angles and pressures tested. The spray sheet angle was observed to increase and then decrease with the increase in the skewness fraction, at low impingement angle. However, the spray sheet angle fluctuated a lot with the increase in the skewness fraction, at high impingement angles and at all the injection pressures tested. The sheet is formed in a plane perpendicular to the jet axis, for a perfect impingement case and the sheet turns as the fraction of skewness increases. That means, the spray fan angle increases with the skewness fraction for all the impingement angles and pressures tested.

Droplet sizes were observed to be more sensitive towards the injection pressure and angle of impingement. At high impingement pressures and high impingement angles, droplet sizes were least sensitive to the fraction of skewness. Linear fitting models were used for the reliability study. It was observed that the spray angle was more sensitive to skewness fraction at higher impingement pressures and the impingement angle was indirectly proportional to the skewness fraction. The spray fan angle linear fits showed that, it is inversely proportional to the impingement angle and is independent of the injection pressures. Also, the spray fan angle was less sensitive to the skewness fraction at high impingement angles. To conclude, the results showed that the maximum allowable skewness fraction is 0.3, considering the optimum performance of the injectors \[36\].

This experiment is a test of yet another geometrical parameter of the like doublet injectors. The current research also includes the geometric miss alignment that is called skewed impingement in the experiment above. But few of the results are contradictory, that can be seen in this thesis.
2.2. Summary

Many theoretical, analytical and experimental works have shown the important role played by the types of fuel injectors and its geometry in the combustion instability that occurs in LREs. Culick et al.,\cite{6,8} derived the expression for frequency shift and the growth or decay rate constant which included a source term. According to Culick et al.,\cite{6,8} the source term was completely dependent on injector type, geometry, dominant process etc. Analytical models found by Rani et al.,\cite{12} stress on the importance of the combustor geometry and the mean flow properties. The mean flow properties are dependent on the type of injectors and injection geometry. Flandro et al.,\cite{9,13,14} derived the energy density equation of the system, which includes a place holder. This place holder is for all the two-phase flow effects, like particle mean flow interactions, spray atomization, etc., that in turn depends on the injector type and geometry. Hutt et al.,\cite{19} explains the passive involvement of the injectors in the combustion process. Sirignano et al.,\cite{20} describes the dependability of combustion instability on intrinsic mechanisms of injection process that are, propellant injection, primary and secondary atomization, boiling of the droplets and vaporization, mixing of the gas phase and heating, combustion and loss of heat to fresh propellant injected. Bennewitz et al,\cite{17} explains that, the symmetric and the asymmetric injection techniques could damp certain modes of combustion instability. Hulka et al.,\cite{23} brief a case study of J2 engine. The case study of J2 engine shows the number of types of injector tested and the importance of the injector design in a LRE. The literature of theoretical works are presented in the Appendix D. However, the theoretical works are summarized here to show the importance of the injectors in the study of combustion instability.

Dombrowski and Hoopers'\cite{35} experiments on laminar and turbulent impinging jets proved that the sheet breakup is solely due to the impact waves and impact waves are independent of
Reynolds number. Anderson et al. [26,29] have done a comparative study of the characteristics of sheet, formed by like doublet impinging jets. The experimental observations of the sheet breakup lengths, droplet sizes and the breakup wavelengths were found to be in agreement with the previous analytical studies. Sweeny et al., [27,30] worked on the effects of jet breakup length to impingement distance ratio on the sheet characteristics, using a like doublet injector setup. It was observed that a steady sheet was formed for the ratio greater than one, unsteady sheet formed for the ration equal to one and no sheet formed for the ratio less than one. Mulmule et al., [31] studied the characteristics of sheet, formed by head-on impingement of like doublet jets, in presence acoustic forces similar to that of pressure waves in the combustion chamber. It was observed that, the liquid sheet responds to the acoustic waves of lower frequencies and as the Weber number of the jet increases, the frequencies to which the sheet responds increases as well. At higher Weber numbers of the flapping regime, the sheet response is very low to any acoustic frequencies. Orifice internal flow effects on the sheet characteristics were studied by Jung et al., [32] using the like doublet impinging injector setup. The experiments on round-edged and sharp-edged orifice flows, showed that the breakup characteristics are affected by both the impact forces and the jet turbulence. The jet turbulence is greatly affected by the orifice entrance shape, injection velocity and liquid properties. Yang et al. [33] studied the velocity distribution of the sheet formed by impingement of gelled fuels, using PIV and compared the results of like doublet and like triplet injectors. It was observed that the high velocity zones were present on the sheets at higher injection pressures and as the impingement angle increases, the high velocity zones decrease but the highest particle velocity still remains the same. Cavitt et al. [34] conducted hot fire experiments on the pentad injectors of different impingement angles, which studied the effects of impingement angle on combustion instability using stability mapping techniques. It was observed that the instability modes were high at low
impingement angles. Subedi et al. [36] studied the sheet characteristics for different partial impingement cases of like doublet jets. The results showed that the optimum injector performance is possible until the skewness fraction is up to 0.3.

From the literature, most of the cold flow experiments are conducted for the fuel injection velocity of range 9 m/s to 30 m/s. It is said that the liquid jets are atomized before the impingement, if the injection velocity of jet is greater than 30 m/s [32]. The fuel injection velocity of F-1 engine was 17 m/s [22]. Hence a safe range of turbulent jet velocities, from 10.5 m/s to 28.5 m/s were selected for the current research. Also, in F-1 Engine, the fuel impingement angle was 30° and the oxidizer impingement angle was 40° [22] and the impingement angle in most of the rocket engines are 60° [30]. Based on all the literature, the impingement angles (2θ) of 30°, 60°, and 90° were selected for the current research.

Table 2.1 summarizes the keywords of the literature review and type of work and corresponding reference numbers that provides detailed explanations of the topics.

<table>
<thead>
<tr>
<th>Keywords</th>
<th>Type</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>Combustion instability coupling the injector design and injector parameters</td>
<td>Theoretical and analytical models</td>
<td>6, 8, 9,12, 13, and 14</td>
</tr>
<tr>
<td>Like-doublet injectors of F-1 engine and Importance of injector design in J2 engine</td>
<td>Case study - experiments</td>
<td>22, and 23</td>
</tr>
<tr>
<td>Passive methods of combustion damping using injector parameters</td>
<td>Theory and experiments</td>
<td>6, 8, 15, 19, and 20</td>
</tr>
<tr>
<td>Laminar and turbulent flow effects on sheet characteristics</td>
<td>Cold flow experiments</td>
<td>26, 29 and 35</td>
</tr>
<tr>
<td>Effects of jet breakup length to impingement length ratio</td>
<td>Cold flow experiments</td>
<td>27 and 30</td>
</tr>
<tr>
<td>Topic</td>
<td>Method</td>
<td>Page</td>
</tr>
<tr>
<td>---------------------------------------------------------</td>
<td>------------------</td>
<td>------</td>
</tr>
<tr>
<td>Acoustic force effects on the liquid sheet</td>
<td>Cold flow experiments</td>
<td>31</td>
</tr>
<tr>
<td>Orifice internal flow effects</td>
<td>Cold flow experiments</td>
<td>32</td>
</tr>
<tr>
<td>Sheet velocity distribution, gelled fuels, PIV</td>
<td>Cold flow experiments</td>
<td>33</td>
</tr>
<tr>
<td>Stability mapping, impingement angles, pentad injectors</td>
<td>Hot fire tests</td>
<td>34</td>
</tr>
<tr>
<td>Partial impingement effects</td>
<td>Cold flow experiments</td>
<td>36</td>
</tr>
</tbody>
</table>

The current research is a cold flow experiment, which includes the study of sheet breakup length and the sheet angle for combined geometrical misalignment cases of the like doublet injector setup. All the cold flow experiments studied earlier, have only one geometrical impingement factor tested at a time. But in the current research, the impingement angle variations, the linear skewed impingements, and partial impingements of jets (discussed in the section 1.3) are studied simultaneous at each set point.

The study of different injection geometries is important because of three reasons. The first reason being most of the literatures shows the symmetric injectors are better with performance but lack to explain the effects of asymmetric injection of the fuel. So, the study of asymmetric injection geometries is yet to be perceived. The second reason is these tests are helpful to understand the effects and concerns of the design and/or manufacturing inaccuracies of the fuel injectors in the rocket engines. The final reason being, the results of these tests can be used in hot firing tests to study the modes of instabilities and the results can be further used to obtain the analytical models of the place holders, seen throughout the literatures. In Chapter 3, the experimental methods and analysis methods used for the current research are discussed.
CHAPTER 3. TEST PLAN, EXPERIMENTAL AND ANALYSIS METHODS

The purpose of this research is to study the sheet breakup lengths and the sheet angles and describe the effects of different geometrical configurations and jet misalignments of like doublet jets. The current research is a series of cold flow experiments, where the propellant simulant used is water. Fully developed turbulent water jets are made to impinge in presence of ambient air. Cold flow experiments at atmospheric conditions, establish baseline data sets that describe the variation of spray characteristics with change in the injector parameters and separate the spray from harsh chemical reactions a combustion chamber. High-speed images of the sprays are obtained and analyzed for the sheet breakup lengths and the sheet angles.

3.1. Test Plan

The sheet breakup lengths and the sheet angles are studied for the sheets formed by three types of geometrical alignments of like-doublet impinging injectors:

**Angularly Skewed Impingement:** The impingement lengths of each jet is varied by rotating one injector with respect to the other.

**Linearly Skewed Impingement:** The impingement length of one jet is fixed and the impingement length of other jet is varied by moving the injector linearly along its axis while holding a fixed impingement angle.

**Laterally Skewed Impingement or partial jet impingements:** One injector is misaligned in the plane of the spray sheet formed, to obtain partial impingement of jets.

Figure 3.1 shows the schematics of angularly skewed, linearly skewed, and laterally skewed or partial jet impingement conditions. The horizontal axis is the conventional X-axis, the vertical axis is the conventional Y-axis and the axis inward and out of the page is the conventional Z-axis. Figure 3.1a illustrates the schematic of the perfect impingement condition. In the perfect
impingement case, both the right and left injectors are at an angle $\theta$ with vertical axis, making the impingement angle $2\theta$. The impingement length is defined as the distance from the outlet of each injector to the geometric intersection point of jets. $L_L$ and $L_R$ are the impingement lengths of left and right jets respectively. Figure 3.1b shows the schematic of the angular skewed impingements where, the left injector is at an angle $\theta_L$ and right injector is at an angle $\theta_R$ with the vertical axis, making the angle of impingement equal to $\theta_L + \theta_R$. $L_L'$ and $L_R'$ are the new impingement lengths of the left and right jets respectively. Figure 3.1c shows the schematic of the linearly skewed

Figure 3.1: Schematic of a) Perfect Impingement Case, b) Angularly Skewed Impingement Case, c) Linearly Skewed Impingement Case, and d) Laterally Skewed or Partial Jet Impingement
impingement conditions. In this case, the right and the left injectors are at an angle $\theta$ with the vertical axis and the angle of impingement is kept constant and it is $2\theta$. Impingement length, $L_R^2$ of the right jet is increased by moving the right injector a distance ‘$y$’ along its axis. Impingement length of the left jet is maintained constant at $L_L^2$. Figure 3.1d) illustrates the schematic of the partial jet impingement conditions along the y-z plane. In the Figure 3.1d, ‘$z$’ is the lateral distance moved by the left injector along the Z-axis that causes the partial impingement of jets. The setup strategy used to obtain the angularly skewed, linearly skewed and partial jets impingement conditions are described below.

The right injector in the like-doublet setup, is mounted on a stepper motor controlled linear stage that allows the right injector to move linearly along its central axis and assists the linearly skewed impingement cases. The right injector and the linear stage assembly is mounted on a stepper motor controlled rotary table that helps to obtain the angular skewed impingement cases. The partial impingements of jets are obtained using a manually operated, horizontal slide that moves the left injector back and forth along the Z-axis. The diameter of like doublet injector orifice is 0.04 in (1.016 mm ± 0.01 mm). A $z$ distance approximately equal to 1/3rd of the diameter (+0.0138 in or +0.35 mm) is moved in positive $z$-direction and 1/3rd of the diameter (-0.0138 in or -0.35 mm) in negative $Z$-direction. Then the two partial impingement cases are compared with the perfect impingement case. The combinations of skewed impingement cases and partial impingement cases are studied along with the perfect impingement cases. The test matrix is detailed in the next subsection.

3.1.1. Experiment Sequence

In this research there are three sets of experiments conducted.
1. The mass flow rate calibration experiments aimed at confirming jet turbulence and cavitation free flows and obtaining the calibration curves for the injector tubes and an orifice.

2. Single jet experiments were carried out to verify that the jet breakup lengths are greater than the impingement lengths studied in all the skewed impingement cases.

3. Impinging jet experiments produced the sprays studied in this research.

3.1.2. Test Matrix

Tables 3.1 shows the test matrix for both mass flow calibration tests and the single jet experiments. For the mass flow rate calibration experiments, water is planned to throttle using an orifice of diameter 0.038in. The pressure upstream the orifice is referred as upstream pressure ($P_1$) in this research. The mass flow rates of the orifice, right and the left injectors are calibrated separately for 7 upstream pressures, and the corresponding jet velocities of both the right and left injectors were obtained. For the same jet velocities single jet experiments were conducted and the high-speed videos were obtained.

Table 3.1: Test Matrix of Mass Flow Rate Tests and Single Jet Experiments

<table>
<thead>
<tr>
<th>Upstream Pressure, $P_1$ (Psig)</th>
<th>Jet Velocities (ft/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>105</td>
<td>34.45</td>
</tr>
<tr>
<td>202</td>
<td>49.21</td>
</tr>
<tr>
<td>303</td>
<td>60.70</td>
</tr>
<tr>
<td>400</td>
<td>70.54</td>
</tr>
<tr>
<td>505</td>
<td>78.74</td>
</tr>
<tr>
<td>606</td>
<td>86.94</td>
</tr>
<tr>
<td>700</td>
<td>93.50</td>
</tr>
</tbody>
</table>

Table 3.2 summaries the test matrix of impinging jets experiments. For the impinging jets experiments, jet velocities of 49.21 ft/s and 82.02 ft/s were chosen. The goal of impinging jet experiments is to study the effects of various geometrical alignments of the injectors. So, only two jet velocities, well within the safe range of turbulent regime were chosen for the study.
Based on literatures, the impingement angles ($\theta$) 30°, 60°, and 90° were selected. At each impingement angle, to obtain the angular skewed impingement, the right injector is rotated in Table 3.2: Test Matrix of Impinging Jets Experiments

<table>
<thead>
<tr>
<th>$\theta_L = \theta_R$</th>
<th>$\theta_L =15^\circ$</th>
<th>$\theta_L =15^\circ$</th>
<th>$\theta_L =15^\circ$</th>
<th>$\theta_L =30^\circ$</th>
<th>$\theta_L =30^\circ$</th>
<th>$\theta_L =30^\circ$</th>
<th>$\theta_L =45^\circ$</th>
<th>$\theta_L =45^\circ$</th>
<th>$\theta_L =45^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2$\theta = 30^\circ$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2$\theta = 35^\circ$</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>2$\theta = 40^\circ$</td>
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<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
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<td>1</td>
</tr>
<tr>
<td>2$\theta = 55^\circ$</td>
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<td>1.5</td>
<td>1.5</td>
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<td>1.5</td>
<td>1.5</td>
<td>1.5</td>
<td>1.5</td>
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</tr>
<tr>
<td>2$\theta = 60^\circ$</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>2$\theta = 65^\circ$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2$\theta = 70^\circ$</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
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</tr>
<tr>
<td>2$\theta = 80^\circ$</td>
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<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
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</tr>
<tr>
<td>2$\theta = 85^\circ$</td>
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<td>1.5</td>
<td>1.5</td>
<td>1.5</td>
<td>1.5</td>
<td>1.5</td>
<td>1.5</td>
<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td>2$\theta = 90^\circ$</td>
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<td>2</td>
<td>2</td>
<td>2</td>
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<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

Injector jet velocities 49.21 ft/s and 82.02 ft/s

37
increments or decrements of 5°. That is, for 30° impingement angle, the angle of right injector ($\theta_R$) is equal to the angle of left injector ($\theta_L$) and both equal to 15°. To obtain the angular skewed impingement, the right injector is rotated such that, $\theta_R = 20°$ and 25°, while $\theta_L$ remained 15°. Similarly, for $2\theta = 60°$ case, $\theta_L$ was maintained at 30° and $\theta_R$ was varied to 25°, 30° and 35° and for $2\theta = 90°$ case, $\theta_L$ was maintained at 45° and $\theta_R$ was varied to 35°, 40° and 45°. In total, 9 impingement angles were tested for each jet velocities.

To study the perfect impingement and partial impingement cases, three points on the Z-axis were selected. For perfect impingement condition, the centers of the left injector and the right injectors are carefully aligned in the X-Z and Y-Z plane and at that point ‘z’ is defined to be equal to zero ($z = 0$). For partial impingement conditions, the left injector is moved 1/3rd of the injector orifice diameter, backward ($z = -0.0138$ in) and forward ($z = 0.0138$ in) along the z-axis. Hence, for each impingement angle, three ‘z’ points were tested.

To observe the effects of linear skewed impingement, the right injector is moved up along its axis in increments of 0.5 in, up to 2 in away from the initial injector position. This is referred as y-offset in this thesis. So, for every partial impingement and perfect impingement cases, there are 5 y-offset conditions, and they are 0 in, 0.5 in, 1 in, 1.5 in, and 2 in. Hence, the total number of set points for this series of experiments were, $2 \times 9 \times 3 \times 5 = 270$. Therefore 270 high-speed videos were captured for the impinging jets experiments.

Table 3.3 shows the impingement lengths of both the right and left jets. For every change in angle of impingement, the injectors are carefully aligned in X-Y plane and X-Z planes and adjusted along X-axis, such that the horizontal distance between the injector tips was 1.1024 in. By geometry, with one known side and all the known angles of a triangle, the other two sides (impingement lengths of the jets) can be determined.
The detailed experimental setup and procedure followed to achieve the objective, is described in the next sections.

### 3.2. Experimental Setup

The series of cold flow experiments are conducted at Atmospheric Spray Facility (ASF) of the Propulsion Research Center, located at the University of Alabama in Huntsville. The ASF

<table>
<thead>
<tr>
<th>Impingement angles (°)</th>
<th>y = 0 in</th>
<th>y = 0.5 in</th>
<th>y = 1 in</th>
<th>y = 1.5 in</th>
<th>y = 2 in</th>
</tr>
</thead>
<tbody>
<tr>
<td>2θ= 30 , θL=15 , θR=15</td>
<td>L_R^1</td>
<td>L_L^1</td>
<td>L_R^1</td>
<td>L_L^1</td>
<td>L_R^1</td>
</tr>
<tr>
<td>2θ= 35 , θL=15 , θR=20</td>
<td>L_R^1</td>
<td>L_L^1</td>
<td>L_R^1</td>
<td>L_L^1</td>
<td>L_R^1</td>
</tr>
<tr>
<td>2θ= 40 , θL=15 , θR=25</td>
<td>L_R^1</td>
<td>L_L^1</td>
<td>L_R^1</td>
<td>L_L^1</td>
<td>L_R^1</td>
</tr>
<tr>
<td>2θ= 55 , θL=30 , θR=25</td>
<td>L_R^1</td>
<td>L_L^1</td>
<td>L_R^1</td>
<td>L_L^1</td>
<td>L_R^1</td>
</tr>
<tr>
<td>2θ= 60 , θL=30 , θR=30</td>
<td>L_R^1</td>
<td>L_L^1</td>
<td>L_R^1</td>
<td>L_L^1</td>
<td>L_R^1</td>
</tr>
<tr>
<td>2θ= 65 , θL=30 , θR=35</td>
<td>L_R^1</td>
<td>L_L^1</td>
<td>L_R^1</td>
<td>L_L^1</td>
<td>L_R^1</td>
</tr>
<tr>
<td>2θ= 80 , θL=45 , θR=40</td>
<td>L_R^1</td>
<td>L_L^1</td>
<td>L_R^1</td>
<td>L_L^1</td>
<td>L_R^1</td>
</tr>
<tr>
<td>2θ= 85 , θL=45 , θR=45</td>
<td>L_R^1</td>
<td>L_L^1</td>
<td>L_R^1</td>
<td>L_L^1</td>
<td>L_R^1</td>
</tr>
<tr>
<td>2θ= 90 , θL=45 , θR=45</td>
<td>L_R^1</td>
<td>L_L^1</td>
<td>L_R^1</td>
<td>L_L^1</td>
<td>L_R^1</td>
</tr>
</tbody>
</table>
consists of a pressure-fed feed system and a benchtop setup. A detailed setup of the feed system and the benchtop instrumentation is explained in the next subsections.

3.2.1. Test Facility Feed System

Figure 3.2 shows the overall schematic of the feed system of the facility. The facility is pressure fed using compressed Nitrogen (N$_2$) K-bottles. The facility accommodates high pressure spray test area, where the spray experiments at high pressures are carried out and the atmospheric spray test area that allows, variety of custom setups and instrumentations for a wide range of research topics. The facility is plumbed in a way, to use water as propellant simulant and air as gas simulant for both high pressure and atmospheric spray experiments. The gas simulant is supplied to all the pneumatic valves, dome pressure regulators and the injectors (if needed) using a 4979.37 ft$^3$ air supply tank. For the current experiments, air supply to the injectors is tapped at the benchtop and only water is used as the liquid simulant.

Figure 3.2 illustrates the facility schematic and the red box shows the part of facility used for this research. An overhead water tank supplies the water to a 8.12 ft$^3$ run tank, from which water is supplied to the injectors. A manual pressure regulator operates on a ball valve to allow the N$_2$ gas to pressurize the system. The N$_2$ gas reaches the dome loader, passing through a filter. Depending on the research requirement, either the gas simulant tank or the liquid simulant tank (run tank) is pressurized, using the dome pressure regulator. For the current experiments, the dome pressure is set by a manually operated pressure regulator, such that the water from the run tank flows at the same pressure towards the injectors and that pressure is approximately equal to the liquid upstream pressure measured before the orifice.

Pneumatic ball valves are used to control the flow of the simulants throughout the system. The simulants are supplied to both the high-pressure facility and the atmospheric spray facility
from the same tanks. Isolation ball valves are used to direct the flow, depending on the experimental requirements. 0.5 in diameter feedlines were used throughout the feed system until the orifice. After the orifice, the 0.5 in diameter feedline is reduced to 0.25 in and then, branched into two lines flowing to the right and left injectors. All the components used for the experiments are numbered in the Figure 3.2. A description of the components used for the current experiments can be seen in the Table 3.4.

### Table 3.4: Component Description Table \[^{[37]}\]

<table>
<thead>
<tr>
<th>Number</th>
<th>Component</th>
<th>Number</th>
<th>Component</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.01</td>
<td>Analog gauge</td>
<td>1.07</td>
<td>Analog gauge – run tank pressure</td>
</tr>
<tr>
<td>9.01</td>
<td>Needle valve- vent</td>
<td>7.04</td>
<td>Manual ball valve – run tank</td>
</tr>
<tr>
<td>14.01</td>
<td>Manual pressure regulator</td>
<td>12.03</td>
<td>Pressure transducer – run tank</td>
</tr>
<tr>
<td>10.01</td>
<td>Pneumatic ball valve - pressurize</td>
<td>16.04</td>
<td>Run tank</td>
</tr>
<tr>
<td>10.02</td>
<td>Pneumatic ball valve -vent</td>
<td>7.06</td>
<td>Manual ball valve – pump</td>
</tr>
<tr>
<td>4.01</td>
<td>Check valve</td>
<td>7.07</td>
<td>Manual ball valve – drain</td>
</tr>
<tr>
<td>1.02</td>
<td>Analog Gauge – feedline</td>
<td>7.08</td>
<td>Manual ball valve – pump</td>
</tr>
<tr>
<td>4.02</td>
<td>Check valve</td>
<td>13.01</td>
<td>Pump -water tank</td>
</tr>
<tr>
<td>6.02</td>
<td>Filter</td>
<td>16.03</td>
<td>Water tank</td>
</tr>
<tr>
<td>5.01</td>
<td>Dome loader - run tank/ vent</td>
<td>7.05</td>
<td>Manual ball valve - liquid line</td>
</tr>
<tr>
<td>8.03</td>
<td>3-way manual driver valve – run tank pressure / vent</td>
<td>12.07</td>
<td>Pressure transducer- liquid simulant</td>
</tr>
<tr>
<td>1.06</td>
<td>Analog gauge – dome loader / run tank</td>
<td>10.08</td>
<td>Pneumatic ball valve – liquid simulant flow</td>
</tr>
<tr>
<td>12.06</td>
<td>Pressure transducer – dome loader/run tank pressure/vent</td>
<td>8.01</td>
<td>3-way manual diverter valve -ASF/HPSF</td>
</tr>
<tr>
<td>10.04</td>
<td>Pneumatic ball valve- dome loader/run tank pressure/ vent</td>
<td>15.01</td>
<td>Relief valve – pressurized run tank</td>
</tr>
</tbody>
</table>
Figure 3.2: Facility Schematic [37]
3.2.2. Bench Top Setup

Two, 4 ft × 4 ft optics tables with a drain attached between the tables is referred as the Atmospheric Spray Facility (ASF) in this research. In the Figure 3.2, the ASF box represent the bench top setup. Figure 3.3 is a schematic of the benchtop setup. The components in the schematic are described in the Table 3.5. This design can support custom setup of variety of injectors and instrumentation for various research projects and allows unobstructed access to the spray field. Liquid simulant pressure is measured using a pressure transducer positioned upstream of a 0.038 in diameter orifice and this pressure referred as the upstream pressure ($P_1$) in this experiment. After the orifice, the 0.25 in feed line branches into two lines, each supplying the water to the right and left injectors. A thermocouple is used to measure the water temperature, at the branch point. These branches are identical in design and same type of fittings are used throughout the branched lines.

![Figure 3.3: Schematic of Benchtop Setup](image)

After the branch point, hand ball valves are used on both the lines that allows to direct the flow through each injector orifice separately, if needed. These ball valves are connected to a custom-made instrumentation cross using a 60 in long stainless-steel braided flex hose on both the
lines. The flex hoses allow the easy movement of the injector orifice. The custom-made instrumentation cross connects the flex hose to the injector orifice and it also holds a static pressure transducer on one side and a high frequency dynamic transducer (not used for this experiment) on the other side. The cross is connected to the injector orifice via 3.5 in long, 0.2 in diameter stainless steel line and a Swagelok UltraTorr fitting. The stainless-steel line is fitted to a rotational platform to avoid the bending stresses on the glass injector orifice.

Table 3.5: Benchtop Component Description Table

<table>
<thead>
<tr>
<th>Number</th>
<th>Component</th>
<th>Number</th>
<th>Component</th>
</tr>
</thead>
<tbody>
<tr>
<td>12.08</td>
<td>High Frequency Pressure Transducer (Not Used)</td>
<td>12.13</td>
<td>Pressure Transducer – Upstream the 0.038 in Orifice</td>
</tr>
<tr>
<td>12.09</td>
<td>High Frequency Pressure Transducer (Not Used)</td>
<td>7.10</td>
<td>Manual Ball Valve – ASF feedline Injector 2</td>
</tr>
<tr>
<td>12.10</td>
<td>Pressure Transduce – Injector 2</td>
<td>7.11</td>
<td>Manual Ball Valve – ASF feedline Injector 1</td>
</tr>
<tr>
<td>12.11</td>
<td>Pressure Transduce – Injector 1</td>
<td>17.01</td>
<td>K-Type Thermocouple - ASF feedline</td>
</tr>
<tr>
<td>12.12</td>
<td>Pressure Transduce – Downstream the 0.038 in Orifice</td>
<td>15.03</td>
<td>Relief Valve – ASF feedline</td>
</tr>
<tr>
<td>3.01</td>
<td>0.038 in Orifice</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

A T-bar, of length 0.7136 ft, width and height of 0.125 ft and thickness of 0.0158 ft, attaches from the rotation platform up to 0.1312 ft upstream the injector orifice exit. The T-bar helps to reduce the vibration of the cantilever injector orifice and increases the structural rigidity of the injectors. The injectors are oriented above the optic tables such that, the plane of the spray sheet formed is at the center of the drain and the normal vector from the plane of the spray sheet directs along the length of the test stand or the optic tables.

The injectors used are, sharp-edged, thick walled and smooth bore borosilicate glass capillary tubes. The glass tubes have an inner diameter of 0.04 in (1.016 mm ± 0.01 mm), an outer diameter of 0.25 in (6.35 mm ± 0.25 mm) and a length of 8 in (203.2 mm ± 1 mm). Hence, the length to diameter ratio is equal to 200. The spray characteristics are affected by many parameters
like impingement angle, injection velocity, turbulence, cavitation, hydraulic flip etc. The current research focuses on the effects of impingement angle, injection velocity and different injection geometries. To avoid the cavitation and turbulence inside of the injector orifice, the liquid simulant is planned to throttle at the orifice, which is of diameter (0.038 in) smaller than that of one injector (0.04 in). To circumvent the effects of hydraulic flips and to obtain the fully developed turbulent flow, the orifice length to diameter ratio of 200 is chosen. Figure 3.4 shows a complete setup of the benchtop design.

![Figure 3.4: Benchtop Setup](image)

The present research is aimed at studying the spray for the skewed impingements and the partial impingements. The angular skewed impingement is obtained by rotating the right injector and the linear skewed impingement is obtained by the linear movement of the right injector. As seen in the Figure 3.4, the right injector held by the T-bar is mounted on Velmex-6 in travel, motorized linear slide. This helped to move the right injector along its axis to any required distance less than 6 in, to obtain the linear skewed impingement conditions. This subassembly of the right
injector and the linear slide is mounted on a 0.5 in Aluminum sheet and then installed on Velmex-B4800 series, motorized rotary table that can hold a cantilever load up to 500 lb-in. The aluminum sheet is used for the alignment purpose.

The motorized rotary table could rotate the right injector subassembly to required degree to obtain the angular skewed impingement. Both the linear slide and rotary table are controlled by single Velmex stepper motor controller, which can control 3 motors at a time. However, the requirement for this experiment was, two motors operation. A joystick was adjoined to the motor controller. The scale of the rotary table is $10^\circ$ for 400 steps of the motor and the scale of the linear slide is 0.5 in for 2000 steps of the motor. Whole subassembly is mounted on right slide of a horizontal rail using optic bases and brackets. The left injector is mounted on a manual rotation platform and that is mounted to a Newport-4 in travel, micrometer guided linear stage using an optic bracket. The 4 in travel linear stage is along the z-axis, described in the experiment and this linear stage is used to obtain the partial impingement conditions.

The left injector subassembly is mounted on the left slide of the horizontal rail. The slides of the horizontal rail could be moved to obtain the required spacing between the injectors. The horizontal rail guide is installed on vertical translating stages so that the primary atomization zone and the impingement points could be moved to the center of camera frame. The alignments are measured using bubble level indicators. Great care was taken to align the left and right injector exits in the same horizontal and vertical planes.

A Phantom V711 high-speed camera was mounted on a rail guide and hence it could translate in the direction normal to the sheet to capture the complete primary breakup zone. The V711 is a 1-megapixel camera. It contains a CMOS image sensor. At its maximum resolution of $1280 \times 800$ pixel, it can capture 7530 frames per second. At its lowest resolution of $128 \times 8$ pixels,
it can capture up to 1,400,000 frames per second with the ‘FAST’ option enabled [40]. For the current research, 10,000 frames per second were captured at a resolution of 608 × 800 pixels. A light source and an optical diffuser are placed right in front of the high-speed camera, on the opposite side of the spray. A 250 W Light Emitting Diode high bay light was used as light source and the light source was placed behind 8 in × 9.125 in size, 220 grit ground glass optical diffuser. The optical diffuser helps to achieve the uniform light intensity level behind the spray, which is not possible with the light source alone. So, the spray is backlit by the light source and the diffuser and the image is captured by the high-speed camera.

Two software programs on two different computers are used to monitor and control the experiment. A computer, connected to the Data Acquisition (DAQ) system, contains the LabView program. The LabView software is used to monitor and save all the feedline pressures and temperature data. The National Instruments developed a graphical programming software called LabView that integrates with the wide selection of DAQ cards and instrumentation modules, which helps to acquire the required data for all types of instrumentation. This capability allows the LabView to be used as embedded control systems, system prototyping, and etc. to obtain the experimental data.

LabView is programmed to measure seven static mean pressures from the transducers located at different points on the feedline and a mean thermocouple temperature, at the branch point. The static thermocouple and pressure data are routed into a NIPCI-6259 DAQ card through NI BNC 2110 instrumentation module. A sample rate of 100 Hz is used for all the measurements by the DAQ card. Another computer is placed on the optic table, next to the high-speed camera, as seen in the figure 3.4. This computer contains the software, Phantom Camera Control (PCC 2.5) which is used to control and record the high-speed videos. The PCC 2.5 software allows the user
to adjust all the camera settings like, frame rate, image tools, shutter speed, trigger options, etc. and it also allows the user to save, view and adjust the high-speed videos captured. Cine viewer, version 3.3 (CV 3.3) is an application from the Phantom, which helps to view and edit phantom cine files and also allows to perform measurements, apply image processing and convert to other file formats. CV 3.3 is used to carry out the analysis of the high-speed videos.

3.3. Experimental Procedure

As described earlier, three sets of experiments were conducted. They are the mass flow rate calibration tests, the single jet experiments and the impinging jets experiments.

3.3.1. Mass Flow Rate Calibration

For the orifice flow calibration experiment, the 0.25 in tubing after the orifice is removed and a 0.5 in L-tube is fitted to the orifice exit, such that the flow, after the orifice is directed to the drain. The calibration method used a bucket, a stopwatch and a weighing scale to determine the mass flowrates. The run tank pressure regulator is operated to set the upstream liquid pressures, \( P_1 \) to 105, 202, 303, 400, 505, 606, and 700 psig in steps. At each pressure, water filled the bucket for 60 seconds and the final weight of was measured. The weight of the empty bucket was zeroed each time before filling the water in it. Each flow condition was repeated three time and averaged. Therefore, 21 points were calibrated for the orifice alone.

The L-tube fittings were replaced with the 0.25 in tube lines that are branched to right and left injectors. The run tank pressure regulator is operated again, to obtain the same upstream pressures in steps. Both the injectors were kept open to flow the water to drain. At each upstream pressure, 3 calibrations points of 60 seconds each, were conducted for both right and left injectors separately, using the same method as described above. In total, 42 calibration points were taken.
for seven pressures and two injectors. The three measurements of water weight at each upstream pressure is averaged, to bring down the uncertainty in the calibration measurements.

Figure 3.5: Mass Flow Rate Calibration Data

Figure 3.5 shows the results of calibration tests. On the plot, the orifice mass flow rate is represented by solid circles, the solid squares represent mass flow rate of right injector, the solid triangles represent left injector mass flow rate and the solid diamonds show the added mass flow rate of right and left injector. The horizontal error bars show the uncertainty in measured upstream pressure $P_1$ and the vertical error bars show the uncertainty in the measured mass flow rate. The mass flow rates of injector right and injector left are very close to each other and the added mass flow rate is close to the orifice mass flow rate. This shows that the experiment is repeatable. The mass flow rate of the orifice flows, and pipe flows are proportional to the square root of the pressure difference.

The equations of trend lines are shown in Figure 3.5. The power of ‘x’ in the equation of the four trend lines is near to 0.5. This confirms that the experiments are reliable. However, the ‘Total of Right and Left Injector’ mass flow rate curve does not coincide on the ‘Orifice’ mass
flow rate curve in the Figure 3.5. This shows that the flow is not choked at the orifice. The purpose of usage of the orifice with 0.038 in diameter, which is less than the injector diameter (0.04 in), is to throttle and control the flow at the orifice. But, the fully developed, turbulent flow can be assumed to exist within the injectors, as the injectors are sharp-edged and long \((l_0/d_0)\). The flow through glass injectors can be modeled as pipe flows as the glass tubes have very large length to diameter \((l_0/d_0)\) ratio \[^{27,30}\].

The velocity of jets \((v_j)\) is found using the Equation 3.1, for all the upstream pressures.

\[
v_j = \frac{4 \times \dot{m}}{\pi d_0^2 \rho}
\]  

(3.1)

To calculate jet velocities of right and left injector, average mass flow rate \((\dot{m})\) of the right and left injectors is calculated at each upstream pressure and \(d_0\) is the diameter of the injector orifice. Temperature of water measured from the thermocouple is 75.2 °F and corresponding density of water \((\rho)\) is 62.3318 lb/ft\(^3\), obtained from the National Institute of Standards and Technology (NIST) Chemistry WebBook \[^{38}\]. The calculated jet velocities corresponding to 105, 202, 303, 400, 505, 606, and 700 psig upstream pressures are 34.45, 49.21, 60.70, 70.54, 78.74, 86.94, and 93.50 ft/s. as described in Section 3.1, the test plan was to study the spray characteristics at 49.21 ft/s and 82.02 ft/s. The corresponding upstream pressures were determined by interpolation of data, given in Table 3.1.

The main objective of the mass flow rate calibration experiments is to calibrate the jet velocities and the mass flow rates with respect to the controllable upstream pressure, and to confirm that the effects to cavitation and hydraulic flips are negligible. The orifice cavitation number \((K_{orifice})\) is found by Equation 3.2.

\[
K_{orifice} = \frac{P_1 - P_v}{P_1 - P_2}
\]  

(3.2)
Here, $P_1$ is the upstream pressure of the orifice, $P_v$ is the vapor pressure of the water at 75.2 °F obtained from the NIST chemistry WebBook [38], and $P_2$ is the orifice downstream pressure. The measured $P_2$ is approximately equal to the liquid pressure at the entrance of each injectors. The cavitation number of each injector ($K_{injctor}$) is found using Equation 3.3.

$$K_{injctor} = \frac{P_{injctor} - P_v}{\frac{1}{2} (\rho v_j^2)}$$

(3.3)

In the Equation 3.3, the $P_{injctor}$ is the pressure at the entrance of right and left injector. The measured pressures at the right and left injectors were approximately equal. $\rho$ is the density of the water at 75.2 °F and $v_j$ is the jet velocity measured. Table 3.6 shows the calculated cavitation number for the orifice and injectors, for the mass flow rate experiments and Table 3.7 shows the injector cavitation number calculated for 49.21 ft/s and 82.02 ft/s.

<table>
<thead>
<tr>
<th>Jet velocity, $v_j$ (ft/s)</th>
<th>$K_{orifice}$</th>
<th>$K_{injctor}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>34.45</td>
<td>2.3216</td>
<td>9.0245</td>
</tr>
<tr>
<td>49.21</td>
<td>2.1723</td>
<td>7.6541</td>
</tr>
<tr>
<td>60.70</td>
<td>2.0919</td>
<td>7.0217</td>
</tr>
<tr>
<td>70.54</td>
<td>2.0419</td>
<td>6.6796</td>
</tr>
<tr>
<td>78.74</td>
<td>1.9867</td>
<td>6.4318</td>
</tr>
<tr>
<td>86.94</td>
<td>1.9484</td>
<td>6.2438</td>
</tr>
<tr>
<td>93.50</td>
<td>1.9204</td>
<td>6.1414</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Jet velocity $v_j$ (ft/s)</th>
<th>$K_{injctor}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>49.21</td>
<td>7.5684</td>
</tr>
<tr>
<td>82.02</td>
<td>6.2808</td>
</tr>
</tbody>
</table>

Table 3.6: Orifice and Injector Cavitation Numbers

Table 3.7: Injector Cavitation Numbers for Impinging Jets Experiment

From Table 3.6, $K_{orifice}$ values are much smaller than the $K_{injctor}$ values. Again, this shows that the liquid simulant is throttled at the orifice, effects of turbulence in the injectors are negligible and fully developed turbulent flow is obtained in the injector orifice. Fixed cavitation in an injector is observed, when the cavitation number is below 1 and if the cavitation number falls below zero,
then it is said to be supercavitation [39]. Hence, from the results, all the experiments are assumed

to be completely free of cavitation effects.

3.3.2. Single Jets Tests

The only objective of single jet experiments, in this research is to verify that, the breakup
length of the jets are longer than all the impingement lengths of linearly skewed impingement
experiments. To achieve this, both the injectors were oriented vertically, such that, there was no
impingement, and the flow was directed to the sink. Both the injectors were kept running for these
experiments.

The V711 high-speed camera is adjusted on the rail guide and the focal length of the lens
were adjusted such that, only the left injector tip is focused. A reference image was captured, by
using a 1.18 in (30 mm) wide scale placed exactly at the left injector tip. The run tank pressure
regulator is operated to achieve range of upstream pressures (105, 202, 303, 400, 505, 606, and
700 psig) in steps. At each set point, the high-speed video is captured at 10,000 frames per second
using PCC 2.5 software. Out of which 201 frames after the trigger is saved with the computer. The

![Reference Image](image1)

![Image of the Jet](image2)

Figure 3.6: Single Jet Experiments a) Reference Image b) Image of the Jet at Velocity 34.45
ft/s
high-speed video files consume a large amount of hard drive memory space in the computer and hence, each video is restricted to 201 frames for this research.

Figure 3.6 shows the reference image of 30 mm wide measuring scale and a sample image of the single jet experiments. The reference image is used to convert pixels to feet in CV 3.3 software and MATLAB image processing tools are used to determine the jet breakup lengths. It was observed that the average breakup lengths of jets were greater than the impingement lengths of all impinging jet experiments that was planned to test. Detailed results are discussed in the Chapter 4.

3.3.3. Impinging Jets Tests

The objective of this research is to study the sprays formed by different angular and linear skewed impingements and the partial impingements of jets. As discussed in Section 3.1.2, two jet velocities 49.21 ft/s and 82.02 ft/s in the turbulent regime, were chosen for the study. The corresponding upstream pressures were 202 psig and 550 psig, obtained by the interpolation of mass flow rate calibration data. Initially, the manually operated run tank pressure regulator was set to read the upstream pressure of 202 psig.

As described in the test matrix Table 3.2, the experiment started with the impingement angle of 30°. The right and left injectors were aligned 15° from the vertical to make the total impingement angle 30°. The right and left injector tips are carefully aligned in horizontal and vertical planes, using the bubble level indicators. The horizontal distance between the injector tips were set to 1.1024 in (28 mm). Perfect impingement of jets were obtained by aligning the injector tips in the Z-axis and Z-axis alignments, which are controlled using the 4-in travel, micrometer guided linear stage of the left injector assembly. The spray fan rotates in clockwise or anticlockwise direction depending on positive or negative misalignments of jets along z- axis and
for a perfect impingement condition, the spray fan is perfectly perpendicular to the length of optic tables.

Once the injectors are aligned, a reference image is taken, by placing the 1.18 in wide scale right at the spray sheet plane. Followed by the reference image, a high-speed video of the spray was captured. Figure 3.7 shows the sample reference image and a sample frame from the high-speed video of the spray. Then the right injector is moved up in steps of 0.5 in up to 2 in, along its axis using the motorized linear stage. At each step, a high-speed video is recorded (10,000 fps and only 201 frames saved). The right injector is moved back to its original position on the linear stage. The motor controlled linear and rotational stages on right injector assembly were of great help to align the injectors accurately in required positions.

Next, partial impingements are achieved by moving the left injector 0.0138 in and -0.0138 in from the perfect impingement, along the Z-axis, using the 4-in travel, micrometer guided linear stage. At each partial impingement condition, the linear axis movement of the right injector is repeated in steps of 0.5 in, as mentioned above and high-speed videos were captured. Then, the angular skewed impingement is achieved by rotating the right injector to 20° and 25°, using the
motorized rotational platform. At each angular skewed impingement, linear skewed conditions and partial impingement conditions were carried out in the exact same procedure. Note that, for every angular skewed impingement, plane of the sheet shifts. So, the camera adjustments and the reference images are necessary. Similar process is carried out for 55°, 60°, 65°, 80°, 85° and 90° impingement angles and the complete procedure is repeated for 82.02 ft/s jet velocity case. As seen above, for every change in impingement angle, the instrumentation and the injector alignments needed some corrections. These are referred as test considerations and they are mentioned below. For every change in the impingement angle,

- First, the Z-axis alignment is adjusted, such that the spray formed is exactly perpendicular to the length of the optic tables. This is the perfect impingement case and note the z position of left injector as zero.
- Second, careful alignment of the injector tips in horizontal and vertical plane is achieved by moving the right injector with the help of motorized linear stage. Bubble level indicators were used to verify the alignments.
- Next, the moveable slides on the horizontal rail guide are adjusted to obtain an exact horizontal distance of 1.1024 in between the injector tips every time.
- The whole injector assembly is moved in vertical direction with the help of vertical translating stages to get the primary breakup zone of the spray into camera frame and camera focus is adjusted.

3.4. Analysis Methods

In this research, data analysis is done using Microsoft Excel workbook, Phantom PCC CV-3.3 software and MATLAB image processing tools. Mass flow rate calibration data and pressures and temperature data from LabView are processed using Excel workbook. Also, the uncertainty
analysis is carried out using Excel, which is discussed in detail in the next chapter. Using the reference images, pixel to feet conversions are done in the Cine Viewer software and the scale is used in the MATLAB code to find the breakup length of sheets. Cine Viewer software is also used to, measure the angle of spray sheets, and convert each frame of the 201 frames high-speed video into separate JPG image files. Each frame or the image file is then processed using the MATLAB code and the sheet breakup lengths are measured using MATLAB image processing tools.

3.4.1. MATLAB Image Processing

An image is considered as a two-dimensional function, $f(x, y)$, where $x$ and $y$ are the spatial coordinates. The intensity or the gray level at a point on the image, is defined as the amplitude of function ‘$f$’ at the pair of coordinates of that point. If the value of intensity, $x$ and $y$ are finite and discrete then the image can be called as the digital image [41]. If the image is divided into $N$ rows and $M$ columns, the intersection of a row and a column is termed as pixel. A digital image is a function of many variables that includes, but not limited to, depth, color and time [42].

MathWorks image processing toolbox is one of the powerful tools for the digital image processing. It provides a comprehensive set of standard algorithms and workflow apps for processing and analysis of the image. The image processing toolbox helps, in automating the image processing workflows and in batch processing a large amount of data. Acquiring and importing data, color threshold measurements, image enhancement, morphological operations, image deblurring, 3D image processing, edge detection, image region analysis, image segmentations, image registrations and many more operations can be performed using the standard algorithms of image processing toolbox in MATLAB [43]. For the current research, a code is developed using some of the standard algorithms of this image processing toolbox to determine the sheet breakup
lengths. Figure 3.8 shows a flow chart of the approach used in this research to find the sheet breakup lengths. The published MATLAB code is in the Appendix A.
As described earlier, this research includes 270 setpoints. At each setpoint, a high-speed video was captured at 10,000 fps and 201 frames after the trigger are saved. Cine Viewer software can batch process 201 frames of all the high-speed videos into individual images and store in a designated folder. The ‘measurements’ application of the Cine Viewer software is used to convert the number of pixels to feet for every reference image at each setpoint. The obtained pixel to feet conversion scale factor is used in the MATLAB code. MATLAB is a powerful tool that can batch process thousands of images without human interruption. Some of the built-in, standard algorithms of MATLAB are very handy in processing these images. A brief description of the standard, built-in functions of MATLAB image processing toolbox, used for the analysis of spray images is presented below.

The built-in function ‘imread’ reads the image from a file. If the file has multiple images, then it reads the first image in sequential order. A ‘for’ loop is used to read each image one by one, from the folder. Each image processed from the Cine Viewer software, is stored in .jpg extension. The ‘imread’ function reads the image in the matrix form, $A(608, 800)$. The ‘im2double’ function converts the image to double precision. If the input image is of class double, the output image will be identical to it. If the input image is not class double, then the ‘im2doble’ functions returns the equivalent image of class double, offsetting or rescaling the data as necessary. The ‘imadjust’ command is a function that adjusts the image intensity value or colormap. It maps the intensity values in the input image to new values in the output image, such that 1% of data is saturated at low or high intensity of input image and hence, the ‘imadjust’ command increases the contrast of the output image.

The function ‘edge’ is used to determine the edge in the intensity image. The ‘edge’ function supports six different edge finding methods. The ‘Sobel’ approximation to derivative is
used to find the edge of spray image and edge is detected at the maximum gradient of the image intensity. The ‘Sobel’ method finds an image intensity threshold converts it into binary image, by making all intensity value higher than the threshold as ‘1’ and lower than the threshold as ‘0’. A correction factor is used to the threshold value found by the ‘Sobel’ method and the chosen threshold correction factor is based on the judgment of the investigator.

A threshold correction factor of 0.48 is chosen for the analysis of the sprays that had a jet velocity of 49.21 ft/s and 0.58 is chosen as the threshold correction factor for the spray that had jet velocity of 82.02 ft/s. All the high-speed videos were captured with the same camera settings and at same atmospheric conditions. But, the images of the sprays with 82.02 ft/s jet velocities had a higher contrast than the images of sprays with 49.21 ft/s jet velocities. Figure 3.9 shows the difference in contrast of 49.21 ft/s spray image and 82.02 ft/s spray image. Hence, after a lot of trials, the threshold correction factors were selected. Sensitivity of the sheet breakup length for these threshold correction factors are checked by increasing and decreasing the correction factor by 10% for 54 setpoints. The variations in the sheet breakup lengths are included in the uncertainty analysis.

![Image showing spray images at different jet velocities](image)

**Figure 3.9: Spray Images at 2θ = 30°**

(a) \( v_J = 49.21 \text{ ft/s}, \) Threshold Correction Factor = 0.49

(b) \( v_J = 82.02 \text{ ft/s}, \) Threshold Correction Factor = 0.58
The binary image is dilated using the ‘imdilate’ function. The ‘imdilate’ function needs an array of structuring elements. The ‘strel’ function is used to create the disk structures of 1-pixel radius that are used in the ‘imdilate’ to dilate the binary image. The ‘imerode’ function is used to erode the highly dilated image. Same sized disk structuring elements are used in the ‘imerode’ function. A function named, ‘bwareaopen’ removes the small objects from the binary image. Desired connectivity of the pixels can be described in the ‘bwareaopen’ functions, such that, the area less than the defined pixel count is removed from the binary image. As the angle of impingement increases, the sheet breakup length significantly decreases. With this fact, pixel count used in the ‘bwareaopen’ function is changed for different impingement angle. After a lot of trials, for 2θ of 30°, 35° and 40° a pixel count of 7000 is used in the ‘bwareaopen’ function, so that the area with pixel count less than 7000 was removed from the binary image. Hence all the unattached ligaments and droplets are removed from the binary image.

Similarly, for 2θ of 55°, 60° and 65° a pixel count of 3500 is used and for remaining angle of impingements, a pixel count of 4500 is used. Note, at impingement angle 80°, 85°, and 90° the sheet formed was shorter, but wider than the sheets formed from the 65°, 60° and 55° angle of impingements. So, the pixel count was increased a little. The gaps within the primary sheet is then filled using the ‘imclose’ function. The ‘imclose’ function morphologically closes the binary image, based on the defined array of 0s or 1s. Next, the sheet breakup length is found using a for-loop, described in the flow chart, figure 3.8. Description of all the image processing, standard built-in functions can be found in MATLAB-help documents.

To check the reliability of the code, out of 270 setpoints, 201 frames of 74 setpoints are made to run through the code one by one (not in the for-loop). The results of the ‘human
interventions’ of 74 setpoints are compared with the ‘no intervention’ results. The difference in results are marginal and the sheet breakup length results are discussed in detail in the Chapter 4.

The jet breakup lengths of the single jet experiments were determined using the same MATLAB code with an additional function. To capture a greater length of the jets, the focal length of the camera lens is increased for the single jet tests. The camera resolutions were set to 128 pixels \(\times\) 800 pixels. From Table 3.3, the maximum jet length needed to be measured is 4.16 in, i.e. 634 pixels for the single jet tests. So, the resolution used for the measurements through MATLAB is 128 pixels \(\times\) 681 pixels. The captured images had the regions at the top and bottom that are poorly illuminated by the backlight. So, once the images of single jet experiments were read and contrast was increased using the ‘imadjust’ function in the code, the ‘imflatfield’ function is used to apply the flat field correction to the image. The ‘imflatfield’ function uses gaussian smoothing with a standard deviation ‘Sigma’ to approximate the shading component of the image. The value of ‘Sigma’ is chosen to be 70 after a lot of trials. Note that the ‘imflatfield’ function is used only for single jet tests, as there were poorly illuminated regions in the images. For the single jet experiments, 8 setpoints were tested. Out of the 8 setpoints, 201 frames of 3 setpoints were made to run through the MATLAB code one by one. This was carried out to check the validity of the code. The average jet breakup lengths determined with human interventions were found to be mostly same as that of the results from the MATLAB code with no interventions. The published MATLAB code and the description of each functions used are given in the Appendix A.

### 3.4.2. Phantom Cine Viewer 3.3 Software.

As mentioned earlier, the Cine Viewer-3.3 is used to convert the frames of the high-speed video into individual JPEG image files. It is also used to obtain the pixel to feet conversion factor and to measure the sheet angles. The Phantom Video Player application allows the user to view
the live spray images on a monitor screen, when the camera is attached to the computer using Ethernet cable. The application is very user friendly and allows to capture, review, edit and/or save the image on the hard drive. The cine files recorded can be quickly edited by selecting the range of images to be played back using video play back buttons. With the help of this user-friendly application, the end user can easily adjust the play back speeds and assign image processing effects to the displayed image \cite{44}. The captured cine files can be saved as a wide variety of files. The frames high-speed video files can be saved as, .jpg, .tif, .dng, .dpx, .exr, .bmp, .pcx, and .tga image files. Also, the high-speed videos can be saved as, .cci, .cine, .avi, .tif, .mp4, and .mov video files. There is a ‘Range option’ available on the save screen that allows to specify the required number of frames to be saved. As mentioned in earlier sections, 201 frames of the each setpoint high-speed

![Figure 3.10: Cine Viewer Measurements](image-url)
video is converted into .jpg files for this research. The files are saved with a filename and ‘.+4’ extension that allows the software to save each frame with a 4 digit, sequentially increasing number (ex: setpoint1.+4 saves each frame as setpoint1.0001.jpg, setpoint1.0002.jpg and so on until setpoint1.0201.jpg image files).

For every change in impingement angle, a reference image is captured. So, one reference image is captured for every 15 setpoints. The known width of 1.18 in (30 mm or 0.0984252 foot) scale is used for capturing the reference image and this width is used to calibrate the image. Figure 3.10 shows the measurements options available in the Cine Viewer. The calibrate button allows the user to measure the number of pixels along 1.18 in wide scale on the reference image and gives a scaling factor. ‘Set to all’ button assigns this scaling factor to all 15 setpoints.

The ‘Set origin’ button allows the user to set the origin point of the spray. The origin point is measured from the top of the image and used as the beginning point in the second for-loop of MATLAB code. The measurements are carried out by activating the ‘instant measurement’ options, shown in the Figure 3.10. There are four different measurement options in the dropdown of ‘instant measurement’. For the current research, ‘Distance & Angle & Speed: Origin + 1point’ option is used for the measurement. This option allows the user to measure the distance of any point from the fixed origin, angle of the point in anticlockwise direction and the speed of the image. The results box shows the distance measured as ‘d’ in feet and angle measured as ‘a’ in radians.

After obtaining the scaling factor and the image origin point, the sheet breakup length is measured using the MATLAB code. However, the sheet angle measurements are done in the Cine Viewer software. Out of 201 frames, zeroth, fiftieth, hundredth, one hundred and fifthieth and the last frames were selected to measure the sheet angle for every setpoint. All the results and their uncertainty analysis are discussed in Chapter 4.
CHAPTER 4. RESULTS

The results of the experimental studies are presented in this chapter. The first section explains the results of the single jet experiments that determined the jet breakup lengths. The second section presents the results of impinging jets including the sheet angle and sheet breakup lengths as a function of the jet velocity and the various alignments of the jets. The sheet characteristics based on the visual observations are also detailed.

4.1. Results of Single Jet Experiments

As described earlier, the aim of the single jet tests is to confirm the jet turbulence, cavitation free flow and determine the average jet breakup lengths for all the seven operating conditions mentioned in the test matrix in order to verify that the average jet breakup lengths are greater than the impingement lengths for all cases investigated in the skewed impingement studies.

<table>
<thead>
<tr>
<th>Jet velocity $v_j$ (ft/s)</th>
<th>$K_{\text{injector}}$</th>
<th>Re</th>
</tr>
</thead>
<tbody>
<tr>
<td>34.45</td>
<td>9.0245</td>
<td>11696.03</td>
</tr>
<tr>
<td>49.21</td>
<td>7.6541</td>
<td>16708.61</td>
</tr>
<tr>
<td>60.70</td>
<td>7.0217</td>
<td>20607.28</td>
</tr>
<tr>
<td>70.54</td>
<td>6.6796</td>
<td>23949.01</td>
</tr>
<tr>
<td>78.74</td>
<td>6.4318</td>
<td>26733.77</td>
</tr>
<tr>
<td>82.02</td>
<td>6.2808</td>
<td>27847.68</td>
</tr>
<tr>
<td>86.94</td>
<td>6.2438</td>
<td>29518.54</td>
</tr>
<tr>
<td>93.50</td>
<td>6.1414</td>
<td>31746.36</td>
</tr>
</tbody>
</table>

Table 4.1: Cavitation Number and Reynolds Number for the Injector

Table 4.1 shows the Reynolds number and the injector cavitation number for all the conditions tested for the single jet experiments. All the Reynolds number in the table are sufficiently greater than 2900. This validates the fully developed turbulent flow assumption in the injectors for all the operating condition. The flow through the injector is observed to be free of cavitation effects. As seen in Section 3.3.1, cavitation numbers of the injectors ($K_{\text{injector}}$) are sufficiently greater than 1, for all the operating conditions. Also, injector cavitation numbers are
much higher than orifice cavitation number ($K_{injector} > K_{orifice}$), supporting the conclusion that effects of cavitation in the injector can be reasonably ignored in the injectors.

**Figure 4.1: Single Jet Snapshots**

a) $v_j = 34.45$ ft/s, b) $v_j = 49.21$ ft/s, c) $v_j = 60.70$ ft/s, d) $v_j = 70.54$ ft/s, e) $v_j = 78.74$ ft/s, f) $v_j = 82.02$ ft/s, g) $v_j = 86.94$ ft/s, h) $v_j = 93.50$ ft/s

Figure 4.1 shows the sample frames of all the operating conditions tested for single jet experiments. The jet breakup length is the distance measured from the injector exit point to the point where the jets breaks or segmented. As mentioned earlier, each operating condition is captured at 10,000 frames per second and 201 frames of data is saved. So, for each operating pressure, 0.0201 seconds of data is processed through the MATLAB code to obtain the jet breakup lengths.
Figure 4.2 illustrates the jet breakup length of 34.45 ft/s and 78.74 ft/s plotted against time. It is observed that, for every operating pressure (and flowrate) tested, the jet breakup length follows a pattern with time. The jet breakup length increases for a fraction of a second, falls suddenly to a minimum point and increases again and this pattern continues. Figure 4.2 shows that, the frequency of this pattern increases with the increase in jet velocity. That is, the lengthening and then sudden shortening of the breakup length cycle occurs more frequently in time as the jet velocity increases. Note that all the operating jet velocities (operating pressures) for the single jet experiments are not plotted against the time, as it creates an ungraceful graph.
Figure 4.3 shows the variation of jet breakup length with time for jet velocities 49.21 ft/s and 82.02 ft/s. In the Figure 4.3, the increase-dip-increase pattern is seen only for the jet velocity 49.21 ft/s and almost a straight horizontal line is seen for the 82.02 ft/s jet velocity. The straight line for jet velocity 82.02 ft/s, only illustrates that the jet breakup lengths are greater than or equal to 0.425962 ft (130mm of the jet or 800 pixels of the images). The reason for the straight line is that the breakup of the jet occurred below the field of view for the camera. The points at 0.43 ft do not represent a breakup length of 0.43 ft but that the breakup length is greater than 0.43 ft. Note that the jet length is measured up to 800 pixels in the MATLAB for single jet experiments. However, the same pattern is followed by the jets of all the jet velocities.

The average or the mean jet breakup length increases with the increase in jet velocity. The standard deviation of the fluctuation in 0.0201 seconds decreases with the increase in the jet velocity. The maximum standard deviation observed for the jet breakup length 0.3133 ft and it is 12.09% of the mean value. The minimum jet breakup lengths are tabulated in the Table 4.2. Note that the jet lengths are measured up to the 800 pixels of the images. For the higher jet velocities, the MATLAB code might not reach to the breakup point of the jet, as they are out of the frame captured. Figure 4.4 illustrates the variation of the minimum jet breakup lengths with respect to the jet velocity.

<table>
<thead>
<tr>
<th>Jet velocity $v_j$ (ft/s)</th>
<th>Minimum Jet Breakup lengths (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>34.45</td>
<td>0.2308</td>
</tr>
<tr>
<td>49.21</td>
<td>0.2652</td>
</tr>
<tr>
<td>60.70</td>
<td>0.2838</td>
</tr>
<tr>
<td>70.54</td>
<td>0.2302</td>
</tr>
<tr>
<td>78.74</td>
<td>0.3024</td>
</tr>
<tr>
<td>82.02</td>
<td>0.3489</td>
</tr>
<tr>
<td>86.94</td>
<td>0.3489</td>
</tr>
<tr>
<td>93.50</td>
<td>0.3696</td>
</tr>
</tbody>
</table>

Table 4.2: Minimum Jet Breakup Lengths of Single Jet Tests
The objective of single jet tests are to verify that the jet breakup lengths are greater than the maximum impingement length of the right injector for the linearly skewed impingement test condition. The maximum length used for the linearly skewed impingement tests is 0.3466 ft (or 105.65 mm, from Table 3.3). As mentioned earlier, the operating jet velocities for the impinging jets tests are 49.21 ft/s and 82.02 ft/s. The mean jet breakup lengths given in Table 4.2, are the average of jet breakup lengths of 201 images. The mean jet breakup lengths obtained for the jet velocities 49.21 ft/s and 82.02 ft/s, are greater than 0.3466 ft. But for the 49.21 ft/s jet velocity operating condition, out of 201 frames, 41 frames have the jet breakup lengths between 0.2652 ft (80.83 mm) and 0.3466 ft (105.65 mm). This shows that there is a 20.4% chance the jets impinge after the primary breakup point of the right jet for the linearly skewed impingement conditions above 0.2625 ft (3.15 in or 80 mm) impingement lengths, operating at a jet velocity of 49.21 ft/s. This is an important point to note while understanding the results of impinging jet experiments for 49.21 ft/s jet velocity operating conditions. However, the minimum jet breakup length measured was 0.3483 ft (106.17 mm), for the 82.02 ft/s s jet velocity condition. This verifies that all the

Figure 4.4: Minimum Jet Breakup Length versus Jet Velocity

The objective of single jet tests are to verify that the jet breakup lengths are greater than
impingements are well within the primary breakup points of jets for 82.02 ft/s jet velocity operating condition of the linearly skewed impingement experiments.

4.2. Like Doublet Impinging Jets Results

The details of the test matrix are presented in Section 3.1.2 and the procedure followed to conduct the experiments are detailed in Section 3.3.3. The sheet angle measurements are obtained using the Phantom Cine Viewer software and the sheet breakup length measurements are obtained using the MATLAB image processing tools. The variation in the sheet angle and the sheet breakup lengths and the variation in the visual sheet characteristics are described in the next subsections.

4.2.1. Sheet Angles

The sheet angle is defined as the angle between the two tangents drawn from the point of impingement and passing through the edges of most of the ligaments, formed before the primary atomization zone of the sheet. The sheet angle measurements are determined using the Phantom Cine Viewer software. Every 0th, 50th, 100th, 150th and 201st frame of the 201 frames high-speed video of each setpoint is analyzed in the Cine Viewer software. The measured sheet angles are averaged and tabulated. Table B.1 and Table B.3 in Appendix B, show the variation of the sheet angle with respect to all the impingement angles, the perfect impingement conditions, the skewed impingement conditions, partial impingement conditions and both the jet velocities. Followed by, 31 graphs that show the variations of sheet angle with respect to all the operating condition.

The investigation of the sheet angles could be considered as one of the pioneer works. Most of the studies from the literature concentrate on the study of the sheet breakup lengths, the droplet sizes, and the ligament wavelengths.
Figure 4.5 illustrates the variation of the sheet angle with respect to the angle of impingements, for perfect impingement condition ($y = 0$ in and $z = 0$ in) and for both the jet velocities ($v_j = 49.21$ ft/s and $82.02$ ft/s). The vertical and the horizontal error bars represent the uncertainty of the sheet angle and impingement angle respectively. It can be noticed that the jet velocities have marginal effect on sheet angle. The sheet angles obtained for both the jet velocities were almost equal. The sheet angle is observed to increase with the increase in the angle of impingement. If the graph above is divided into three regions, the first region is between $30^\circ$ and $40^\circ$ impingement angles, the second region between $55^\circ$ and $65^\circ$ impingement angles and the third region between $80^\circ$ and $90^\circ$ impingement angles. The slope of the curve in the second region is greater than that of the first and the third region. That is the increase in the sheet angle between $55^\circ$ and $65^\circ$ is greater. It can be inferred from the Figure 4.5 that the sheet angle is more sensitive to impingement angle between $55^\circ$ and $65^\circ$. Similarly, there is a minimum increase of the sheet angle for the increase in the impingement angle from $65^\circ$ to $80^\circ$. This trend is consistent for all the linearly skewed impingement cases ($y = 0$ in, $0.5$ in, $1$ in, $1.5$ in, and $2$ in) and all the partial impingement conditions ($z = -0.0138$ in, $0$ in, $+0.0138$ in). Figure B.1 to Figure B.15 in Appendix

Figure 4.5: The Sheet Angles versus Impingement Angles
B illustrate the variation of sheet angle with respect to the angle of impingement for all the operating conditions.

![Figure 4.6: The Sheet Angles versus the Partial Impingement Conditions](image)

Figure 4.6 illustrates the variation of the sheet angle for the partial impingement conditions (z = -0.0138 in and +0.0138 in) and the perfect impingement condition (z = 0 in), for all the impingement angles, jet velocity \( v_j = 82.02 \text{ ft/s} \) and \( y\)-offset = 0 in. The horizontal and the vertical errors bars in the graph represent the uncertainty in the measured \( z \)-distance and the sheet angle respectively. As expected, the sheet angle for the partial impingement conditions is less than that of the perfect impingement condition for all the impingement angles, jet velocities and \( y\)-offset conditions. For the partial impingement conditions, \( 2/3 \)\(^{rd} \) of both the jets impinge and remaining \( 1/3 \)\(^{rd} \) of each jet, just pass by the side without impingement. This causes the sheet formed to be of lower sheet angle compared to that of the perfect impingement condition. The images of partial impingements are presented in the next subsections. The sheet angle variation with respect to \( z \) – axis movement of the injector follows the similar trend for all the linearly skewed impingement conditions and jet velocities tested. Figure B.16 to Figure B.25 in Appendix B illustrates the sheet angle variation with respect to \( Z\)-axis movement of the injector.
Figure 4.7 illustrates the variation of the sheet angle with respect to the y-offsets or the linearly skewed impingements tested, for all the angle of impingements, the jet velocity $v_j = 82.02$ ft/s and the $z = 0$ in. The horizontal and the vertical error bars in the graph represent the uncertainty in the y-offset and the sheet angles respectively. The sheet angle remains mostly constant with the increase in the y-offsets. From the graph, it can be inferred that the linearly skewed impingements have minimal effect on the sheet angle. The sheet angle variation trend with respect to y-offset is consistent for all the z-axis movement of the injector and the jet velocities tested. Figure B.26 to Figure B.31 in Appendix B illustrate the variation of the sheet angle with respect to y-offsets for all the impingement angles, partial impingement conditions and the jet velocities tested.

![Sheet Angle vs Linearly Skewed Impingements](image)

**Figure 4.7: The Sheet Angles versus Linearly Skewed Impingements**

### 4.2.2. Sheet Breakup Lengths

The sheet breakup length is defined as the distance from the point of impingement to the point on the sheet where the sheet disintegrates into ligaments and large droplets. The sheet breakup lengths of all the setpoints are determined using the MATLAB image processing tools. 201 frames of the high-speed video of each operating condition are batch processed into 201 .jpg image files and then all the images are processed through the standard MATLAB image processing
functions to determine the average sheet breakup length of each setpoints. The complete process is detailed in Section 3.4.1. The results are tabulated in Table B.5 and Table B.7 of Appendix B which are followed by the graphs that illustrate the variation of sheet breakup lengths with respect to all the operating conditions.

![Graph showing sheet breakup lengths versus angle of impingement]

**Figure 4.8: The Sheet Breakup Lengths versus the Angle of Impingement**

Figure 4.8 illustrates the variation of the sheet breakup length with respect to the angle of impingement for both the jet velocities tested and for the perfect impingement condition (y-offset = 0 in and z = 0 in). The horizontal and vertical error bars represent the uncertainty measured in impingement angle and the sheet breakup lengths respectively. The sheet breakup lengths for jet velocity 82.02 ft/s are marginally higher than that of the 49.21 ft/s jet velocity for most of the impingement angles. The sheet breakup length decreases as the angle of impingement increases. If the Figure 4.8 is divided into three regions, first region from 30° to 40° angle of impingements, the second region from 55° to 65° angle of impingements and the third region from 80° to 90° angle of impingements. The slope of the curve in the second region is comparatively less than that of the first and the third region. From the graph it can be inferred that the sheet breakup lengths decrease greatly from 30° to 40° and 80° to 90° angle of impingements. Also, the change in the
sheet breakup length from 65° angle of impingement to 80° angle of impingement is less. This trend is consistent for all the setpoint tested. Figure B.32 to Figure B.46 in Appendix B shows the variation of the sheet breakup lengths with respect to the impingement angle for all the jet velocities tested, linearly skewed impingement conditions and partial impingement conditions.

The results that the sheet breakup lengths decrease with the increase in the impingement angle and minimal effects of the jet velocity on the sheet breakup lengths are similar to the results seen in the literature. However, relatively lower sensitivity of the sheet breakup lengths for the impingement angles between 55° and 65° and least change in the sheet breakup lengths for the impingement angle between 65° to 80° are the discrete outcomes of this research.

Figure 4.9 shows the variation of sheet breakup length with respect to the partial impingement conditions ($z = -0.0138$ in and $+0.0138$ in) and the perfect impingement condition ($z = 0$ in), for the jet velocity $v_j = 82.02$ ft/s and $y$-offset = 0 in. The horizontal and vertical error bars represent the uncertainties in the measured $z$– distance and the sheet breakup lengths respectively. It can be seen in the graph that the sheet breakup lengths are higher for the partial impingement conditions than that of the perfect impingement condition. For the partial impingements, $1/3^\text{rd}$ of

![Figure 4.9: The Sheet Breakup Lengths versus Partial Impingement Conditions](image-url)
each jet do not impinge on each other causing the sheet formed to be lean and slightly longer than the sheets formed due to perfect impingements of jets. The images presented in the next subsection describe the variation of the sheet breakup lengths and the sheet angles with respect to the partial impingement conditions. The sheet breakup variation with respect to partial impingement conditions follow the similar trend for all the linearly skewed impingements and the jet velocities tested. Figure B.47 to Figure B.56 in Appendix B illustrate the variation of the sheet breakup length with respect to z- axis movement of the injector for all the operating conditions.

![Graph showing Sheet Breakup Lengths versus Linearly Skewed Impingement Conditions](image)

**Figure 4.10: The Sheet Breakup Lengths versus Linearly Skewed Impingement Conditions**

Figure 4.10 shows the variation of the sheet breakup lengths with respect to linearly skewed impingement conditions for all the impingement angles tested, jet velocity $v_j = 82.02$ ft/s and $z = 0$ in. The horizontal and vertical error bars represent the uncertainties in the measured y-offset and the sheet breakup lengths respectively. As it can be seen in the graph, at higher angle of impingements, y-offsets or the linearly skewed conditions have minimal effect on the sheet breakup length and at lower angle of impingements, the sheet breakup lengths vary slightly as the y-offset increases. This trend is consistent for all the partial impingement and perfect impingement conditions and jet velocities tested. Figure B.57 to Figure B.62 in Appendix B illustrate the variation of sheet breakup length with respect to the y-offsets for all the operating conditions.
4.2.3. Visual Sheet Characteristics

<table>
<thead>
<tr>
<th>Angle</th>
<th>Image of Jet Velocity v_j</th>
</tr>
</thead>
<tbody>
<tr>
<td>20°</td>
<td><img src="image1" alt="" /></td>
</tr>
<tr>
<td>30°</td>
<td><img src="image2" alt="" /></td>
</tr>
<tr>
<td>60°</td>
<td><img src="image3" alt="" /></td>
</tr>
<tr>
<td>90°</td>
<td><img src="image4" alt="" /></td>
</tr>
</tbody>
</table>

- **v_j = 49.21 ft/s**
- **v_j = 82.02 ft/s**

**Figure 4.11: Spray Images of Perfect Impingement Conditions for Jet Velocities, 49.21 ft/s and 82.02 ft/s**

Based on the high-speed images observations, certain sheet characteristics can be described. Figure 4.11 presents the spray images of jet velocity \( v_j = 49.21 \) ft/s and 82.02 ft/s, 30°, 60°, and 90° angle of impingements and perfect impingement conditions (y-offset = 0 in and z = 0 in). Figure 4.11 illustrates that the sheet formed at the higher jet velocity \( v_j = 82.02 \) ft/s is more chaotic and disturbed, with a greater number of droplets around, compared to the sheet formed at the jet velocity \( v_j = 49.21 \) ft/s. The ligaments are comparatively flat for the sheets formed at the jet velocity \( v_j = 82.02 \) ft/s and the ligaments of the sheet formed at 49.21 ft/s jet velocity are more like U-shape. From Figure 4.11, it can be observed that the jet velocities have a least effect on the average sheet angles and the average sheet breakup lengths. But the analysis showed that the sheet breaks more rapidly, meaning lengthening and then sudden shortening of the sheet breakup length cycle occurs more frequently in time at the higher jet velocity \( v_j = 82.02 \) ft/s compared to the jet velocity 49.21 ft/s.
Figure 4.12: Spray Images of Angularly Skewed Impingements for the Jet Velocity 82.02 ft/s

Figure 4.12 illustrates the spray sheets for all the angularly skewed impingement conditions for the jet velocity $v_j = 82.02$ ft/s. The terms $\theta_L$ and $\theta_R$ represent impingement angle of the left and the right jets respectively from the plane of the sheet formed, the terms $L_R$ and $L_L$ represent the impingement lengths of right and left jet respectively and the superscript ‘2’ represents angularly skewed impingement angle $\theta_L = 15^\circ$, $\theta_R = 15^\circ$, $\theta_L = 15^\circ$, $\theta_R = 20^\circ$, $\theta_L = 15^\circ$, $\theta_R = 25^\circ$, $\theta_L = 20^\circ$, $\theta_R = 30^\circ$, $\theta_L = 25^\circ$, $\theta_R = 35^\circ$, $\theta_L = 30^\circ$, $\theta_R = 40^\circ$, $\theta_L = 35^\circ$, $\theta_R = 45^\circ$, $\theta_L = 40^\circ$, $\theta_R = 45^\circ$, $\theta_L = 45^\circ$, $\theta_R = 55^\circ$, $\theta_L = 45^\circ$, $\theta_R = 60^\circ$, $\theta_L = 50^\circ$, $\theta_R = 65^\circ$, $\theta_L = 55^\circ$, $\theta_R = 70^\circ$, $\theta_L = 60^\circ$, $\theta_R = 75^\circ$, $\theta_L = 65^\circ$, $\theta_R = 90^\circ$. The impingement lengths (in) for these conditions are shown in the table below:

<table>
<thead>
<tr>
<th>Impingement angle</th>
<th>Impingement lengths (in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_L = 15^\circ$, $\theta_R = 15^\circ$</td>
<td>$L_R = 2.16$, $L_L = 2.16$</td>
</tr>
<tr>
<td>$\theta_L = 15^\circ$, $\theta_R = 20^\circ$</td>
<td>$L_R^2 = 1.88$, $L_L^2 = 1.91$</td>
</tr>
<tr>
<td>$\theta_L = 15^\circ$, $\theta_R = 25^\circ$</td>
<td>$L_R^2 = 1.68$, $L_L^2 = 1.71$</td>
</tr>
<tr>
<td>$\theta_L = 20^\circ$, $\theta_R = 30^\circ$</td>
<td>$L_R = 2.16$, $L_L = 2.16$</td>
</tr>
<tr>
<td>$\theta_L = 25^\circ$, $\theta_R = 35^\circ$</td>
<td>$L_R^2 = 1.88$, $L_L^2 = 1.91$</td>
</tr>
<tr>
<td>$\theta_L = 30^\circ$, $\theta_R = 40^\circ$</td>
<td>$L_R^2 = 1.68$, $L_L^2 = 1.71$</td>
</tr>
<tr>
<td>$\theta_L = 35^\circ$, $\theta_R = 45^\circ$</td>
<td>$L_R = 2.16$, $L_L = 2.16$</td>
</tr>
<tr>
<td>$\theta_L = 40^\circ$, $\theta_R = 55^\circ$</td>
<td>$L_R^2 = 1.88$, $L_L^2 = 1.91$</td>
</tr>
<tr>
<td>$\theta_L = 45^\circ$, $\theta_R = 60^\circ$</td>
<td>$L_R^2 = 1.68$, $L_L^2 = 1.71$</td>
</tr>
</tbody>
</table>

The terms $L_R$ and $L_L$ represent the impingement lengths of right and left jet respectively.
<table>
<thead>
<tr>
<th>$2\theta$</th>
<th>30°</th>
<th>60°</th>
<th>90°</th>
</tr>
</thead>
<tbody>
<tr>
<td>Impingement lengths (in)</td>
<td>$L_R$</td>
<td>$L_L$</td>
<td>$L_R$</td>
</tr>
<tr>
<td>$y$-offset = 0 in</td>
<td>2.16</td>
<td>2.16</td>
<td>1.27</td>
</tr>
<tr>
<td>Impingement lengths (in)</td>
<td>$L_R$</td>
<td>$L_L$</td>
<td>$L_R$</td>
</tr>
<tr>
<td>$y$-offset = 0.5 in</td>
<td>2.66</td>
<td>2.16</td>
<td>1.77</td>
</tr>
<tr>
<td>Impingement lengths (in)</td>
<td>$L_R$</td>
<td>$L_L$</td>
<td>$L_R$</td>
</tr>
<tr>
<td>$y$-offset = 1 in</td>
<td>3.16</td>
<td>2.16</td>
<td>2.27</td>
</tr>
<tr>
<td>Impingement lengths (in)</td>
<td>$L_R$</td>
<td>$L_L$</td>
<td>$L_R$</td>
</tr>
<tr>
<td>$y$-offset = 1.5 in</td>
<td>3.66</td>
<td>2.16</td>
<td>2.77</td>
</tr>
<tr>
<td>Impingement lengths (in)</td>
<td>$L_R$</td>
<td>$L_L$</td>
<td>$L_R$</td>
</tr>
<tr>
<td>$y$-offset = 2 in</td>
<td>4.16</td>
<td>2.16</td>
<td>3.27</td>
</tr>
</tbody>
</table>

Figure 4.13: Spray Images of Linearly Skewed Impingements for the Jet Velocity 82.02 ft/s
skewed impingement conditions (refer to Table 3.3). The angularly skewed impingements result in shift of the plane of the sheet formed. This is the main effect of the angularly skewed impingement conditions. Care has to be taken about the plane of sheets, when a combination of like doublet injectors are used. As it can be seen in Figure 4.12, the sheet angles increase, and the sheet breakup lengths decrease as the total impingement angle increases.

Figure 4.13 illustrates the spray images of the linearly skewed impingement conditions for 82.02 ft/s jet velocity and 30°, 60°, and 90° angle of impingements. The terms $L_R$ and $L_L$ represent the impingement lengths of the right jet and the left jet respectively and the superscript ‘1’ represents the linearly skewed impingement cases (refer to Table 3.3). As observed in the previous sections, the linearly skewed impingement conditions have the least effect on the sheet breakup lengths and the sheet angles. This is evident in the high-speed spray images as well. However, a close observation of the spray images shows that the distance between the ligaments on the sheet increase and the number of ligaments on the sheet gradually decrease as the y-offset increases.

Figure 4.14 presents the spray images for the partial and the perfect impingement conditions for 82.02 ft/s jet velocity and 30°, 60°, and 90° angle of impingements. For all the partial impingement conditions, two third of both the jets are made to impinge against each other and the remaining one third of each jet pass without impingement, forming sheets with diagonal ligaments. This results in smaller sheet angles and longer sheets than that of the perfect impingement conditions. Figure 4.14 illustrates that, at the higher impingement angles, the effect of partial impingement of jets is comparatively low.
Figure 4.1 illustrates the side view (captured by the high-speed camera) and the front view of the spray for the perfect impingement condition and the partial impingement condition \((z = -0.0138 \text{ in})\), formed at 60° angle of impingement and 82.02 ft/s jet velocity. The spray images on 2\(\theta\) = 30°

2\(\theta\) = 60°

2\(\theta\) = 90°

Figure 4.14: Spray Images of Partial Impingements for 82.02 ft/s Jet Velocity

Figure 4.15: Spray Images of the Perfect Impingement and Partial Impingement of Jets at 82.02 ft/s Jet Velocity and 60° Angle of Impingement a) The Side View b) The Front View

Figure 4.15 illustrates the side view (captured by the high-speed camera) and the front view of the spray for the perfect impingement condition and the partial impingement condition \((z = -0.0138 \text{ ft})\), formed at 60° angle of impingement and 82.02 ft/s jet velocity. The spray images on
the left of Figure 4.15a) and Figure 4.15b) are the images of spray for perfect impingement condition (y-offset = 0 in and z = 0 in). The side view of the spray sheet for the perfect impingement is wider and the front view of the spray sheet is lean. The spray images on the right of Figure 4.15a) and Figure 4.15b) are the spray images for the partial impingement condition (z = -0.0138 in) where just the two third of the jets impinge against each other and the one third of the jets pass without impingement. The side view of the spray for the partial impingement shows a thinner sheet than the perfected impingement condition with the diagonal ligaments. The front view of the spray for the partial impingement conditions shows the wider sheet since the one third of each jet pass without impingement. The plane of the sheet seems to rotate for the partial jet impingement conditions, when it is viewed from the top. This is because, the one third of each jet passes the edge of the sheet without impingement. But it is important to note that the sheet is formed only by the portion of jets that impinge against each other. So it is inferred that the sheet plane rotation and the position of the high speed camera has the least effects on the sheet angle and the sheet breakup length measurements.

As described in Section 3.1, the combination of the angularly skewed impingement conditions, linearly skewed impingement conditions and partial impingement conditions were tested. The results obtained for the sheet angles, the sheet breakup lengths and the visual sheet characteristics are like the results described in this section. All the graphs of the sheet angles and the sheet breakup lengths can be seen in Appendix B.
CHAPTER 5. CONCLUSIONS

The literature provides enough evidence that the spray characteristics and the injectors play a particularly important role in the performance of liquid rocket engines. With that motive, a set of cold flow experiments were conducted at the atmospheric pressure to study the spray sheet characteristics for the different geometrical misalignments of the like-doublet injectors. Two jet velocities, nine angularly skewed impingement cases, five linearly skewed impingement cases, three partial impingement cases of jets and their combinations were tested. The high-speed images of each test conditions were analyzed using the MATLAB image processing tools and the Phantom Cine Viewer 3.3 software to obtain the empirical sheet breakup lengths and the empirical sheet angles.

5.1. Mass Flow Rate Calibration and Single Jet Tests

The results of mass calibration tests showed that the simulant is not throttled at the 0.038 in orifice but throttled in the injectors. The cavitation numbers of the orifice and the glass injectors were sufficiently greater than one for all the mass flowrates tested. This is shows that the flow was free of cavitation for all the mass flow rates tested. The results of the mass flow calibration tests supported the belief that a fully developed turbulent flow that is free of effects of cavitation and hydraulic flips, is formed for all the mass flowrates tested.

The results of single jet experiments showed that the average jet breakup length increases as the velocity of the jet increases and the jet breaks more rapidly for the higher jet velocities. The main objective is to verify that the jet breakup lengths for 49.21 ft/s and 82.02 ft/s jet velocities are greater than the all the impingement lengths of the jets chosen for linearly skewed impingement experiments. Though the average jet breakup lengths are greater than all the impingement lengths of jets of linearly skewed impingement studies, 41 high-speed images out of 201 images for 49.21
ft/s jet velocity tests showed the jet breakup lengths between 3.18 in (80.83 mm) and 4.16 in (105.65 mm). This indicated that there is 20.4% chance that the jets impinge after the primary breakup point of the right jet for the linearly skewed impingement conditions with the impingement lengths greater than 3.15 in (80 mm), operating at a jet velocity of 49.21 ft/s. However, all the jet breakup lengths for the jet velocity 82.02 ft/s, were found to be greater than all the impingement lengths of linearly skewed tests.

5.2. Impinging Jets Tests

The results of impinging jet experiments show that the sheet angles increase, and the sheet breakup lengths decrease with the increase in the impingement angles. For the conditions investigated at $\gamma$-offset = 0 in and $z$-distance = 0 in, the sheet angles increased from 39° to 110° and the sheet breakup length decreased from 2.16 in to 1.15 in, as the impingement angle increased from 30° to 90°. The sheet angles are relatively more sensitive, and the sheet breakup lengths are relatively less sensitive to the change in impingement angles between 55° and 65°. There is a marginal change in both the sheet angle and the sheet breakup lengths for the impingement angles between 65° and 80°.

Partial impingement of jets results in the decreased sheet angles and increased sheet breakup lengths. This is due to the formation of diagonal ligaments when, only the part of the jets are impinged.

It is observed that the linearly skewed impingements and the jet velocities have the least effects on both the sheet angle angles and the sheet breakup lengths.

The observations of the high-speed images showed that the, more disturbed and chaotic sheet are formed with a greater number of droplets for the jet velocity of 82.02 ft/s compared to that of the jet velocity 49.21 ft/s. The ligaments on the sheets are observed to be comparatively flat
for the jet velocities 82.02 ft/s and the ligaments on the sheets are U-shaped for the jet velocity 49.21 ft/s respectively. Though the jet velocities have minimum effect on the average sheet breakup lengths, the lengthening and then sudden shortening of the sheet breakup length cycle occurs more rapidly at the jet velocity 82.02 ft/s compared to that of the jet velocity 49.21 ft/s.

The angularly skewed impingements shift the plane of the sheet formed and its effect on the sheet characteristics is same as that of the change in the angle of impingements. It can be noticed in the images that the number of ligaments decrease and the distance between the ligaments gradually increase as the linear y-offset increases.

5.3. Future Work Considerations

Observations from the graphs of sheet angle versus impingement angle show that, the increase in the sheet angle for the impingement angle between 55° and 65° is greater than the other regions of the graph and the change in the sheet angle for the impingement angles between 65° and 80° is very small. Similarly, observations from the sheet breakup length versus impingement angle show that the decrease in the sheet angle for the impingement angle between 55° and 65° is lesser compared to the other regions of the graph and the change in the sheet breakup length for the impingement angle between 65° and 80° is minimum. This is one of the important findings of this research and further work is recommended to confirm these results.

These results could be used as preliminary data for hot firing tests to determine the optimal mixing and atomization conditions. The results of this experiments can also aid the development of analytical models for the like doublet impinging injectors.
APPENDICES
APPENDIX A. MATLAB CODE AND DEMONSTRATION

A.1 Published MATLAB code

clear;
clc;

srcFiles = dir('C:\Users\kvswa\Desktop\single jets\frames\P1 701.6\*.jpg');
% the folder in which your images exists
for i = 1 : length(srcFiles)
  % Reading images
  filename = strcat('C:\Users\kvswa\Desktop\single jets\frames\P1 701.6\',srcFiles(i).name);
  A = imread(filename);
  B = im2double(imread(filename,'jpg'));
  % Convert the image to double precision
  % figure, imshow(B);
  % title ('The original spray image');
  % Enhance the image
  C = imadjust(B);
  D = imadjust(C);
  % figure, imshow(D);
  % h = imdistline;
  % title ('enhanced spray image');
  % flat field correction is only for single jet tests
  sigma = 70;
  Iflatfield = imflatfield(D,sigma);
  % 2-D color and grayscale image flat-field correction
  % figure, imshow(Iflatfield);
  % h = imdistline;
  % title ('Flat field correction');
  % Image segmentation and binary conversion
  [~,threshold] = edge(D,'sobel');
  % Finding the threshold of intensity using Edge-sobel method
  ThresholdfudgeFactor = 0.52;
  % Further intensity corrections by a factor of 0.5
  BWs = edge(D,'sobel',threshold * ThresholdfudgeFactor);
  % Edge detection and conversion to binary image using Edge-sobel
  % figure, imshow(BWs);
  % title ('Segmented Binary-sobel spray');
  % Dilate the Image
  % se90 = strel('line',2,90);
  % Create a line structure vertically using strel-line
  se0 = strel('disk',1);
  % BWsdil = imdilate(BWs,[se90 se0]);
  % Dilate the binary image
  BWsdil = imdilate(BWs,se0);
  % figure, imshow(BWsdil);
  % title ('Dilated binary spray');
% Smoothen the spray
seD = strel('disk',1,4); % Create diamond structures of size 1
BWfinal = imerode(BWsdil,seD); % Erode the spray image with diamond structure

% Figure, imshow(BWfinal);
% title ('Gaps filled smooth spray image');

% Extract the spray sheet, neglecting the water droplets and broken ligaments
BWao = bwareaopen(BWfinal,950,8); % Extrat the spray and delete unwanted noise

% Figure, imshow(BWao);
% title ('Extracted binary sheet');

% Smooth the edges and close any minor holes
nhood = true(9);
closeBWao = imclose(BWao,nhood); % Smoothens the edges and closes the any minor holes in 9 by 9 neighborhood

% Figure, imshow(closeBWao);
% title ('Smooth sheet');

% Make sure the holes are filled
% roughMask = imfill(closeBWao,'holes'); % Fill the holes using imfill

% Figure, imshow(roughMask);
% title ('Completely filled binary image of the sheets');

% Remove unconnected ligaments, if any
% mask2 = bwareaopen(roughMask,1000,8); % Extract the first sheet

% Figure, imshow(mask2);
% title ('Completely filled binary image of the first sheet');

% Place the binary Sheet on the original image to verify
I2 = B;
I2(mask2) = 0;
% Figure, imshow(I2);
% title ('Created binary sheet on the original image');

% Pixel count for Total Breakup length
count = 0;
for j=22:681
    semicount = 0;
    for i=1:128
        semicount = semicount + mask2(j,i);
    end
    if (semicount==0)
        break
    end
    if (semicount>0)
        count = count + 1;
    end
end

Total_breakup_length = count * 0.00054680666667; % Multiply with the scale
A.2 Setpoint specific details

Variables used for the each setpoint of the experiments are listed below.

A.2.1 Single jet experiments

For all the jet velocities ($v_j$) (34.45 ft/s, 49.21 ft/s, 60.70 ft/s, 70.54 ft/s, 78.74 ft/s, 82.02 ft/s, 86.94 ft/s, 93.50 ft/s)

- Scale obtained from the Cine Viewer software: 0.00054680666667 ft/pixel
- The injector tip from the top of the image: $j = 22$ pixels
- Sigma used for the ‘imflatfield’ function (MATLAB code): 70
- Threshold correction factor used for the edge detection and binary conversion: 0.52
- Desired connectivity of the ‘bwareaopen’ function: 950

A.2.2 Impinging jets experiments

For the jet velocities ($v_j$): 49.21 ft/s and the angle of impingement ($2\theta$): 30°. For all the linearly skewed impingements ($y = 0$in, 0.5in, 1in, 1.5in and 2in) and for all the partial impingements and perfect impingement ($z = -0.0138$ in, 0 in and +0.0138 in).

- Scale obtained from the Cine Viewer: 0.00036453777778 ft/pixel
- Point of impingement from the top of the image: $j = 45$ pixels
- Threshold correction factor used for the edge detection and binary conversion: 0.49
- Desired connectivity of the ‘bwareaopen’ function: 7000

For the jet velocities ($v_j$): 49.21 ft/s and the angle of impingement ($2\theta$): 35°. For all the linearly skewed impingements ($y = 0$in, 0.5in, 1in, 1.5in and 2in) and for all the partial impingements and perfect impingement ($z = -0.0138$ in, 0 in and +0.0138 in).
• Scale obtained from the Cine Viewer: 0.00035026539770 ft/pixel
• Point of impingement from the top of the image: j = 45 pixels
• Threshold correction factor used for the edge detection and binary conversion: 0.49
• Desired connectivity of the ‘bwareaopen’ function: 7000

For the jet velocities \( v_j \): 49.21 ft/s and the angle of impingement \( 2\theta \): 40°. For all the linearly skewed impingements \( y = 0\text{in}, 0.5\text{in}, 1\text{in}, 1.5\text{in} \) and 2in) and for all the partial impingements and perfect impingement \( z = -0.0138 \text{in}, 0 \text{in} \) and +0.0138 in).  

• Scale obtained from the Cine Viewer: 0.00033251566999 ft/pixel
• Point of impingement from the top of the image: j = 43 pixels
• Threshold correction factor used for the edge detection and Binary conversion: 0.49
• Desired connectivity of the ‘bwareaopen’ function: 7000

For the jet velocities \( v_j \): 49.21 ft/s and the angle of impingement \( 2\theta \): 55°. For all the linearly skewed impingements \( y = 0\text{in}, 0.5\text{in}, 1\text{in}, 1.5\text{in} \) and 2in) and for all the partial impingements and perfect impingement \( z = -0.0138 \text{in}, 0 \text{in} \) and +0.0138 in).  

• Scale obtained from the Cine Viewer: 0.000372822727273 ft/pixel
• Point of impingement from the top of the image: j = 43 pixels
• Threshold correction factor used for the edge detection and Binary conversion: 0.49
• Desired connectivity of the ‘bwareaopen’ function: 3500

For the jet velocities \( v_j \): 49.21 ft/s and the angle of impingement \( 2\theta \): 60°. For all the linearly skewed impingements \( y = 0\text{in}, 0.5\text{in}, 1\text{in}, 1.5\text{in} \) and 2in) and for all the partial impingements and perfect impingement \( z = -0.0138 \text{in}, 0 \text{in} \) and +0.0138 in).  

• Scale obtained from the Cine Viewer: 0.000340569587479 ft/pixel
• Point of impingement from the top of the image: j = 96 pixels
• Threshold correction factor used for the edge detection and Binary conversion: 0.49
• Desired connectivity of the ‘bwareaopen’ function: 3500

For the jet velocities ($v_j$): 49.21 ft/s and the angle of impingement ($2\theta$): 65°. For all the linearly skewed impingements ($y = 0\text{in}, 0.5\text{in}, 1\text{in}, 1.5\text{in} \text{and} 2\text{in}$) and for all the partial impingements and perfect impingement ($z = -0.0138 \text{in}, 0 \text{in} \text{and} +0.0138 \text{in}$).

• Scale obtained from the Cine Viewer: 0.000340571626298 ft/pixel
• Point of impingement from the top of the image: $j = 68$ pixels
• Threshold correction factor used for the edge detection and Binary conversion: 0.49
• Desired connectivity of the ‘bwareaopen’ function: 3500

For the jet velocities ($v_j$): 49.21 ft/s and the angle of impingement ($2\theta$): 80°. For all the linearly skewed impingements ($y = 0\text{in}, 0.5\text{in}, 1\text{in}, 1.5\text{in} \text{and} 2\text{in}$) and for all the partial impingements and perfect impingement ($z = -0.0138 \text{in}, 0 \text{in} \text{and} +0.0138 \text{in}$).

• Scale obtained from the Cine Viewer: 0.00038597820856 ft/pixel
• Point of impingement from the top of the image: $j = 90$ pixels
• Threshold correction factor used for the edge detection and Binary conversion: 0.49
• Desired connectivity of the ‘bwareaopen’ function: 4500

For the jet velocities ($v_j$): 49.21 ft/s and the angle of impingement ($2\theta$): 85°. For all the linearly skewed impingements ($y = 0\text{in}, 0.5\text{in}, 1\text{in}, 1.5\text{in} \text{and} 2\text{in}$) and for all the partial impingements and perfect impingement ($z = -0.0138 \text{in}, 0 \text{in} \text{and} +0.0138 \text{in}$).

• Scale obtained from the Cine Viewer: 0.000381493023256 ft/pixel
• Point of impingement from the top of the image: $j = 106$ pixels
• Threshold correction factor used for the edge detection and Binary conversion: 0.49
• Desired connectivity of the ‘bwareaopen’ function: 4500
For the jet velocities \((v_j)\): 49.21 ft/s and the angle of impingement \((2\theta)\): 90°. For all the linearly skewed impingements \((y = 0\text{ in}, 0.5\text{ in}, 1\text{ in}, 1.5\text{ in and } 2\text{ in})\) and for all the partial impingements and perfect impingement \((z = -0.0138 \text{ in}, 0 \text{ in and } +0.0138 \text{ in})\).

- Scale obtained from the Cine Viewer: 0.000364535277544 ft/pixel
- Point of impingement from the top of the image: \(j = 80\) pixels
- Threshold correction factor used for the edge detection and Binary conversion: 0.49
- Desired connectivity of the ‘bwareaopen’ function: 4500

For the jet velocities \((v_j)\): 82.02 ft/s and the angle of impingement \((2\theta)\): 30°. For all the linearly skewed impingements \((y = 0\text{ in}, 0.5\text{ in}, 1\text{ in}, 1.5\text{ in and } 2\text{ in})\) and for all the partial impingements and perfect impingement \((z = -0.0138 \text{ in}, 0 \text{ in and } +0.0138 \text{ in})\).

- Scale obtained from the Cine Viewer: 0.000361854907455 ft/pixel
- Point of impingement from the top of the image: \(j = 64\) pixels
- Threshold correction factor used for the edge detection and Binary conversion: 0.58
- Desired connectivity of the ‘bwareaopen’ function: 7000

For the jet velocities \((v_j)\): 82.02 ft/s and the angle of impingement \((2\theta)\): 35°. For all the linearly skewed impingements \((y = 0\text{ in}, 0.5\text{ in}, 1\text{ in}, 1.5\text{ in and } 2\text{ in})\) and for all the partial impingements and perfect impingement \((z = -0.0138 \text{ in}, 0 \text{ in and } +0.0138 \text{ in})\).

- Scale obtained from the Cine Viewer: 0.000349025531915 ft/pixel
- Point of impingement from the top of the image: \(j = 41\) pixels
- Threshold correction factor used for the edge detection and Binary conversion: 0.58
- Desired connectivity of the ‘bwareaopen’ function: 7000
For the jet velocities ($v_j$): 82.02 ft/s and the angle of impingement ($2\theta$): 40°. For all the linearly skewed impingements ($y = 0$ in, 0.5 in, 1 in, 1.5 in and 2 in) and for all the partial impingements and perfect impingement ($z = -0.0138$ in, 0 in and $+0.0138$ in).

- Scale obtained from the Cine Viewer: 0.000334779591837 ft/pixel
- Point of impingement from the top of the image: $j = 40$ pixels
- Threshold correction factor used for the edge detection and Binary conversion: 0.58
- Desired connectivity of the ‘bwareaopen’ function: 7000

For the jet velocities ($v_j$): 82.02 ft/s and the angle of impingement ($2\theta$): 55°. For all the linearly skewed impingements ($y = 0$ in, 0.5 in, 1 in, 1.5 in and 2 in) and for all the partial impingements and perfect impingement ($z = -0.0138$ in, 0 in and $+0.0138$ in).

- Scale obtained from the Cine Viewer: 0.000387497784299 ft/pixel
- Point of impingement from the top of the image: $j = 72$ pixels
- Threshold correction factor used for the edge detection and Binary conversion: 0.58
- Desired connectivity of the ‘bwareaopen’ function: 3500

For the jet velocities ($v_j$): 82.02 ft/s and the angle of impingement ($2\theta$): 60°. For all the linearly skewed impingements ($y = 0$ in, 0.5 in, 1 in, 1.5 in and 2 in) and for all the partial impingements and perfect impingement ($z = -0.0138$ in, 0 in and $+0.0138$ in).

- Scale obtained from the Cine Viewer: 0.000374237598952 ft/pixel
- Point of impingement from the top of the image: $j = 68$ pixels
- Threshold correction factor used for the edge detection and Binary conversion: 0.58
- Desired connectivity of the ‘bwareaopen’ function: 3500
For the jet velocities \( (v_j) \): 82.02 ft/s and the angle of impingement \((2\theta)\): 65°. For all the linearly skewed impingements \((y = 0\text{in}, 0.5\text{in}, 1\text{in}, 1.5\text{in} \text{and} 2\text{in})\) and for all the partial impingements and perfect impingement \((z = -0.0138 \text{ in}, 0 \text{ in and} +0.0138 \text{ in})\).

- Scale obtained from the Cine Viewer: 0.000355323316333 ft/pixel
- Point of impingement from the top of the image: \( j = 37 \) pixels
- Threshold correction factor used for the edge detection and Binary conversion: 0.58
- Desired connectivity of the ‘bwareaopen’ function: 3500

For the jet velocities \( (v_j) \): 82.02 ft/s and the angle of impingement \((2\theta)\): 80°. For all the linearly skewed impingements \((y = 0\text{in}, 0.5\text{in}, 1\text{in}, 1.5\text{in} \text{and} 2\text{in})\) and for all the partial impingements and perfect impingement \((z = -0.0138 \text{ in}, 0 \text{ in and} +0.0138 \text{ in})\).

- Scale obtained from the Cine Viewer: 0.000377108045977 ft/pixel
- Point of impingement from the top of the image: \( j = 67 \) pixels
- Threshold correction factor used for the edge detection and Binary conversion: 0.6
- Desired connectivity of the ‘bwareaopen’ function: 4500

For the jet velocities \( (v_j) \): 82.02 ft/s and the angle of impingement \((2\theta)\): 85°. For all the linearly skewed impingements \((y = 0\text{in}, 0.5\text{in}, 1\text{in}, 1.5\text{in} \text{and} 2\text{in})\) and for all the partial impingements and perfect impingement \((z = -0.0138 \text{ in}, 0 \text{ in and} +0.0138 \text{ in})\).

- Scale obtained from the Cine Viewer: 0.000381467235374 ft/pixel
- Point of impingement from the top of the image: \( j = 55 \) pixels
- Threshold correction factor used for the edge detection and Binary conversion: 0.58
- Desired connectivity of the ‘bwareaopen’ function 4500
For the jet velocities ($v_j$): 82.02 ft/s and the angle of impingement ($2\theta$): 90°. For all the linearly skewed impingements ($y = 0$ in, 0.5 in, 1 in, 1.5 in and 2 in) and for all the partial impingements and perfect impingement ($z = -0.0138$ in, 0 in and +0.0138 in).

- Scale obtained from the Cine Viewer: 0.00035921658394 ft/pixel
- Point of impingement from the top of the image: $j = 31$ pixels
- Threshold correction factor used for the edge detection and Binary conversion: 0.58
- Desired connectivity of the ‘bwareaopen’ function: 4500
A.3. Demonstration of image processing used

A.3.1. Single jet image processing

A.3.2. Impinging jets image processing

APPENDIX B. RESULTS AND GRAPHS

B.1. The Sheet Angle Result Graphs

B.1.1. Sheet Angle versus Angularly Skewed Impingements

Figure B.1: Sheet Angle versus Impingement Angle for \( y\)-offset = 0 in, \( z = 0 \) in

Figure B.2: Sheet Angle versus Impingement Angle for \( y\)-offset = 0.5 in, \( z = 0 \) in

Figure B.3: Sheet Angle versus Impingement Angle for \( y\)-offset = 1 in, \( z = 0 \) in

Figure B.4: Sheet Angle versus Impingement Angle for \( y\)-offset = 1.5 in, \( z = 0 \) in
Figure B.5: Sheet Angle versus Impingement Angle for y-offset = 2 in, z = 0 in

Figure B.6: Sheet Angle versus Impingement Angle for y-offset = 0 in, z = -0.0138 in

Figure B.7: Sheet Angle versus Impingement Angle for y-offset = 0.5 in, z = -0.0138 in

Figure B.8: Sheet Angle versus Impingement Angle for y-offset = 1 in, z = -0.0138 in
Figure B.9: Sheet Angle versus Impingement Angle for $y$-offset = 1.5 in, $z = -0.0138$ in

Figure B.10: Sheet Angle versus Impingement Angle for $y$-offset = 2 in, $z = -0.0138$ in

Figure B.11: Sheet Angle versus Impingement Angle for $y$-offset = 0 in, $z = 0.0138$ in

Figure B.12: Sheet Angle versus Impingement Angle for $y$-offset = 0.5 in, $z = 0.0138$ in
Figure B.13: Sheet Angle versus Impingement Angle for y-offset = 1 in, $Z = 0.0138$ in

Figure B.14: Sheet Angle versus Impingement Angle for y-offset = 1.5 in, $Z = 0.0138$ in

Figure B.15: Sheet Angle versus Impingement Angle for y-offset = 2 in, $Z = 0.0138$ in
B.1.2. Sheet Angle versus Partial Skewed Impingements

Figure B.16: Sheet Angle versus $z$ – Distance for $y$-offset = 0 in, $v_j = 49.21$ ft/s

Figure B.17: Sheet Angle versus $z$ – Distance for $y$-offset = 0.5 in, $v_j = 49.21$ ft/s

Figure B.18: Sheet Angle versus $z$ – Distance for $y$-offset = 1 in, $v_j = 49.21$ ft/s

Figure B.19: Sheet Angle versus $z$ – Distance for $y$-offset = 1.5 in, $v_j = 49.21$ ft/s
Figure B.20: Sheet Angle versus z – Distance for y-offset = 2 in, \( V_j = 49.21 \) ft/s

Figure B.21: Sheet Angle versus z – Distance for y-offset = 0 in, \( V_j = 82.02 \) ft/s

Figure B.22: Sheet Angle versus z – Distance for y-offset = 0.5 in, \( V_j = 82.02 \) ft/s

Figure B.23: Sheet Angle versus z – Distance for y-offset = 1 in, \( V_j = 82.02 \) ft/s
B.1.3. Sheet Angle versus Linearly Skewed Impingements

Figure B.24: Sheet Angle versus \( z \) – Distance for \( y \)-offset = 1.5 in, \( v_j = 82.02 \) ft/s
Figure B.25: Sheet Angle versus \( z \) – Distance for \( y \)-offset = 2 in, \( v_j = 82.02 \) ft/s

Figure B.26: Sheet Angle versus \( y \) – Offset for \( z = 0 \) in, \( v_j = 49.21 \) ft/s
Figure B.27: Sheet Angle versus \( y \) – Offset for \( z = -0.0138 \) in, \( v_j = 49.21 \) ft/s
B.2. Sheet Angle Results

Figure B.28: Sheet Angle versus y – Offset for \( z = 0.0138 \text{ in}, \ v_j = 49.21 \text{ ft/s} \)

Figure B.29: Sheet Angle versus y – Offset for \( z = 0 \text{ in}, \ v_j = 82.02 \text{ ft/s} \)

Figure B.30: Sheet Angle versus y – Offset for \( z = -0.0138 \text{ in}, \ v_j = 82.02 \text{ ft/s} \)

Figure B.31: Sheet Angle versus y – Offset for \( z = 0.0138 \text{ in}, \ v_j = 82.02 \text{ ft/s} \)
Table B.1: Sheet Angle (SA) Results (°)

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Table B.2: Sheet Angle Uncertainty (SAU) Results (°)

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### Table B.3: Sheet Angle (SA) Results (°)

Injector jet velocities 82.02 ± 2.81 ft/s

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B.3. The Sheet Breakup Length Results Graphs

B.3.1. Sheet Breakup Length versus Angularly Skewed Impingements

Figure B.32: Sheet Breakup Length versus Impingement Angle for y-offset = 0 in, z = 0 in

Figure B.33: Sheet Breakup Length versus Impingement Angle for y-offset = 0.5 in, z = 0 in

Figure B.34: Sheet Breakup Length versus Impingement Angle for y-offset = 1 in, z = 0 in

Figure B.35: Sheet Breakup Length versus Impingement Angle for y-offset = 1.5 in, z = 0 in
Figure B.36: Sheet Breakup Length versus Impingement Angle for \( y \)-offset = 2 in, \( z = 0 \) in

Figure B.37: Sheet Breakup Length versus Impingement Angle for \( y \)-offset = 0 in, \( z = -0.0138 \) in

Figure B.38: Sheet Breakup Length versus Impingement Angle for \( y \)-offset = 0.5 in, \( z = -0.0138 \) in

Figure B.39: Sheet Breakup Length versus Impingement Angle for \( y \)-offset = 1 in, \( z = -0.0138 \) in
Figure B.40: Sheet Breakup Length versus Impingement Angle for $y$-offset = 1.5 in, $z = -0.0138$ in

Figure B.41: Sheet Breakup Length versus Impingement Angle for $y$-offset = 2 in, $z = -0.0138$ in

Figure B.42: Sheet Breakup Length versus Impingement Angle for $y$-offset = 0 in, $z = 0.0138$ in

Figure B.43: Sheet Breakup Length versus Impingement Angle for $y$-offset = 0.5 in, $z = 0.0138$ in
Figure B.44: Sheet Breakup Length versus Impingement Angle for y-offset = 1 in, \( z = 0.0138 \) in

Figure B.45: Sheet Breakup Length versus Impingement Angle for y-offset = 1.5 in, \( z = 0.0138 \) in

Figure B.46: Sheet Breakup Length versus Impingement Angle for y-offset = 2 in, \( z = 0.0138 \) in
B.3.2. Sheet Breakup Length versus Partial Impingement Conditions

Figure B.47: Sheet Breakup Length versus $z$ – Distance for $y$-offset = 0 in, $v_j = 49.21$ ft/s

Figure B.48: Sheet Breakup Length versus $z$ – Distance for $y$-offset = 0.5 in, $v_j = 49.21$ ft/s

Figure B.49: Sheet Breakup Length versus $z$ – Distance for $y$-offset = 1 in, $v_j = 49.21$ ft/s

Figure B.50: Sheet Breakup Length versus $z$ – Distance for $y$-offset = 1.5 in, $v_j = 49.21$ ft/s
Figure B.51: Sheet Breakup Length versus $z$ – Distance for $y$-offset = 2 in, $v_j = 49.21$ ft/s

Figure B.52: Sheet Breakup Length versus $z$ – Distance for $y$-offset = 0 in, $v_j = 82.02$ ft/s

Figure B.53: Sheet Breakup Length versus $z$ – Distance for $y$-offset = 0.5 in, $v_j = 82.02$ ft/s

Figure B.54: Sheet Breakup Length versus $z$ – Distance for $y$-offset = 1 in, $v_j = 82.02$ ft/s
Figure B.55: Sheet Breakup Length versus \( z \) – Figure B.56: Sheet Breakup Length versus \( z \) – Distance for \( y \)-offset = 1.5 in, \( v_j = 82.02 \text{ ft/s} \) Distance for \( y \)-offset = 2 in, \( v_j = 82.02 \text{ ft/s} \)

B.3.3. Sheet Breakup Length versus Linearly Skewed Impingements

Figure B.57: Sheet Breakup Length versus \( y \) – Figure B.58: Sheet Breakup Length versus \( y \) – Offset for \( z = 0 \) in, \( v_j = 49.21 \text{ ft/s} \) Offset for \( z = -0.0138 \) in, \( v_j = 49.21 \text{ ft/s} \)
B.4. The Sheet Breakup Length Results

Figure B.59: Sheet Breakup Length versus y – Offset for $z = 0.0138$ in, $v_j = 49.21$ ft/s

Figure B.60: Sheet Breakup Length versus y – Offset for $z = 0$ in, $v_j = 82.02$ ft/s

Figure B.61: Sheet Breakup Length versus y – Offset for $z = -0.0138$ in, $v_j = 82.02$ ft/s

Figure B.62: Sheet Breakup Length versus y – Offset for $z = 0.0138$ in, $v_j = 82.02$ ft/s
Table B.5: Sheet Breakup Length (SBL) Results (ft)

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Table B.6: Sheet Breakup Length Uncertainty (SBLU) Results (ft)

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<td>±0.0093</td>
<td>±0.0084</td>
<td>±0.0081</td>
<td>±0.0078</td>
<td>±0.0073</td>
<td>±0.0069</td>
<td>±0.0067</td>
</tr>
<tr>
<td></td>
<td>1.5</td>
<td>±0.0107</td>
<td>±0.0104</td>
<td>±0.0098</td>
<td>±0.0086</td>
<td>±0.0082</td>
<td>±0.0077</td>
<td>±0.0072</td>
<td>±0.0072</td>
<td>±0.0064</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>±0.0101</td>
<td>±0.0105</td>
<td>±0.0093</td>
<td>±0.0086</td>
<td>±0.0083</td>
<td>±0.0076</td>
<td>±0.0075</td>
<td>±0.0066</td>
<td>±0.0066</td>
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</table>
Table B.7: Sheet Breakup Length (SBL) Results (ft)

<table>
<thead>
<tr>
<th>y-offset ± 0.0002 (in)</th>
<th>2θ = 30° ± 0.5°</th>
<th>2θ = 35° ± 0.5°</th>
<th>2θ = 40° ± 0.5°</th>
<th>2θ = 55° ± 0.5°</th>
<th>2θ = 60° ± 0.5°</th>
<th>2θ = 65° ± 0.5°</th>
<th>2θ = 80° ± 0.5°</th>
<th>2θ = 85° ± 0.5°</th>
<th>2θ = 90° ± 0.5°</th>
</tr>
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<tr>
<td>0</td>
<td>0.1842</td>
<td>0.1650</td>
<td>0.1552</td>
<td>0.1343</td>
<td>0.1337</td>
<td>0.1228</td>
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<td>0.1661</td>
<td>0.1534</td>
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<td>0.1410</td>
<td>0.1243</td>
<td>0.1266</td>
<td>0.1234</td>
<td>0.1078</td>
</tr>
<tr>
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<td>0.1381</td>
<td>0.1259</td>
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<td>0.1217</td>
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<tr>
<td>1.5</td>
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<td>0.1658</td>
<td>0.1504</td>
<td>0.1434</td>
<td>0.1378</td>
<td>0.1256</td>
<td>0.1239</td>
<td>0.1237</td>
<td>0.1078</td>
</tr>
<tr>
<td>2</td>
<td>0.1818</td>
<td>0.1709</td>
<td>0.1559</td>
<td>0.1411</td>
<td>0.1406</td>
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<td>0.1243</td>
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<tr>
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<td>0.1618</td>
<td>0.1496</td>
<td>0.1306</td>
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<td>0.1149</td>
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<td>0.1521</td>
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<tr>
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<td>0.1778</td>
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<td>0.1509</td>
<td>0.1388</td>
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<td>0.1644</td>
<td>0.1527</td>
<td>0.1387</td>
<td>0.1346</td>
<td>0.1263</td>
<td>0.1205</td>
<td>0.1145</td>
</tr>
<tr>
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<td>2</td>
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<td>0.1556</td>
<td>0.1470</td>
<td>0.1395</td>
<td>0.1329</td>
<td>0.1211</td>
<td>0.1220</td>
<td>0.1151</td>
</tr>
</tbody>
</table>
**Table B.8: Sheet Breakup Length Uncertainty (SBLU) Results (ft)**

Injector jet velocities  82.02 ± 2.81 ft/s

| z ± 0.0002 (in) | y-offset ± 0.0003 (in) | 2θ = 30°  
|                |                        | ± 0.5° | 2θ = 35°  
|                |                        | ± 0.5° | 2θ = 40°  
|                |                        | ± 0.5° | 2θ = 55°  
|                |                        | ± 0.5° | 2θ = 60°  
|                |                        | ± 0.5° | 2θ = 65°  
|                |                        | ± 0.5° | 2θ = 80°  
|                |                        | ± 0.5° | 2θ = 85°  
|                |                        | ± 0.5° | 2θ = 90°  
|                |                        |       | SBLU (ft) | SBLU (ft) | SBLU (ft) | SBLU (ft) | SBLU (ft) | SBLU (ft) | SBLU (ft) | SBLU (ft) |
| 0              | 0                       | ±0.0117 ±0.0105 ±0.0098 ±0.0085 ±0.0083 ±0.0077 ±0.0078 ±0.0074 ±0.0072 ±0.0067 |
| 0              | 0.5                     | ±0.0116 ±0.0105 ±0.0099 ±0.0088 ±0.0088 ±0.0078 ±0.0079 ±0.0078 ±0.0078 ±0.0068 |
| 1              | 0                       | ±0.0119 ±0.0107 ±0.0097 ±0.0088 ±0.0086 ±0.0079 ±0.0078 ±0.0076 ±0.0076 ±0.0068 |
| 1              | 0.5                     | ±0.0116 ±0.0106 ±0.0097 ±0.0090 ±0.0086 ±0.0079 ±0.0077 ±0.0077 ±0.0077 ±0.0068 |
| 1.5             1.5 | ±0.0116 ±0.0109 ±0.0099 ±0.0089 ±0.0088 ±0.0078 ±0.0078 ±0.0078 ±0.0078 ±0.0068 |
| 2              | 0                       | ±0.0117 ±0.0105 ±0.0095 ±0.0083 ±0.0084 ±0.0076 ±0.0074 ±0.0072 ±0.0067 |
| 2              | 0.5                     | ±0.0116 ±0.0107 ±0.0097 ±0.0088 ±0.0082 ±0.0077 ±0.0075 ±0.0072 ±0.0067 |
| 0              | 1                       | ±0.0119 ±0.0105 ±0.0098 ±0.0085 ±0.0086 ±0.0078 ±0.0074 ±0.0072 ±0.0067 |
| 1              | 0.5                     | ±0.0113 ±0.0106 ±0.0100 ±0.0086 ±0.0084 ±0.0077 ±0.0074 ±0.0073 ±0.0068 |
| 1              | 1.5                     | ±0.0115 ±0.0105 ±0.0097 ±0.0087 ±0.0085 ±0.0076 ±0.0074 ±0.0072 ±0.0068 |
| 1.5             2 | ±0.0118 ±0.0104 ±0.0098 ±0.0089 ±0.0084 ±0.0082 ±0.0082 ±0.0077 ±0.0072 ±0.0068 |
| 2              | 0                       | ±0.0119 ±0.0104 ±0.0100 ±0.0088 ±0.0085 ±0.0080 ±0.0075 ±0.0072 ±0.0068 |
| 2              | 0.5                     | ±0.0117 ±0.0104 ±0.0097 ±0.0087 ±0.0084 ±0.0079 ±0.0078 ±0.0072 ±0.0067 |
| 0              | 1                       | ±0.0118 ±0.0105 ±0.0097 ±0.0087 ±0.0084 ±0.0079 ±0.0079 ±0.0075 ±0.0072 ±0.0066 |
| 1              | 0.5                     | ±0.0116 ±0.0102 ±0.0095 ±0.0087 ±0.0084 ±0.0077 ±0.0076 ±0.0073 ±0.0067 |
APPENDIX C. UNCERTAINTY ANALYSIS

C.1. Uncertainty in Pressure

Uncertainty in upstream pressure, right and left injector pressure readings are calculated as follows. The 95% uncertainty estimate for the Pressure Readings is given by

\[ U_{\Delta P} = \left( B_{P\text{cal}}^2 + B_{P\text{repeat}}^2 + S_P^2 \right)^{\frac{1}{2}} \]

Where, \( B_{P\text{cal}} \) = Systematic calibration uncertainty determined using first order linear least square regression uncertainty method (Ref. Chapter 8 of Coleman and Steele) \[46\].

\[ B_{P\text{cal}} = \left( \frac{\sum_{i=1}^{N_1} (P_i - mV_i - c)^2}{N_1 - 2} \right)^{0.5} \]

Where, \( N_1 \) is number of calibration points, \( V_i \) is the calibration voltage, \( m \) is the slope of calibration curve equation and \( c \) is the constant of calibration curve equation.

\[ B_{P\text{repeat}} = 0.2\% \times \text{Full Scale} \]

95% confidence random Uncertainty is given by,

\[ S_P = \frac{1.96\sigma_P}{\sqrt{N}} \]

\( N \) is the number of pressure readings taken.

C.2. Uncertainty in Mass Flow Rate

Uncertainty in mass flow rate is calculated using the Monte Carlo Analysis in MATLAB. The Monte Carlo simulations used for the uncertainty analysis, generate random sampling to simulate the physical phenomena. The MATLAB code used to determine the mass flow rate uncertainty is attached.
Import data from Injector Cal text file

Parse Data from the two matrices imported into vectors of Data for each measurement

Curve Fit the data

Monte Carlo Uncertainty for Curvefit of mass flow vs injector Pressure and Orifice Pressure

Estimate the residual standard deviation for the difference between predicted and actual

clc
clear
close all

Import data from Injector Cal text file

opts = delimitedTextImportOptions("NumVariables", 8);
 opts.dataLines = [1, Inf];
 opts.delimiter = ";"
 opts.ColumnName = ["VarName1", "VarName2", "VarName3", "VarName4", "VarName5", "VarName6", "VarName7", "VarName8"];
 opts.variableTypes = ["double", "double", "double", "double", "double", "double", "double", "double"];
 opts.ExtraColumnsRule = "ignore";
 opts.EmptyLineRule = "read";
 INJ = readtable("C:\Users\kvswa\Desktop\my thesis\Uncertainty\Swarna Injector Cal.txt", opts);
INJ = table2array(INJ);
clear opts

Parse Data from the two matrices imported into vectors of Data for each measurement

I_P1 = INJ(:,1);
I_P2 = INJ(:,2);
I_Pleft = INJ(:,4);
I_Pr = INJ(:,3);
I_m_left = INJ(:,7);
I_m_right = INJ(:,6);
I_m_orifice = INJ(:,8);

% O_P1 = Orif(:,1);
% O_P1_abs = Orif(:,1)+14.7;
% O_m = Orif(:,2);

Curve Fit the data

Here use the average values from the measurements

% Left Injector only vs P_left
coef_mdotL_PL = polyfit(log(I_Pleft),log(I_m_left/60),1);
coef_mdotL_PL(2) = exp(coef_mdotL_PL(2));
% Right Injector only vs P_Right
coef_mdotR_PR = polyfit(log(I_Pright),log(I_m_right/60),1);
coef_mdotR_PR(2) = exp(coef_mdotR_PR(2));
% Orifice only vs P_upstream
coef_mdotO_PO = polyfit(log(I_P1),log(I_m_orifice/60),1);
coef_mdotO_PO(2) = exp(coef_mdotO_PO(2));

syms x
% Functions for mass flows based on pressures
f_PL_Left = coef_mdotL_PL(2) * x^(coef_mdotL_PL(1));
f_PR_Right = coef_mdotR_PR(2) * x^(coef_mdotR_PR(1));
f_PO_Orifice = coef_mdotO_PO(2) * x^(coef_mdotO_PO(1));

fig2 = figure;
plot(I_Pleft,I_m_left/60,'x','color','b')
hold on
plot(I_Pright,I_m_right/60,'o','color','r')
hold on
plot(I_P1, I_m_orifice/60,'^','color','g')
fpplot(f_PL_Left,[xlim],'--b')
fpplot(f_PR_Right,[xlim],'--r')
fpplot(f_PO_Orifice,[xlim],'--g')

txt3 = ['mdot = ' num2str(coef_mdotL_PL(2),3) ' P_{injector} ^{'} num2str(coef_mdotL_PL(1),3) ']};
text(200, .0325, txt3);
txt4 = ['mdot = ' num2str(coef_mdotR_PR(2),3) ' P_{injector} ^{'} num2str(coef_mdotR_PR(1),3) ']};
text(450, .080, txt4);
txt5 = ['mdot = ' num2str(coef_mdotO_PO(2),3) 'P_{orifice} ^{'} num2str(coef_mdotO_PO(1),3) ']};
text(30, .0725, txt5);
legend('Left Injector','Right Injector','Orifice','Location','NorthWest')
ylabel('Injector Mass Flow [lb/s]')
xlabel('Pressure [psig]')
Monte Carlo Uncertainty for Curve Fit of mass flow vs injector Pressure and Orifice Pressure

%Establish the Array of possible values based on the uncertainty sources

% Assign estimates of uncertainty for measurements
b_P1 = 3.182516454; %systematic uncertainty associated with Pressure P1 (calibration and repeatability uncertainty from First Order linear least square regression)
b_PRight = 1.802891774; %systematic uncertainty associated with Pressure P_Right (calibration and repeatability uncertainty from First Order linear least square regression)
b_PLeft = 1.383648411; %systematic uncertainty associated with Pressure P_Left (calibration and repeatability uncertainty from First Order linear least square regression)
s_P1 = 0.001863694; %random uncertainty associated with Pressure P1 (assumed 95% confidence and averaged)
s_PRight = 0.000807653; %random uncertainty associated with Pressure PRight (assumed 95% confidence and averaged)
s_PLeft = 0.000736273; %random uncertainty associated with Pressure P1 (assumed 95% confidence and averaged)

b_m = .0025; %systematic uncertainty associated with mass 1/2 of resolution of Scale
s_mOrifice = 0.000514099; %random uncertainty of mass assuming 95% confidence and averaged.
s_mRight = 0.000140952; %random uncertainty of mass assuming 95% confidence and averaged.
s_mLeft = 0.000205459; %random uncertainty of mass assuming 95% confidence and averaged.

b_t = .5; %systematic uncertainty associated with time (conservative estimate of human reaction time)
s_t = .25; %random uncertainty associated with time (assumed 95%)

N = 10000; %Number of iterations for monte carlo analysis

%Systematic Uncertainties : These are the same for all measurements, so they are generated once and applied to all measurements
MC.B_P1 = randn(N,1)*b_P1/2;
MC.B_Rinj_P = randn(N,1)*b_PRight/2;
MC.B_Linj_P = randn(N,1)*b_PLeft/2;
MC.B_Linj_m = (rand(N,1)-.5)*2*b_m;
MC.B_Rinj_m = (rand(N,1)-.5)*2*b_m;
MC.B_Orif_m = (rand(N,1)-.5)*2*b_m;
MC.B_Linj_t = (rand(N,1)-.5)*2*b_t;
MC.B_Rinj_t = (rand(N,1)-.5)*2*b_t;
MC.B_Orif_t = (rand(N,1)-.5)*2*b_t;

%Random Uncertainties : These are independent for each measurement, so there is a set of random uncertainties for each measured value of the variable
M = length(I_P1);
MC.S_P1 = randn(N,M)*s_P1/2;
MC.S_Rinj_P = randn(N,M)*s_PRight/2;
MC.S_Linj_P = randn(N,M)*s_PLeft/2;
MC.S_Rinj_t = randn(N,M)*s_t/2;
MC.S_Linj_t = randn(N,M)*s_t/2;
MC.S_Orif_t = randn(N,M)*s_t/2;

%Assuming the random uncertainty on weight is a percentage of the total
%mass. (i.e. proportional to flow rate)
MC.S_Linj_m = zeros(N,M);
MC.S_Rinj_m = zeros(N,M);
MC.S_Orif_m = zeros(N,M);
for i = 1:M
    MC.S_Linj_m(:,i) = (randn(N,1)*(s_mLeft*I_m_left(i))/2);
    MC.S_Rinj_m(:,i) = (randn(N,1)*(s_mRight*I_m_right(i))/2);
    MC.S_Orif_m(:,i) = (randn(N,1)*(s_mOrifice*I_m_orifice(i))/2);
end

% Generate total Errors
% Total Error for each simulated measurement in Monte Carlo is the sum of
% the systematic and the random uncertainties.
MC.P1_error = MC.B_P1+MC.S_P1;
MC.Rinj_P_error = MC.S_Rinj_P+MC.B_Rinj_P;
MC.Linj_P_error = MC.S_Linj_P+MC.B_Linj_P;
MC.Rinj_m_error = MC.S_Rinj_m+MC.B_Rinj_m;
MC.Linj_m_error = MC.S_Linj_m+MC.B_Linj_m;
MC.Orif_m_error = MC.S_Orif_m+MC.B_Orif_m;
MC.Rinj_t_error = MC.S_Rinj_t+MC.B_Rinj_t;
MC.Linj_t_error = MC.S_Linj_t+MC.B_Linj_t;
MC.Orif_t_error = MC.S_Orif_t+MC.B_Orif_t;

% Monte Carlo Data Sets
% Add the average value to the error to produce random distributions for
% each measurement based on the estimated random and systematic
% uncertainties.
MC.P1 = (MC.P1_error'+I_P1)';
MC.Rinj_P = (MC.Rinj_P_error'+I_Pright)';
MC.Linj_P = (MC.Linj_P_error'+I_Pleft)';
MC.Rinj_m = (MC.Rinj_m_error'+I_m_right)';
MC.Linj_m = (MC.Linj_m_error'+I_m_left)';
MC.Orif_m = (MC.Orif_m_error'+I_m_orifice)';
MC.Rinj_t = (MC.Rinj_t_error'+60)';
MC.Linj_t = (MC.Linj_t_error'+60)';
MC.Orif_t = (MC.Orif_t_error'+60)';

% Calculate important parameters
% mass flow
MC.Linj_mdot = MC.Linj_m./MC.Linj_t;
MC.Rinj_mdot = MC.Rinj_m./MC.Rinj_t;
MC.Orif_mdot = MC.Orif_m./MC.Orif_t;

MC.mdot_tot = (MC.Linj_mdot+MC.Rinj_mdot);
MC.P_inj_avg = (MC.Rinj_P+MC.Linj_P)/2;

% Predicted mass flow based on pressure
U.mdot_left_predict = coef_mdotL_PL(2) * I_left.^((coef_mdotL_PL(1)));
U.mdot_Right_predict = coef_mdotR_PR(2) * I_Pright.^(coef_mdotR_PR(1));
U.mdot_Orifice_predict = coef_mdotO_PO(2) * I_P1.^(coef_mdotO_PO(1));

% Standard error of regression
for i = 1:length(U.mdot_Left_predict)
    U.Left_sigma(i) = (sum((MC.Linj_mdot(:,i)-U.mdot_Left_predict(i)).^2)./
        (length(MC.Linj_mdot(:,i)))).^0.5;
    U.Right_sigma(i) = (sum((MC.Rinj_mdot(:,i)-U.mdot_Right_predict(i)).^2)./
        (length(MC.Rinj_mdot(:,i)))).^0.5;
    U.Orifice_sigma(i) = (sum((MC.Orif_mdot(:,i)-U.mdot_Orifice_predict(i)).^2)./
        (length(MC.Orif_mdot(:,i)))).^0.5;
end

U.coef_L = polyfit(transpose(I_Pleft),U.Left_sigma,1);
f_U_Left = 2*(x*U.coef_L(1) + (U.coef_L(2)));
U.coef_R = polyfit(transpose(I_Pright),U.Right_sigma,1);
f_U_Right = 2*(x*U.coef_R(1) + (U.coef_R(2)));
U.coef_O = polyfit(transpose(I_P1),U.Orifice_sigma,1);
f_U_Orifice = 2*(x*U.coef_O(1) + (U.coef_O(2)));

figure
plot(I_Pleft,2*U.Left_sigma,'o','color','b')
hold on
plot(I_Pright,2*U.Right_sigma,'x','color','r')
hold on
plot(I_P1,2*U.Orifice_sigma,'^','color','g')
fplot(f_U_Left,[xlim],'color','b')
fplot(f_U_Right,[xlim],'color','r')
fplot(f_U_Orifice,[xlim],'color','g')
txt3 = ['U_{mdot left} = ' num2str(U.coef_L(1),3) ' P_{injector left} + { ' num2str(U.coef_L(2),3) ' }'];
text(150, .0006, txt3);
txt4 = ['U_{mdot right} = ' num2str(U.coef_R(1),3) ' P_{injector right} + { ' num2str(U.coef_R(2),3) ' }'];
text(100, .0013, txt4);
txt5 = ['U_{mdot orifice} = ' num2str(U.coef_O(1),3) ' P_{orifice} + { ' num2str(U.coef_O(2),3) ' }'];
text(300,.0025,txt5);
legend('Left Injector','Right Injector','Orifice', 'Location','NorthWest')
ylabel('Mass Flow Uncertainty [lb/s]')
xlabel('Pressure [psig]')
C.3. Uncertainty in Jet Velocity

The data reduction equation of the jet velocity is given by,

\[ v_j = \frac{4 \times \dot{m}}{\pi d_0^2 \rho} \]

Where \( \dot{m} \) is the mass flow rate of injectors, \( d_0 \) is the diameter of the injector and \( \rho \) is the density of the simulant at the measured temperature. The uncertainty in the jet velocity \( v_j \) is given by \( U_{v_j} \) in the equation below,

\[ U_{v_j} = \sqrt{\left( \frac{\partial v_j}{\partial \dot{m}} \times U_{\dot{m}} \right)^2 + \left( \frac{\partial v_j}{\partial d_0} \times U_{d_0} \right)^2 + \left( \frac{\partial v_j}{\partial \rho} \times U_{\rho} \right)^2} \]

Where, \( U_{\dot{m}} \) is the mass flow rate (\( \dot{m} \)) uncertainty obtained from the Monte Carlo analysis. \( U_{d_0} \) is the injector diameter (\( d_0 \)) uncertainty and is equal to 0.00039 in (Given by the manufacturer). \( U_{\rho} \) is the uncertainty in density (\( \rho \)) \( 2.6042 \times 10^{-6} \text{ lbm/in}^3 \) (from NIST Web Book). The partial differentiation of \( v_j \) is given by the following equations.
\[
\frac{\partial v_j}{\partial \dot{m}} = \frac{4}{\pi d_0^2 \rho}
\]
\[
\frac{\partial v_j}{\partial d_0} = -\frac{8 \dot{m}}{\pi d_0^3 \rho}
\]
\[
\frac{\partial v_j}{\partial \rho} = -\frac{4 \dot{m}}{\pi d_0^2 \rho^2}
\]

C.4. Uncertainty in Sheet Angle

The total Sheet Angle Uncertainty (SAU) is given by the equation below.

\[U_{SAU} = \sqrt{B_{SA}^2 + S_{SA}^2}\]

In the equation above, \(S_{SA}\) is the random uncertainty in the empirical sheet angle and it is given by the equation below.

\[S_{SA} = \frac{t \sigma_{SA}}{\sqrt{N_2}}\]

Where, \(t = 2.776\) for the 95\% confidence interval from the t-distribution table, \(\sigma_{SA}\) is the standard deviation of the measurement and \(N_2 = 5\), the number of measurement made for each setpoint.

\(B_{SA}\) is the systematic uncertainty determined from the method described below.

Figure C.1: Sheet Angle Measurement Technique
The sheet angle is measured in the Phantom CV 3.3 software. The size of each frame measured is 608×800 pixels. If the left top corner of the frame is considered as the origin (0, 0) of the frame, then the impingement point can be considered as (x1, y1), the left tangent to the sheet is considered to end at point (x2, y2) and the right tangent is considered to end at point (x3, y3). Using the trigonometry, the sheet angle $\theta_{SA}$ can be determined.

The data reduction equations used to find uncertainty in the sheet angle are described below.

\[
\theta_1 = \tan^{-1}\left(\frac{y_2}{x_2}\right) - \tan^{-1}\left(\frac{y_1}{x_1}\right)
\]

\[
\theta_2 = \tan^{-1}\left(\frac{y_3}{x_3}\right) - \tan^{-1}\left(\frac{y_1}{x_1}\right)
\]

By Cosine Rule,

\[
a = \sqrt{x_2^2 + y_2^2 + x_1^2 + y_1^2 - 2 \sqrt{x_1^2 + y_1^2} \sqrt{x_2^2 + y_2^2} \cos \theta_1}
\]

By Sine Rule,

\[
\frac{a}{\sin \theta_1} = \frac{\sqrt{x_2^2 + y_2^2}}{\sin A}
\]

Similarly, by Cosine Rule.

\[
b = \sqrt{x_3^2 + y_3^2 + x_1^2 + y_1^2 - 2 \sqrt{x_1^2 + y_1^2} \sqrt{x_3^2 + y_3^2} \cos \theta_2}
\]

Again by Sine Rule,

\[
\frac{b}{\sin \theta_2} = \frac{\sqrt{x_3^2 + y_3^2}}{\sin B}
\]

Hence the sheet angle $\theta_{SA}$ is,

\[
\theta_{SA} = B - A
\]
The uncertainty in the selected points \(x_1, y_1, x_2, y_2, x_3\) and \(y_3\) is assumed to be \(\pm 10\) pixels.

\[
U_{x_1} = U_{y_1} = U_{x_2} = U_{y_2} = U_{x_3} = U_{y_3} = 10 \times \text{Scale}
\]

The Scale for each setpoints are mentioned in Appendix A. The uncertainties of each data reduction equations are mentioned below. The uncertainty in \(\theta_1\) is,

\[
U_{\theta_1} = \sqrt{\left(\frac{\partial \theta_1}{\partial x_1} U_{x_1}\right)^2 + \left(\frac{\partial \theta_1}{\partial y_1} U_{y_1}\right)^2 + \left(\frac{\partial \theta_1}{\partial x_2} U_{x_2}\right)^2 + \left(\frac{\partial \theta_1}{\partial y_2} U_{y_2}\right)^2}
\]

The uncertainty in \(\theta_2\) is,

\[
U_{\theta_2} = \sqrt{\left(\frac{\partial \theta_1}{\partial x_1} U_{x_1}\right)^2 + \left(\frac{\partial \theta_1}{\partial y_1} U_{y_1}\right)^2 + \left(\frac{\partial \theta_1}{\partial x_3} U_{x_3}\right)^2 + \left(\frac{\partial \theta_1}{\partial y_3} U_{y_3}\right)^2}
\]

The uncertainty in length ‘\(a\)’ is,

\[
U_a = \sqrt{\left(\frac{\partial a}{\partial x_1} U_{x_1}\right)^2 + \left(\frac{\partial a}{\partial y_1} U_{y_1}\right)^2 + \left(\frac{\partial a}{\partial x_2} U_{x_2}\right)^2 + \left(\frac{\partial a}{\partial y_2} U_{y_2}\right)^2 + \left(\frac{\partial a}{\partial \theta_1} U_{\theta_1}\right)^2}
\]

And,

\[
\frac{\partial a}{\partial x_1} = \frac{1}{a} \left( x_1 - \left( \frac{x_1^2 + y_2^2 \cos \theta_1}{\sqrt{x_1^2 + y_1^2}} \right) \right)
\]
\[
\frac{\partial a}{\partial y_1} = \frac{1}{a} \left( y_1 - \left( \frac{x_1^2 + y_1^2}{\sqrt{x_1^2 + y_1^2}} \cos \theta_1 \right) \right)
\]

\[
\frac{\partial a}{\partial x_2} = \frac{1}{a} \left( x_2 - \left( \frac{x_1^2 + y_1^2}{\sqrt{x_1^2 + y_1^2}} \cos \theta_1 \right) \right)
\]

\[
\frac{\partial a}{\partial y_2} = \frac{1}{a} \left( y_2 - \left( \frac{x_1^2 + y_1^2}{\sqrt{x_1^2 + y_1^2}} \cos \theta_1 \right) \right)
\]

\[
\frac{\partial a}{\partial \theta_1} = \frac{1}{a} \left( \frac{x_1^2 + y_1^2}{x_1^2 + y_1^2} \right) \sin \theta_1
\]

The uncertainty in angle ‘A’ is,

\[
U_A = \sqrt{\left( \frac{\partial A}{\partial \theta_1} U_{\theta_1} \right)^2 + \left( \frac{\partial A}{\partial a} U_a \right)^2 + \left( \frac{\partial A}{\partial x_2} U_{x_2} \right)^2 + \left( \frac{\partial A}{\partial y_2} U_{y_2} \right)^2}
\]

And partial differentiations are given by the equations below.

\[
\frac{\partial A}{\partial \theta_1} = \frac{\sqrt{x_1^2 + y_1^2} \cos \theta_1}{a \sqrt{1 - \sin^2 \theta_1} \frac{x_1^2 + y_1^2}{a^2}}
\]

\[
\frac{\partial A}{\partial a} = \frac{-\sqrt{x_1^2 + y_1^2} \sin \theta_1}{a^2 \sqrt{1 - \sin^2 \theta_1} \frac{x_1^2 + y_1^2}{a^2}}
\]

\[
\frac{\partial A}{\partial x_2} = \frac{\sin \theta_1 \cdot x_2}{a \cdot \sqrt{x_1^2 + y_1^2} \cdot \sqrt{1 - \sin^2 \theta_1} \frac{x_1^2 + y_1^2}{a^2}}
\]

\[
\frac{\partial A}{\partial y_2} = \frac{\sin \theta_1 \cdot y_2}{a \cdot \sqrt{x_1^2 + y_1^2} \cdot \sqrt{1 - \sin^2 \theta_1} \frac{x_1^2 + y_1^2}{a^2}}
\]

The uncertainty in the length ‘b’ is,
\[ U_b = \sqrt{\left( \frac{\partial b}{\partial x_1} U_{x_1} \right)^2 + \left( \frac{\partial b}{\partial y_1} U_{y_1} \right)^2 + \left( \frac{\partial b}{\partial x_3} U_{x_3} \right)^2 + \left( \frac{\partial b}{\partial y_3} U_{y_3} \right)^2 + \left( \frac{\partial b}{\partial \theta_2} U_{\theta_2} \right)^2} \]

And,

\[ \frac{\partial b}{\partial x_1} = \frac{1}{b} \left( x_1 - \left( \frac{\sqrt{x_1^2 + y_1^2} \cos \theta_2}{\sqrt{x_1^2 + y_1^2}} \right) \right) \]

\[ \frac{\partial b}{\partial y_1} = \frac{1}{b} \left( y_1 - \left( \frac{\sqrt{x_1^2 + y_1^2} \cos \theta_2}{\sqrt{x_1^2 + y_1^2}} \right) \right) \]

\[ \frac{\partial b}{\partial x_3} = \frac{1}{b} \left( x_3 - \left( \frac{\sqrt{x_3^2 + y_3^2} \cos \theta_2}{\sqrt{x_3^2 + y_3^2}} \right) \right) \]

\[ \frac{\partial b}{\partial y_3} = \frac{1}{b} \left( y_3 - \left( \frac{\sqrt{x_3^2 + y_3^2} \cos \theta_2}{\sqrt{x_3^2 + y_3^2}} \right) \right) \]

\[ \frac{\partial b}{\partial \theta_2} = \frac{1}{b} \left( \sqrt{x_1^2 + y_1^2} \right) \frac{x_1}{\sqrt{x_3^2 + y_3^2}} \sin \theta_2 \]

The uncertainty in angle ‘B’ is,

\[ U_B = \sqrt{\left( \frac{\partial B}{\partial \theta_2} U_{\theta_2} \right)^2 + \left( \frac{\partial B}{\partial b} U_b \right)^2 + \left( \frac{\partial B}{\partial x_3} U_{x_3} \right)^2 + \left( \frac{\partial B}{\partial y_3} U_{y_3} \right)^2} \]

And partial differentiations are given by the equations below.

\[ \frac{\partial B}{\partial \theta_2} = \frac{\sqrt{x_3^2 + y_3^2} \cos \theta_2}{b \sqrt{1 - \sin^2 \theta_2 \frac{x_1^2 + y_1^2}{b^2}}} \]

\[ \frac{\partial B}{\partial b} = \frac{-\sqrt{x_3^2 + y_3^2} \sin \theta_2}{b^2 \sqrt{1 - \sin^2 \theta_2 \frac{x_1^2 + y_1^2}{b^2}}} \]
\[
\frac{\partial B}{\partial x_3} = \frac{\sin \theta_2 \cdot x_3}{b \cdot \sqrt{x_3^2 + y_3^2} \cdot \sqrt{1 - \sin^2 \theta_2 \frac{x_3^2 + y_3^2}{b^2}}}
\]

\[
\frac{\partial B}{\partial y_3} = \frac{\sin \theta_2 \cdot y_3}{b \cdot \sqrt{x_3^2 + y_3^2} \cdot \sqrt{1 - \sin^2 \theta_2 \frac{x_3^2 + y_3^2}{b^2}}}
\]

Therefore the systematic uncertainty in the sheet angle is,

\[B_{SA} = \sqrt{U_B^2 + (-U_A)^2}\]

C.5. Uncertainty in Sheet Breakup Length

The total uncertainty in the Sheet Breakup Lengths is given by the equation below.

\[U_{SBL} = \sqrt{(B_{SBL}^2 + S_{SBL}^2)}\]

In the equation above, \(S_{SBL}\) is the random uncertainty, measured assuming 95% confidence in measurement.

\[S_{SBL} = \frac{t \sigma_{SBL}}{\sqrt{N_{frames}}}\]

\(N_{frames}\) is equal to 201 frames of the high speed video. \(\sigma_{SBL}\) is the standard deviation in the measured Sheet breakup Length of 201 frames and \(t\) is equal to 1.962 for 95% confidence interval.

\(B_{SBL}\) is the systematic uncertainty in the sheet breakup length. The data reduction equation of the Sheet Breakup Length (SBL) is,

\[SBL = \text{Number of pixels} \times \text{Scale}_{SBL}\]

\(\text{Scale}_{SBL}\) is the scale obtained by measuring the reference image in the Phantom CV 3.3 software. \(\text{Scale}_{SBL}\) is given by the equation below.

\[\text{Scale}_{SBL} = \frac{l_{reference}}{N_{pixel}}\]
$l_{\text{reference}}$ is the length of the metal scale used to capture the reference image and is equal to 0.09843 ft (30 mm) and the uncertainty associated in measuring $l_{\text{reference}}$, $U_{l_{\text{ref}}}$ is 0.00033 ft (0.01 mm, caliper resolution). $N_{\text{pixel}}$ is the number of pixel in the reference image and it is equal to 329 pixels. Uncertainty associated with $N_{\text{pixel}}$, $U_N$ is assumed to be $\pm$ 10 pixels. Therefore percentage uncertainty is scale is given by the equation below.

$$U_{\text{scale}} = Scale \times \sqrt{\left( \frac{U_{l_{\text{ref}}}}{l_{\text{reference}}} \right)^2 + \left( \frac{U_N}{N_{\text{pixel}}} \right)^2}$$

Therefore, the Systematic uncertainty in Sheet Breakup Length is found the equation below.

$$B_{\text{SBL}} = SBL \times \sqrt{\left( \frac{U_{\text{Num-of-pixel}}}{\text{Number of pixel}} \right)^2 + \left( \frac{U_{\text{scale}}}{Scale} \right)^2}$$

In the equation above, number of pixel is obtained from the MATLAB code for each setpoint. The threshold correction factor (fudge factor) is the variable responsible for the uncertainty in the measurement of the SBL. For fifty four setpoints out of two hundred and seventy setpoints, the sheet breakup lengths were measured using the MATLAB code with $+10\%$ threshold correction factor and $-10\%$ threshold correction factor. The sheet breakup lengths for these fifty four setpoints are compared with the original sheet breakup lengths. The maximum % difference in the sheet breakup length is 6.1125% and this value is assumed as the uncertainty in the measurement of number of pixel, $U_{\text{Num-of-pixel}}$. The total % systematic uncertainty of SBL is converted to the unit feet. Then the total uncertainty of the sheet breakup length is calculated.
APPENDIX D. LITERATURE - II

COMBUSTION INSTABILITY AND THE EFFECTS OF INJECTORS

The combustion instabilities are the inevitable phenomenon in the LREs. It is important to understand the types and modes of combustion instabilities, factors effecting the combustion stability, instability damping techniques and the role of the injectors in combustion stability/instability. This chapter is a review of all the topics mentioned above and few experimental works carried out to study the injector parameters of the LREs.

D.1. Combustion Instability

Combustion instability is defined as, the severe pressure oscillations, vibrations and heat release fluctuation due to the coupling between the thermo-fluid parameters and the engine hardware, that could effectively damage or destroy the engine. Complicated feedback mechanism between the heat releases is used to maintain the combustion instability in the engine, which is actually controlled by the injection process and the acoustic within the chamber [5]. Combustion instabilities were discovered in solid and liquid rocket engines in late 1930’s. Most of the studies were focused on the solid propellant motor instabilities and their passive treatment methods until the end of World War II. Though the work on liquid rocket engine instabilities started in 1940’s, it got significance, when the large Intercontinental Ballistic Missiles (ICBMs) were developed after the World War II. In 1960’s, Apollo program stressed more on the LREs instability study, particularly because, the astronauts were onboard the rocket. Not much work about LREs instabilities were documented from 1970 to 1980. Lower frequency longitudinal oscillations in liquid fuel ramjets received greater attention in 1980’s, which led to a quite good understanding of
the causes and the treatments of the instabilities in the LREs\textsuperscript{[6]}. Figure D.1 shows the chronology of the combustion instability study from 1940 to 2000.

![Figure D.1: Chronology of Combustion Instability\textsuperscript{[8]}](image)

**D.1.1. Characteristics of Combustion Instabilities**

When the system’s fluid dynamics couple with the combustion process and the engine modes, combustion instabilities occur. These instabilities are characterized with the self-sustained periodic pressure oscillations, the substantial heat release and the velocity supported by complicated feedback loops between the fluid dynamics, the pressure oscillations, the combustor resonant frequencies and the combustion energy\textsuperscript{[5,7]}. A substantial energy release in a comparatively smaller chamber volume with minimum energy loss is a good environment for the instabilities to excite and sustain. A little interaction and/or feedback between the combustion process and the fluid dynamics is enough to excite the pressure oscillation in the chamber\textsuperscript{[8]}. In 1950s and 1960s, a profound attention was given to the study of linear and nonlinear behavior of these pressure oscillations\textsuperscript{[6]}.

**D.1.1.1. Linear Combustion Instability**

Any linear disturbances are considered as an infinite series of harmonic motions, which allows to calculate the complex wave number for each acoustic mode considering the classical
acoustic modes as the terms in the series. The wave number is given by equation (D.1). The real part of this wave number ($K_n$) gives the frequency shift ($\Omega_n$) and the imaginary part gives the growth rate or the decay rate constant ($\alpha_n$) of each mode.

$$K_n = (\Omega_n - i\alpha_n)/\bar{a}$$

(D.1)

$\alpha_n = 0$, acts as the stability boundary. The natural motion is unstable if $\alpha_n < 0$. $\alpha_n$ is also called as response factor$^6$. To consider the instability in the chamber to be linear, the following assumptions were made. First one is that, the small amplitude pressure oscillations are because of low speed mean flow. This expands the governing equations asymptotically both in the surface Mach number of the mean injected flow and the wave amplitude. Second assumption is, reacting surface layer is permeable and thin. This allows to express the complex surface reaction effects including pressure and combustion coupling, using simple acoustic boundary conditions foisted at the chamber surface. Last assumption is the chamber acoustic modes drives the oscillatory flow field. These assumptions suppress all unsteady rotational flow effects and simplifies the time dependent model$^9$.

Rayleigh’s criterion states that, heat release rate fluctuations ($q'$) due to propellant burning must be in phase with the pressure oscillations ($p'$) to obtain a resonant interaction between the combustion process and the acoustic field. On the other hand, unsteady motions are damped if the heat release rate and the pressure oscillations are out of phase$^6$. Mathematically$^{10}$,

$$\int_V \int_t p'(V, t)q'(V, t)dtdV > 0$$

(D.2)

If $p'$ and $q'$ are harmonic perturbations of pressure and heat release rate, then they can be expressed as$^{10}$,

$$p'(t) = p'_{max}\sin(\omega t)$$

(D.3)
\[ \dot{q}'(t) = \dot{q}'_{\text{max}} \sin(\omega t + \varphi) \quad (D.4) \]

When \(0 < |\varphi| < 90^\circ\), flame response to pressure oscillations will be positive (excitation). If \(|\varphi| = 90^\circ\), flame response is neutral. If \(90^\circ < |\varphi| < 180^\circ\), negative flame response (damping) \(^{[10]}\). To have a linear combustion instability, it is important to have the heat release rate in phase with the pressure oscillations, but that may not excite the acoustic oscillations. Practically, there exists a dissipation mechanism. If acoustic oscillation energy is greater than the dissipation energy, then oscillations grows and leads to unstable system. If acoustic oscillation energy is less than dissipation energy, then net damping occurs. If oscillation energy balances the dissipation energy, system remains stable with no growth and/or no decay. A quantitative extension of Rayleigh criterion is given by the equation \((D.5)\), where \(\gamma\) is the specific heat ratio, \(\rho\) is the density of the fluid, \(c\) is the speed of sound, and \(T\) is the time period of acoustic oscillation \(^{[11]}\).

\[
\frac{\gamma - 1}{\rho T c^2} \int \int p'(V, t) \dot{q}'(V, t) dt dV > \int_A \vec{F} \cdot \hat{n} dA + \int_V D dV \quad (D.5)
\]

In 1950s, the concept of time lag was introduced to represent the frequency shift and the growth or the decay constant of the linear combustion instability by Gunder and Friant. Time interval between the time at which the propellant enters the combustion chamber and the time, it burns or combusts is called as the time lag \((\tau)\). Using Fourier analysis, with the assumption that no external forces acts on the combustion chamber, Culick and Yang derived the equations for the growth or the decay constant and the frequency shift of the normal mode \(^{[6]}\). These are given by the equations \((D.6)\) and \((D.7)\) respectively.

\[
\alpha_n = C_1 \int \psi_n \bar{\omega}_t^{(r)} dV - C_2 \quad (D.6)
\]

\[
\Omega_n = \omega_n + C_3 \int \psi_n \bar{\omega}_t^{(l)} dV - C_4 \quad (D.7)
\]
Where, $\hat{\omega}_l$ is the source term of the Fourier expansion of $\alpha_n$ and $\Omega_n$ that represents the fluctuation in conversion of liquid to gas. This source term $\hat{\omega}_l$ depends on the time lag ($\tau$) and the interaction index ($n$). The index ‘$n$’ represents the sensitivity of the combustion to pressure oscillation. The ‘$\tau$’ and the ‘$n$’ in turn depends on the parameters defining the system based on the injector type, geometry, dominant process and so forth. It is clearly mentioned in Culick’s work that, by varying these injector parameters, loci of values can be found such that, a small change in one direction produces the unstable oscillations and changes in the other side of the locus which produces the disturbances to be attenuated. With the known stability boundary (loci points), time lag can be solved and there by growth rate and frequency shifts can be determined $^6$. This is one of the strongest bases to the current research.

It is worth mentioning here that, a reduced order analytical model to determine the combustion stability characteristics of a 2D Cartesian dump combustor was developed by Rani and Rani. Results of this acoustically consistent linear modal analysis are quite interesting. One of the results is that the uniform or the non-uniform nature of the velocity profile has a little impact on the duct acoustics. This is contradicting the assumptions that the non-uniform velocity profile was the source term in the wave equation and a factor in the heat release source term. The demonstrations of the instabilities for the fundamental transverse mode and the fundamental longitudinal modes and its harmonics, reveal that the instabilities are principally affected by the combustor geometry and the mean flow properties. The flame length and such parameters have minor effects on the instability. This is contradicting the general principle of the combustion instability, the Rayleigh’s Criterion $^{12}$. But the point to be noted is, the results of this analytical model stresses on the importance of the combustor geometry and the mean flow properties. The mean flow properties are dependent on the type of injectors and injection geometry.
D.1.1.2. Nonlinear Combustion Instability

Most of the studies on combustion instability and their modes in LREs assume the linearity of the combustion instabilities to solve the problems. According to Culick et al., determining the condition for existence and stability of the limit cycle and the condition at which the linearly stable system becomes unstable due to a sufficiently large disturbance are the only two basic nonlinear problems in combustion instability \(^{[6,8]}\). However, there are a few researchers that advocate the importance of the nonlinearity considerations in the combustion instability problems. Flandro et al., demonstrated an experimental and an analytical model which focus mainly on travelling steep-fronted shocked pressure waves, rotational flow corrections’ effects, heat transfer, frictional losses and the other surface effects, the detonation wave phenomenon and the other combustion coupling issues \(^{[9,13,14]}\). Flandro derives the system’s energy density equation using the base of Navier-Stokes continuity, momentum and energy balance equations. The key assumption to solve the nonlinearity problem is that, fully steepened travelling pressure wave \((p(r, t))\) is a composite of the chamber normal modes. Avoiding the assumption of isentropic flow limitations and including the effects of heat transfer and viscosity, for a calorically perfect gas, the system’s energy density \((E)\) equation is derived \(^{[9, 13]}\). The system’s energy density is given by the equation (D.8).

\[
\frac{\partial E}{\partial t} = -\nabla \left[ \rho u \left( \frac{T}{\gamma (\gamma - 1)} + \frac{1}{2} u \cdot u \right) \right] + \left\{ -\frac{1}{\gamma} \nabla \cdot (\rho u) + \rho u \cdot (u \times \omega) + \delta^2 \left[ \omega \cdot \omega - u \cdot \nabla \times \omega \right] \right. \\
+ \left. \frac{\delta^2}{(\gamma - 1) Pr} \nabla^2 T + \delta^2 \left[ (\nabla \cdot u)^2 + u \cdot \nabla (\nabla \cdot u) \right] + \dot{Q} + u \cdot F \right\} \tag{D.8}
\]

Where, \(\gamma\) is the ratio of specific heats, \(\rho\) is the density, \(u\) is the oscillatory velocity vector, and \(\delta\) is the reciprocal of the square root of the acoustic Reynolds number. The above model represents a wave system that consists of super imposed wave of compressibility \((\delta_d)\), vorticity
\((\omega)\), and entropy. In the above energy density equation, \(F\) is the body force, a place holder for all the two-phase flow effects like particle mean flow interactions, spray atomization, etc. \([9,13]\). Further, the spatial averaging, the linear growth rate equations and the limit cycle amplitude derivation using fluid dynamics’ approach can be seen in Flandro’s work. Note that, there is a place holder \((F)\) in Flandro’s nonlinear combustion instability analytical model, which insists on the importance of the propellant injection and the injector geometry.

D.1.2. Modes of Combustion Instability

Multiple types and modes of combustion instabilities can coexist in a combustion chamber. These are classified as thermos-acoustic instabilities, low frequency combustion instabilities and high frequency combustion instabilities. Combustion heat release is treated as the source of the thermos-acoustic combustion instability that makes the sound disturbance to propagate throughout the thrust chamber. When these heat waves hit the acoustic boundaries, they reflect back towards the flame. In the vicinity of the injector plate, the combination of these reflected waves creates acoustic velocity and pressure variations that could effectively alter the incoming propellant stream and create perturbations in the unsteady heat release rate. If the burning rate is altered with correct phasing by these acoustic oscillations, the intensity of the flame oscillations and the instabilities of the reaction zone can be increased. This in turn creates higher amplitude heat waves that leads to the growth of the combustion instabilities. This type of instabilities is called as thermos-acoustic instability \([15]\).

The combustion instabilities of frequency of order 100 Hz or lower are referred to the low frequency combustion instabilities. These instabilities usually have a linear growth rate from low to high amplitudes. Coupling of the unsteady heat release from the combustion with the propellant feed system is the primary reason for these type of combustion instabilities. The fluctuation within
the propellant feed system create the unsteady heat release that then couples with the acoustic modes of the chamber, generating a feedback sufficient to sustain the growth rate. This type of instabilities are also called as “chugging” \cite{5,15}.

A destructive type of combustion instabilities of frequency range 1,000 Hz to 10,000 Hz are referred to high frequency combustion instability. These are also called as “screech” mode instabilities. These are characterized with highly localized large acoustic velocity and pressure fluctuations throughout the chamber. Due to the faster mixing of the propellant, the localized heat release rate increases. If the localized heat release is too large, the chamber can be destructed. These instabilities are further classified as longitudinal modes and transverse modes. The longitudinal mode instabilities oscillate in the axial direction of the chamber. These are directed by the chamber boundary conditions and longitudinal mode instabilities have some amount of natural damping within the chamber due to presence of the nozzle and spatial axial combustion distribution. These are comparatively less harmful than the transverse mode. The instability modes in the direction perpendicular to the injector surface are called as transverse modes. These are mainly affected by the wall boundary conditions and the cross-sectional area of the combustion chamber. Transverse mode instabilities are of three types and they are, tangential modes, radial modes and combined tangential-radial modes \cite{5,15,16}. Tangential pressure nodes split the chamber in circumferential direction. If the tangential nodes spin around the chamber with the frequency equal to the instability frequency, they are called spinning nodes. If these tangential modes have a fixed orientation, then they are called standing nodes. The spinning and the standing nodes can transform into each other spontaneously and hence, it is possible to have hybrid tangential modes in the combustor \cite{5,15,17}. The pressure nodes which are orthogonal to the radial axis of the combustion chamber are called radial nodes. The combined tangential-radial mode has both the
tangential component and the radial component, where nodes split the chamber circumferentially as well as radially \cite{15}. Figure D.2 shows the schematic of different transverse instability modes.

![Figure D.2: Schematic of Common Transverse Instability Modes \cite{15}](image)

**D.1.3. Factors Affecting Combustion Instability**

The five main processes that take place in the combustion chamber are injection, atomization, vaporization, mixing and combustion. Propellant injection plays a very important role in the combustion chamber. The Injection process controls the propellant original droplet size and there by affects the atomization. This in turn affects the vaporization, mixing and then combustion process. The atomization is not a rate controlling process. Like injection process, atomization also has an indirect effect on the unsteady heat release in the chamber. Acoustic velocity and the pressure oscillations influence the primary droplet sizes and vice versa. The process of vaporization involves boiling of the propellant droplet and the evaporation. Characteristic time of the droplet boiling is higher than that of the evaporation process and this is when the vaporization process interacts with the acoustic oscillations. Hence, vaporization is the primary coupling process that drives the high frequency oscillations and it is also considered to be the rate controlling process in a subcritical environment. In a subcritical injection, mixing occurs more rapidly than
the vaporization and it does not have a great effect on the burn rate. However, mixing affects the spatial distribution of heat release. But in a supercritical injection process, the dispersion and turbulent mixing of the threadlike structure from the jet core is attributed as the driving force of combustion instability modes. In a supercritical injection, mixing is also considered as rate controlling process, since the primary atomization and vaporization do not occur. Once the propellants are sufficiently chemically mixed, the process of combustion occurs. Generally, the characteristic time of the combustion process is less than that of the vaporization process, hence they are not considered as the rate controlling processes. However, in fuel rich combustion, characteristic time of the combustion may reach to 1ms that allows combustion process to couple with the acoustic oscillations. Depending on presence of the fuel rich combustion environment, chemical kinetics (combustion) are considered as rate controlling process or driving force of the combustion instabilities \cite{15}.

Injection is the primary process that dictates all the rate controlling processes in a LREs. Injector is considered as the “heart of the thrust chamber” \cite{1}. In a combustion chamber, there exists a mixture of liquid spray and gaseous components. The liquid propellant spray is confined to a small region near the injector face and remaining chamber is occupied with the gaseous combustion products. Therefore, the density gradient varies along the axis of the chamber. Acoustic pressure gradient is larger in the higher density regions and hence the amplitudes of the acoustic oscillations tend to accumulate in the vicinity of the injector face \cite{18}. Therefore, the injector plays an important role in both intrinsic processes of combustion and injection-coupled feedback mechanism of the combustion instability. Injectors are passively involved in the instability excitations, but they control the spray characteristics (droplet sizes and breakup lengths) that direct the other processes involved in the chamber \cite{19}. The instability excitation depends on
some of the intrinsic mechanisms of injection process and they are, propellant injection, primary and secondary atomization, boiling of the droplets and vaporization, mixing of the gas phase and heating, combustion and loss of heat to fresh propellant injected \[^{[20]}\].

Injector designs are broadly classified as pressure sensitive design and velocity sensitive design. Pressure sensitive injector couples with the chamber pressure oscillations and exhibit appreciable heat release oscillations as a function of pressure fluctuations. This type of designs has a large recess or a mixing cup below the injector face. So that the critical injection and initial combustion process occur within the element and is shielded from the transverse instability oscillations. Velocity sensitive designs have a small recess or no recess and the critical injection. In velocity sensitive designs, the initial combustions are affected by transverse mode combustions stabilities \[^{[5,10,21]}\].

Constant flow rate through the injector is desired for a steady state engine operation. The flow rate through the injector is greatly influenced by the chamber and the injector manifold pressure fluctuations and the injector impedance. Generally, engine stability is increased by decreasing the volume of the injector and increasing the injector impedance. Therefore, injector impedance is one more factor that affects the instability of the engine \[^{[5]}\].

**D.1.4. Combustion Instability Damping Method**

Over the course of time, various instability control techniques have been developed to suppress the different types of instabilities in the LREs. These techniques are broadly classified as passive control and active control. Devising the external means of control which damps the combustion instabilities in a non-responsive way are known as passive methods. Whereas the techniques that can alter the combustion instability responses in real time are called as active control techniques. The LREs mostly include the passive control techniques as they involve non-
movable parts \cite{15}. Some examples of the passive control techniques are baffles, acoustic liners, resonators and symmetric and asymmetric fuel injector distribution. Axially extended baffles suppress certain instability modes in the LREs. A single baffle along the diameter of the chamber suppresses the spinning tangential mode instabilities. Symmetrically placed radial baffles control the tangential modes. But the baffles are limited to certain length of the chamber to maintain the structural integrity and flow losses. Resonators used in the LREs convert the high amplitude instabilities into wider amplitudes \cite{6}. Symmetric fuel injector distribution methods were developed by Russians. They stimulate the baffle effect by operating certain order of injectors at a time. This helps to remove the baffles from the chamber and the engine becomes more reliable. An asymmetric injection technique is the one where, few injectors can change the spray characteristics independent of remaining injectors. The study revealed that the first spinning tangential modes are suppressed by the asymmetric fuel injection technique \cite{15}. Including the appropriate acoustic fields in the chamber to cancel the unwanted noise is one of the active control methods \cite{6}. Fuel line flow modulation is another active control technique. Pulsating the fuel flow rate in a way that produces the feedback between the acoustic pressure and the unsteady combustion heat release suppresses the combustion instability and this called fuel flow modulation technique \cite{15}. Active control methods seem promising. However, time lags are unavoidable in the active control methods \cite{6}. The current research is the study of spray characteristics for different geometrical misalignment cases of the impinging injectors. This is one of the passive techniques. But, the scope of this research is to study only the spray characteristics.

D.1.5. **Case Studies of Combustion Instabilities and Damping Techniques.**

D.1.5.1. **Combustion Instability of F1 Engine**
The combustion instabilities tests were carried out on the F1 engine for more than seven years. The preliminary tests of first F1 engine was carried out from January 1959 to May 1960. Out of 44 tests, 20 of them showed a combustion instability of peak amplitudes almost equal to or more than the average chamber pressure. Chamber erosion and the injector face burning showed the presence of huge radial and tangential modes. These preliminary tests suggested the use of baffles to control the dynamic instabilities [6]. To get around these instability problems, a program called “Project First” was initiated in October 1962. The span of this program was from 1962 to 1966 and in 1966, the F1 engine was qualified for the manned mission. The Project Frist program developed in three steps and they are Preliminary Flight Rating Tests (PFRT), Flight Rating Test (FRT) and flight qualification test. A total of 3200 full scale tests were performed during the development of F1 engine. 2000 tests were performed during the Project First program. During PFRT, 207 full scale tests with 11 different injectors. 422 tests with 46 types of injector in FRT and in qualifying stage, 703 tests with 51 injectors were tested. Most of the tests were conducted with 5U and modified 5U injectors with or without baffles. The final design evolved with a modified 5U injector design with 13 compartments by 7.62 cm baffle [6, 22]. The final design was able to damp not only the self-excited oscillations, but also the finite nonlinear disturbances. The tests during Project First revealed that most of the instabilities occur at the injector face containing spray fans, 10in downstream the injector face, where, evaporation takes place and further downstream the chamber where both fuel and oxidizer are in gaseous state. In the F1 engine, baffles damped the instabilities in the first region, the exhaust nozzle damped some of transverse and longitudinal modes and the type of film cooling used eliminated the resurging problems. The two main conclusions drawn from F1 engine tests are, the physical and chemical processes near the injector face are more sensitive to the velocity fluctuations parallel to the face than the unsteady
oscillations normal to the injector face and the action of the exhaust nozzle provides damping for the longitudinal modes more than the tangential modes \[^6\].

D.1.5.2. Combustion Instability of J2 Engine

Under the contract to NASA, the development of J2, a liquid oxygen/hydrogen engine started at Rocketdyne in 1960. Initially, full size ring manifold injectors were tested at a simulated condition similar to the liquid hydrogen temperature expected at the start sequence of the engine. This test generated high frequency and spontaneous combustion instabilities. Then the radial baffles were installed which eliminated most of the instabilities, but the cooling of the baffles was of big concern. In 1961, the first concentric orifice element with transpiration cooled faceplate without baffles were tested. The engine was comparatively more stable than other orifice element engines at low hydrogen injection temperatures. However, at high injection temperature of the hydrogen, the engine was still unstable. Later, oxygen tube flushed with the injector face of the concentric orifice element and with mixing cup designs were tested. The stability of the engine was improved, but cooling was difficult. Next, recessed concentric orifice were tested. This design increased the characteristic velocity performance of the injectors and decreased the liquid hydrogen injection temperature. This design was selected during PFRT. During FRT, oxygen tube was inserted, which improved chugging stability margin. The engine stability testing revealed that, the region of 30% to 50% of the full thrust of J2 engine is found to be dynamically unstable, with most of the injector designs. During the development of J2 engine, different injector elements are extensively tested at low mixture ration with liquid hydrogen at engine start transient temperatures. It was observed that all the tests were unstable. When the hydrogen injection temperature was reduces to nominal operating conditions, all the tests, including bomb stability tests exhibited stability, even with no baffle pattern. Results showed that the engine can operate stably over the
hydrogen injection temperature range of 28 K to 167 K\textsuperscript{[23]}. Major conclusion drawn from J2 engine tests shows the importance of propellant density and injection temperature.

D.1.5.3. **Lunar Module Decent Engine**

For the Lunar Excursion Module (LEM) a single engine with redundant feed system and throttling ratio was needed. Two engines were developed in parallel. They are, Helium injection throttled engine and Variable Area Pintle injector throttle able engine. The helium injection throttled engine injected the helium to the propellant that reduced the mass flow rate of the propellant while maintaining the pressure drop across the injector. A silica tape wrap was made as ablative liner in the chamber and 165 FOF injectors design was used. This engine was a redundant throttle-able engine. But, from 35\% to 70\% of the full thrust of the engine experienced a significant chugging instability. In addition to chugging, isolated pops pressure peaks of 0.698 MPa and resurging occurred. Because of these problems, this engine was not selected. In Pintle injector, the propellant is injected to the chamber radially. The oxidizer flows through the center of the Pintle and ejected out through the radial slots. The fuel flows in the outer sleeve of the Pintle through an axial annular sheet. This engine operated with a non-redundant throttling mechanism. The trust could be varied at a constant \textit{I}_sp. The propellant flow was also controlled by the feed system valves along with the Pintle sleeve. Since this engine used an active instability control mechanism, this was relatively free of stability problems. The only problem detected with this engine was occasional rough combustion with unpredictable frequencies. This engine was selected for the LEM decent program\textsuperscript{[24]}.

These case studies are very relevant, as they provide better insight to different types of passive and active instability control techniques and show the importance of the injection process and the injectors in damping the combustion instabilities.
D.2. Effects of Impingement Angle on High Frequency Combustion Instability

The hot firing tests of three pentad injectors, with different impingement angles, were conducted at University of Alabama in Huntsville by Cavitt et al., to study the combustion stability as a function of impingement angle. A unique type of pentad injectors, typically used in Russian military engines with hypergolic propellants, was tested. The propellant is injected to the central oxidizer flow through four radial holes at an impingement angle. Three such injectors with 45°, 30° and 60° angle of impingement were tested. The purpose was to produce spontaneous high frequency combustion instability with each injector and study the effects of impingement angle.

Pre-existing rocket test stand was used. The facility was operated remotely from the control room. Combustion chamber was fire face sealed, 8 in cylinder with no nozzle. Injector manifold allowed different types of injector fixture and injector covers were flushed to the fire face. Each injector for these tests were placed 2.5 in from the chamber wall. Pure gaseous Methane and gaseous oxygen were used as propellants and the oxidizer respectively. 9 Methane mass flow rates were selected and for each fuel flow rate, 6 oxy rich, 6 lean and 1 stoichiometric combustion were tested. Figure D.3 shows the stability mapping test matrix. For each test firing duration was 2 minutes and the equivalence ratio varied consistently from 2 to 0.5. Repeatability was tested by

![Figure D.3: Stability Mapping Test Matrix][34]
conducting the experiments at 4 different occasions. The chamber was fitted with five thermocouples and a dynamic pressure transducer. The pressure and temperature data were read by Data Acquisition system. Gaseous Hydrogen-Oxygen igniter was used to ignite the fuel. Each test case video was recorded to monitor the flame structure and instability modes. Each test data from the dynamic transducer was sampled at 150 kHz and channeled through an anti-aliasing Butterworth filter. Then the time domain signals converted to frequency domain by Fast Fourier Transforms (FFT). Maximum amplitudes, peak frequencies and power densities could be obtained from the FFT data [34].

The dynamic pressure signals recorded, were the smooth oscillations of unstable pressures. That indicated that, no steep front waves were present, and the sampling rate was sufficient. Peak to peak of the pressure signals were approximately 24% of the mean chamber pressure which is beyond the unstable thresholds. When the system became unstable, the chamber temperature was almost doubled and injector manifold showed a huge variation, of about 33% of the total pressure drop. By the illustrations of the video clip, it was observed that the system became unstable at 2500 Hz and 3800 Hz. At 2500 Hz, the flame was short and bent towards the center of the chamber, i.e. a radial mode instability occurred. At 3800 Hz, the flame was almost 90° to the previous mode, i.e. a combined first radial and second tangential mode instabilities were observed. For each test condition, peak amplitudes of FFT was considered in mapping the stability graphs. Figure D.4 shows the stability mapping of the test matrix at different impingement angles. The high amplitude instabilities corresponding to 2500 Hz is steep and can be seen at the center region of the 30° injector. The instabilities corresponding to 3800 Hz are at the right corner, at high flow rates of the 30° injector’s plot. No instabilities observed in the rich region. The trend is similar in case of 45° injector, except that the instabilities were spread wider. For 60° injector, the instabilities are
shifted up and right of the plots and instabilities corresponding to 3800 Hz did not exist at the flow rates tested. This showed that the 30° injector is prone to radial and combined mode instabilities.

Since the instability regions were widespread in 45° injector case, it is said that 45° injectors are the transition from stable to unstable operations. While 60° injectors exhibited radial mode instabilities and combined modes are said to be at higher amplitudes than the other injectors [34].

Most of the research on the impinging injectors are cold flow experiments. In the pentad injector tested the impingement angle was measured to the central axis of the injector. This hot fire experiment shows that the 45° injectors had a smooth transition from stable to unstable operating conditions and at higher angle of impingements only radial modes were observed at the flow rates tested.

Figure D.4: Maximum Amplitude Stability Maps a) 30° Impingement Angle b) 45° Impingement Angle c) 60° Impingement Angle [34]

Figure 2.10: Maximum Amplitude Stability Maps A) 30° Impingement Angle B) 45° Impingement Angle C) 60° Impingement Angle [34].
REFERENCE


