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Resolving a Conflict Between Coherence Theory and Classical Radiometry, Incoherent Planar Sources are Lambertian

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ABSTRACT.

Using the classical definition of radiant intensity, I derive a unique formula for the radiant intensity of an incoherent source. This new formula differs from the standard equation used in coherence theory, by a factor of $\cos\theta$ [1]. From this I will show that the radiant intensity of an incoherent source follows Lambert's law. A spatially incoherent source is therefore Lambertian. It radiates flux as a Lambertian source in accordance with classical radiometry, and a directionally independent radiance.

This paper also includes a discussion of the basics of radiometry, and the derivation of the standard radiant intensity equation used in coherence theory.

If you divide these equations by R^2 you get the solid angle of the source as viewed from the detector, $\delta\Omega_1$ and the solid angle of the detector as viewed from the source, $\delta\Omega_0$,

$$\delta\Omega_1 = \frac{\delta A_0^{proj}\cos\theta_0}{R^2} \quad \text{and} \quad \delta\Omega_0 = \frac{\delta A_1^{proj}\cos\theta_1}{R^2}. \tag{4}$$

If we multiply $\delta\Omega_0$ by $\cos\theta_1$ we get the projected solid angle of the detector as viewed by the source. Multiplying $\delta\Omega_1$ by $\cos\theta_0$, we get the projected solid angle of the source as viewed by the detector,

$$\delta\Omega_1^{proj} = \frac{\delta A_0 \cos\theta_0 \cos\theta_1}{R^2} \quad \text{and} \quad \delta\Omega_0^{proj} = \frac{\delta A_1 \cos\theta_1 \cos\theta_0}{R^2}. \tag{5}$$

These concepts are illustrated in figure (2).

The radiance $L_\nu(\hat{s})$ can now be expressed in many different ways using the above geometric terms,

$$L_\nu(\hat{s},\vec{r}_0) = \frac{d^2\Phi_\nu}{dA_0^{proj} d\Omega_0} = \frac{d^2\Phi_\nu}{dA_0 d\Omega_0^{proj}} = \frac{d^2\Phi_\nu}{dA_1^{proj} d\Omega_1} = \frac{d^2\Phi_\nu}{dA_1 d\Omega_1^{proj}}. \tag{6}$$

In this equation, $\hat{s}$ is the direction of the radiant flux and $\vec{r}_0$ refers to the location on the source. In free space propagation, the radiance function is conserved along the straight line path of an optical ray[2]. From the radiance function we can derive the three other radiometric quantities by integrating over only one parameter of equation (6).

The spectral radiant exitance is the total spectral flux that leaves the source per unit area,

$$M_\nu(\vec{r}_0) = \frac{d\Phi^2}{dA_0} = \iint_{\text{hemisphere}} L_\nu(\vec{r}_0,\hat{s})d\Omega_0^{proj} = \iint_{\text{hemisphere}} L_\nu(\vec{r}_0,\hat{s})\cos(\theta_0)d\Omega_0. \tag{7}$$

Next is the radiant spectral intensity, $I_\nu(\hat{s})$. this is the total spectral flux emitted by the source in the $\hat{s}$ direction per unit solid angle,

$$\delta U_{\nu} = \frac{dQ_{\nu}}{dV} = \tfrac{1}{c} L_0. \tag{12}$$

Equation(12) expresses a very important relationship. For this collimated or plane-wave-like source, its radiance L_0, is the flux density(flux per unit area) over the beam in the direction of propagation. The vector $\hat{s}_0 L_0$ is analogous to the magnitude of the Poynting or flux density vector of a plane wave[3].

To find the radiant energy density for a general source, we need to add the contributions from each composite plane wave, or integrate equation(12) over all solid angles from which there is radiance(i.e. over the solid angle subtended by the source). This leads to[3]

$$U_{\nu}^{(r)}(\vec{r}_1) = \tfrac{1}{c} \iint_{\text{source}} L_{\nu}(r_0, \hat{s}) d\Omega_1 = \tfrac{1}{c} \iint_{\text{source}} L_{\nu}(r_0, \hat{s}) \frac{\cos(\theta_0)}{R^2} dA_0. \tag{13}$$

Although we have written equation (13) explicitly in terms of the source, the important issue is that the integral is evaluated over all directions for which the radiant function is not zero. Equivalently, we can write equation (13)

$$U_{\nu}^{(r)}(\vec{r}_1) = \tfrac{1}{c} \oiint L_{\nu}(\vec{r}_1, \hat{s}) d\Omega \tag{14}$$

noting that $L_{\nu}(\vec{r}_1, \hat{s})$ is non-zero only over the ray directions that trace back to the source. Recalling that the radiance function is conserved along a ray path, allows the conversion between Equations (13) and (14).

Equation (13) leads to another definition of radiance that is valid everywhere. *The radiance function, $L_{\nu}(\vec{r}_1, \hat{s})$, is c times the contribution to the energy density at the point $\vec{r}_1$ of the radiant flux traveling in the $\hat{s}$ -direction per unit solid angle*[3]. Similarly, the spectral flux density vector $\vec{S}_{\nu}(\vec{r}_1)$ is

$$\vec{S}_\nu(\vec{r}_1) = \oint\!\!\oint \hat{s} L_\nu(\vec{r}_1, \hat{s}) d\Omega. \tag{15}$$

In the limit that R is large , the spectral energy density , Equation (13), asymptotically approaches

$$\lim_{R\to\infty} U_\nu^{(r)}(\hat{s}R) = \frac{1}{c}\frac{\cos(\theta_\nu)}{R^2} \iint L_\nu(\vec{r}_0, \hat{s}) dA_0 = \frac{1}{c}\frac{I_\nu(\hat{s})}{R^2}, \tag{16}$$

and equation (15) the flux density vector approaches,

$$\lim_{R\to\infty} \vec{S}_\nu(\hat{s}R) = \frac{I_\nu(\hat{s})}{R^2}\hat{s}. \tag{17}$$

1.3 Lambertian sources

A Lambertian source is by definition one whose radiance is completely independent of viewing angle, in other words, $L_\nu(\hat{s})$ is constant[2].

For a lambertian source of area A_0 ,and radiance L_0 equation (8) gives,

$$I = \int_{source} L_0 \cos\theta \, dA = L_0 A_0 \cos\theta = I_0 \cos\theta , \tag{18}$$

which is known as Lambert's cosine law. This formula shows that the decrease in intensity associated with increased observation angle ,θ, is due only to the decreased projected area of the source.

The formulas for radiant flux and radiant exitance also simplify from equations (1) and (7) for lambertian sources to

$$\delta^2\Phi = \frac{L_0 dA_0 dA_1}{r^2} \quad \text{and} \quad M = \pi L. \tag{19}$$

2.COHERENCE THEORY.

2.1 Spacial and temporal coherence.

Coherence is a description of the statistical properties, or correlation functions of radiation fields[4]. Coherence can be divided into two classifications, even though the two are interrelated, as temporal and spatial.

Temporal coherence is directly related to the finite bandwidth of the source. A quasi-monochromatic beam of light can be pictured as a series of randomly phased wave trains, as in figure (4). The average constituent wave train exists for a time, Δt_c, which is known as the coherence time. The phase relationship between two points P_1 and P_2, located along the wave and separated by a distance r is the temporal coherence. If r is less than $c\Delta t_c$, the coherence length, the points are highly coherent. If r is much greater that the coherence length then the phases at the two points are unrelated, therefore they would be considered highly uncorrelated, or incoherent.

Spatial coherence is related to the sources finite extent in space. It measures the phase difference between two points P_1 and P_2, which are the same distance away from the source, but do not lie upon the same path. This is illustrated in Young's double split experiment, shown in figure (5). If the two pinholes P_1 and P_2, are illuminated by a primary monochromatic source, the phases at P_1 and P_2, are the same and fringes of light and dark are formed on a distant observation plane. The phases at these points are highly coherent. If the two slits were to be illuminated by separate sources, no fringes would be formed on the screen and the two points would be completely uncorrelated, or incoherent.

One goal of modern coherence theory has been to clarify the relationships between classical radiometry, and the electromagnetic and quantum theories of light[5]. In order to do this we must first define the correlation functions and then we must relate these functions to the measured quantities of radiometry.

2.2 Defining the mutual coherence function and its Fourier transforms.

Light is an electromagnetic wave that can be described by $\bar{E} = \tilde{E}_0 e^{i(kx-\omega t+\alpha)}$,where $\tilde{E}_0$ is the complex amplitude, k is the wave number, ω is the frequency, and α is the phase angle. If two waves of this form $\bar{E}_1$ and $\bar{E}_2$ are combined (i.e. They are incident on the same surface, or travel along the same path) the resultant wave is

$$\bar{E}_R = \bar{E}_1 + \bar{E}_2 = \tilde{E}_{R0} e^{i(k_R x-\omega_R t+\alpha_R)}. \tag{20}$$

An important property of light is the energy density which is proportional to the squared modulus of the complex amplitude. Therefore

$$U_R = |\tilde{E}_{0R}|^2 = |\tilde{E}_{01} + \tilde{E}_{02}|^2 = |\tilde{E}_{01}|^2 + |\tilde{E}_{02}|^2 + 2\,\mathrm{Re}(\tilde{E}_{01}^* \cdot \tilde{E}_{02}). \tag{21}$$

The first two terms are the energy densities of the two component waves and the last term is the interference term. This term describes the correlation of the phases between the radiation fields, or the coherence of the two fields. Complex patterns of light and dark regions are the visual results of this term and the visibility, or contrast of these patterns reflect the coherence properties of the fields.

To describe and quantify the coherence properties of the optical field we define the mutual coherence function to be[6]

$$\Gamma(\vec{r}_1,\vec{r}_2,\tau) = \left\langle \widetilde{E}^*(\vec{r}_1,t)\widetilde{E}(\vec{r}_2,t+\tau)\right\rangle = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\widetilde{E}^*(\vec{r}_1,t)\widetilde{E}(\vec{r}_2,t+\tau)dt. \quad (22)$$

This equation represent the statistical average of the field amplitudes at points $\vec{r}_1$and $\vec{r}_2$ with a time offset of τ. This correlation function is the statistical generalization of the interference term in equation (21). If we set $\vec{r}_1$=$\vec{r}_2$ and τ=0 we get the self mutual coherence function ,which describes the local energy density

$$\Gamma(r,r,0) = \left\langle |E(r,t)|^2\right\rangle = \lim_{T\to\infty}\frac{1}{2T}\int|\widetilde{E}(\vec{r},t)|^2\,dt. \quad (23)$$

The normalized mutual coherence function is given by

$$\gamma(\vec{r}_1,\vec{r}_2,\tau) = \frac{\Gamma(\vec{r}_1,\vec{r}_2,\tau)}{\sqrt{\Gamma(\vec{r}_1,\vec{r}_1,0)\Gamma(\vec{r}_2,\vec{r}_2,0)}}. \quad (24)$$

This function is called the complex degree of coherence and equation (21) can then be written as

$$U_T = U_1 + U_2 + 2\,\mathrm{Re}\{\gamma(\vec{r}_1,\vec{r}_2,\tau)\}\sqrt{U_1U_2}. \quad (25)$$

If we take the temporal Fourier transform of equation (22) we get the cross spectral density,

$$W(\vec{r}_1,\vec{r}_2,\nu) = \int_{-\infty}^{+\infty}\Gamma(\vec{r}_1,\vec{r}_2,\tau)e^{i2\pi\nu t}dt. \quad (26)$$

The self-cross-spectral density, $W(\vec{r},\vec{r},\nu)$, is the diagonal element of the cross-spectral density which is equivalent to the energy density spectrum,

$$W(\vec{r},\vec{r},\nu) = U_\nu(\hat{s},\vec{r}) \quad (27)$$

where,

$$U(\hat{s},\vec{r}) = \int_0^{\infty} U_\nu(\hat{s},\vec{r}) d\nu . \tag{28}$$

According to the van Cittern-Zernike theorem, the cross-spectral density satisfies the two Helmholtz equations[7],

$$\begin{aligned} [\nabla_1^2 + K^2] W(\vec{r}_1,\vec{r}_2,\nu) = 0 \\ [\nabla_2^2 + K^2] W(\vec{r}_1,\vec{r}_2,\nu) = 0 \end{aligned} \tag{29}$$

where $K = \frac{2\pi\nu}{c} = \frac{2\pi}{\lambda}$.

From these relations we can solve for the self-cross-spectral density using standard Green's function techniques, outlined in Goodman[8]. The self-cross-spectral density at a point $\vec{r}_k$ in the upper half space is (assuming $\vec{r}_k >> \lambda$.),

$$W(\vec{r}_k,\vec{r}_k,\nu) = \frac{1}{\lambda^2} \iint_{\substack{\text{source}\\\text{plane}}} d^2 r_1 \iint_{\substack{\text{source}\\\text{plane}}} d^2 r_2 W(\vec{r}_1,\vec{r}_2,\nu) \frac{\exp(iK(r_{2k} - r_{1k}))}{r_{1k} r_{2k}} \cos(\theta_1)\cos(\theta_2), \tag{30}$$

where, r_{1k} and r_{2k} are the distances from the source points $\vec{r}_1$ and $\vec{r}_2$ to $\vec{r}_k$, and θ_1 and θ_2 are the corresponding angles with respect to the surface normal of the source. In the limit that the point $\vec{r}_k$ is a large distance, R, from the source, equation (30) approaches the asymptotic limit[1],

$$\lim_{R\to\infty} W(\vec{r}_k,\vec{r}_k,\nu) = W^{(r)}(\vec{r}_k,\vec{r}_k,\nu) = \frac{\cos^2\theta}{\lambda^2 R^2} \tilde{w}_0(-f_\perp, f_\perp, \nu), \tag{31}$$

where $\vec{f}_\perp$ is the transverse spatial frequency associated with a plane wave traveling in the direction of $\vec{r}_k$ from the source and,

$$\tilde{w}_0(f_{1\perp}, f_{2\perp}, \nu) = \iint d^2 r_1 \int d^2 r_2 W(\vec{r}_1,\vec{r}_2,\nu) \exp(-i2\pi (f_{1\perp}\cdot r_1 + f_{2\perp}\cdot r_2). \tag{32}$$

Equation (32) is the dual spatial Fourier transform of the cross spectral density, or the projected-spatial frequency correlation function in the z=0 plane.

It is important to understand the physical meaning of $\tilde{w}_0(-f_\perp, f_\perp, \nu)$. In the coherent limit, we see that

$$\tilde{w}_0(-f_\perp, f_\perp, \nu) \xrightarrow{\text{coherent limit}} \lim_{T\to\infty} \frac{1}{2T} \left|\tilde{\xi}_\nu(f_\perp)\right|^2 \tag{33}$$

where

$$\tilde{\xi}_\nu(\bar{f}_\perp) = \iint E_\nu(\bar{r}_0) \exp(-i2\pi(f_x x_0 + f_y y_0) dx_0 dy_0. \tag{34}$$

In equation (34) $E_\nu(\hat{r}_0)$ is the temporal Fourier transform of the electric field.

2.3 Deriving the general radiant intensity from coherence theory.

We can now derive the radiant intensity using the cross-diagonal elements of the projected-spatial frequency correlation function and the self-cross spectral density. From the relationship given in equation(23), we know that

$$I_\nu(\hat{s}) = cR^2 \lim_{R\to\infty} U_\nu^{(r)}(\hat{s}R). \tag{35}$$

By combining this with the relationship given in equation (27) and equation (31) we get the equation for radiant intensity in the far field,

$$I_\nu(\hat{s}) = \frac{c\cos^2\theta}{\lambda^2} \tilde{w}_0(-f_\perp, f_\perp, \nu). \tag{36}$$

This is the standard equation for radiant intensity used in coherence theory[9], outside of factors that may differ due to choice of units.

2.4 Radiant intensity of an incoherent source.

According to coherence theory, an incoherent source is one in which the cross-spectral density is delta correlated[1]. In general the cross-spectral density can be described as the energy density spectrum multiplied by some function of the positions $\vec{r}_1$ and $\vec{r}_2$,

$$W(\vec{r}_1,\vec{r}_2,\nu) = U_\nu\{\tfrac{1}{2}(\vec{r}_1+\vec{r}_2)\}F(\vec{r}_2-\vec{r}_1). \tag{37}$$

If this function is delta correlated the equation can only be non-zero for one set of points $\vec{r}_1$ and $\vec{r}_2$[10]. In other words a source is incoherent if

$$W(\vec{r}_1,\vec{r}_2,\nu) = U_\nu\{\vec{r}_1\}\delta(\vec{r}_2-\vec{r}_1). \tag{38}$$

Substituting this equation into equation (32) we get

$$\tilde{w}_0(-\vec{f}_\perp,\vec{f}_\perp,\nu) = \iint\limits_{\substack{\text{source}\\\text{plane}}} d^2r_1 \iint\limits_{\substack{\text{source}\\\text{plane}}} d^2r_2 U_\nu(\vec{r}_1)\delta(\vec{r}_2-\vec{r}_1)\exp\{-i2\pi\vec{f}_\perp(\vec{r}_1-\vec{r}_2)\}$$

$$= \iint\limits_{\substack{\text{source}\\\text{plane}}} d^2r_1 U_\nu(\vec{r}_1) = Q_\nu, \tag{39}$$

where Q_ν is a constant. Returning to equation (36) we see that for an incoherent source

$$I_\nu(\hat{s}) = \frac{d\Phi_\nu}{d\Omega} = c\cdot\cos^2\theta\cdot Q_\nu. \tag{40}$$

This implies that an incoherent source is not a Lambertian source, since as we recall from equation (18), I is proportional to $\cos\theta$,for a Lambertian source, not $\cos^2\theta$.

3. THE NEW CORRELATION FUNCTIONS BASED ON OPTICAL FLUX.

3.1 Relating radiant flux to electromagnetic quantities.

In electromagnetics the Poynting vector,

$$\bar{S} = {}^{c}\!/_{4\pi}\left(\vec{E}x\vec{H}\right) \quad , \tag{41}$$

is defined to be the rate at which electromagnetic energy passes through a unit area whose normal is in the direction of propagation $\hat{s}$. According to Poynting's theorem the conservation of energy for a combined system of charged particles and electromagnetic fields is expressed as[1]

$$\frac{dQ}{dt} = -\oint_{\substack{\text{detector}\\\text{area}}} \vec{S}\cdot\hat{\eta}\, dA, \tag{42}$$

which from radiometry is the radiant flux $\Phi = \frac{dQ}{dt}$. We also know from equation. (9) that the flux is related to the irradiance by the integral

$$\Phi = \int E_{\nu}\, dA_{1}\,. \tag{43}$$

Combining these three equations we find the relationship between optical flux and the electromagnetic fields to be,

$$\Phi = \iint_{\substack{\text{detector}\\\text{area}}} E\, dA = \iint_{\substack{\text{detector}\\\text{area}}} \vec{S}\cdot\hat{\eta}\, dA = \frac{c}{4\pi} \iint_{\substack{\text{detector}\\\text{area}}} \left(\vec{E}x\vec{H}\right)\cdot\hat{\eta}\, dA\,. \tag{44}$$

3.2 Derivation of correlation function based on flux density.

If two beams with measured optical fluxes Φ_1 and Φ_2, respectively, are combined then the resultant flux Φ_T will be

$$\Phi_T = \Phi_1 + \Phi_2 + \Phi_{12}. \tag{45}$$

The third term that results is the interference term. The interference is the result of the wave nature of light. The phase difference between the two waves determine how the waves interfere at each point along the wave, either adding constructively or destructively.

From equation (44) we can write Φ_T as

$$\Phi = \iint_{\substack{\text{detector}\\ \text{area}}} \vec{S}_T \cdot \hat{\eta} dA = \left(\frac{c}{4\pi}\right) \iint_{\substack{\text{detector}\\ \text{area}}} \left(\vec{E} \; x \vec{H}\right) \cdot \hat{\eta} dA \tag{46}$$

For simplicity we will solve for the total Poynting vector, so for $\vec{E} = \vec{E}_1 + \vec{E}_2$ and $\vec{H} = \vec{H}_1 + \vec{H}_2$

$$\vec{S}_T(\vec{r}_1, \vec{r}_2, t_1, t_2) = \left(\frac{c}{4\pi}\right) \left[\left(\vec{E}^*(\vec{r}_1, t_1) + \vec{E}^*(r_2, t_2)\right) \times \left(\vec{H}(r_1, t_1) + \vec{H}(r_2, t_2)\right)\right] \tag{47}$$

Working out the cross product we get

$$\vec{S}_T(\vec{r}_1, \vec{r}_2, t_1, t_2) = \left(\frac{c}{4\pi}\right) \Big(\left[\vec{E}^*(r_1, t_1) \times \vec{H}(r_1, t_1)\right] + \left[\vec{E}^*(r_2, t_2) \times \vec{H}(r_2, t_2)\right)$$

$$+ \left[\vec{E}^*(r_1, t_1) \times \vec{H}(r_2, t_2) + \vec{E}^*(r_2, t_2) \times \vec{H}(r_1, t_1)\right] \Big) \tag{48}$$

The first two terms of this equation correspond to the Pointing vectors for the two individual beams, $\vec{S}\,(\vec{r}_1, t_1)$ and $\vec{S}(r_2, t_2)$ leaving the last term as the interference term, we get

$$\vec{S}_{12}(r_1, r_2, \tau, \hat{\eta}) \cdot \hat{\eta} = \left(\frac{c}{4\pi}\right)\left\{\vec{E}^*(r_1, t) \times \vec{H}(r_2, t+\tau) - \vec{H}^*(r_1, t) \times \vec{E}(r_2, t+\tau)\right\} \cdot \hat{\eta} . \quad (49)$$

From this we can define our new correlation function, $M(\vec{r}_1, \vec{r}_2, \tau, \hat{\eta})$by taking the statistical average of equation (49) with a time offset of τ,

$$M(\vec{r}_1, \vec{r}_2, \tau, \hat{\eta}) = \left(\frac{c}{4\pi}\right)\left\langle \vec{E}^*(\vec{r}_1, t) \times \vec{H}(\vec{r}_2, t+\tau) - \vec{H}^*(\vec{r}_1, t) \times \vec{E}(\vec{r}_2, t+\tau)\right\rangle_{ave} \cdot \hat{\eta} . \quad (50)$$

We will call this function the flux density correlation function. We obtain its cross-spectral flux density function, $M_\nu(\vec{r}_1, \vec{r}_2, \hat{\eta})$,as before, by taking the temporal Fourier transform of $M(\vec{r}_1, \vec{r}_2, \tau, \hat{\eta})$,

$$M_\nu(\vec{r}_1, \vec{r}_2; \hat{\eta}) = \int M(\vec{r}_1, \vec{r}_2, \tau, \hat{\eta}) \exp(i2\pi\nu\tau) d\tau$$

$$= \frac{c}{4\pi}\left\{\vec{E}_\nu^*(\vec{r}_1) \times \vec{H}_\nu(\vec{r}_2) - \vec{H}_\nu^*(\vec{r}_1) \times \vec{E}_\nu(\vec{r}_2)\right\} \cdot \hat{\eta} , \quad (51)$$

where $\vec{E}_\nu$ and $\vec{H}_\nu$ represent the temporal Fourier transforms of the electric and magnetic fields respectively. To obtain the associated projected-spatial frequency correlation function, we take the dual two -dimensional spatial Fourier transforms of $M_\nu(\vec{r}_1, \vec{r}_2, \hat{\eta})$,

$$\tilde{m}_\nu(\vec{f}_{1\perp}, \vec{f}_{2\perp}) = \iint\limits_{\substack{source\\plane}} d^2 r_1 \iint\limits_{\substack{source\\plane}} d^2 r_2\, M_\nu(\vec{r}_1, \vec{r}_2; \hat{\eta}) \exp(-i2\pi(\vec{f}_{1\perp} \cdot r_1 + \vec{f}_{2\perp} \cdot r_2)$$

$$= \frac{c}{4\pi}\left\{\tilde{\xi}_\nu^*(\vec{f}_{1\perp}) \times \mathrm{H}_\nu(\vec{f}_{2\perp}) - \mathrm{H}_\nu^*(\vec{f}_{1\perp}) \times \tilde{\xi}_\nu(\vec{f}_{2\perp})\right\} \cdot \eta , \quad (52)$$

where $\tilde{\xi}_\nu$ and $\tilde{\mathrm{H}}_\nu$ represent the spatial Fourier transforms of E_ν and H_ν. Remembering that $\hat{s} \times \tilde{\xi}_\nu = \tilde{\mathrm{H}}_\nu$ we can rewrite equation (52) as

$$\tilde{m}_\nu(-f_\perp, f_\perp) = \frac{c}{2\pi}\left[\tilde{\xi}_\nu^*(\vec{f}_\perp) \cdot \tilde{\xi}_\nu(\vec{f}_\perp)\right] \cdot \left[\hat{s} \cdot \hat{\eta}\right] . \quad (53)$$

We can see that the first term is equal to $\tilde{w}_0(-f_\perp, f_\perp, \nu)$, and from geometry that the second term is $\cos\theta$, where θ is the angle between $\hat{s}$ and $\hat{\eta}$. This yields a simple relationship between $\tilde{m}_\nu(\vec{f}_{1\perp}, \vec{f}_{2\perp})$ and $\tilde{w}_0(\vec{f}_{1\perp}, \vec{f}_{2\perp}; \nu)$,

$$\tilde{m}_\nu(\vec{f}_{1\perp}, \vec{f}_{2\perp}) = \frac{c}{2\pi}\tilde{w}_0(\vec{f}_{1\perp}, \vec{f}_{2\perp}; \nu)\cos(\theta). \tag{54}$$

If we apply the definition of incoherence to the cross-spectral flux density, we obtain

$$M_\nu(\vec{r}_1, \vec{r}_2, \hat{\eta}) = P_\nu\{\vec{r}_1\}\delta(\vec{r}_2 - \vec{r}_1) \quad , \tag{55}$$

which describes the measured spatially incoherent source.

From this we find that

$$\tilde{m}_\nu(-\vec{f}_\perp, \vec{f}_\perp) = \iint_{\substack{\text{source}\\\text{plane}}} d^2 r_1 \iint_{\substack{\text{source}\\\text{plane}}} d^2 r_2 P_\nu(\vec{r}_1)\delta(\vec{r}_2 - \vec{r}_1)\exp\{-i2\pi\vec{f}_\perp(\vec{r}_1 - \vec{r}_2)\}$$

$$= \iint_{\substack{\text{source}\\\text{plane}}} d^2 r_1 P_\nu(\vec{r}_1) = \Psi_\nu, \tag{56}$$

where Ψ_ν is a constant.

4. DERIVATION OF RADIANT INTENSITY FROM NEW CORRELATION FUNCTION.

Recall from equation(36) that

$$I_\nu(\hat{s}) = \frac{d\Phi_\nu}{d\Omega} = \frac{c\cdot\cos^2\theta}{\lambda^2}\cdot\tilde{w}_0(-f_\perp, f_\perp, \nu). \tag{57}$$

Using the relationship between $\tilde{m}_\nu(\vec{f}_{1\perp}, \vec{f}_{2\perp})$ and $\tilde{w}_0(\vec{f}_{1\perp}, \vec{f}_{2\perp}; \nu)$,from equation (53), we can rewrite this as

Using the relationship between $\tilde{m}_\nu(\vec{f}_{1\perp}, \vec{f}_{2\perp})$ and $\tilde{w}_0(\vec{f}_{1\perp}, \vec{f}_{2\perp}; \nu)$,from equation (53), we can rewrite this as

$$I_\nu(\hat{s}) = \frac{2\pi}{\lambda^2}\tilde{m}_\nu(-\vec{f}_\perp, \vec{f}_\perp)\cos\theta = \frac{2\pi}{\lambda^2}\Phi_\nu \cos\theta\,. \tag{58}$$

This shows that $\tilde{w}_0(-f_\perp, f_\perp, \nu)\cos\theta$ is a constant, and that the radiant intensity of an incoherent source is proportional to $\cos\theta$,which is the same dependence exhibited by a Lambertian source.

5. CONCLUSIONS.

In this paper I have merged the principals of classical radiometry with the modern theory of partial coherence. I have derived a new equation describing the radiant intensity of an incoherent source to be proportional to $\tilde{w}_0(-f_\perp, f_\perp, \nu)$and $\cos\theta$. This new formula corrects the standard coherence equation, which was proportional to $\cos^2\theta$. In doing this I have shown that the radiant intensity of an incoherent source exhibits the same $\cos\theta$ dependence as a Lambertian source.

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Figure 1: Geometry for definition of radiance.

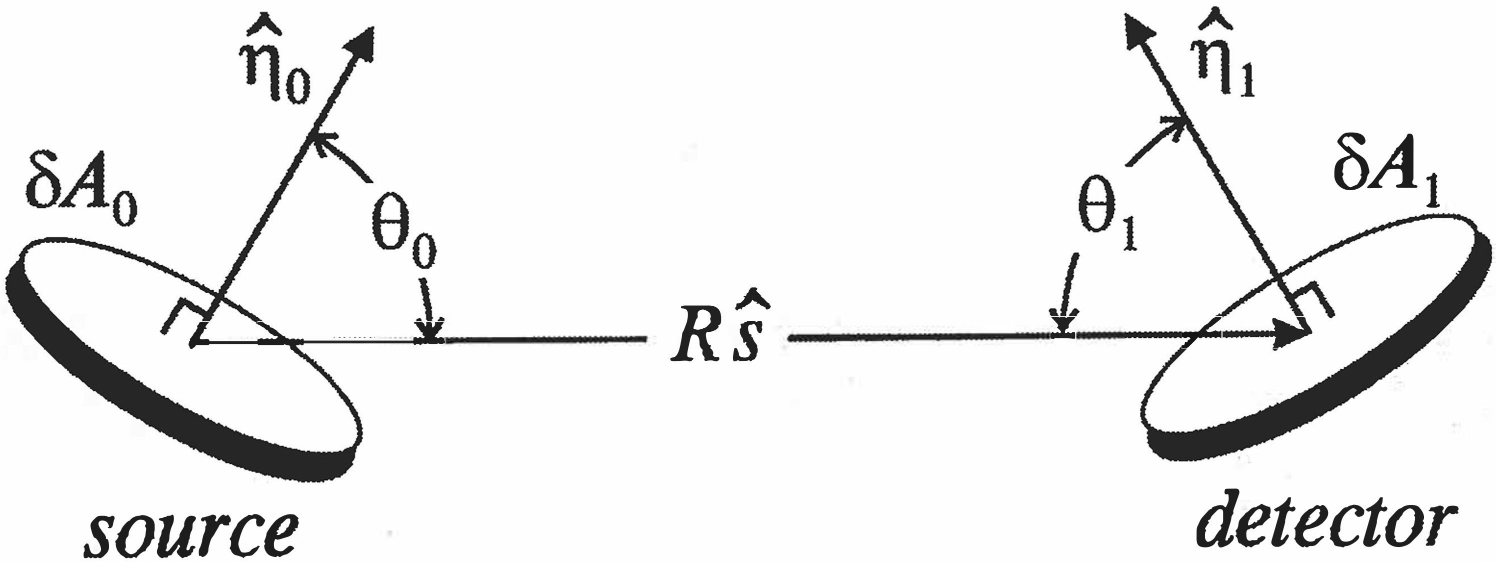

Figure 2: Solid Angle and Projected Solid Angle

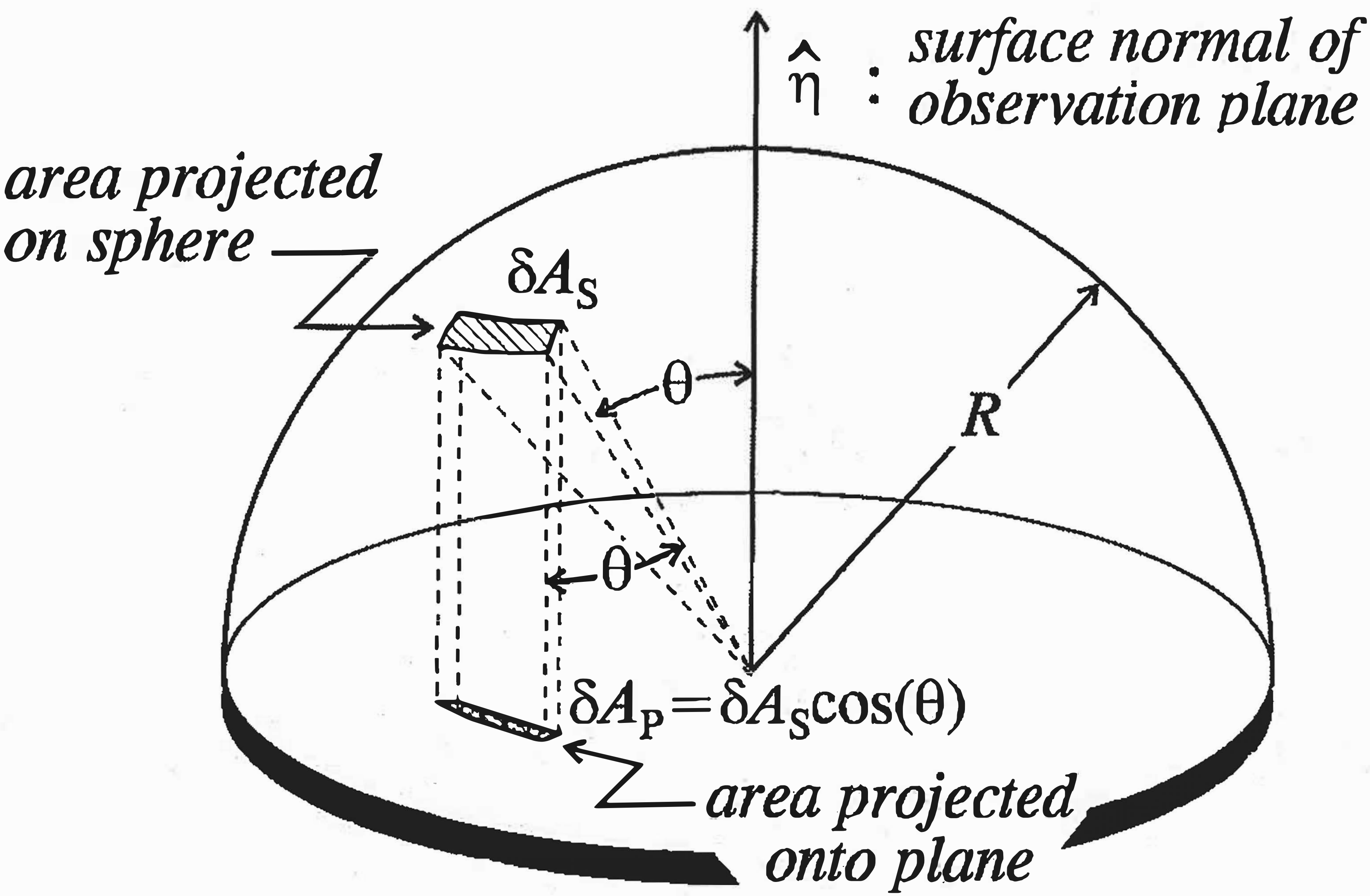

Figure 3: Geometry for a perfectly collimated source

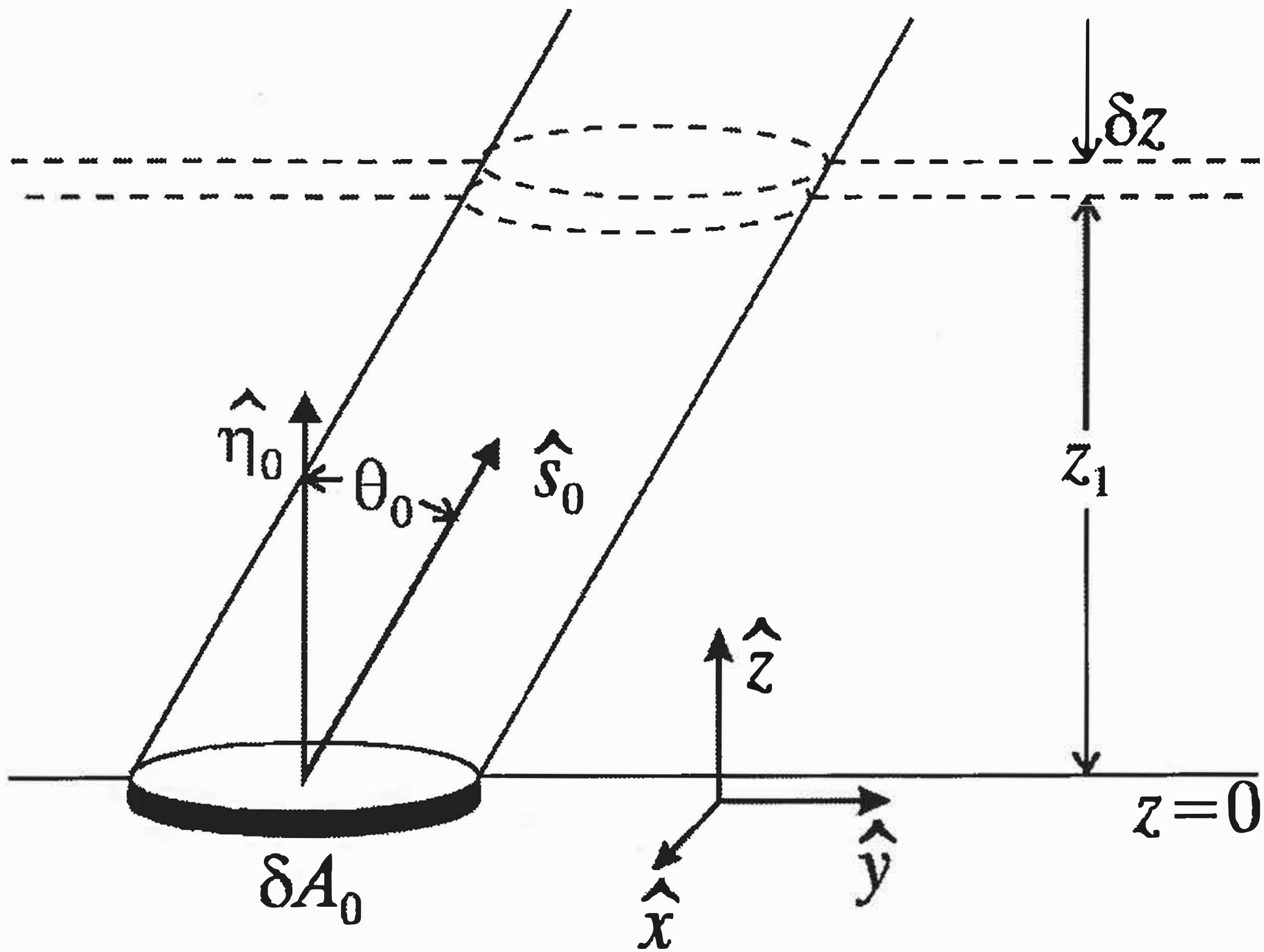

Figure 4: Temporal Coherence, randomly phased wave train

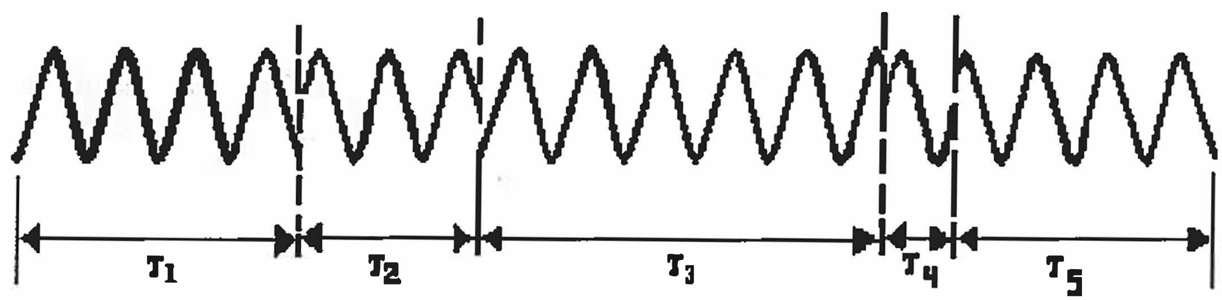

Figure 5 : Young's Double Split experiment