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Asset Allocation in the S&P 500 Index: Optimizing Investments Using the Efficient Frontier

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Abstract

This research project was conceived for two primary reasons. First, it was to determine whether asset allocation decisions could be improved by using certain tools from Modern Portfolio Theory. Second, it was to test data that is relevant to the UAHuntsville Capital Management Group (CMG), in order to help the student portfolio managers make better asset allocation decisions for their portfolios. The CMG is comprised of students in the College of Business Administration that manage a portion of TVA's Nuclear Decommissioning Trust Fund for the TVA Investment Challenge, in addition to managing the group's own portfolio, the Charger Fund. The CMG's TVA portfolio is comprised of large capitalization stocks, which are representative of the S&P 500 Index. The index is comprised of 10 economic sectors, each of which is weighted differently in the index. The objective of this research is to investigate whether using the efficient frontier to allocate investments among these sectors will provide better portfolio performance as measured by overall risk and return.

The key questions posed to test the research hypothesis are as follows:

1. Can the portfolio's risk-to-return ratio be improved by making asset allocation decisions according to the efficient frontier?
2. What effect does the choice of timeframe for calculating the efficient frontier have on asset allocation decisions?
3. Does this method provide better portfolio return or risk in absolute terms?

The methods developed in this thesis were back-tested historically to determine their utility. It became clear as the research progressed that the risk and return profile of the test portfolios were improved significantly. The results provide insight into what methods might be used to better make sector allocation decisions within a portfolio that is designed to outperform the S&P 500 Index.

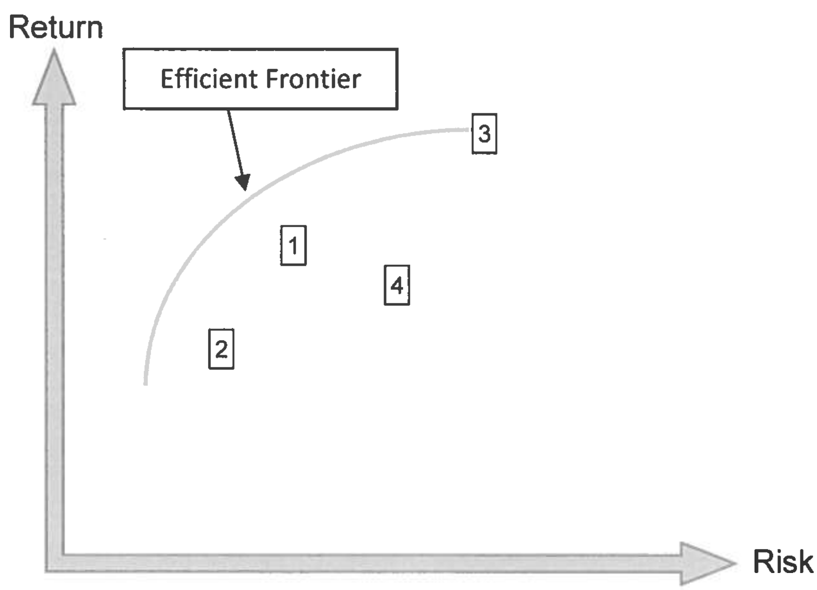
Introduction

Theory

Modern Portfolio Theory (MPT) is a financial model that seeks to explain the relationships between risk and return for investment portfolios. Since most investors are risk averse, meaning they desire to avoid the possibility of losing investments, this model tries to systematically manage the problem of reducing risk in portfolios while maximizing return potential. One common saying that broadly describes the risk reduction goal is "don't put all of your eggs in one basket." This is the principle of diversification, which reduces overall risk by reducing the chance that an entire portfolio will be hurt in the event that one security fails. Diversification, however, is only one focus of the broader investment decision of asset allocation. Asset allocation includes diversification, but deals more generally with making allocations among asset classes, which carry varying levels of expected risk and return. A typical asset allocation decision would involve selecting the balance of domestic and international investments, asset classes (stocks/equities, bonds, cash, commodities, etc.), and industries or economic sectors.

The portfolio management world was revolutionized with the publication of the seminal article, "Portfolio Selection", by Harry Markowitz in *The Journal of Finance* in 1952 (Markowitz, 1952). In the article, Markowitz developed the efficient frontier, a mathematical process that finds and plots a group of unique portfolios that are said to be efficient. An efficient portfolio is a portfolio that has been optimized for both return (mean) and risk (variance) by adjusting the proportion of investment in each of the underlying securities. Each efficient portfolio on the frontier represents the maximum possible expected return for a given level of risk, and, conversely, represents a minimum risk level for a given expected return. It would thus be impossible, given the underlying securities, to achieve a higher return portfolio without adding additional risk. The plot of the efficient frontier is in mean-standard deviation space and is typically represented by a convex curve when more than two underlying securities are present. Figure 1 illustrates a graph of a typical efficient frontier. Points 1 through 4 depict the individual assets that, when combined in varying proportions, make up the efficient portfolios along the efficient frontier.

Figure 1



Hypothesis

This research seeks to answer multiple questions related to improving portfolio management. Broadly, the tested hypothesis is the assertion that future asset allocation decisions can be improved by analyzing historical data with the efficient frontier. More specifically, this hypothesis led to the formation of a series of related questions:

1. Can a portfolio's risk-to-return ratio be improved by making asset allocation decisions using the efficient frontier?
2. What effect does the length of the historical estimation period for calculating the efficient frontier have on asset allocation decisions?
3. Does the efficient frontier provide better portfolio return or risk in absolute terms?

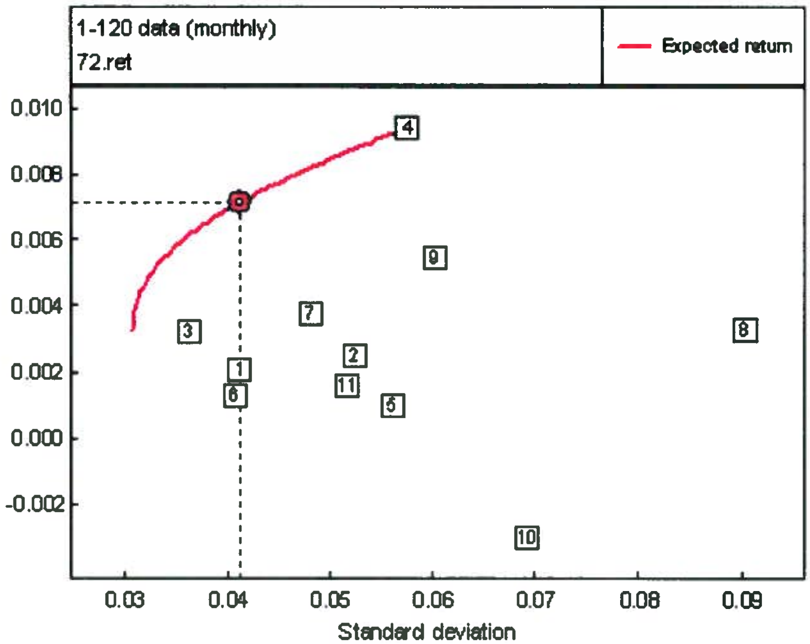
The two primary purposes for this research are to evaluate whether the efficient frontier is a useful tool for making asset allocation decisions among broad economic sectors and to provide insight into sector allocation decisions for the student-managed UAHuntsville Capital Management Group.

Methods/Software

To test the hypothesis while also focusing on the information that would most benefit the UAHuntsville Capital Management Group, primary research was conducted to determine if the efficient frontier would be relevant in making asset allocation decisions among the ten economic sectors represented in the S&P 500 Index.

To systematically test the validity of the hypothesis, a series of monthly test portfolios were constructed, back-tested, and compared with the control portfolio over time. As the performance benchmark, the S&P 500 Index was selected as the control portfolio. Each test portfolio consisted of some mixture of the 10 economic sectors within the index, according to the two methods described in the section below. The proportion of investment in each sector for the individual test portfolios was determined by the efficient frontier. The efficient frontier, however, gives a set of many efficient portfolios. So, to select the most comparable efficient portfolio, the portfolio that exhibited the same level of risk as the index was selected in each period. Graphically, this is illustrated in Figure 2 as the portfolio (red circle) that lies directly above the index (labeled 1) on the frontier.

Figure 2



In each case, because the index itself is not an efficient portfolio, this process produced a portfolio with a similar risk profile, but higher historical returns than the index.

The construction of each efficient frontier was completed using *VisualMVO*, a standalone software package developed by Efficient Solutions, Incorporated. *VisualMVO* uses monthly return data to calculate the efficient frontier. Once the proper efficient portfolio was selected from the plot, the software estimated the corresponding proportions that should be used for each of the 10 sectors during the test period.

The daily prices of the S&P 500 Index and each of its 10 sectors were obtained from Yahoo Finance and Standard & Poor's, and were transformed into monthly return data for input into *VisualMVO*.

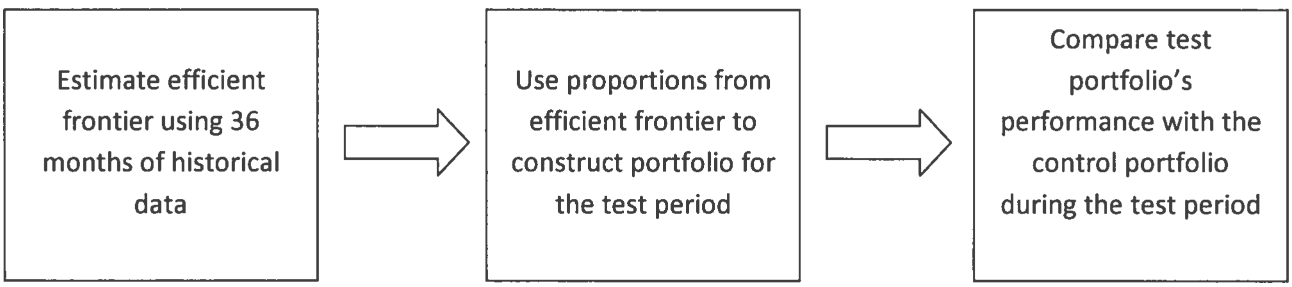
Return and Risk Calculations

Return during period *n-1* to *n* was calculated using:

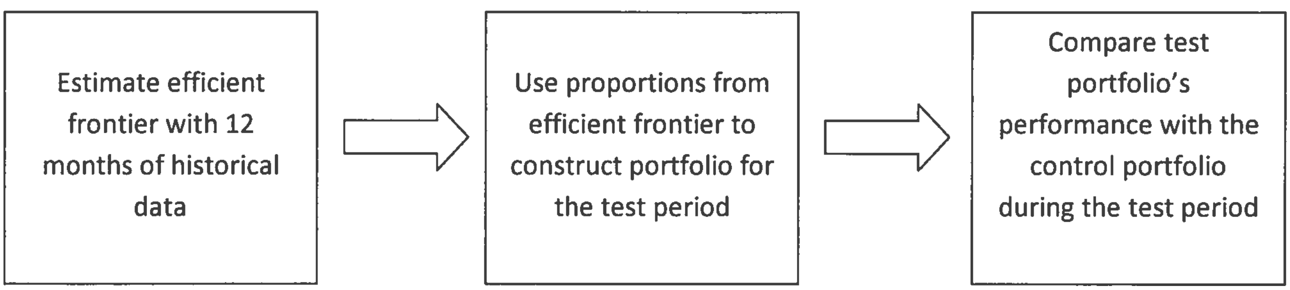
$$\frac{p_n - p_{n-1}}{p_{n-1}}$$

Where p_{n-1} is the price of an asset at the beginning of the period, and p_n is the price of the asset at the end of the period. Then, the standard deviation of the returns was calculated and served as a proxy for risk.

Method 1 used three years of return data for estimating the efficient frontier. Therefore, the test period portfolio was allocated according to the efficient frontier derived from the 36 months directly preceding the test period. The following diagram depicts the process used in Method 1.



Method 2 used one year of return data for estimating the efficient frontier. Therefore, the test period portfolio was allocated according to the efficient frontier derived from the 12 months directly preceding the test period. The following diagram depicts the process used in Method 2.



Assumptions

There are several important assumptions within this model that are critical for evaluating the hypothesis accurately.

- The first is that the standard deviation of returns, which is a measure of volatility, serves as a proxy for risk.
- Second, sector-based securities, such as exchange traded funds (ETFs), are available and have infinite share divisibility. This means that any proportion of shares can be purchased. Additionally, these securities must track the value of sectors exactly.
- The third assumption is that there are no transaction costs, which can vary widely depending on brokerage arrangements. By eliminating these variables, the hypothesis can be tested on its own merits, without regard to investors' individual situations.

Including certain variables that are investor-specific would skew the results. While some of these assumptions are not realistic for individual investors, many of them become so for managers of large portfolios, due to the scale of investment. These assumptions would be adjusted on a case-by-case basis for actual investment decisions, such as those for the UAHuntsville Capital Management Group.

Overview of Data

A table of daily sector prices from October 1998 to August 2008 was provided by Standard & Poor's, and the concurrent daily index prices were obtained from Yahoo Finance. From this combined data set, monthly return calculations were made according to the method previously described and placed in a format appropriate for importing into the *VisualMVO* software package. This provided 119 periods (6 years and 11 months) of monthly returns from which to test the hypothesis. One important note, which will be discussed in the data analysis section below, is that the stock market was experiencing the "tech bust" from 2001 to February of 2003. This was the greatest loss in value the stock market had experienced in many years. This extended market downturn was actually helpful for this research, because the hypothesis was tested through both bear and bull markets, which is important for drawing accurate conclusions about its validity.

Results and Data Analysis

Figure 3 summarizes the results of the test portfolios from October 2001 to August 2008, and compares them to the control portfolio. A complete table of monthly return data is located in Appendix 1.

Figure 3

	Control Portfolio (Index)	Method 1 (3 Year Est. Period)	Method 2 (1 Year Est. Period)
Annual Return	3.07%	5.28%	6.30%
Standard Deviation	3.71%	4.54%	3.89%
Risk-to-Return Ratio	1.21	0.86	0.62

During the 83 months tested, the annual return of the control portfolio was 3.07%. In contrast, portfolios constructed using Method 1, which used a 36 month historical estimation period, achieved 5.28% annual returns, outperforming the control portfolio by 2.21 percentage points. Method 2, which used only a 12 month estimation period, produced the best results. The return on portfolios constructed using Method 2 was 3.23 percentage points higher than the control portfolio, more than doubling its returns.

Figure 4 shows the cumulative returns of each method based on a $10,000 initial investment.

Figure 4

In addition to analyzing the returns of the test methods, it is also critical to evaluate their risk. The risk-to-return ratio is a measure of the level of risk taken for every unit of return, thus, the lower this ratio, the better the relative performance. Method 1 produced approximately one additional percentage point of standard deviation, but still resulted in a significant reduction in the risk-to-return ratio, from 1.21 to 0.86. Method 2 produced a 0.62 risk-to-return ratio, which is nearly half of the 1.21 characteristic of the control portfolio. This outperformance is noteworthy because it more than doubled the returns without significantly increasing the risk level when compared to the control portfolio. The risk and return profile of Method 2 actually mirrored the long run performance of the S&P 500 (1950 to present), which produced 6.72% returns and a standard deviation of 4.18%. Achieving this performance during a period that contained one of the top five market downturns in United States history shows that this asset allocation strategy may not only bolster growth during bull markets, but may also provide promise for portfolio protection during bear markets.

Because the test period contained both bear and bull markets, it is interesting to analyze the performance of the methods during each type of market individually. October 2001 to February 2003, the first 17 periods of the data set, were in declining markets. This historic drop is commonly referred to as the "tech bust." The remaining 66 periods until August 2008 contain positive overall growth. Figure 5 gives a summary of the average monthly performance of both test methods compared to the control portfolio during these market environments.

Figure 5

	Control Portfolio (Index)	Method 1 (3 Year Est. Period)	Method 2 (1 Year Est. Period)
Overall	0.32%	0.52%	0.58%
Bull Market	0.68%	1.11%	0.92%
Bear Market	-1.10%	-1.76%	-0.76%

As Figure 5 shows, test Method 1 and Method 2 on average outperformed the market by 0.20% and 0.26% per month, respectively. When broken down into bear and bull markets, however, it becomes clear that the two methods perform very differently. Method 1 outperformed by a larger margin during up markets, while it significantly underperformed during down markets. On the contrary, Method 2 produced more consistent results. It outperformed the market by 0.24% during bull markets, and outperformed even stronger, by 0.34%, during the bear market. So, even though Method 1 is stronger during the much longer bull market, Method 2 produces better overall results by protecting its value during the 17 month bear market. Method 1 results in more risk, so it appears that it functions as a market lever, increasing positive returns, and worsening negative returns.

It would seem that Method 2, which contains higher returns, would outperform the benchmark more consistently than does Method 1. Interestingly, as Figure 6 shows, this is not the case.

Figure 6

Number of months of outperformance and underperformance

	Overall Period		Bull Market		Bear Market	
	Outperform	Underperform	Outperform	Underperform	Outperform	Underperform
Method 1	45	38	36	30	9	8
Method 2	43	40	33	33	10	7

Method 1 outperforms the index seven more periods than it underperforms (45 months versus 38 months), while the gap is reduced to three periods for Method 2 (43 months versus 40 months). Looking at the trend in both bull and bear markets, Method 1 nearly cancels its over- and underperformance in bear markets, but outperforms six more periods than it underperforms in bull markets. The opposite is true for Method 2. Its net outperformance is 3 periods during the bear market, and completely balances occurrences during the bull market. This obviously ignores the magnitude of each over- and underperformance, but illustrates the persistence of each method during different market environments.

Since the two methods perform differently in bear and bull markets, it is interesting to look at how a hybrid method, one that follows Method 1 during bull markets and Method 2 during bear markets, would have performed. Not surprisingly, the results were far more impressive than one method alone. Figure 7 summarizes the data for the hybrid method and compares it to the test methods and the control portfolio.

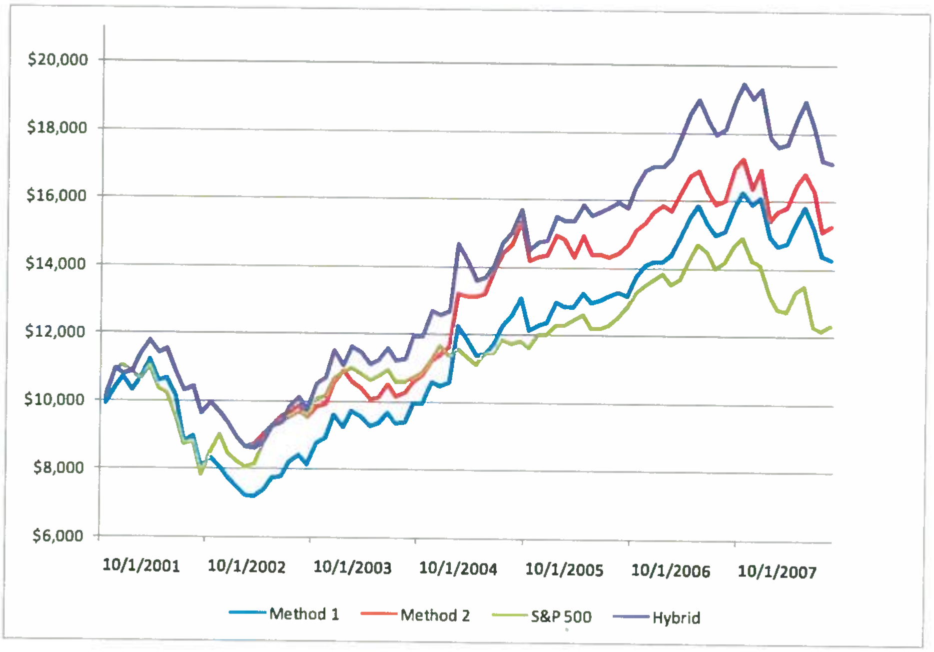
Figure 7

	Control Portfolio (Index)	Method 1 (3 Year Est. Period)	Method 2 (1 Year Est. Period)	Hybrid Method
Annual Return	3.07%	5.28%	6.30%	8.06%
Standard Deviation	3.71%	4.54%	3.89%	3.94%
Risk-to-Return Ratio	1.21	0.86	0.62	0.49

These results are impressive. The hybrid method produced a similar risk profile to the index and Method 2 during this period, but beat the market's returns by 4.99 percentage points. It also resulted in a risk-to-return ratio that beats even the long run values for the S&P 500. Figure 8 illustrates a $10,000 investment in the hybrid strategy compared with the initial strategies.

It should be noted that the hybrid method was constructed in retrospect. On a real-time basis, it is impossible to determine when bull and bear markets begin and end. Therefore, the hybrid method would be very difficult to employ for actual portfolio managers. Nevertheless, the results provide another glimpse into what might be possible with improved methods.

Figure 8



Conclusion

After testing the hypothesis, it seems possible that asset allocation decisions can be improved using the efficient frontier. The data also gives insight into which methods work better in different market environments. This research has shown that it is possible to use the efficient frontier as a framework for making asset allocation decisions, but opens the door to many more questions about which methods would be best to more frequently beat the market by greater margins.

The following three questions, which were posed in the hypothesis, have been answered by the research.

1. **Can a portfolio's risk-to-return ratio be improved by making asset allocation decisions according to the efficient frontier?**

   Based on the results of applying Methods 1, 2, and the hybrid method, it is clear that the risk-to-return ratio can be improved significantly by using the efficient frontier. In the case of Method 2, which could easily be implemented, the risk-to-return ratio was practically cut in half.

2. **What effect does the length of the historical estimation period for calculating the efficient frontier have on asset allocation decisions?**

   While all three methods tested provided portfolios that consistently outperformed the S&P 500 Index, it is clear that the historical estimation period has a significant impact on the overall performance of a strategy and performance within varying market environments. Due to the complexity of determining changes in the market environment real-time, it could be argued that using a single model, such as Method 2, that outperforms more on average should be used to eliminate subjectivity. This places the investment decision in the hands of an algorithm that systematically improves performance.

3. **Does this method provide better portfolio return or risk in absolute terms?**

   Method 1, which used a 36 month estimation period, produced both greater risk and greater returns than the control method. Method 2 did not significantly increase the level of risk, but more than doubled the control method's returns.

Recommendations for further research

This research not only answered several key questions, but laid the foundation for the next level of research into making asset allocation decisions using similar methods. There are three separate issues that need to be investigated to determine which factors most contribute to the optimal application of this strategy.

1. Further research should be conducted to iteratively test these methods over longer periods of time. Market data is available for most of the 20th century, which should be tested to discover whether this model is persistent over time and through various market situations.
2. It is clear that the length of the historical estimation period affects the overall performance of the asset allocation model. What level is optimal? Does the optimal level really depend on the market environment? Is it likely that there is a tradeoff between shortening the length of the estimation period and whether a representative sample can be obtained from the shorter estimation period?
3. Can better results in terms of risk and reward be obtained by using a different method to select a portfolio on the efficient frontier? Does constructing the capital market line (CML) by adding a risk free security allow a proxy for the market portfolio to be found? Would this "market portfolio" be the best portfolio to beat the market?

Acknowledgements

First, I would like to thank Dr. John Burnett, Associate Professor of Finance. It was he who initiated the curiosity for this research topic, and it has led in numerous interesting directions. I look forward to exploring this more, and appreciate his direction in the formation of this thesis.

I would also like to acknowledge the contribution of Standard & Poor's Index Services. Without the assistance of a representative I will call Tony, my search for historical data would have been endless.

Dr. Harry Markowitz's contribution to portfolio theory shook the portfolio management world in 1952. My hope is that I can but reach high enough to his tree of influence to enjoy some of the low-hanging fruit.

As always, I can do nothing without the strength of my Lord Jesus Christ.

Works Referenced

Historical price data for the S&P 500 Index was obtained from finance.yahoo.com

Historical price data for the 10 sectors of the S&P 500 Index was obtained from Standard & Poor's Index Services

MARKOWITZ, HARRY. "PORTFOLIO SELECTION." Journal of Finance 7.1 (Mar. 1952): 77-91. Business Source Premier. EBSCO. M. Louis Salmon Library, Huntsville, AL. 21 Apr. 2009 <http://libsys.uah.edu:2054/login.aspx?direct=true&db=buh&AN=6635932&site=bsi-live>.

Appendix 1

Monthly Return Data

	2001			2002			2003			2004		
	Index	Method 1	Method 2	Index	Method 1	Method 2	Index	Method 1	Method 2	Index	Method 1	Method 2
January				-1.557%	-3.543%	0.860%	-2.741%	-3.675%	-4.656%	1.728%	-3.628%	3.359%
February				-2.077%	3.286%	4.814%	-1.700%	-2.903%	-2.903%	1.221%	5.063%	-2.630%
March				3.674%	5.440%	3.284%	0.836%	-0.591%	1.128%	-1.636%	-1.617%	-2.067%
April				-6.142%	-5.683%	-2.899%	8.104%	2.893%	3.388%	-1.679%	-2.902%	-2.940%
May				-0.908%	0.821%	0.821%	5.090%	4.708%	2.798%	1.208%	0.950%	0.386%
June				-7.246%	-5.411%	-6.096%	1.132%	0.735%	2.909%	1.799%	3.321%	3.900%
July				-7.900%	-12.745%	-4.631%	1.622%	5.395%	0.997%	-3.429%	-3.122%	-3.352%
August				0.488%	1.476%	0.959%	1.787%	2.586%	2.232%	0.229%	0.137%	0.920%
September				-11.002%	-10.061%	-7.501%	-1.194%	-3.354%	-3.648%	0.936%	6.148%	3.407%
October	1.810%	-0.989%	0.646%	8.645%	3.431%	3.431%	5.496%	7.710%	3.442%	1.401%	0.083%	1.521%
November	7.518%	5.007%	8.675%	5.707%	-3.022%	-3.022%	0.713%	1.727%	0.553%	3.859%	6.297%	4.252%
December	0.757%	2.837%	-1.537%	-6.033%	-4.272%	-3.207%	5.077%	7.426%	6.435%	3.246%	-1.084%	1.274%
Average Returns	3.362%	2.285%	2.595%	-2.029%	-2.524%	-1.099%	2.018%	1.888%	1.056%	0.740%	0.804%	0.669%
Standard Deviation	3.637%	3.036%	5.378%	5.949%	5.590%	4.003%	3.287%	3.991%	3.284%	2.093%	3.598%	2.809%

Continued on next page

Appendix 1 (continued)

Monthly Return Data

	2005			2006			2007			2008		
	Index	Method 1	Method 2	Index	Method 1	Method 2	Index	Method 1	Method 2	Index	Method 1	Method 2
January	-2.529%	0.900%	2.424%	2.547%	4.820%	3.995%	1.406%	-0.025%	1.476%	-6.116%	-7.293%	-8.975%
February	1.890%	15.852%	13.406%	0.045%	-0.960%	-0.703%	-2.185%	1.451%	-1.230%	-3.476%	-1.650%	1.916%
March	-1.912%	-3.250%	-0.648%	1.110%	0.129%	-3.432%	0.998%	3.640%	3.313%	-0.596%	0.507%	0.666%
April	-2.011%	-4.192%	0.095%	1.216%	2.919%	4.378%	4.329%	4.038%	3.199%	4.755%	4.257%	4.281%
May	2.995%	0.634%	0.378%	-3.092%	-1.943%	-3.734%	3.255%	2.086%	0.904%	1.067%	2.774%	2.077%
June	-0.014%	2.780%	5.599%	0.009%	0.722%	-0.002%	-1.782%	-3.317%	-3.598%	-8.596%	-4.067%	-3.260%
July	3.597%	4.371%	3.348%	0.509%	0.932%	-0.498%	-3.198%	-2.114%	-2.223%	-0.986%	-5.235%	-7.107%
August	-1.122%	2.334%	1.938%	2.127%	0.893%	0.660%	1.286%	0.814%	0.610%	1.219%	-0.645%	0.973%
September	0.695%	4.303%	4.497%	2.457%	-1.018%	1.773%	3.579%	4.768%	6.105%			
October	-1.774%	-7.377%	-7.384%	3.151%	4.144%	3.059%	1.482%	2.637%	1.573%			
November	3.519%	1.420%	0.656%	1.647%	2.751%	1.566%	-4.404%	-2.273%	-5.029%			
December	-0.095%	0.488%	0.500%	1.262%	0.706%	1.849%	-0.863%	1.408%	3.371%			
Average Returns	0.270%	1.522%	2.067%	1.082%	1.175%	0.743%	0.325%	1.093%	0.706%	-1.591%	-1.419%	-1.179%
Standard Deviation	2.256%	5.723%	4.830%	1.645%	2.097%	2.590%	2.796%	2.605%	3.225%	4.304%	3.968%	4.755%